

Instructor's Resource Binder

Elementary Algebra

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Section Teasers:

1.4 Adding Real Numbers; Properties of Addition

2.7 Solving Inequalities

3.2 Graphing Linear Equations

4.4 Problem Solving Using Systems of Equations

5.2 Zero and Negative Exponents

6.6 A Factoring Strategy

7.4 Addition and Subtraction
of Rational Expressions with Unlike Denominators

Assessment Teasers:

5A Diagnostic and Tally Sheet (*Exponents and Polynomials*)

6D Metacognitive Skills Assessment (*Factoring and Quadratic Equations*)

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Teaching Guide

Section 1.4: Adding Real Numbers; Properties of Addition

■ Preparing for Your Class

Suggested Class Time: 45-60 minutes

Materials Needed: Student activities, guided learning activity, and classroom overhead (if desired)

Vocabulary

- Commutative Property of Addition, Associative Property of Addition
- Parentheses (used for grouping terms)
- Addition property of 0, identity element
- Opposites, additive inverses
- Addition Property of Opposites (Inverse Property of Addition)

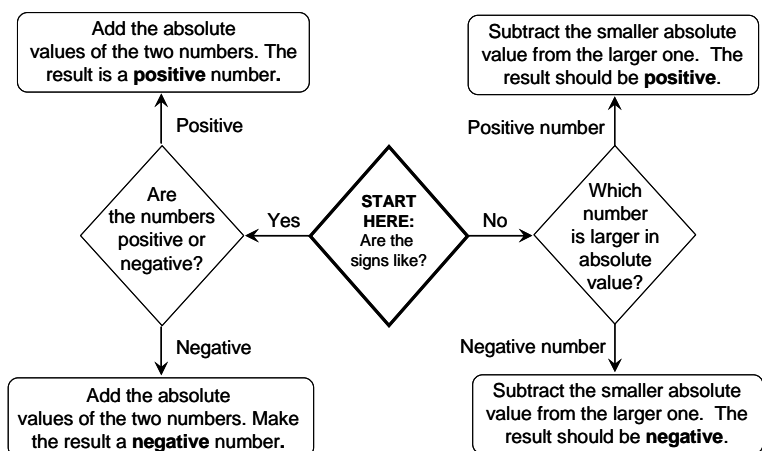
Instruction Tips

- Parentheses are always used around a trailing negative number in sums, like $2 + (-3)$ or $-2 + (-2)$. Consider that with many operations, it would not make sense to allow two adjacent operations, for example, we would not write $8 \div \div 2$ or $2 \cdot \div 4$. So in the instance of addition, it is not proper to have two adjacent operations unless they are separated by parentheses.
- You may want to choose from Student Activities A or B depending on what your students will like. If you want your class to have lots of practice, do both!

■ Teaching Your Class

Guided Learning Activity: *Using Addition Models.* This is a guided lecture to help the students understand the concepts of adding signed numbers. The worksheet uses the colored counter model and the number line model to help the student develop their conceptual understanding. Extensive explanation is provided in case students (or instructor) are not familiar with the models.

A Decision Chart for Addition of Real Numbers:



Classroom Overhead: *Decision Chart for Addition of Real Numbers; Decision Chart for Multiplication and Division of Real Numbers.* This can be copied to an overhead, projected from a portable video projector, or projected from a computer and ceiling projector to a whiteboard. This overhead has the decision charts for Section 1.4 and Section 1.6.

Properties of Addition:

- Commutative Property of Addition: $a + b = b + a$
(the word commute means to go back and forth, like *commuting* between work and home)
- Associative Property of Addition: $a + (b + c) = (a + b) + c$
(the word associate means to join a group, like the friends you *associate* with)
- Addition Property of 0: $a + 0 = a$ (0 is the additive identity, think of *identical* twins)
- Addition Property of Opposites: $a + (-a) = 0$ and $-a + a = 0$
- NOTE: Be careful to not use subtraction in your examples, or you could get yourself into trouble having to explain the distributive property or subtraction of negatives a bit early.

Examples: (identify the property)

- $(3 + \frac{1}{4}) + (-\frac{3}{4}) = 3 + (\frac{1}{4} + (-\frac{3}{4}))$ (associative)
- $500 + 57 + 300 = 500 + 300 + 57$ (commutative)
- $-4 + 4 = 0$ (opposites)
- $(-2 + 3) + 10 = 10 + (-2 + 3)$ (commutative)
- $-5 + (3 + 0) = -5 + 3$ (property of zero)
- $6 + 0 = 6 + (-2 + 2)$ (opposites)
- $-4 + (-2 + 5) + 8 = (-4 + (-2)) + (5 + 8)$ (associative)

Student Activity A: *Match up on Addition of Real Numbers.* This activity works well in groups or with students working in pairs at a whiteboard (have students draw an empty grid on the board for their answers). This activity also involves fractions, so students will get more practice with this topic.

Student Activity B: *Signed Numbers Magic Puzzles.* In this activity, students get practice with addition of signed numbers in an interesting set of puzzles. Because they also have to work with finding the “missing” numbers for a sum, they will also begin to lay the foundation for subtraction of signed numbers. Note: If you’re familiar with Magic Squares, these puzzles differ in that only the rows and columns need to sum to the magic number (not the diagonals).

Guided Learning Activity

Section 1.4: Using Addition Models

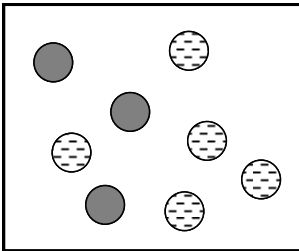
Part I: The first model for addition of real numbers that we look at is called the “colored counters” method. Traditionally, this is done with black and red counters, but we make a slight modification here to print in black and white.



Solid counters (black) represent positive integers, +1 for each counter.

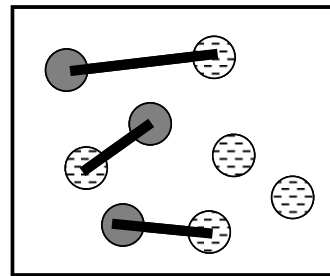


Dashed counters (red), represent negative integers, -1 for each counter.

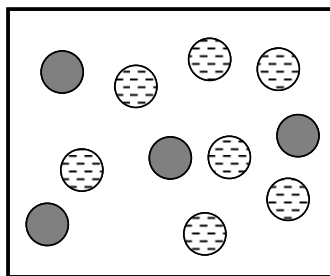


When we look at a collection of counters (inside each rectangle) we can write an addition problem to represent what we see. We do this by counting the number of solid counters (in this case 3) and counting the number of dashed counters (in this case 5). So the addition problem becomes $3 + (-5) = \underline{\quad}$.

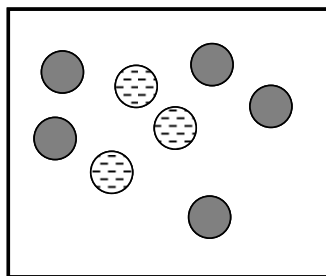
To perform the addition, we use the Additive Inverse Property, specifically, that $1 + (-1) = 0$. By matching up pairs of positive and negative counters until we run out of matched pairs, we can see the value of the remaining result. In this example, we are left with two dashed counters, representing the number -2. So the collection of counters represents the problem $3 + (-5) = -2$.



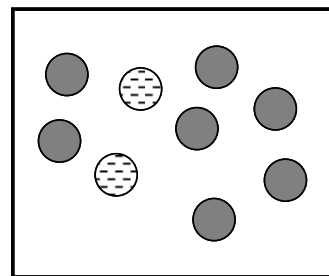
Now try to write the problems that represent the collections below.



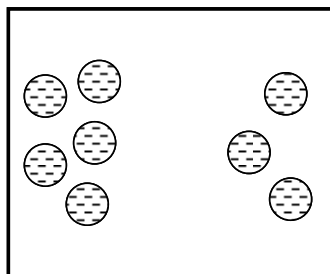
a. $\underline{\quad} + \underline{\quad} = \underline{\quad}$



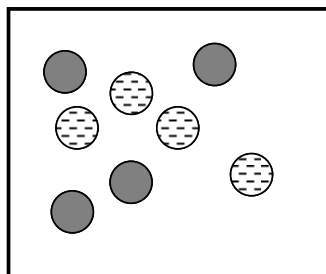
b. $\underline{\quad} + \underline{\quad} = \underline{\quad}$



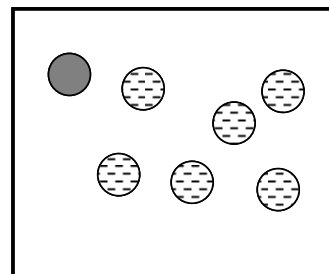
c. $\underline{\quad} + \underline{\quad} = \underline{\quad}$



d. $\underline{\quad} + \underline{\quad} = \underline{\quad}$



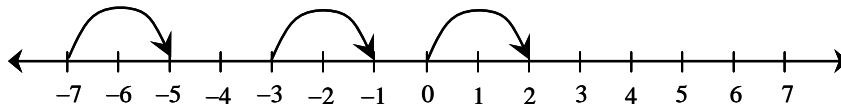
e. $\underline{\quad} + \underline{\quad} = \underline{\quad}$



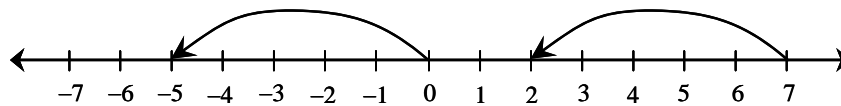
f. $\underline{\quad} + \underline{\quad} = \underline{\quad}$

Part II: The second model for addition of real numbers that we look at is called the “number line” method. We use directional arcs to represent numbers that are positive and negative. The length that the arc represents corresponds to the magnitude of the number.

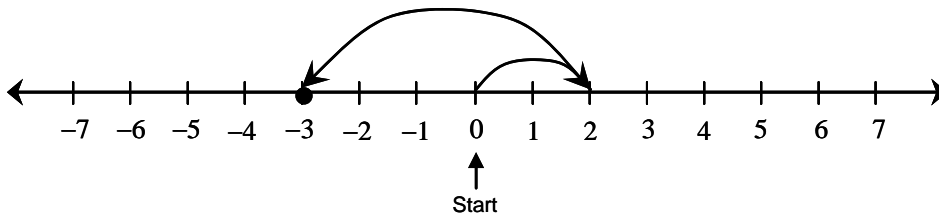
When a directional arc indicates a positive direction (to the right), it represents a positive number. In the diagram below, each arc represents the number 2, because each arc represents a length of two and each arrow points to the right.



In the next diagram, each directional arc represents the number -5 , since each arc represents a length of five and each arrow points to the left.

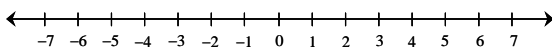


When we want to represent an addition problem, **we start at zero**, and travel from each number to the next using a new directional arc. Thus, the following number line diagram represents $2 + (-5) = -3$. The final landing point is the answer to the addition problem.

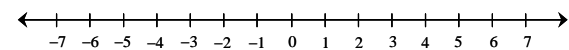


Now try to solve these addition problems on a number line using directional arcs.

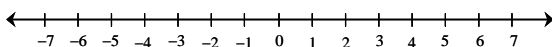
a. $-3 + 7 = \underline{\hspace{2cm}}$



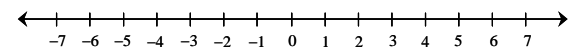
d. $-6 + 2 = \underline{\hspace{2cm}}$



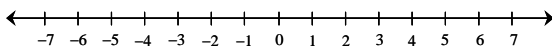
b. $-2 + (-3) = \underline{\hspace{2cm}}$



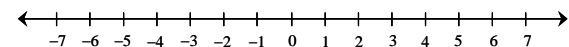
e. $-2 + (-2) + (-1) = \underline{\hspace{2cm}}$



c. $5 + (-5) = \underline{\hspace{2cm}}$

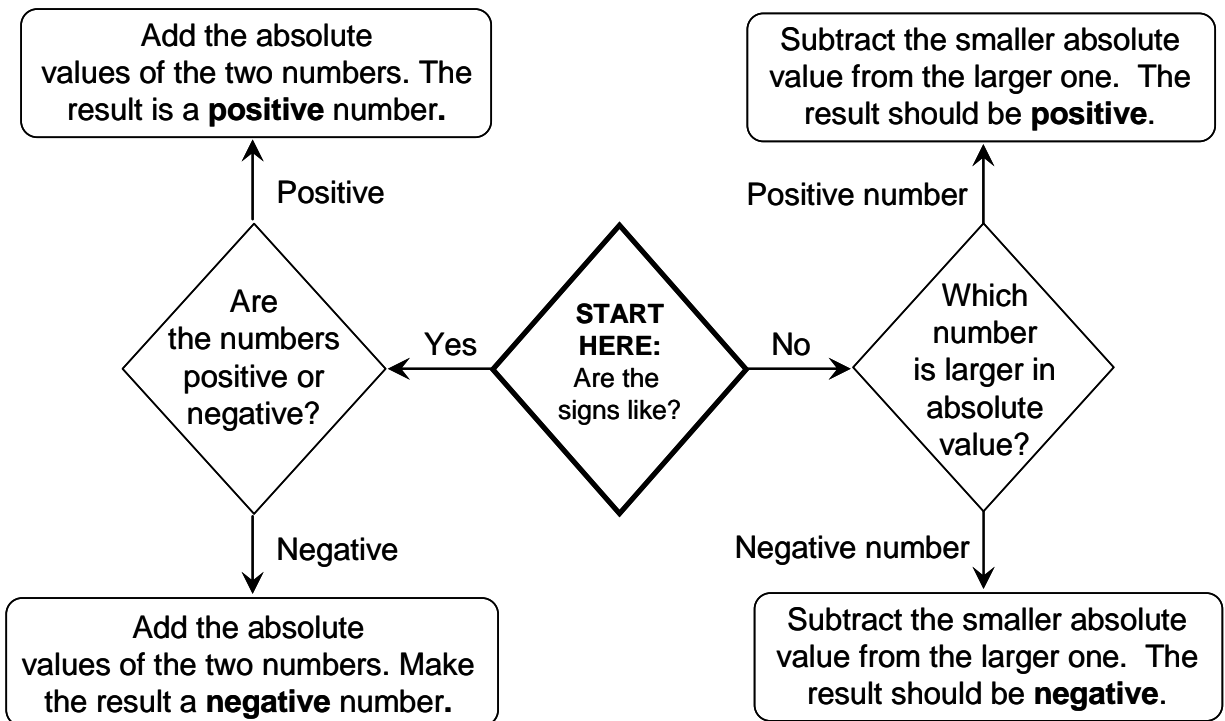


f. $4 + (-8) + 3 = \underline{\hspace{2cm}}$



Classroom Overhead:

Section 1.4: Decision Chart for Addition of Real Numbers



Classroom Overhead:

Section 1.6: Decision Chart for Multiplication and Division of Real Numbers



Student Activity A

Section 1.4: Match Up on Addition of Real Numbers

Match-up: Match each of the expressions in the squares of the table below with its simplified value at the top. If the solution is not found among the choices A through D, then choose E (none of these).

A 3

B -6

C -1

D 1

E None of these

$-8+2$	$-9+8$	$-7+10$	$-3+(-3)$
$-2+(-1)+6$	$3+(-3)$	$-10+(-4)+8$	$-\frac{1}{2}+\left(-\frac{1}{2}\right)$
$6+(-7)$	$5+(-6)$	$-1+(-1)+(-1)+4$	$-11+12$
$-4+\left(-\frac{1}{2}\right)+\frac{1}{2}$	$8+(-5)+(-9)$	$10+(-6)+(-1)$	$-6+12$
$0+(-6)$	$-1+7$	$30+(-27)$	$1+\frac{1}{4}+\frac{1}{2}+\left(-\frac{3}{4}\right)$

Student Activity B

Section 1.4: Signed Numbers Magic Puzzles

Directions: In these “magic” puzzles, each row and column adds to be the same “magic” number. Fill in the missing squares in each puzzle so that the rows and columns each add up to be the given magic number.

Magic Puzzle #1

-2	8	
	-9	4

Magic Number = 5

Magic Puzzle #2

		4
-5		
8		-6

Magic Number = 0

Magic Puzzle #3

		$2\frac{1}{4}$
	$2\frac{1}{2}$	$-2\frac{1}{2}$
$\frac{1}{4}$		

Magic Number = $\frac{1}{2}$

Magic Puzzle #4

-2		9	-7
	-9		4
8	-4		-5
		-6	

Magic Number = 1

Magic Puzzle #5

-8	-3		7
	-11	2	6
8	-2		-7

Magic Number = -2

Solutions to Activities

Section 1.4 Adding Real Numbers; Properties of Addition

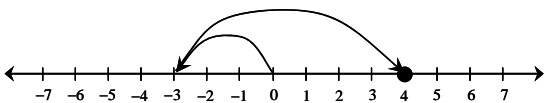
Solution to 1.4 Guided Learning Activity: Using Addition Models

Part I:

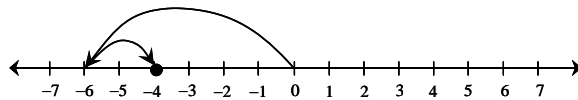
- a. $4 + (-7) = -3$ or $-7 + 4 = -3$ b. $5 + (-3) = 2$ or $-3 + 5 = 2$
 c. $7 + (-2) = 5$ or $-2 + 7 = 5$ d. $-5 + (-3) = -8$ or $0 + (-8) = -8$
 e. $4 + (-4) = 0$ or $-4 + 4 = 0$ f. $1 + (-6) = -5$ or $-6 + 1 = -5$

Part II:

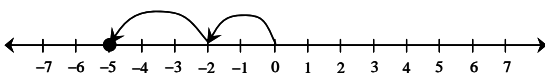
a. $-3 + 7 = 4$



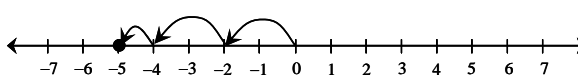
d. $-6 + 2 = -4$



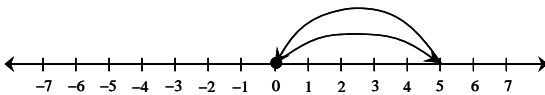
b. $-2 + (-3) = -5$



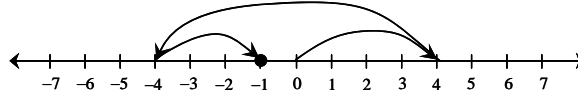
e. $-2 + (-2) + (-1) = -5$



c. $5 + (-5) = 0$



f. $4 + (-8) + 3 = -1$



Solution to 1.4 Student Activity A: Match Up on Addition of Real Numbers

B	C	A	B
A	E	B	C
C	C	D	D
E	B	A	E
B	E	A	D

Solution to 1.4 Student Activity B: Signed Number Magic Puzzles

Magic Puzzle #1

-2	8	-1
10	-9	4
-3	6	2

Magic Number = 5

Magic Puzzle #2

-3	-1	4
-5	3	2
8	-2	-6

Magic Number = 0

Magic Puzzle #3

$-\frac{1}{4}$	$-1\frac{1}{2}$	$2\frac{1}{4}$
$\frac{1}{2}$	$2\frac{1}{2}$	$-2\frac{1}{2}$
$\frac{1}{4}$	$-\frac{1}{2}$	$\frac{3}{4}$

Magic Number = $\frac{1}{2}$

Magic Puzzle #4

-2	1	9	-7
4	-9	-4	10
8	-4	2	-5
-9	13	-6	3

Magic Number = 1

Magic Puzzle #5

-8	-3	2	7
-3	14	-5	-8
1	-11	2	6
8	-2	-1	-7

Magic Number = -2

Teaching Guide

Section 2.7: Solving Inequalities

■ Preparing for Your Class

Suggested Class Time: 60-90 minutes

Materials Needed: Student activities, guided learning activity, colored pencils for students to use, and colored markers (or colored chalk) for instructor to use

Vocabulary

- Inequality ($<$, $>$, \leq , \geq)
- Solution of an inequality, graphing the inequality, equivalent inequalities
- Open circle, closed circle, parentheses, brackets
- Interval notation, intervals, positive infinity, negative infinity, ∞ , $-\infty$
- Addition and subtraction property of inequality,
- Multiplication and division property of inequality
- Compound inequality (like $-3 < x \leq 5$)

Instruction Tips

- Often students try to get by without ever learning that $<$ is less than and $>$ is greater than, so really stress the language here.
- Students have a hard time understanding why $-10 < -3$ is true, to them, -10 has a greater magnitude than -3 , so stress that the **lesser number** (on a number line) is on the **left**.
- Some students really have a hard time understanding why $3 \leq 3$ is true or why $2 < 2$ is false. Student Activity A will help make sure that the students learn this concept.
- It is helpful to do many examples with interval notation; using the Guided Learning Activity will save you and your students a lot of time drawing endless number lines.
- Many students do not discern that a parentheses and a bracket are vastly different in interval notation. This is not surprising, after all, up to this point, we treated parentheses and brackets as equivalent grouping symbols. The expression $3(x-5)$ and $3[x-5]$ are equivalent. However, the intervals $(3,5)$ and $[3,5]$ are **not** equivalent. You have to stress that in interval notation, parentheses and brackets cannot be used interchangeably.
- No matter how many times you remind them, many students will forget to reverse the direction of the inequality when they encounter multiplication or division by a negative number. Some students will incorrectly decide that subtraction causes reversal of the inequality, since the subtracted number looks negative.
- Like in sections 2.1, 2.2, and 2.4, it will be helpful to use colored pencils to illustrate the step where you apply the inequality properties to both sides of the inequality. Bring colored pencils for your students to use in their notes.
- The distinction between an interval with open ends and an ordered pair is based solely on context in which you find the information, and without a great deal of practice understanding different mathematical contexts, it can be easy for students to get confused. If an interval contains $-\infty$ or ∞ , it is easily identified as such. However, for a statement like: "The solution is $(3,5)$," it is really not clear at all whether $(3,5)$ is an interval or an ordered pair.

■ Teaching Your Class

What is an inequality?

- $<$ is less than and \leq is less than or equal to (note that the $<$ looks like a slightly tipped capital L)
- $>$ is greater than and \geq is greater than or equal to
- We read a compound inequality with the word *between*. For example $3 < x < 5$ is read as “ x is between 3 and 5.”
- Give some examples of true and false inequalities.
- Show paired examples with inequalities facing opposite directions. For example, $-2 < 3$ and $3 > -2$ give the same relationship.
- It is helpful to draw up a large number line on the board to use in your examples.

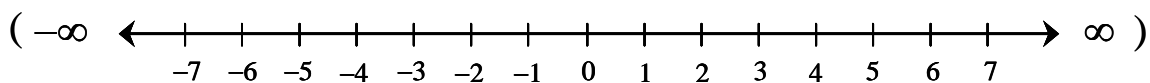
Student Activity A: *Tic-Tac-Toe with Inequalities.* The first game is deciding whether an inequality is true or false. The second game is deciding whether a number is a solution of an inequality. It may help the students to draw a number line on their page to help them make good decisions.

Graphing Inequalities

- Two methods for graphing $<$ or $>$: you can use an open circle or a parenthesis that opens in the direction where the inequality is true.
- Two methods for graphing \leq or \geq : you can use a closed circle or a bracket that opens in the direction where the inequality is true.
- If students get used to graphing with parentheses or brackets, it is a bit easier to teach them interval notation.

Interval Notation

- Interval notation is always written from left to right as it appears on a number line. If the interval you are graphing extends indefinitely to the left, that is designated $-\infty$. If the interval you are graphing extends indefinitely to the right, that is designated ∞ .
- In interval notation, $-\infty$ and ∞ always have parentheses (never brackets, since infinity is approached but never reached).
- Often students write intervals like this: $[3, -\infty)$ or $[4, \infty]$, misinterpreting either the location of the ∞ or $-\infty$, or the proper symbol to use $[$ or $($. On the number line below, the $-\infty$ and ∞ symbols have been added with the appropriate parentheses, to remind students of how it would look when properly written in interval notation (it is also this way in the guided learning activity).



Guided Learning Activity: *Graphing Inequalities and Using Interval Notation.* Students get practice both with graphing inequalities and going back and forth between graphs, set notation, and interval notation.

Solving Inequalities using Addition or Subtraction

- Addition and Subtraction Properties of Inequality:
For a, b, c are real numbers, if $a < b$ then $a + c < b + c$ and $a - c < b - c$.
(this one is parallel to the property for equations)
- Have students write the property for multiplication by a negative number ($c < 0$) using $a > b$, $a \leq b$ or $a \geq b$.

Examples: (Solve and write the answer in interval notation.)

$x - 5 < 7$	$3 + x > 4$	$5 \geq 2 + x$	$-3 < x + 2 \leq 5$
$(-\infty, 12)$	$(1, \infty)$	$(-\infty, 3]$	$(-5, 3]$

Solving Inequalities using Multiplication or Division

- It is important that students understand the multiplication and division property for inequalities, so the following set of examples will illustrate it. We try to apply the rules for equations to inequalities. In each example we start with a statement that is true.

$-5 < 10$ (subtract 2 on both sides)	$-5 < 10$ (add 2 on both sides)
$-7 < 8$ (still true)	$-3 < 12$ (still true)
$-5 < 10$ (multiply by 8 on both sides)	$-5 < 10$ (multiply by -8 on both sides)
$-40 < 80$ (still true)	$40 < -80$ (NO LONGER TRUE)
$-5 < 10$ (divide both sides by 5)	$-5 < 10$ (divide both sides by -5)
$-1 < 2$ (still true)	$1 < -2$ (NO LONGER TRUE)

- Multiplication and Division Properties of Inequality
For a, b, c are real numbers, where c is positive, if $a < b$, then $a \cdot c < b \cdot c$.
For a, b, c are real numbers, where c is negative, if $a < b$, then $a \cdot c > b \cdot c$.
(this one is very different from the property for equations)
- Summarize this property: *When you multiply or divide by a negative number, reverse the direction of the inequality.*
- Have students write the property for $a > b$, $a \leq b$ or $a \geq b$.

Examples: (Solve and write the answer in interval notation.)

$5x < 25$	$8 > -2x$	$-6 \geq \frac{x}{3}$	$-x \leq 5$	$-3 \leq 3x < 12$
$(-\infty, 5)$	$(-4, \infty)$	$(-\infty, -18]$	$[-5, \infty)$	$[-1, 4]$
$2x - 1 > 8$	$4 - x \leq 8$	$\frac{x}{5} - \frac{1}{2} < 3$	$-1 < 2x - 1 \leq 5$	
$(\frac{9}{2}, \infty)$	$[-4, \infty)$	$(-\infty, \frac{35}{2})$	$(0, 3]$	

Student Activity B: *Match up on Solving Inequalities.* Students will get lots of practice with solving fairly simple inequalities involving all the properties of inequalities. They also get practice with interpreting an expression like $x < 2$ as $2 > x$ in order to do the matching.

Student Activity A

Section 2.7: Tic-Tac-Toe with Inequalities

Tic-tac-toe #1: If the inequality in the square is true, then put an **O** on the square. If it is false, then put an **X** on the square.

$-5 < -1$	$8 > 8$	$-6 > -2$
$2 - 3 < -1$	$\frac{1}{2} < \frac{1}{3}$	$7 \leq 7$
$a \geq a$	$0 < -4$	$ -3 > 1$

Tic-tac-toe #2: If the number in the square is a solution to the inequality, then put an **O** on the square. If it is false, then put an **X** on the square.

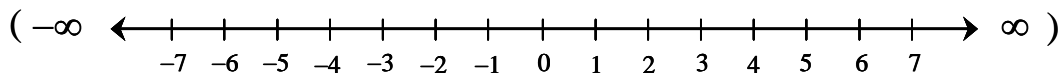
$x - 2 < 7$ 9	$x + 4 \geq -2$ -7	$x - 3 \leq -1$ 1
$-2x < 4$ -1	$\frac{1}{2}x \geq 2$ 6	$-1 \leq 2 - x$ 3
$8 > 4 - x$ -5	$-x < 4$ -3	$2x + 3 < 3x$ 2

Guided Learning Activity

Section 2.7: Graphing Inequalities and Using Interval Notation

Think of interval notation as a way to tell someone how to draw the graph, from left to right, giving them only a “begin” value and an “end” value for each interval.

- Use $-\infty$ and ∞ to denote the “ends” of the number line (as shown in the number line below) Remember that it would read $(-\infty, \dots$ or $\dots, \infty)$ at the ends of an interval.



- Always give intervals from LEFT to RIGHT on the number line.
- Use (or) to denote an endpoint that is approached, but not included.
- Use [or] to denote an endpoint that may be included.
- The parenthesis or bracket needs to open in the direction of the true part of the inequality.

	Set notation	Graph	Interval Notation
1.	$x > 2$		
2.	$x \leq -3$		
3.	$x \geq -1$		
4.			
5.			
6.	$-2 < x \leq 3$		
7.			
8.			$(-\infty, 4)$
9.			$(-2, 0]$
10.			$(-\infty, \infty)$

Student Activity B

Section 2.7: Match up on Solving Inequalities

Match-up: Match each of the inequalities in the squares of the table below with an equivalent inequality from the top. If the solution is not found among the choices A through D, then choose E (none of these).

A $x > 2$

B $x < 3$

C $x > -1$

D $x < -3$

E None of these

$2x < -6$	$-3x < -6$	$-5x < 5$	$x + 2 < 5$
$1 > x - 1$	$-1 < x$	$8 < 4x$	$15 < -5x$
$\frac{3}{2}x > 3$	$6 > -6x$	$2x + 3 < -3$	$\frac{x}{2} + 1 < \frac{5}{2}$
$5 - 2x < 11$	$2 - 3x < 5$	$\frac{x}{3} < 1$	$x - 7 > -8$
$2x < -2$	$\frac{x+1}{2} > \frac{1}{2}$	$1 - x > 4$	$-9 > x - 6$

Solutions to Activities

Section 2.7: Solving Inequalities

Solution to 2.7 Student Activity A: Tic-Tac-Toe with Inequalities

O	X	X
X	X	O
O	X	O

X	X	O
O	O	O
X	O	X

Solution to 2.7 Classroom Worksheet:

Graphing Inequalities and Using Interval Notation

1.	$x > 2$	(----- begin on 2	$(2, \infty)$
2.	$x \leq -3$	-----] end on -3	$(-\infty, -3]$
3.	$x \geq -1$	[----- begin on -1	$[-1, \infty)$
4.	$x < 0$	-----) end on 0	$(-\infty, 0)$
5.	$x \geq -2$	[----- begin on -2	$[-2, \infty)$
6.	$-2 < x \leq 3$	(-----] begin on -2, end on 3	$(-2, 3]$
7.	$-2 < x < 2$	(-----) begin on -2, end on 2	$(-2, 2)$
8.	$x < 4$	-----) end on 4	$(-\infty, 4)$
9.	$-2 < x \leq 0$	(-----] begin on -2, end on 0	$(-2, 0]$
10.	$x \in \mathbb{R}$	----- the whole line	$(-\infty, \infty)$

Solution to 2.7 Student Activity B: Match up on Solving Inequalities

D	A	C	B
E	C	A	D
A	C	D	B
E	C	B	C
E	E	D	D

Teaching Guide

Section 3.2: Graphing Linear Equations

■ Preparing for Your Class

Suggested Class Time: 45-60 minutes

Materials Needed: Student activities, guided learning activity, colored pencils for the students and colored markers (or chalk) for the instructor, large straight-edge for graphing lines, classroom overheads from Section 3.1.

Vocabulary

- Linear equation in one variable, linear equation in two variables
- Solution of an equation in two variables, table of solutions (or table of values)
- Graph of the equation, infinitely many solutions
- Standard form (general form)

Instruction Tips

- Some students have a great deal of difficulty creating a table of solutions that is correct. Often this is because they refuse to write down any work as they find each new entry in the table of values – thinking that the work is easy enough to do correctly in their heads.
- When students begin solving equations for y , if there is division involved, they often forget to divide *all* the terms by the divisor.

For example (incorrect work): $2x + 4y = 8$

$$4y = -2x + 8$$

$$y = \frac{-2x}{4} + 8$$

$$y = -\frac{1}{2}x + 8$$

Corrected: $2x + 4y = 8$

$$4y = -2x + 8$$

$$** \quad y = \frac{-2x + 8}{4}$$

$$** \quad y = \frac{-2x}{4} + \frac{8}{4}$$

$$y = -\frac{1}{2}x + 2$$

Showing the two starred (**) steps are crucial here in your instruction.

- As you solve equations for y , mention that on the right side, it is common practice to write the terms in descending powers of x (in other words, the x -term first) – this sets the student up to learn the $y = mx + b$ form later on.

■ Teaching Your Class

Student Activity A: *Tic-Tac-Toe with Ordered Pairs.* This activity gives students practice with checking whether an ordered pair is a solution to a linear equation. The first game works with an equation that is in $y = mx + b$ form, the second game works with an equation that is in standard form.

Guided Learning Activity: *Graphing Linear Equations.* This guided lecture/discussion gives you the opportunity to “force” students to show their work as they calculate the missing value for each ordered pair (there are two examples). Plotting the resulting values and drawing the line will begin to show them what is meant by a *linear* equation.

Creating a Table of Values

- Practice with creating a table of values from scratch using the examples below
- You can use the overhead graphs from section 3.1 to show the graphs of these equations.

Examples:

- $y = \frac{1}{2}x$ (discuss “good” points to put in the table of values... x 's divisible by 2)
- $y = -10x + 100$ (this one will foster a discussion of scale on graphs)
- $y = \frac{x}{3} + 1$ (again, good points to use would be those with x 's divisible by 3)

Student Activity B: *Out-of-line Detective.* In this activity, students plot someone else's data and identify any suspicious-looking points (points that are “out of the line” and most likely incorrect). The student then has to find the correct point and plot the new point on the graph.

Solving for y

- It is easier to create a table of solutions if the equation has the variable y isolated.
- Remind students to isolate the y -term first; then isolate the y variable.
- You may want to use colored markers (or chalk) to show each new step on both sides of the equation – bring some for your students to use in their notes too!

Examples: (solve each equation for y , then graph using a table of values)

- $3x + 2y = -6$ (becomes $y = -\frac{3}{2}x - 3$)
- $x - y = 4$ (becomes $y = x - 4$, students often lose the $-$ on the $-y$ term)
- $4x + 4y = 2$ (becomes $y = -x + \frac{1}{2}$, students are tempted to ignore the 2)

Application Example:

A church group is going to hold a scrapbooking party as a fundraiser for their mission trip. They estimate that it will cost them \$100 to create posters and flyers to advertise the event, and \$300 to rent a large room for the event. They estimate that they will spend \$10 on food for each participant. The organizer writes an equation for their costs: $C = 400 + 10n$ and she can spend a total of \$1,000 on the event.

- a) Assuming that the room is large enough, how many participants could be invited?
- b) If each participant pays \$25 for the event, will they make any money? If so, how much?

Student Activity C: *The Cost of College – A Linear Function.* In this activity, students use their own tuition and fees to develop an equation for a linear function. You may have to do a little research of your own first to find out what kind of fees and tuition oddities exist at your college. Tuition bills may have surprises on them. For example, some colleges charge no additional tuition between 15 and 18 credits and sometimes a new student will have to pay a new student registration fee that a returning student will not have to pay.

Student Activity A

Section 3.2: Tic-Tac-Toe with Ordered Pairs

Tic-tac-toe #1: If the ordered pair in the square **is** a solution to the equation $y = 2x - 5$, then circle it (thus placing an **O** on the square). If the ordered pair is **not** a solution, then put an **X** over it.

$(-10, -30)$	$(-10, -25)$	$\left(\frac{2}{3}, -\frac{11}{3}\right)$
$(26, 47)$	$(0, -5)$	$(0, 0)$
$(5, 0)$	$\left(-\frac{1}{2}, -4\right)$	$(1, 3)$

Tic-tac-toe #2: If the ordered pair in the square **is** a solution to the equation $4x - 6y = 12$, then circle it (thus placing an **O** on the square). If the ordered pair is **not** a solution, then put an **X** over it.

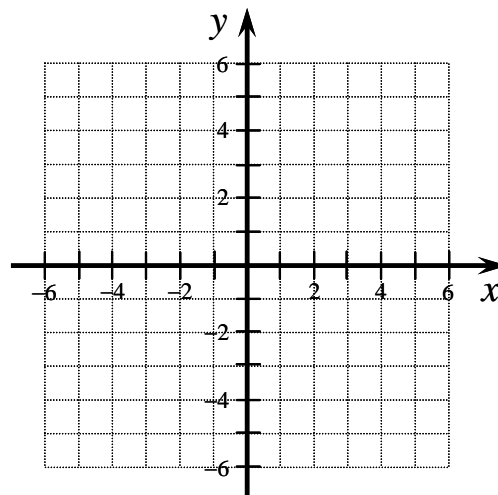
$(-9, -8)$	$\left(1, -\frac{4}{3}\right)$	$(3, 0)$
$(-2, 0)$	$(0, 0)$	$\left(-\frac{3}{2}, -3\right)$
$(1, 1)$	$\left(\frac{2}{3}, -\frac{14}{9}\right)$	$(9, -4)$

Guided Learning Activity

Section 3.2: Graphing Linear Equations

Example 1: Fill in the missing values in the table of solutions for the linear equation, $y = -x + 4$, plot the points, and then draw the line through the points.

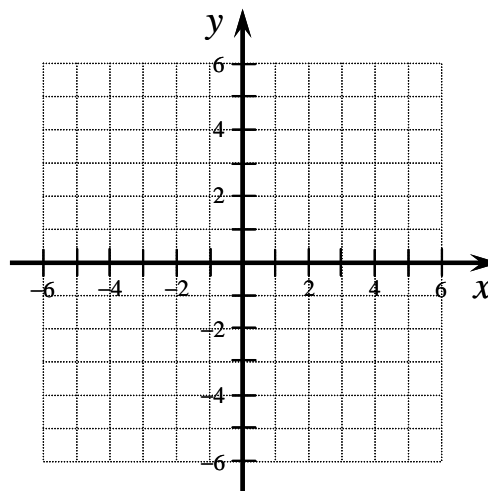
x	y
3	
	3
0	
-2	
	5



If $x = 3$, find y .	If $y = 3$, find x .	If $x = 0$, find y .	If $x = -2$, find y .	If $y = 5$, find x .

Example 2: Fill in the missing values in the table of solutions for the linear equation, $3x - y = -2$, plot the points, and then draw the line through the points.

x	y
	5
0	
	-1
-2	
	3



If $y = 5$, find x .	If $x = 0$, find y .	If $y = -1$, find x .	If $x = -2$, find y .	If $y = 3$, find x .

Student Activity B

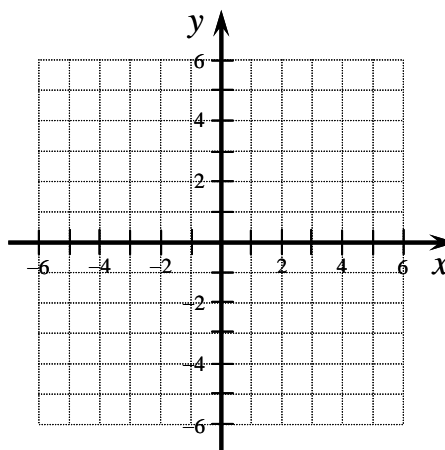
Section 3.2: Out-of-line Suspects

Directions: A student has created a table of solutions for each **linear** equation below.

- 1) Plot the points that the student has found.
- 2) Use the graphed points to identify which points (if any) are most likely incorrect.
- 3) Circle the incorrect points in the table of values.
- 4) Find the correct ordered pairs for any that are circled.
- 5) Plot the corrected ordered pairs in a different color.

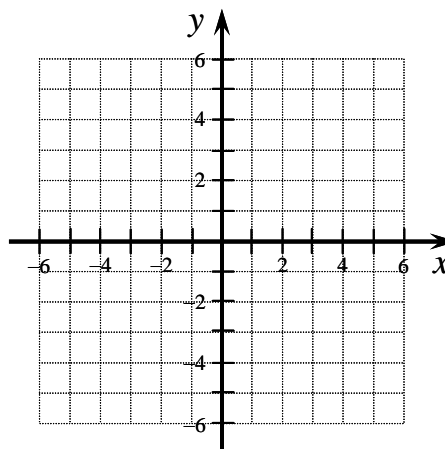
1. Linear equation: $y = 3x - 1$

x	y
-1	2
-2	3
0	-1
$\frac{1}{3}$	0
2	5



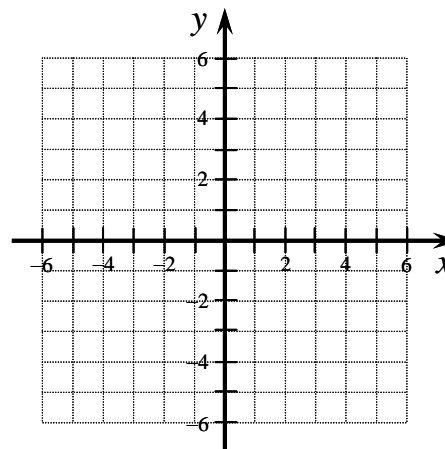
2. Linear equation: $y = \frac{1}{2}x + 2$

x	y
-2	0
-1	$\frac{3}{2}$
2	5
0	2
4	4



3. Linear equation: $y = 4 - x$

x	y
-2	6
-6	1
1	3
2	-4
6	-2



Student Activity C

Section 3.2: The Cost of College – A Linear Function

When you pay for your college education, you have typically have three different types of charges:

- 1) **Variable cost** – Tuition, which is paid per credit hour (or unity of study). There may also be fees that vary depending on the number of credit hours (for example, an additional \$6 per credit hour).
- 2) **Fixed cost** – Student fees, which are paid regardless of how many or what types of classes you take, (for example, a \$25 Registration Fee). These are fixed costs.
- 3) Course fees or lab fees, which are paid for certain courses only (we will ignore these in this model). Your instructor may tell you to ignore some other fee or tuition oddities to allow a linear model to be used.

Our linear model of college costs will only consider the first two types of charges. Your instructor will help you locate the information necessary to build the linear model – perhaps they have asked you to bring in a copy of your tuition bill!

How much do you pay in **fixed** costs for your enrollment at your college? _____

Per credit hour, how much do you pay in tuition (and **variable** fees)? _____

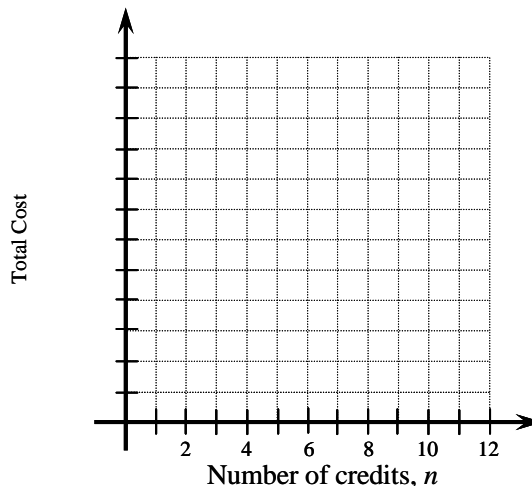
Fill in the table below for a student at your college:

Credits (or units) taken by student, n	Fixed student fees	Tuition and other variable costs for this number of credits (or units)	Total cost for this number of credits, C
4			
8			
12			
16			

Create a graph containing the data points above.

Write a linear equation to model the cost C , for taking a certain number of credits n .

Show how you could use the linear equation you just wrote to estimate the cost of taking 9 credits (or units).



Solutions to Activities

Section 3.2: Graphing Linear Equations

Solution to 3.2 Student Activity A: Tic-Tac-Toe with Ordered Pairs

Game #1

X	O	O
O	O	X
X	X	X

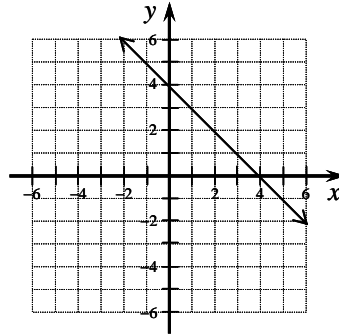
Game #2

O	O	O
X	X	O
X	O	X

Solution to 3.2 Guided Learning Activity: Graphing Linear Equations

Example 1:

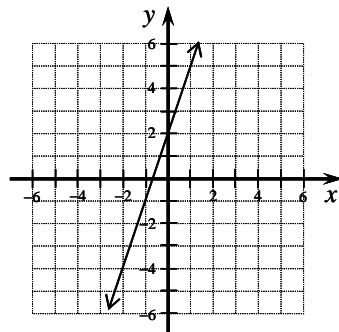
x	y
3	1
1	3
0	4
-2	6
-1	5



If $x=3$, find y .	If $y=3$, find x .	If $x=0$, find y .	If $x=-2$, find y .	If $y=5$, find x .
$y = -(\quad) + 4$	$(\quad) = -x + 4$	$y = -(\quad) + 4$	$y = -(\quad) + 4$	$(\quad) = -x + 4$
$y = -(3) + 4$	$(3) = -x + 4$	$y = -(0) + 4$	$y = -(-2) + 4$	$(5) = -x + 4$
$y = 1$	$-1 = -x$	$y = 4$	$y = 2 + 4$	$1 = -x$
	$1 = x$		$y = 6$	$-1 = x$

Example 2:

x	y
1	5
0	2
-1	-1
-2	-4
$\frac{1}{3}$	3



If $y=5$, find x .	If $x=0$, find y .	If $y=-1$, find x .	If $x=-2$, find y .	If $y=3$, find x .
$3x - (\quad) = -2$	$3(\quad) - y = -2$	$3x - (\quad) = -2$	$3(\quad) - y = -2$	$3x - (\quad) = -2$
$3x - (5) = -2$	$3(0) - y = -2$	$3x - (-1) = -2$	$3(-2) - y = -2$	$3x - (3) = -2$
$3x = 3$	$-y = -2$	$3x + 1 = -2$	$-6 - y = -2$	$3x = 1$
$x = 1$	$y = 2$	$3x = -3$	$-y = 4$	$x = \frac{1}{3}$
		$x = -1$	$y = -4$	

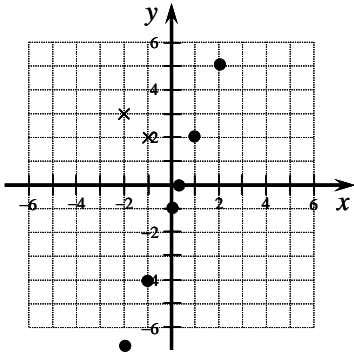
Solutions to Activities

Section 3.2: Graphing Linear Equations

Solution to 3.2 Student Activity B: *Out-of-line Suspects*

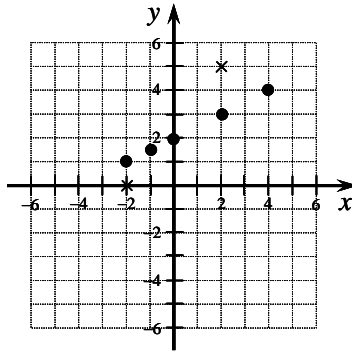
1. Incorrect points:

$(-1, 2)$, $(-2, 3)$



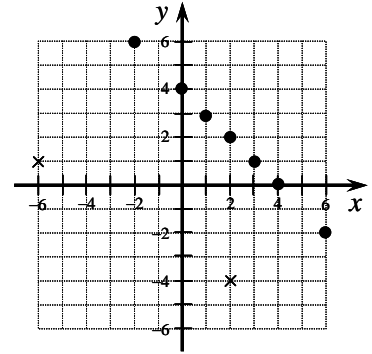
2. Incorrect points

$(-2, 0)$, $(2, 5)$



3. Incorrect points:

$(-6, 1)$, $(2, -4)$



Solutions to 3.2 Student Activity C: *The Cost of College – A Linear Equation*

The solutions for this activity will vary depending on the college.

Teaching Guide

Section 4.4: Problem Solving Using Systems of Equations

■ Preparing for Your Class

Suggested Class Time: 60-90 minutes

Materials Needed: Student activities, guided learning activity

Vocabulary

- Complementary, supplementary

Instruction Tips

- Students rarely like doing application problems. Many will simply look at an application problem and give up (often before they even read it). The more practice you can do in class with application problems, letting the *students* guide the problem-solving strategies, the more comfortable your students will feel on their own.
- Often students do not see that there is one distinct advantage associated with application problems: the chance to ask “does this answer make sense?” It is a good habit for students to examine application problems and try to guess what the solutions should look like, before they attempt the problems. This way, they can compare the results they get at the end, and see if these results are realistic.
- Because variables represent the unknown quantities in problems, most of the time your students can determine what the variables should be simply by looking for the question in the problem (which is usually the last sentence). Although this seems obvious to us, students often don’t understand this simple truth. It is very difficult to construct a system of equations if the student does not know what the variables should be, and once they practice, it should not be hard for them to correctly declare variables. Student Activity A is designed specifically for students to practice with variable declaration.
- A variety of application problems are included in this section, but you may want to emphasize specific categories of problems to avoid running out of time. Here is a rough classification of the application problems included in this section:
 - Angle problems (supplementary, complementary)
 - Related quantity problems (length-width, salary figures, etc.)
 - Distance-Rate-Time problems (rate of current, rate of wind)
 - Value mixture problems
 - Percent solution problems, percent alloy problems
 - Investment problems
- The majority of this section can be taught with the activities and worksheets.

■ Teaching Your Class

Problem-Solving Strategy

1. Analyze the problem. *What is the given information? What are you trying to solve for? Will a diagram or table help here?*
2. Declare variables. Write equations.
3. Solve the system of equations.
4. State the conclusion. *Don’t forget units.*
5. Check the result. *Does it make sense? Is it reasonable?*

Student Activity A: *The Last Sentence.* Students often want to skip the variable declaration step of these application problems, and then they end up with a faulty system of equations. This activity forces students to work with the variable declaration step specifically, using the context of the examples from this section for practice.

Student Activity B: *Working with the Smaller Pieces.* If your students are weak on application problems in general, you may first want to make sure that they understand the concepts that underlie these system-of-equations application problems. In this activity, we go through small problems related to distance-rate-time, investment problems, value mixture problems and solution mixture problems.

Guided Learning Activity: *Using the Problem Solving Strategy.* This worksheet provides three examples, similar to the ones in the book, that the instructor and students can work through together using the problem-solving strategy. Note: Linden dollars (sometimes called “ellz”) are the currency of the virtual world Second Life and are pegged to the U.S. Dollar.

Student Activity C: *Setting Up the System.* In this activity, students practice with declaring the variables *and* setting up the system of equations for a variety of application problems. This is typically what students have the most difficulty with in this section. Once the system of equations is written, they will not usually struggle as much with solving it.

Student Activity D: *The Moving Walkway Problem.* Students with little experience of flying planes or driving boats do not necessarily understand the concepts of upstream and downstream or flying with or against the wind. But many students have either seen people-movers in action movies or ridden on a people mover at an amusement park, museum exhibit, zoos, or airport. So here’s a new twist on an old problem.

Student Activity A

Section 4.4: The Last Sentence

Directions: What you see below is the last sentence from each of the example problems in this section of the textbook. Just from this last sentence, you can make a fairly good guess about what the two variables should be for the problem. The first one has been done for you.

Example 1: *Find the height of the figure and the height of the base.*

Let f = the height of the figure.

Let b = the height of the base.

Example 2: *Find the measure of each angle.*

Example 3: *Find the length and width of the poster.*

Example 4: *Find the cost of a class picture and the cost of an individual wallet-size picture.*

Example 5: *How much money was invested in each account?*

Example 6: *Find the speed of the boat in still water and the speed of the current.*

Example 7: *How many fluid ounces of each batch should she use?*

Example 8: *How many ounces of each ingredient should be used to create a 20-ounce box of raisin bran cereal that can be sold for \$0.15 an ounce?*

Now go back and read all the example problems in this section. Can you add any detail to your variable declarations after reading the complete problems.

Student Activity B

Section 4.4: Working with the Smaller Pieces

Practice with Distance-Rate-Time Problems

Distance, rate and time can be related using the formula: $d = rt$

1. If you travel 8 miles from your house to visit a friend, what is your round-trip distance? _____
2. Will you run faster when you run *with* the wind or *against* it? _____
3. How far can you drive in 90 minutes at a rate of 70 miles per hour? _____
4. If you work 30 miles away from your home, how long will it take you to get there if your average rate is 45 miles per hour? _____
5. If you drive for 3 hours and travel 210 miles, what was your average rate?

6. If you are in a boat that can travel 4 miles per hour in still water, will you be traveling *faster* or *slower* than 4 miles per hour when traveling upstream? _____

Practice with Value Mixture and Percent Mixture Problems

7. If a store owner mixes peanuts that cost \$3 per pound with cashews that cost \$8 per pound, a pound of mixed nuts should cost
 - a. less than \$3
 - b. between \$3 and \$8
 - c. more than \$8
8. Simon has to mix two solutions in his chemistry class. One contains 3% sulfuric acid, and the other contains 10% sulfuric acid. How much sulfuric acid will the resulting mixture contain?
 - a. less than 3%
 - b. between 3% and 10%
 - c. between 10% and 13%
 - d. more than 13%
9. Lucy has to combine plain water with a solution that contains 15% ammonia. The resulting solution will be
 - a. less than 15% ammonia
 - b. more than 15% ammonia
10. If you wanted to obtain 5 liters of a solution that is 4% nitric acid, which solution could you **not** add?
 - a. 3 liters of a 3% solution
 - b. 2 liters of a 5% solution
 - c. 6 liters of a 2% solution
11. At Meg's coffee shop, frozen coffees cost \$3.50 each. If Meg took in a total of \$42 on Tuesday, what is the maximum number of frozen coffees she could have sold? _____

Practice with Investment Problems

Interest earned, principle invested, interest rate per year, and time invested (years) can be related using the formula: $I = Prt$

12. If you invest \$2,500 for one year at an annual rate of 6%, how much interest does the investment earn? _____
13. If you invested a total of \$15,000 in two different accounts, and you put \$6,000 in the first account, how much did you put in the second account? _____
14. If you invest \$1,000 in an account that pays 5% annual interest, how much money will you have after one year? _____
15. Martin invests \$2,000 in a bank CD. If the interest after one year is \$160, what was the annual interest rate? _____

Guided Learning Activity

Section 4.4: Using the Problem Solving Strategy

Problem A: At a scrapbooking party, customers can purchase two special packages, the vacation scrapbook package and the baby album package. Shaude purchases 2 vacation packages and one baby album package for a total of \$93.50. Stephan purchases one vacation package and three baby album packages for a total of \$138. Find the cost of each of the special packages that were offered at the party.

1. Analyze the problem.

2. Define two variables. Write two equations.

3. Solve the system of equations.

4. State the conclusion.

5. Check the result.

Problem B: Jasper invests a total of 250,000 L\$ (Linden dollars). He splits his investment between a 30-day Linden Bank CD paying 7% interest and a 30-day Second Life real estate investment scheme organized by his brother that ends up losing 4% of its value. At the end of 30 days, Jasper makes 2,100 L\$. How much did Jasper invest in the CD and how much did he invest in the real estate scheme?

1. Analyze the problem.

2. Define two variables. Write two equations.

3. Solve the system of equations.

4. State the conclusion.

5. Check the result.

Problem C: Technical grade hydrogen peroxide is 35% H_2O_2 . Common household hydrogen peroxide is only 3% H_2O_2 . Imogen wants to mix 500 mL of a 10% H_2O_2 solution to use in removing a toxic mold in a city building. What amount of the technical grade solution and what amount of the household solution (to the nearest 0.1 mL) should she mix to create the solution she desires?

1. Analyze the problem.

2. Define two variables. Write two equations.

3. Solve the system of equations.

4. State the conclusion.

5. Check the result.

Student Activity C

Section 4.4: Setting Up the System

Directions: Read each problem carefully.

a) **Define the variables** you would use to solve the problem.

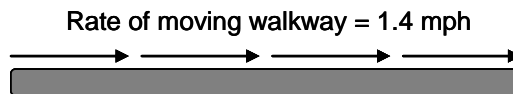
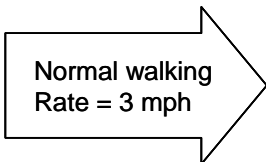
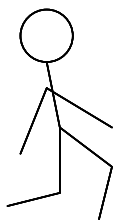
b) **Write a system** of two equations that could be used to solve each problem. You do not need to solve the application problems.

1. The elevation in Denver, Colorado, is 13 times greater than that of Waikapu, Hawaii. The sum of their elevations is 5,600 feet. Find the elevation of each city.
2. The owner of a pet food store wants to mix birdseed that costs \$1.25 per pound with sunflower seeds that cost \$0.75 per pound to make 50 pounds of a mixture that costs \$1.00 per pound. How many pounds of each type of seed should he use?
3. An airplane can fly 500 miles in 2 hours when flying with the wind. The same journey takes 4 hours when flying against the wind. Find the speed of the wind and the speed of the plane.
4. Barb invested a total of \$25,000 in two accounts. One account has an annual interest rate of 3.5%, while the other has an annual rate of 6%. She earned \$1,257 in interest in one year. How much did she invest in each account?
5. Two angles are supplementary. The measure of the first angle is 10 degrees more than three times the second angle. Find the measure of each angle.

Student Activity D

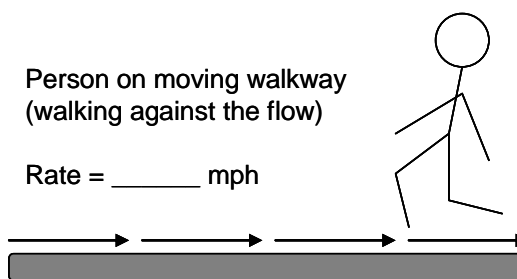
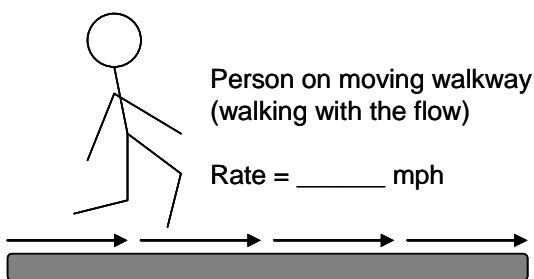
Section 4.4: The Moving Walkway Problem

A moving walkway is like an escalator (transporting you from one place to another), only it is a flat belt that you can walk on. Moving

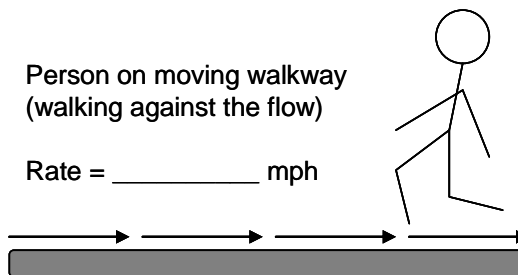
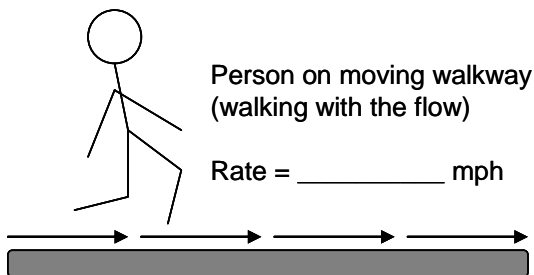


walkways can commonly be found now in airports, museums, amusement parks, or zoos. When you walk onto a moving walkway going in your direction and continue to walk, your walking speed is increased by the speed of the walkway.

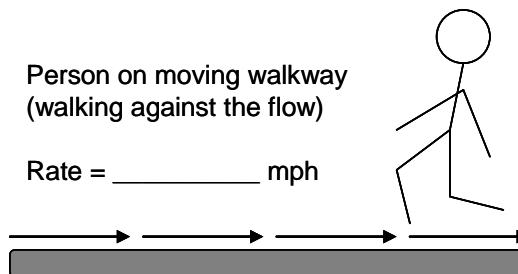
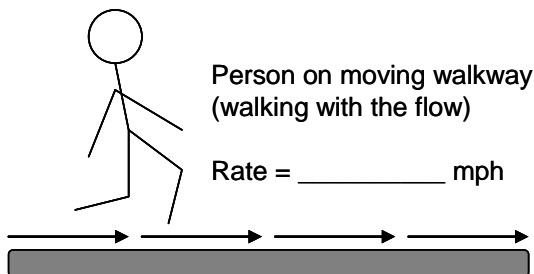
1. Using the information from the diagram above, fill in the missing rates.



2. Often, you do not know one of the rates. In these cases, you have to express the rate as an algebraic expression. Assume the person is walking at a rate of 2.7 mph and the rate of the moving walkway is x mph. Fill in the missing rates.



3. Now the rate of the person is R mph and the rate of the walkway is 2.3 mph. Fill in the missing rates.



4. At the Montparnasse station in Paris, there is a high-speed moving walkway that was designed to help commuters get through the station faster. This moving walkway, which is 0.18 km long, moves at a speed of 9 km/hr (originally the walkway was designed to go 11 km/hr, but there were too many accidents). David walks at his normal pace in the direction of flow on the walkway – it takes 54 seconds to go the full distance of the walkway. How fast was David walking?

5. While on a layover at the Detroit International Airport, a bored math teacher decides to calculate the speed of one of the many moving walkways in the airport. She knows (from years of treadmill walking) that her normal walking speed is 2.5 mph (or 2.2 ft/sec). Walking with the flow of the walkway, it takes 43 seconds to go the distance of the walkway. Walking against the flow of the walkway, it takes 2 minutes and 47 seconds. What is the approximate rate of the moving walkway in feet per second and what is the length of the walkway in feet?

	Rate	Time	Distance
With Flow of Walkway			
Against Flow of Walkway			

6. The Central-Mid-Levels Escalator in Hong Kong is the world's longest covered escalator, stretching 800m (2,625 ft). A trip on the escalator, from top to bottom, takes about 20 minutes when a person is stands still. What is the approximate rate of the escalator in feet per second? How long would the trip take if the person walked downward at a rate of 2 ft/sec?

Solutions to Activities

Section 4.4: Problem Solving Using Systems of Equations

Solution to 4.4 Student Activity A: *The Last Sentence*

Note: Information in parentheses is from going back to read the complete problem.

Example 2: *Find the measure of each angle.*

Let x = degree measure of the first angle

Let y = degree measure of the second angle

Example 3: *Find the length and width of the poster.*

Let L = the length of the poster (in inches)

Let W = the width of the poster (in inches)

Example 4: *Find the cost of a class picture and the cost of an individual wallet-size picture.*

Let c = cost of a class picture

Let w = cost of a wallet-size picture

Example 5: *How much money was invested in each account?*

Let x = money invested in first account (Swiss bank account paying 8%)

Let y = money invested in second account (Cayman Islands account paying 7%)

Example 6: *Find the speed of the boat in still water and the speed of the current.*

Let b = speed of the boat in still water (in miles per hour)

Let c = speed of the current (in miles per hour)

Example 7: *How many fluid ounces of each batch should she use?*

Let x = fluid ounces of first batch (weak batch, 40% alcohol)

Let y = fluid ounces of second batch (strong batch, 60% alcohol)

Example 8: *How many ounces of each ingredient should be used to create a 20-ounce box of raisin bran cereal that can be sold for \$0.15 an ounce?*

Let x = ounces of first ingredient (raisins)

Let y = ounces of second ingredient (bran flakes)

Solution to 4.4 Student Activity B: *Working with the Smaller Pieces*

1. 16 miles

2. with the wind

3. 105 miles

4. 40 minutes

5. 70 miles per hour

6. slower

7. b

8. b

9. a

10. c

11. 12 frozen coffees

12. \$150

13. \$9,000

14. \$1,050

15. 8%

Solution to 4.4 Guided Learning Activity: *Using the Problem Solving Strategy*

Note: For each problem, the solution process is just roughly outlined below.

Problem A: Let x = Price of vacation package

y = Price of baby album package

$$\begin{cases} 2x + y = \$93.50 \\ x + 3y = \$138 \end{cases}$$

$$x = \$28.50$$

$$y = \$36.50$$

Problem B: Let x = Amount invested in 30-day CD

y = Amount invested in real estate scheme

$$\begin{cases} x + y = 250,000 \text{ L\$} \\ 0.07x + (-0.04)y = 2,100 \text{ L\$} \end{cases}$$

$$x = 110,000 \text{ L\$}$$

$$y = 140,000 \text{ L\$}$$

Problem C: Let x = Amount of 35% H_2O_2 solution (in mL)

y = Amount of 3% H_2O_2 solution (in mL)

$$\begin{cases} x + y = 500 \text{ mL} \\ 0.35x + 0.03y = 0.10(500 \text{ mL}) \end{cases}$$

$$x \approx 109.4 \text{ mL}$$

$$y \approx 390.6 \text{ mL}$$

Solution to 4.4 Student Activity C: *Setting Up the System*

1. d = elevation of Denver, Colorado (in feet)

w = elevation of Waikapu, Hawaii (in feet)

$$d = 13w$$

$$d + w = 5600$$

2. b = pounds of birdseed

s = pounds of sunflower seeds

$$\$1.25b + \$0.75s = \$1.00(50)$$

$$b + s = 50$$

3. p = speed of plane in miles per hour

w = speed of wind in miles per hour

$$500 = 2p + 2w$$

$$500 = 4p - 4w$$

4. x = amount invested at 3.5%

y = amount invested at 6%

$$0.035x + 0.06y = \$1,275$$

$$x + y = \$25,000$$

5. x = degree measure of first angle

y = degree measure of second angle

$$x + y = 180^\circ$$

$$x = 3y + 10$$

Solution to 4.4 Student Activity D: *The Moving Walkway Problem*

1. With: 4.4 mph; Against: 1.6 mph
2. With: $2.7 + x$; Against: $2.7 - x$
3. With: $R + 2.3$; Against: $R - 2.3$
4. 3 km/hr
5. Walkway speed ≈ 1.3 ft/sec
Walkway length ≈ 150.5 ft
6. Escalator speed ≈ 2.2 ft/sec
Time to bottom when walking
 ≈ 625 sec or 10.4 min

Teaching Guide

Section 5.2: Zero and Negative Exponents

■ Preparing for Your Class

Suggested Class Time: 45-60 minutes

Materials Needed: Classroom activity, student activities, guided learning activity, classroom overhead

Vocabulary

- Zero exponent, negative exponent

Instruction Tips

- In the more complicated exponential expressions, it may be helpful to students if you indicate the change from one step to the next in color. If students do this in their notes too, it will be easier for them to identify the individual steps as they progress through reading their notes.
- Once students learn the rule for negative exponents, most of them have a very difficult time discerning between expressions like -3 and 3^{-1} . Expressions like these two seem to be naturally categorized “equivalent” when they are not. For students, 3^{-1} looks similar to $3 \cdot -1$, and so it is probable that they are both initially processed the same way during the learning process. Do your best to emphasize the difference at *every* opportunity, as this is a hard concept to fortify.
- Natural instincts lead students to believe that x^0 should be 0 (although it is not). This is an unfortunate remnant of their learning the multiplication property of 0. Since we learn first that any number times zero is zero, we begin to associate any expression with a zero in it that involves multiplication with an answer of zero. Consider that x^0 is an expression with zero in it, and we define exponents in terms of the number of factors (multiplication), so it is not surprising that students default to this idea that x^0 should be 0. Every time this mistake arises in class, remind students of the pattern development on the Guided Learning Activity that clearly shows that x^0 has to be 1.
- After we teach students that the expressions with negative exponents can be moved across the fraction bar, they inevitably want to take the coefficients along for the ride. For example, the majority of students will rewrite $2x^{-1}$ as $\frac{1}{2x}$ instead of $\frac{2}{x}$. To replace this faulty thinking, make sure to emphasize the exponents on *every* factor; thus $2^1 x^{-1}$ becomes $\frac{2^1}{x^1}$.
- Why do we rewrite expressions without negative exponents? First, and of most importance for algebra students, it is much more intuitive to evaluate an expression like $\frac{1}{2^3}$ than it is to evaluate the expression 2^{-3} . The second reason applies to the future classes that students might take; students in Calculus *must* be able to convert *flawlessly* between expressions that do and do not involve negative exponents in order to apply the power rule for derivatives. Consider these examples from a calculus section:

$$\circ \frac{d}{dx} \left(\frac{1}{3x^4} \right) = \frac{d}{dx} \left(\frac{1}{3} x^{-4} \right) = \frac{1}{3} (-4x^{-5}) = -\frac{4}{3x^5}$$

$$\circ \frac{d}{dx} \left(\frac{2}{x} \right) = \frac{d}{dx} (2x^{-1}) = -2x^{-2} = -\frac{2}{x^2}$$

■ Teaching Your Class

Guided Learning Activity: *Zero and Negative Exponents.* The concepts of zero and negative exponents are not naturally intuitive. Because it is best for learning if students discover these rules for themselves, this Guided Learning Activity will guide students to the rules for zero exponents and negative exponents. After the class completes this worksheet, you should be able to discuss the more difficult ideas in this section.

Another set of learning examples for zero and negative exponents

Simplify each example two ways:

- $\frac{x^4}{x^2} = x^{4-2} = x^2$ and $\frac{x^4}{x^2} = \frac{x \cdot x \cdot x \cdot x}{x \cdot x} = \frac{\cancel{x^1} \cdot \cancel{x^1} \cdot x \cdot x}{\cancel{x^1} \cdot \cancel{x^1}} = x \cdot x = x^2$
- $\frac{x^3}{x^2} = x^{3-2} = x^1 = x$ and $\frac{x^3}{x^2} = \frac{x \cdot x \cdot x}{x \cdot x} = \frac{\cancel{x^1} \cdot \cancel{x^1} \cdot x}{\cancel{x^1} \cdot \cancel{x^1}} = x$
- $\frac{x^2}{x^2} = x^{2-2} = x^0$ and $\frac{x^2}{x^2} = \frac{x \cdot x}{x \cdot x} = \frac{\cancel{x^1} \cdot \cancel{x^1}}{\cancel{x^1} \cdot \cancel{x^1}} = 1$ (thus $x^0 = 1$)
- $\frac{x}{x^2} = x^{1-2} = x^{-1}$ and $\frac{x}{x^2} = \frac{x}{x \cdot x} = \frac{\cancel{x^1}}{\cancel{x^1} \cdot x} = \frac{1}{x}$ (thus $x^{-1} = \frac{1}{x}$)
- $\frac{x}{x^3} = x^{1-3} = x^{-2}$ and $\frac{x}{x^3} = \frac{x}{x \cdot x \cdot x} = \frac{\cancel{x^1}}{\cancel{x^1} \cdot x \cdot x} = \frac{1}{x^2}$ (thus $x^{-2} = \frac{1}{x^2}$)

Other simplification involving negative exponents

- Consider this example: $\frac{1}{2^{-3}} = \frac{1}{\frac{1}{2^3}} = 1 \div \frac{1}{2^3} = 1 \cdot \frac{2^3}{1} = 2^3$ so $\frac{1}{2^{-3}} = 2^3$
- A general rule: $\frac{1}{x^{-n}} = \frac{1}{\frac{1}{x^n}} = 1 \div \frac{1}{x^n} = 1 \cdot \frac{x^n}{1} = x^n$ so $\frac{1}{x^{-n}} = x^n$
- Consider this example: $\left(\frac{3}{5}\right)^{-2} = \frac{3^{-2}}{5^{-2}} = \frac{\frac{1}{3^2}}{\frac{1}{5^2}} = \frac{1}{3^2} \div \frac{1}{5^2} = \frac{1}{3^2} \cdot \frac{5^2}{1} = \frac{5^2}{3^2} = \left(\frac{5}{3}\right)^2$
- A general rule: $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$ (with $x \neq 0$ and $y \neq 0$, n a natural number)

Rewrite without negative exponents (simplify if possible)

- $x^3 y^{-4}$
- $6x^2 y^{-3}$
- $6x^{-2} y$
- $(10x)^{-1}$ vs. $10x^{-1}$
- $(-6)^{-2}$ vs. -6^{-2}
- $\frac{x^4}{y^{-2}}$
- $\left(\frac{2}{b}\right)^{-3}$
- $\frac{1}{x^{-2} y^{-3}}$
- $\frac{1}{2x^{-2} y^{-3}}$
- $\frac{1}{(2x)^{-2}}$

Classroom Overhead: *A Summary of Exponent Rules.* This can be copied to an overhead, projected from a portable video projector, or projected from a computer and ceiling projector to a whiteboard. The rules are numbered to make it easier to ask students which rule they applied in a particular step of a problem.

Simplifying expressions that involve many exponent rules

- For most of these examples, there are at least two ways to show the simplification. Because students all learn differently, it is important to show multiple ways to perform the simplifications. Some students will naturally gravitate to one method over the others.

- $\frac{x^{-3}}{x^2} = x^{-3-2} = x^{-5} = \frac{1}{x^5}$ or $\frac{x^{-3}}{x^2} = \frac{1}{x^2 x^3} = \frac{1}{x^{2+3}} = \frac{1}{x^5}$
- $\frac{-3x^2}{x^7} = -3x^{2-7} = -3x^{-5} = -\frac{3}{x^5}$ or $\frac{-3x^2}{x^7} = \frac{-3}{x^5}$ (by replacing factors of $\frac{x}{x}$ with 1)
- $(x^2)^{-4} = x^{2(-4)} = x^{-8} = \frac{1}{x^8}$ or $(x^2)^{-4} = \frac{1}{(x^2)^4} = \frac{1}{x^{2 \cdot 4}} = \frac{1}{x^8}$
- $(3a^2b^{-3})^2 = 3^2 a^4 b^{-6} = \frac{9a^4}{b^6}$ or $(3a^2b^{-3})^2 = \left(\frac{3a^2}{b^3}\right)^2 = \frac{3^2 a^4}{b^6} = \frac{9a^4}{b^6}$
- $\left(\frac{2x^{-2}}{y^3}\right)^{-3} = \frac{2^{-3} x^6}{y^{-9}} = \frac{x^6 y^9}{2^3} = \frac{x^6 y^9}{8}$ or $\left(\frac{2x^{-2}}{y^3}\right)^{-3} = \left(\frac{2}{x^2 y^3}\right)^{-3} = \left(\frac{x^2 y^3}{2}\right)^3 = \frac{x^6 y^9}{2^3} = \frac{x^6 y^9}{8}$

Student Activity A: *Match Up on Trickier Exponent Rules.* This activity works well in groups or with students working in pairs at a whiteboard (have students draw an empty grid on the board for their answers). Some of the boxes in the grid involve remembering how to add or subtract like terms involving exponents.

Student Activity B: *Double the Fun on Exponent Rules.* In this activity, students practice simplifying exponential expressions using direct application of exponent rules **and** by moving factors to avoid working with negative exponents. Simplifying the expressions both ways is an excellent way for students to learn to check their work and truly understand exponent rules.

Student Activity C: *Mathematical Heteronyms.* In this activity, students practice simplifying sets of expressions that look very similar, but yield very different results. Students really struggle with this activity if they have not thoroughly separated similar-looking mathematical concepts in their minds.

Student Activity D: *Exponents using a Calculator.* Students practice entering expressions that involve exponents on their calculator.

Guided Learning Activity

Section 5.2: Zero and Negative Exponents

To develop the concepts of zero and negative exponents, we will first look at several patterns of values.

1. Fill in the blanks to complete each sequence below:

81, 27, 9, ____, ____, ____, ____ How do you get the next term? _____

16, 8, 4, ____, ____, ____, ____ How do you get the next term? _____

$x^4, x^3, x^2, _, _, _$ How do you get the next term? _____

$4^4, 4^3, 4^2, _, _, _$ How do you get the next term? _____

$\frac{1}{125}, \frac{1}{25}, \frac{1}{5}, _, _, _$ How do you get the next term? _____

2. Now complete each table below by finishing the patterns:

81	3^4
27	3^3
9	3^2

16	2^4
8	2^3
4	2^2

$\frac{1}{125}$	
$\frac{1}{25}$	
$\frac{1}{5}$	
	5^2
	5^3

64	4^3
	4^2
	4^1

3. Based on your observations in the patterns above, finish the exponent rules below:

Rule for Zero Exponents: $x^0 = _$ (for $x \neq 0$)

Rule for Negative Exponents: $x^{-1} = _$ (for $x \neq 0$)

$$x^{-2} = _$$

$$x^{-3} = _$$

$$x^{-n} = _$$

4. Apply the rules you have just developed to write each of these exponential expressions without zero or negative exponents.

$$6^{-2} = \quad 10^0 = \quad y^{-4} = \quad 2^{-5} = \quad z^{-1} = \quad (4a)^0$$

Classroom Overhead

Section 5.2: A Summary of Exponent Rules

Let m and n represent integers and let there be no divisions by zero.

1. Product Rule: $x^m \cdot x^n = x^{m+n}$

2. Quotient Rule: $\frac{x^m}{x^n} = x^{m-n}$

3. Power Rule: $(x^m)^n = x^{m \cdot n}$

4. Power of a Quotient: $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

5. Power of a Product: $(xy)^n = x^n y^n$

6. Exponent of 0 or 1: $x^0 = 1$ and $x^1 = x$

7. Negative Exponent: $x^{-n} = \frac{1}{x^n}$

8. Negative Exponent in the Denominator: $\frac{1}{x^{-n}} = x^n$

9. Negative Exponent on a Quotient: $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$

10. Moving Exponents Across the Fraction Bar:

$$\frac{x^{-m}}{y^{-n}} = \frac{y^n}{x^m}$$

Student Activity A

Section 5.2: Match up on Trickier Exponent Rules

Match-up: Match each of the expressions in the squares of the grid below with an equivalent simplified expression from the top. If an equivalent expression is not found among the choices A through D, then choose E (none of these).

A 1

B $\frac{4}{x^2}$

C $9x^2y^3$

D $\frac{-9x^4}{y^3}$

E None of these

$(4x)^{-2}$	$(4x^{-2})^0$	$4x^{-2}$	$4x^0$
$\frac{-(3x^2y)^2}{y^5}$	$\frac{(-3xy)^2}{y^{-1}}$	$\frac{3^{-2}y^{-3}}{x^{-4}}$	$\left(\frac{x}{2}\right)^{-2}$
$8x^2\left(\frac{x^{-2}}{8}\right)$	$(9x^2y^3)^0$	$\left(\frac{x}{2}\right)^2$	$y^7\left(\frac{y^2}{3x}\right)^{-2}$
$4\left(\frac{1}{x^2}\right)^0$	$(2x)^{-2}$	$3(x^2y^2)(3x^2y^2)^{-1}$	$\frac{(2x^{-1}z^2)^2}{z^4}$
$\left(\frac{-y}{a^4b^4}\right)\left(\frac{3xab}{y}\right)^4$	$\left(\frac{100x^{27}y^{35}}{a^4b^5}\right)^0$	$(2yz)^2(xyz)^{-2}$	$\frac{-12x^4}{5}\left(\frac{5}{-12x^4}\right)$

Student Activity B

Section 5.2: Double the Fun on Exponent Rules

Directions: For many problems involving the simplification of exponents, there are at least two ways to tackle the problem. For each of the problems below, try the problem both ways – your answers should be the same for both methods. If they are not, you'll have to go back and look for a mistake. The first one has been done for you.

Simplify this expression	Apply exponent rules directly	First move factors to avoid working with negative exponents
$\frac{x^{-2}y^5}{x^{-6}y^{-1}}$	$\frac{x^{-2}y^5}{x^{-6}y^{-1}} = x^{-2-(-6)}y^{5-(-1)}$ $= x^{-2+6}y^{5+1} = x^4y^6$	$\frac{x^{-2}y^5}{x^{-6}y^{-1}} = \frac{x^6y^5y^1}{x^2} = x^{6-2}y^{5+1} = x^4y^6$
$\frac{16a^3b^{-2}}{8a^{-1}b^5}$		
$3x^{-2}x^3x^{-1}$		
$(2x)^{-2}(-6x^3)$		
$\left(\frac{a^4b}{ab^2}\right)^{-2}$		
$\left(\frac{x^{-2}y^5}{x^{-1}y^6}\right)^3$		
$\frac{(xy^3)^{-1}}{2x^4y^{-1}}$		

Student Activity C

Section 5.2: Mathematical Heteronyms

Directions: In writing, there are words that are spelled the same but have different pronunciations and different definitions; these are called heteronyms. Many mathematical expressions look similar but are really very different (almost like mathematical heteronyms). In each set of expressions below, pay close attention to the use of parentheses and the mathematical operations and notation.

1.

$3-1$	$3(-1)$	3^{-1}	$3-(-1)$	-3^{-1}	$3\div(-1)$

2.

$2-5$	$2(-5)$	2^{-5}	$2^{-1}-5$	$2^{-1}-5^{-1}$	$2-5^{-1}$

3.

x^2x^{-3}	$x^2(-x^3)$	x^2x^3	$x^{-2}x^3$	x^2-x^3	$(x^2)^{-3}$

4.

$2a^{-3}$	$(-2a)^3$	$2^3(-a)$	$2-a^3$	$(2a)^{-3}$	$(-2a)\cdot 3$

5.

$\left(\frac{3}{4}\right)^{-2}$	$\frac{3^{-2}}{4}$	$\left(\frac{3}{4}\right)(-2)$	$\frac{3}{4}-2$	$\frac{3}{4^{-2}}$	$\left(-\frac{3}{4}\right)^2$

Student Activity D

Section 5.2: Exponents Using a Calculator

When you input exponents into a calculator, you must be careful to tell the calculator which part is the base and which part is the exponent. Each calculator requires a specific set of keystrokes to evaluate exponential expressions and this can become quite complicated when the expression also involves negatives. We will fill out the table below, first performing each calculation by hand, then finding the proper keystrokes to do the evaluation on your calculator.

First you need to locate your exponent button. It may look like $\boxed{\wedge}$ or $\boxed{x^y}$.

Reminders: There is a difference between the minus key and the negative key on your calculator. Fractions need to be placed inside parentheses for proper evaluation.

	Expression	Evaluate by hand	Calculator Keystrokes to get equivalent result
a.	2^3		
b.	2^{-1}		
c.	5^{-2}		
d.	$\left(\frac{1}{2}\right)^3$		
e.	$\left(\frac{2}{3}\right)^{-2}$		
f.	-2^4		
g.	$(-2)^4$		
h.	-4^{-2}		
i.	$(-4)^{-2}$		
j.	$2^{-1} + 3^{-1}$		
k.	$(2+3)^{-1}$		
l.	$5 - 2^{-1}$		

Solutions to Activities

Section 5.2: Negative and Zero Exponents

Solutions to 5.2 Guided Learning Activity: Zero and Negative Exponents

1. $81, 27, 9, 3, 1, \frac{1}{3}, \frac{1}{9}$ Next term? Divide by 3
- $16, 8, 4, 2, 1, \frac{1}{2}, \frac{1}{4}$ Next term? Divide by 2
- $x^4, x^3, x^2, x^1, 1, \frac{1}{x}$ Next term? Divide by x or subtract 1 from the power
- $4^4, 4^3, 4^2, 4^1, 4^0, 4^{-1}$ Next term? Subtract 1 from the power
- $\frac{1}{125}, \frac{1}{25}, \frac{1}{5}, 1, 5, 25$ Next term? Multiply by 5

- 2.
- | | |
|---------------|----------|
| 81 | 3^4 |
| 27 | 3^3 |
| 9 | 3^2 |
| 3 | 3^1 |
| 1 | 3^0 |
| $\frac{1}{3}$ | 3^{-1} |
| $\frac{1}{9}$ | 3^{-2} |
- | | |
|---------------|----------|
| 16 | 2^4 |
| 8 | 2^3 |
| 4 | 2^2 |
| 2 | 2^1 |
| 1 | 2^0 |
| $\frac{1}{2}$ | 2^{-1} |
| $\frac{1}{4}$ | 2^{-2} |
- | | |
|-----------------|----------|
| $\frac{1}{125}$ | 5^{-3} |
| $\frac{1}{25}$ | 5^{-2} |
| $\frac{1}{5}$ | 5^{-1} |
| 1 | 5^0 |
| 5 | 5^1 |
| 25 | 5^2 |
| 125 | 5^3 |
- | | |
|----------------|----------|
| 64 | 4^3 |
| 16 | 4^2 |
| 4 | 4^1 |
| 1 | 4^0 |
| $\frac{1}{4}$ | 4^{-1} |
| $\frac{1}{16}$ | 4^{-2} |
| $\frac{1}{64}$ | 4^{-3} |

3. **Rule for Zero Exponents:** $x^0 = 1$ (for $x \neq 0$)

Rule for Negative Exponents: $x^{-1} = \frac{1}{x^1}$ $x^{-2} = \frac{1}{x^2}$ $x^{-3} = \frac{1}{x^3}$ $x^{-n} = \frac{1}{x^n}$

4. $6^{-2} = \frac{1}{36}$ $10^0 = 1$ $y^{-4} = \frac{1}{y^4}$ $2^{-5} = \frac{1}{32}$ $z^{-1} = \frac{1}{z}$ $(4a)^0 = 1$

Solution to 5.2 Student Activity A: Match Up on Trickier Exponent Rules

E	A	B	E
D	C	E	B
A	A	E	C
E	E	A	B
D	A	B	A

Solution to 5.2 Student Activity B: Double the Fun on Exponent Rules

Simplify this expression	Apply exponent rules directly	First move factors to avoid working with negative exponents
$\frac{x^{-2}y^5}{x^{-6}y^{-1}}$	$\frac{x^{-2}y^5}{x^{-6}y^{-1}} = x^{-2-(-6)}y^{5-(-1)}$ $= x^{-2+6}y^{5+1} = x^4y^6$	$\frac{x^{-2}y^5}{x^{-6}y^{-1}} = \frac{x^6y^5y^1}{x^2} = x^{6-2}y^{5+1} = x^4y^6$
$\frac{16a^3b^{-2}}{8a^{-1}b^5}$	$\frac{16a^3b^{-2}}{8a^{-1}b^5} = \frac{16}{8}a^{3-(-1)}b^{-2-5}$ $= 2a^{3+1}b^{-2-5} = 2a^4b^{-7} = \frac{2a^4}{b^7}$	$\frac{16a^3b^{-2}}{8a^{-1}b^5} = \frac{16a^3a^1}{8b^5b^2} = \frac{2a^4}{b^7}$
$3x^{-2}x^3x^{-1}$	$3x^{-2}x^3x^{-1} = 3x^{-2+3+(-1)}$ $= 3x^0 = 3(1) = 3$	$3x^{-2}x^3x^{-1} = \frac{3x^3}{x^2x^1} = \frac{3x^3}{x^3}$ $= 3x^{3-3} = 3x^0 = 3(1) = 3$
$(2x)^{-2}(-6x^3)$	$(2x)^{-2}(-6x^3) = 2^{-2}x^{-2}(-6x^3)$ $= \frac{1}{4}(-6)x^{-2+3} = -\frac{3}{2}x$	$(2x)^{-2}(-6x^3) = \frac{-6x^3}{(2x)^2}$ $= \frac{-6x^3}{4x^2} = \frac{-3}{2}x^{3-2} = -\frac{3}{2}x$
$\left(\frac{a^4b}{ab^2}\right)^{-2}$	$\left(\frac{a^4b}{ab^2}\right)^{-2} = \frac{a^{4(-2)}b^{-2}}{a^{-2}b^{2(-2)}} = \frac{a^{-8}b^{-2}}{a^{-2}b^{-4}}$ $= a^{-8-(-2)}b^{-2-(-4)} = a^{-6}b^2 = \frac{b^2}{a^6}$	$\left(\frac{a^4b}{ab^2}\right)^{-2} = \frac{a^{4(-2)}b^{-2}}{a^{-2}b^{2(-2)}} = \frac{a^{-8}b^{-2}}{a^{-2}b^{-4}}$ $= \frac{a^2b^4}{a^8b^2} = a^{2-8}b^{4-2} = a^{-6}b^2 = \frac{b^2}{a^6}$
$\left(\frac{x^{-2}y^5}{x^{-1}y^6}\right)^3$	$\left(\frac{x^{-2}y^5}{x^{-1}y^6}\right)^3 = \frac{x^{-2(3)}y^{5(3)}}{x^{-1(3)}y^{6(3)}} = \frac{x^{-6}y^{15}}{x^{-3}y^{18}}$ $= x^{-6-(-3)}y^{15-18} = x^{-3}y^{-3} = \frac{1}{x^3y^3}$	$\left(\frac{x^{-2}y^5}{x^{-1}y^6}\right)^3 = \left(\frac{x^1y^5}{x^2y^6}\right)^3 = \frac{x^{1(3)}y^{5(3)}}{x^{2(3)}y^{6(3)}} = \frac{x^3y^{15}}{x^6y^{18}}$ $= x^{3-6}y^{15-18} = x^{-3}y^{-3} = \frac{1}{x^3y^3}$
$\frac{(xy^3)^{-1}}{2x^4y^{-1}}$	$\frac{(xy^3)^{-1}}{2x^4y^{-1}} = \frac{x^{-1}y^{3(-1)}}{2x^4y^{-1}} = \frac{x^{-1}y^{-3}}{2x^4y^{-1}}$ $= \frac{1}{2}x^{-1-4}y^{-3-(-1)} = \frac{1}{2}x^{-5}y^{-2} = \frac{1}{2x^5y^2}$	$\frac{(xy^3)^{-1}}{2x^4y^{-1}} = \frac{y^1}{2x^4(xy^3)} = \frac{y^1}{2x^{4+1}y^3}$ $= \frac{1}{2x^5}y^{1-3} = \frac{1}{2x^5}y^{-2} = \frac{1}{2x^5y^2}$

Solution to 5.2 Student Activity C: Mathematical Heteronyms

1.	$3-1$	$3(-1)$	3^{-1}	$3-(-1)$	-3^{-1}	$3\div(-1)$
	2	-3	$\frac{1}{3}$	4	$-\frac{1}{3}$	-3

2.	$2-5$	$2(-5)$	2^{-5}	$2^{-1}-5$	$2^{-1}-5^{-1}$	$2-5^{-1}$
	-3	-10	$\frac{1}{32}$	$-\frac{9}{2}$	$\frac{3}{10}$	$\frac{9}{5}$

3.	x^2x^{-3}	$x^2(-x^3)$	x^2x^3	$x^{-2}x^3$	x^2-x^3	$(x^2)^{-3}$
	$\frac{1}{x}$	$-x^5$	x^5	x	x^2-x^3	$\frac{1}{x^6}$

4.	$2a^{-3}$	$(-2a)^3$	$2^3(-a)$	$2-a^3$	$(2a)^{-3}$	$(-2a)\cdot 3$
	$\frac{2}{a^3}$	$-8a^3$	$-8a$	$2-a^3$	$\frac{1}{8a^3}$	$-6a$

5.	$\left(\frac{3}{4}\right)^{-2}$	$\frac{3^{-2}}{4}$	$\left(\frac{3}{4}\right)(-2)$	$\frac{3}{4}-2$	$\frac{3}{4^{-2}}$	$\left(-\frac{3}{4}\right)^2$
	$\frac{16}{9}$	$\frac{1}{36}$	$-\frac{3}{2}$	$-\frac{5}{4}$	48	$\frac{9}{16}$

Solutions to 5.2 Student Activity D: Exponents Using a Calculator

a. 8 b. $\frac{1}{2}$ c. $\frac{1}{25}$ d. $\frac{1}{8}$ e. $\frac{9}{4}$ f. -16 g. 16 h. $-\frac{1}{16}$

i. $\frac{1}{16}$ j. $\frac{5}{6}$ k. $\frac{1}{5}$ l. $\frac{9}{2}$

a-l. Calculator keystrokes will vary depending on calculator.

Teaching Guide

Section 6.6: A Factoring Strategy

■ Preparing for Your Class

Suggested Class Time: 30-60 minutes

Materials Needed: Classroom overhead, guided learning activity, student activities

Vocabulary

- There is not any new vocabulary in this section, but you may wish to review the accumulated vocabulary from the first five sections.

Instruction Tips

- Often, students do not remember to check for the GCF first.
- Once students go through the factoring strategy once, many do not look for further factoring possibilities. Try to catch this mistake in class using the class examples.

■ Teaching Your Class

Steps for Factoring a Polynomial

1) Is there a common factor? If so, factor out the GCF or the opposite of the GCF so that the lead coefficient is positive.

2) How many terms does the polynomial have?

If it has two terms, look for the following problem types:

- The difference of two squares.
- The sum of two squares.
- The difference of two cubes.
- The sum of two cubes.

If it has three terms, look for the following problem types:

- A perfect-square trinomial.
- A trinomial of the form $x^2 + bx + c$.
- A trinomial of the form $ax^2 + bx + c$.

If it has four terms, look for a way to factor by grouping.

3) Can any factors be factored further? If so, factor them completely.

4) Does the factorization check? Check by multiplying.

Classroom Overhead: *A General Factoring Strategy*. This is a flowchart for decision-making that includes all of the factoring strategies.

Guided Learning Activity: *How Recognizing a GCF Can Make Your Life Easier*.

Here is a paired example to teach students exactly why they should always look for a possible GCF as the *first* step of a factoring strategy. Students then get a bit of practice factoring out the GCF and factoring the remaining expression. Students who are not good at finding the *greatest* common factor will probably find this one a bit difficult; it might be best to have students work on this one in pairs.

Examples that use the General Factoring Strategy: (factor)

- $9x^3 + 18x^2 - 27x = 9x(x+3)(x-1)$
- $4y^3 - 32 = 4(y-2)(y^2 + 2y + 4)$
- $8w^2 - 30w - 27 = (4w+3)(2w-9)$
- $4x^2 - 25y^2 = (2x+5y)(2x-5y)$
- $x^3 - 3x^2 + 5x - 15 = (x^2 + 5)(x-3)$
- $a^3 - 7a^2 - 9a + 63 = (a+3)(a-3)(a-7)$
- $m^2 + 8mp + 16p^2 = (m+4p)^2$
- $4x^2 + 100 = 4(x^2 + 25)$
- $8z^3 + 125 = (2z+5)(4z^2 - 10z + 25)$

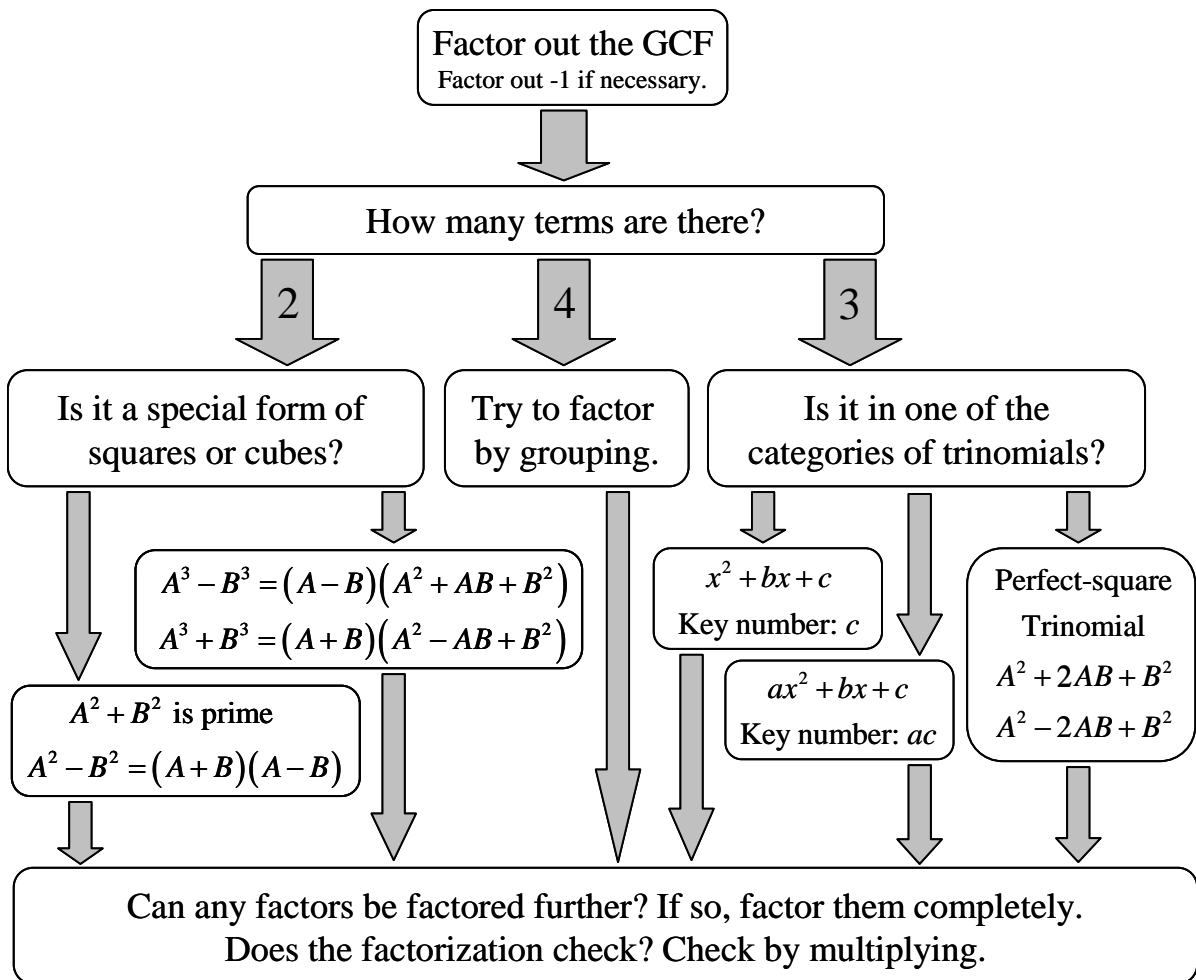
Student Activity A: *Factoring Strategizing.* In this activity, students practice with the decision-making process in factoring any expression. You can put up the overhead of a general factoring strategy for students to look at while they are strategizing.

Student Activity B: *Factoring Variations on a Number.* In this activity, all of the expressions to be factored have the same key number (positive or negative). Some are binomials and some are trinomials – it should provide a good opportunity to use the general factoring strategy.

Student Activity C: *Checking the Factoring with a Calculator.* Another way for students to check their work is to substitute a value into the given polynomial and the factored form and make sure that these results are equal. There is a note in the directions about avoiding the use of the numbers 0, 1, and 2 for such checks. If you want to show why, use the value of 2 as the check value in the last problem.

Classroom Overhead:

Section 6.6: A General Factoring Strategy



Guided Learning Activity

Section 6.6: How Recognizing a GCF Can Make Your Life Easier

Often it is tempting to start factoring a polynomial expression by immediately setting up a factor table or two sets of parentheses without first looking for a GCF. While you can still factor out a GCF at the end (as long as you remember to look for it), consider how much easier some factoring problems could be if you look for the GCF first.

Example 1: Factor $60x^2 + 120x + 60$

Method 1: Finding GCF Last

First we find the key product and key sum.

Key Product 3600		Key Sum 120
1	3,600	3,601
2	1,800	1,802
3	1,200	1,203
⋮	⋮	⋮
48	75	123
50	72	122
60	60	120

There are 23 factor pairs for the key product before we get to the correct one.

Then we rewrite the middle term using the two terms from the table, and factor by grouping.

$$\begin{aligned} &60x^2 + 120x + 60 \\ &60x^2 + 60x + 60x + 60 \\ &60x(x+1) + 60(x+1) \\ &(60x+60)(x+1) \\ &60(x+1)(x+1) \\ &60(x+1)^2 \end{aligned}$$

Notice that if we stop here, the expression is not fully factored.

Method 2: Finding the GCF First

$$\begin{aligned} &60x^2 + 120x + 60 \\ &60(x^2 + 2x + 1) \end{aligned}$$

You may notice that the polynomial portion is a perfect square and go directly to the skeleton:

$$60[(x)^2 + 2(1)(x) + (1)^2] = 60(x+1)^2.$$

Or, you could use the factor table:

Key Product	Key Sum
1	2
1 1	2

In this case, the factoring is significantly easier:

$$\begin{aligned} &60(x^2 + 2x + 1) \\ &60(x+1)(x+1) \\ &60(x+1)^2 \end{aligned}$$

You can see that the second method is much shorter and leaves much less room for arithmetic error. When factoring, it is always best to check for a GCF first!

Match-up: In each box of the grid, you will find an expression that needs to be factored. Find and factor out the GCF first. If you don't, you'll regret it! Once you have factored the expression, look to see whether any of the factors appear in the list at the top. If none of the factors are listed, then choose E (none of these are factors). Some boxes may have more than one answer. The first one has been done for you.

A $x+1$ **B** $2x+3$ **C** $4x-1$ **D** $x-5$ **E** None of these are factors

$40x^2 + 100x + 60$ $20(2x^2 + 5x + 3)$ $20(2x^2 + 2x + 3x + 3)$ $20[2x(x+1) + 3(x+1)]$ $20(2x+3)(x+1)$ A, B	$120x^2 + 90x - 30$	$70 - 70x^2$	$-30x^2 + 15x + 90$
$-60x^2 + 315x - 75$	$40x^2 + 80x + 40$	$8x^2 + 10x - 3$	$-600x^2 - 750x + 225$
$60x^2 - 260x - 200$	$-45x^2 + 195x + 150$	$-15x^2 - 45x - 30$	$x^2 - 4x - 5$
$11x^2 - 44x - 55$	$40x^2 - 140x - 300$	$20x^2 - 40x + 20$	$320x^2 - 160x + 20$

Student Activity A

Section 6.6: Factoring Strategizing

For each type of factoring problem, describe the classification of the factoring problem and the strategies and key steps (like the parentheses skeleton) to remember while doing the problem. You do not have to complete the problems.

Choose from these classifications (more than one may apply):

Factor out a GCF

Trinomial $ax^2 + bx + c$

Sum of Cubes

Factor by Grouping

Sum of Squares

Difference of Cubes

Trinomial $x^2 + bx + c$

Difference of Squares

Perfect Square Trinomial

Expression to be factored	Classification	Strategies and Key Steps
1. $9x^2 - 4$		
2. $4u^4 + 32u$		
3. $t^2 - 12t + 35$		
4. $x^2 + 49$		
5. $x^3 + 5x^2 - 3x - 15$		
6. $15y^2 - 36y + 12$		
7. $x^2 + 14x + 49$		
8. $64z^3 - 27$		
9. $a^2 + 15ab + 54b^2$		
10. $4x^4 + 4x^3 - 80x^2$		

Student Activity B

Section 6.6: Factoring Variations on a Number

Directions: In each set there are six expressions to be factored, all ending with the same constant (positive or negative). If the expression is factorable, factor it. If it is not factorable, write "PRIME."

Variations on 1	$x^2 - 1$	$x^3 - 1$	$x^2 - 2x + 1$
	$x^2 + 1$	$x^3 + 1$	$x^2 + 2x + 1$

Variations on 64	$x^2 - 64$	$x^2 + 64$	$x^2 - 16x + 64$
	$x^3 - 64$	$x^3 + 64$	$x^2 + 16x + 64$

Variations on 36	$x^2 - 36$	$9x^2 + 36$	$x^2 + 12x + 36$
	$4x^2 - 36$	$9x^2 - 36$	$x^2 - 12x + 36$

Student Activity C

Section 6.6: Checking the Factoring with a Calculator

Directions: One way you can check your factoring is to evaluate both the original expression and the factored expressions for the same value of the variable. It is not wise to use the values of 0, 1, or 2 for these types of checks since these three numbers have some quirky properties:

$$0+0=0 \text{ and } 0 \cdot 0=0 \qquad 1 \cdot 1=1 \qquad 2+2=4 \text{ and } 2 \cdot 2=4$$

If a check by evaluation results in different answers, then you should reexamine the factoring. Note that getting the same result from evaluation does not **ensure** that your factors are correct, but it is likely to find a mistake in your work if there is one.

In each table below, check the student's factoring by evaluating for the given values with a calculator. It will help to write out the skeleton with the value substituted first. Then input the numerical expression into your calculator to get the result. If you find a factoring mistake, correct it! The first one has been started for you.

1.

	Evaluate for $x = 3$.	Evaluate for $x = 2.4$.
$3x^2 + 7x + 2$	$3(3)^2 + 7(3) + 2 = 50$	
$(x + 2)(3x + 1)$	$(3 + 2)(3(3) + 1) = 50$	

2.

	Evaluate for $x = 4$.	Evaluate for $x = 1/3$.
$x^2 + 4x - 21$		
$(x - 7)(x + 3)$		

3.

	Evaluate for $x = 1/2$.	Evaluate for $x = -3$.
$4x^2 - 25$		
$(2x + 5)(2x - 5)$		

4.

	Evaluate for $x = 3.5$.	Evaluate for $x = -5/4$.
$4x^2 + 20x + 25$		
$(4x + 5)^2$		

5.

	Evaluate for $x = 5$.	Evaluate for $x = 3$.
$x^3 - 8$		
$(x - 2)(x^2 + 4x + 4)$		

Solutions to Activities

Section 6.6: A Factoring Strategy

Solution to 6.6 Guided Learning Activity:

How Recognizing a GCF Can Make Your Life Easier

A,B	A,C	A	B
C,D	A	B,C	B,C
D	D	A	A,D
A,D	B,D	E	C

Solution to 6.6 Student Activity A: Factoring Strategizing

1. $9x^2 - 4$	Difference of Squares	Skeleton: $(3x)^2 - (2)^2$
2. $4u^4 + 32u$	Factor out a GCF, Sum of Cubes	GCF step: $4u(u^3 + 8)$ Skeleton: $4u[(u)^3 + (2)^3]$ $4u(+)(- +)$
3. $t^2 - 12t + 35$	Trinomial $x^2 + bx + c$	Key number: 35 Factors that work: $-5(-7)$
4. $x^2 + 49$	Sum of Squares	Skeleton: $(x)^2 + (7)^2$ A sum of squares is prime.
5. $x^3 + 5x^2 - 3x - 15$	Factor by Grouping	First step: $x^3 + 5x^2 - 3x - 15$ Grouping step: $x^2(x+5) - 3(x+5)$
6. $15y^2 - 36y + 12$	Factor out a GCF, Trinomial $ax^2 + bx + c$	GCF step: $3(5y^2 - 12y + 4)$ Key number: 20 Factors that work: $-10(-2)$
7. $x^2 + 14x + 49$	Perfect Square Trinomial	Skeleton: $(x)^2 + 2(x)(7) + (7)^2$
8. $64z^3 - 27$	Difference of Cubes	Skeleton: $(4z)^3 - (3)^3$ $(-)(+ +)$
9. $a^2 + 15ab + 54b^2$	Trinomial $x^2 + bx + c$	Key number: 54 Factors that work: $9 \cdot 6$ Don't forget the b factors
10. $4x^4 + 4x^3 - 80x^2$	Factor out a GCF, Trinomial $x^2 + bx + c$	GCF step: $4x^2(x^2 + x - 20)$

Solutions to 6.6 Student Activity B: Factoring Variations on a Number

Variations on 1	$x^2 - 1$ $(x+1)(x-1)$	$x^3 - 1$ $(x-1)(x^2 + x + 1)$	$x^2 - 2x + 1$ $(x-1)^2$
	$x^2 + 1$ Prime	$x^3 + 1$ $(x+1)(x^2 - x + 1)$	$x^2 + 2x + 1$ $(x+1)^2$

Variations on 64	$x^2 - 64$ $(x+8)(x-8)$	$x^2 + 64$ Prime	$x^2 - 16x + 64$ $(x-8)^2$
	$x^3 - 64$ $(x-4)(x^2 + 4x + 16)$	$x^3 + 64$ $(x+4)(x^2 - 4x + 16)$	$x^2 + 16x + 64$ $(x+8)^2$

Variations on 36	$x^2 - 36$ $(x+6)(x-6)$	$9x^2 + 36$ $9(x^2 + 4)$	$x^2 + 12x + 36$ $(x+6)^2$
	$4x^2 - 36$ $4(x+3)(x-3)$	$9x^2 - 36$ $9(x+2)(x-2)$	$x^2 - 12x + 36$ $(x-6)^2$

Solution to 6.6 Student Activity C: Checking the Factoring with a Calculator

1.	Evaluate for $x = 3$.	Evaluate for $x = 2.4$.
$3x^2 + 7x + 2$	$3(3)^2 + 7(3) + 2 = 50$	$3(2.4)^2 + 7(2.4) + 2 = 36.08$
$(x+2)(3x+1)$	$(3+2)(3(3)+1) = 50$	$(2.4+2)(3(2.4)+1) = 36.08$

The factoring seems correct.

2.	Evaluate for $x = 4$.	Evaluate for $x = 1/3$.
$x^2 + 4x - 21$	$(4)^2 + 4(4) - 21 = 11$	$(\frac{1}{3})^2 + 4(\frac{1}{3}) - 21 \approx -19.56$
$(x-7)(x+3)$	$(4-7)(4+3) = -21$	$((\frac{1}{3})-7)((\frac{1}{3})+3) \approx -22.22$

This one is not factored correctly. Correct factoring: $(x+7)(x-3)$.

3.	Evaluate for $x = 1/2$.	Evaluate for $x = -3$.
$4x^2 - 25$	$4(\frac{1}{2})^2 - 25 = -24$	$4(-3)^2 - 25 = 11$
$(2x+5)(2x-5)$	$(2(\frac{1}{2})+5)(2(\frac{1}{2})-5) = -24$	$(2(-3)+5)(2(-3)-5) = 11$

The factoring seems correct.

4.

	Evaluate for $x = 3.5$.	Evaluate for $x = -5/4$.
$4x^2 + 20x + 25$	$4(3.5)^2 + 20(3.5) + 25 = 144$	$4\left(\frac{-5}{4}\right)^2 + 20\left(\frac{-5}{4}\right) + 25 = 6.25$
$(4x+5)^2$	$(4(3.5)+5)^2 = 361$	$\left(4\left(\frac{-5}{4}\right)+5\right)^2 = 0$

This one is not factored correctly. Correct factoring: $(2x+5)^2$.

5.

	Evaluate for $x = 5$.	Evaluate for $x = 3$.
$x^3 - 8$	$(5)^3 - 8 = 117$	$(3)^3 - 8 = 19$
$(x-2)(x^2 + 4x + 4)$	$(5-2)((5)^2 + 4(5) + 4) = 147$	$(3-2)((3)^2 + 4(3) + 4) = 25$

This one is not factored correctly. Correct factoring: $(x-2)(x^2 + 2x + 4)$.

Teaching Guide

Section 7.4: Addition and Subtraction with Unlike Denominators

■ Preparing for Your Class

Suggested Class Time: 45-60 minutes

Materials Needed: Student activities, colored pencils for students, colored markers (or chalk) for instructor

Vocabulary

- Unlike denominators, opposites

Instruction Tips

- It might be helpful to write in the factors that are equal to one using color, so that it is clear that this was the new step in the procedure. Make sure to provide your students with colored pencils too so that it is also clear in their notes. For example, in $\frac{5}{6} - \frac{3}{8} = \frac{5}{6} \cdot \frac{4}{4} - \frac{3}{8} \cdot \frac{3}{3} = \dots$, you

would write $\cdot \frac{4}{4}$ and $\cdot \frac{3}{3}$ in a different color.

- When students have built their equivalent rational expressions, they often have a strong desire to simplify them again before performing the addition. For example, a student might add $\frac{3}{x+2} + \frac{4}{x+5}$, build the expressions $\frac{3(x+5)}{(x+2)(x+5)} + \frac{4(x+2)}{(x+2)(x+5)}$ and then do this

$\frac{3(x+5)^1}{(x+2)(x+5)_1} + \frac{4(x+2)^1}{(x+2)_1(x+5)}$, which brings them back to where they started. The strong

desire to re-simplify is a remnant of all that practice that they put in factoring and simplifying rational expressions in the previous three sections. One solution for this problem is to avoid writing any rational expressions that look like they need to be simplified. You can have

students go through the steps like this: $\frac{3}{x+2} \cdot \frac{x+5}{x+5} + \frac{4}{x+5} \cdot \frac{x+2}{x+2}$, then insert parentheses

where necessary for multiplication: $\frac{3}{(x+2)} \cdot \frac{(x+5)}{(x+5)} + \frac{4}{(x+5)} \cdot \frac{(x+2)}{(x+2)}$, then distribute the

numerators, $\frac{3x+15}{(x+2)(x+5)} + \frac{4x+8}{(x+2)(x+5)}$ etc.

- The urge for students to skip steps is very strong with these problems. There are so many steps to write, and students grow weary of the problems very quickly.

■ Teaching Your Class

Student Activity A: *The Missing Form of 1.* In this activity, student practice building equivalent rational expressions by filling in the missing form of 1. This would be a good warm-up activity.

Adding and subtracting rational expressions with unlike denominators

Start with a few examples of numeric fractions:

$$\bullet \quad \frac{5}{6} - \frac{3}{8} = \frac{5}{6} \cdot \frac{4}{4} - \frac{3}{8} \cdot \frac{3}{3} = \frac{20}{24} - \frac{9}{24} = \frac{11}{24}$$

LCD: 24

$$\bullet \quad \frac{3}{10} + \frac{7}{30} = \frac{3}{10} \cdot \frac{3}{3} + \frac{7}{30} = \frac{9}{30} + \frac{7}{30} = \frac{16}{30} = \frac{8}{15}$$

LCD: 30

Then some simple rational expressions with integer denominators:

$$\bullet \quad \frac{2x}{3} - \frac{x}{5} = \frac{2x}{3} \cdot \frac{5}{5} - \frac{x}{5} \cdot \frac{3}{3} = \frac{10x}{15} - \frac{3x}{15} = \frac{7x}{15}$$

LCD: 15

$$\bullet \quad \frac{x+1}{2} - \frac{x+3}{6} = \frac{(x+1)}{2} \cdot \frac{3}{3} - \frac{(x+3)}{6} = \frac{(3x+3) - (x+3)}{6} = \dots = \frac{2x}{6} = \frac{x}{3}$$

LCD: 6

Procedure for adding and subtracting rational expressions with unlike denominators:

1. Find the LCD.
2. Rewrite each rational expression as an equivalent expression with the LCD as the denominator. To do so, build each fraction using a form of 1 that involves any factor(s) needed to obtain the LCD.
3. Add or subtract the numerators and write the sum or difference over the LCD.
4. Simplify the result, if possible.

Progress to a few examples with monomial denominators involving variables:

$$\bullet \quad \frac{2}{3x} + \frac{1}{4x^2} = \frac{2}{3x} \cdot \frac{4x}{4x} + \frac{1}{4x^2} \cdot \frac{3}{3} = \frac{8x}{12x^2} + \frac{3}{12x^2} = \frac{8x+3}{12x^2}$$

LCD: $12x^2$

$$\bullet \quad \frac{4}{5xy^2} - \frac{2}{15x^2y} = \frac{4}{5xy^2} \cdot \frac{3x}{3x} - \frac{2}{15x^2y} \cdot \frac{y}{y} = \dots = \frac{12x-2y}{15x^2y^2}$$

LCD: $15x^2y^2$

Progress to examples with binomial and trinomial denominators:

$$\bullet \frac{3}{x+4} + \frac{5}{x-2} = \frac{3}{(x+4)} \cdot \frac{(x-2)}{(x-2)} + \frac{5}{(x-2)} \cdot \frac{(x+4)}{(x+4)} = \dots = \frac{8x+14}{(x-2)(x+4)}$$

LCD: $(x+4)(x-2)$

$$\bullet \frac{14}{2x+10} - \frac{4}{x+5} = \frac{14}{2(x+5)} - \frac{4}{(x+5)} \cdot \frac{2}{2} = \dots = \frac{3}{x+5} \text{ (last step simplifies)}$$

LCD: $2(x+5)$

$$\bullet \frac{x}{x+3} - \frac{2x+12}{x^2+9x+18} = \frac{x}{(x+3)} \cdot \frac{(x+6)}{(x+6)} - \frac{(2x+12)}{(x+3)(x+6)} = \dots = \frac{x-2}{x+3}$$

LCD: $(x+3)(x+6)$
(last step simplifies)

$$\bullet \frac{x+2}{x^2-25} - \frac{x}{3x+15} = \frac{x+2}{(x+5)(x-5)} \cdot \frac{3}{3} - \frac{x}{3(x+5)} \cdot \frac{(x-5)}{(x-5)} = \dots = \frac{-x^2+8x+6}{3(x+5)(x-5)}$$

LCD: $3(x+5)(x-5)$

$$\bullet \frac{x+8}{x+4} + \frac{4-x^2}{x^2+x-12} = \frac{(x+8)}{(x+4)} \cdot \frac{(x-3)}{(x-3)} + \frac{(4-x^2)}{(x+4)(x-3)} = \dots = \frac{5x-20}{(x+4)(x-3)}$$

LCD: $(x+4)(x-3)$

Adding and subtracting a rational expressions and a polynomial

$$\bullet \frac{3}{4} - 2 = \frac{3}{4} - \frac{2}{1} = \frac{3}{4} - \frac{2 \cdot 4}{1 \cdot 4} = \frac{3}{4} - \frac{8}{4} = -\frac{5}{4}$$

LCD: 4

$$\bullet 6 + \frac{2x}{y} = \frac{6}{1} + \frac{2x}{y} = \frac{6 \cdot y}{1 \cdot y} + \frac{2x}{y} = \frac{6y+2x}{y}$$

LCD: y

$$\bullet \frac{4}{y-2} + 3 = \frac{4}{y-2} + \frac{3}{1} = \frac{4}{(y-2)} + \frac{3}{1} \cdot \frac{(y-2)}{(y-2)} = \dots = \frac{3y-2}{y-2}$$

LCD: $y-2$

$$\bullet x+2 + \frac{x}{x-5} = \frac{x+2}{1} + \frac{x}{x-5} = \frac{(x+2)}{1} \cdot \frac{(x-5)}{(x-5)} + \frac{x}{(x-5)} = \dots = \frac{x^2-2x-10}{x-5}$$

LCD: $x-5$

Adding and subtracting rational expressions with denominators that are opposites

Multiply the rational expression with an opposite denominator by $\frac{-1}{-1}$.

$$\bullet \quad \frac{5}{x-2} + \frac{4}{2-x} = \frac{5}{(x-2)} + \frac{4}{(2-x)} \cdot \frac{-1}{-1} = \frac{5}{x-2} + \frac{-4}{x-2} = \frac{1}{x-2}$$

$$\bullet \quad \frac{2x-3}{x^2+3x-4} - \frac{x+2}{4-3x-x^2} = \frac{(2x-3)}{(x^2+3x-4)} - \frac{(x+2)}{(4-3x-x^2)} \cdot \frac{-1}{-1} = \dots = \frac{3x-1}{x^2+3x-4}$$

Student Activity B: *The Reunion.* All four mathematical operations reunite here in one activity. Students use the same two rational expressions to practice addition, subtraction, multiplication, and division in three sets of problems.

Student Activity C: *Tempting Expressions.* In all the expressions in this activity, students will find themselves tempted to do the mathematics incorrectly by seeing visually-pleasing expressions. This is an attempt to re-train the mind to *not* necessarily go with the first instinct.

Student Activity A

Section 6.4: The Missing Form of 1

Directions: In each equation below, there is a missing multiple with a form of 1 and a missing numerator. Fill in the empty spaces to complete each expression.

Example: $\frac{x-2}{x+3} \cdot \frac{\boxed{}}{\boxed{}} = \frac{}{(x+3)(x-2)}$ becomes $\frac{x-2}{x+3} \cdot \frac{x-2}{x-2} = \frac{x^2-4x+4}{(x+3)(x-2)}$.

Hint for problems 6-10: Try factoring the denominator on the right hand side first.

1. $\frac{x+1}{x+2} \cdot \frac{\boxed{}}{\boxed{}} = \frac{}{3x(x+2)}$

6. $\frac{3x}{x+1} \cdot \frac{\boxed{}}{\boxed{}} = \frac{}{x^2-1}$

2. $\frac{x-5}{x+5} \cdot \frac{\boxed{}}{\boxed{}} = \frac{}{(x+5)(x-5)}$

7. $\frac{2x+3}{x+7} \cdot \frac{\boxed{}}{\boxed{}} = \frac{}{x^2+14x+49}$

3. $\frac{x}{2-x} \cdot \frac{\boxed{}}{\boxed{}} = \frac{}{x-2}$

8. $\frac{16x}{4-x} \cdot \frac{\boxed{}}{\boxed{}} = \frac{}{16-8x-x^2}$

4. $\frac{x^2+3x}{3x+2} \cdot \frac{\boxed{}}{\boxed{}} = \frac{}{(3x+2)(x+3)}$

9. $\frac{x^2}{5x+4} \cdot \frac{\boxed{}}{\boxed{}} = \frac{}{30x^3+24x^2}$

5. $\frac{4}{x+3} \cdot \frac{\boxed{}}{\boxed{}} = \frac{}{(x+3)(x+4)(x+5)}$

10. $\frac{8+x}{4x+3} \cdot \frac{\boxed{}}{\boxed{}} = \frac{}{20x^2+7x-6}$

Student Activity B

Section 7.4: The Reunion

Directions: Now that we've made it through addition, subtraction, multiplication, and division, it seems only fair to bring the whole gang back together for a reunion. Simplify each expression according to its operation!

$\frac{x-1}{x+2} + \frac{x+3}{x+2}$	
$\frac{x-1}{x+2} - \frac{x+3}{x+2}$	
$\frac{x-1}{x+2} \cdot \frac{x+3}{x+2}$	
$\frac{x-1}{x+2} \div \frac{x+3}{x+2}$	

$\frac{x}{x-3} + \frac{x-1}{3-x}$	
$\frac{x}{x-3} - \frac{x-1}{3-x}$	
$\frac{x}{x-3} \cdot \frac{x-1}{3-x}$	
$\frac{x}{x-3} \div \frac{x-1}{3-x}$	

$\frac{12}{x+2} + \frac{4x+8}{x+1}$	
$\frac{12}{x+2} - \frac{4x+8}{x+1}$	
$\frac{12}{x+2} \cdot \frac{4x+8}{x+1}$	
$\frac{12}{x+2} \div \frac{4x+8}{x+1}$	

Student Activity C

Section 7.4: Tempting Expressions

Directions: In all the expressions below, you will find yourself tempted to do incorrect mathematics by visually pleasing expressions. Of course, just because it looks good, that doesn't mean it is the right thing to do. ☺ Think carefully about the steps involved in each expression.

1. Add: $\frac{5}{x} + \frac{x}{5}$

6. Multiply: $\frac{x^2 + 4x - 12}{x^2 + 4x - 32} \cdot \frac{x^2 + 10x + 16}{x^2 + 10x + 24}$

2. Subtract: $\frac{2}{x+2} - \frac{1}{x}$

7. Subtract: $\frac{3x}{x+2} - \frac{x+2}{3x+3}$

3. Add: $\frac{x}{x+4} + \frac{x+4}{x}$

8. Simplify: $\frac{5x^2 + 25x}{5x^2 - 125}$

4. Divide: $\frac{x-1}{x+6} \div \frac{x+6}{x^2-36}$

9. Subtract: $\frac{x^2}{x^2-9} - 9$

5. Simplify: $\frac{2x+8}{2x+4}$

10. Add: $\frac{2}{x+1} + \frac{2}{1-x}$

Solutions to Activities

Section 7.4: Addition and Subtraction of Rational Expressions with Unlike Denominators

Solution to 7.1 Guided Learning Activity: Undefined Expressions

Expression	Parentheses Skeleton	Evaluate the expression for ...				
		$x=1$	$x=4$	$x=0$	$x=-2$	$x=\frac{1}{3}$
$\frac{3x-12}{x^2+2x}$	$\frac{3(\)-12}{(\)^2+2(\)}$	-3	0	undefined	undefined	$-\frac{99}{7}$
$\frac{3x^2+8x-3}{x^2-5x+4}$	$\frac{3(\)^2+8(\)-3}{(\)^2-5(\)+4}$	undefined	undefined	$-\frac{3}{4}$	$-\frac{7}{18}$	0
$\frac{x^2-16}{x^2+2x}$	$\frac{(\)^2-16}{(\)^2+2(\)}$	-5	0	undefined	undefined	$-\frac{143}{7}$

Expression	Factored Form of the Expression	Evaluate the expression for ...				
		$x=1$	$x=4$	$x=0$	$x=-2$	$x=\frac{1}{3}$
$\frac{3x-12}{x^2+2x}$	$\frac{3(x-4)}{x(x+2)}$	-3	0	undefined	undefined	$-\frac{99}{7}$
$\frac{3x^2+8x-3}{x^2-5x+4}$	$\frac{(x+3)(3x-1)}{(x-4)(x-1)}$	undefined	undefined	$-\frac{3}{4}$	$-\frac{7}{18}$	0
$\frac{x^2-16}{x^2+2x}$	$\frac{(x+4)(x-4)}{x(x+2)}$	-5	0	undefined	undefined	$-\frac{143}{7}$

Expression	Factored Form of the Expression	Set the denominator = 0. Solve the resulting equation.	Where is the expression undefined?
$\frac{3x-12}{x^2+2x}$	$\frac{3(x-4)}{x(x+2)}$	$x(x+2)=0$ $x=0$ OR $x+2=0$ $x=0$ $x=-2$	$x=0, x=-2$
$\frac{3x^2+8x-3}{x^2-5x+4}$	$\frac{(x+3)(3x-1)}{(x-4)(x-1)}$	$(x-4)(x-1)=0$ $x-4=0$ OR $x-1=0$ $x=4$ $x=1$	$x=4, x=1$
$\frac{x^2-16}{x^2+2x}$	$\frac{(x+4)(x-4)}{x(x+2)}$	$x(x+2)=0$ $x=0$ OR $x+2=0$ $x=0$ $x=-2$	$x=0, x=-2$

Solution to 7.4 Student Activity A: The Missing Form of 1

$$1. \frac{x+1}{x+2} \cdot \frac{3x}{3x} = \frac{3x^2+3x}{3x(x+2)}$$

$$6. \frac{3x}{x+1} \cdot \frac{x-1}{x-1} = \frac{3x^2-3x}{x^2-1}$$

$$2. \frac{x-5}{x+5} \cdot \frac{x-5}{x-5} = \frac{x^2-10x+25}{(x+5)(x-5)}$$

$$7. \frac{2x+3}{x+7} \cdot \frac{x+7}{x+7} = \frac{2x^2+17x+21}{x^2+14x+49}$$

$$3. \frac{x}{2-x} \cdot \frac{-1}{-1} = \frac{-x}{x-2}$$

$$8. \frac{16x}{4-x} \cdot \frac{4-x}{4-x} = \frac{64x-16x^2}{16-8x-x^2}$$

$$4. \frac{x^2+3x}{3x+2} \cdot \frac{x+3}{x+3} = \frac{x^3+6x^2+9x}{(3x+2)(x+3)}$$

$$9. \frac{x^2}{5x+4} \cdot \frac{6x^2}{6x^2} = \frac{6x^4}{30x^3+24x^2}$$

$$5. \frac{4}{x+3} \cdot \frac{(x+4)(x+5)}{(x+4)(x+5)} = \frac{4x^2+36x+80}{(x+3)(x+4)(x+5)}$$

$$10. \frac{8+x}{4x+3} \cdot \frac{5x-2}{5x-2} = \frac{5x^2+38x-16}{20x^2+7x-6}$$

Solution to 7.4 Student Activity B: The Reunion

$\frac{x-1}{x+2} + \frac{x+3}{x+2}$	$\frac{2x+2}{x+2}$
$\frac{x-1}{x+2} - \frac{x+3}{x+2}$	$\frac{-4}{x+2}$
$\frac{x-1}{x+2} \cdot \frac{x+3}{x+2}$	$\frac{x^2+2x-3}{x+2}$
$\frac{x-1}{x+2} \div \frac{x+3}{x+2}$	$\frac{x-1}{x+3}$

$\frac{x}{x-3} + \frac{x-1}{3-x}$	$\frac{1}{x-3}$
$\frac{x}{x-3} - \frac{x-1}{3-x}$	$\frac{2x-1}{x-3}$
$\frac{x}{x-3} \cdot \frac{x-1}{3-x}$	$\frac{x^2-x}{-x^2+6x-9}$
$\frac{x}{x-3} \div \frac{x-1}{3-x}$	$\frac{-x}{x-1}$

$\frac{12}{x+2} + \frac{4x+8}{x+1}$	$\frac{4x^2+28x+28}{x^2+3x+2}$
$\frac{12}{x+2} - \frac{4x+8}{x+1}$	$\frac{-4x^2-4x-4}{x^2+3x+2}$
$\frac{12}{x+2} \cdot \frac{4x+8}{x+1}$	$\frac{48}{x+1}$
$\frac{12}{x+2} \div \frac{4x+8}{x+1}$	$\frac{3x+3}{x^2+4x+4}$

Solution to 7.4 Student Activity C: Tempting Expressions

$$1. \frac{25+x^2}{5x}$$

$$5. \frac{x+4}{x+2}$$

$$9. \frac{-8x^2-81}{x^2-9}$$

$$2. \frac{x-2}{x^2+2x}$$

$$6. \frac{x^2-4}{x^2-16}$$

$$10. \frac{4}{1-x^2}$$

$$3. \frac{2x^2+8x+16}{x^2+4x}$$

$$7. \frac{8x^2+5x-4}{3x^2+9x+6}$$

$$4. \frac{x^2-7x+6}{x+6}$$

$$8. \frac{x}{x-5}$$

Assessment 5A

Pretest and Diagnostic Tool: Exponents and Polynomials

Directions: Complete this assessment without looking back at your notes or your book. **Do not use a calculator for this assessment.**

- Multiply: $3 \cdot 3 \cdot 3$. _____
- Multiply: $(-2)(-2)$. _____
- Simplify: -4^2 . _____
- Write 2^3 using multiplication.

- Simplify: $\frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 3}$.
- Write $(4 \cdot 4)(4 \cdot 4 \cdot 4)$ as 4 to some power. _____
- Fill in the blank to complete this pattern: 8, 4, 2, _____, $\frac{1}{2}$, $\frac{1}{4}$.
- Fill in the blanks to complete this pattern: 27, 9, 3, 1, _____, _____.
- -3 is the same as $\frac{1}{3}$. True or false?

- Evaluate: $3 - (-4)$. _____
- $47,000 = 4.7(1000)$ True or false?

- $0.00058 = 5.8(0.001)$ True or false?

- $\frac{1}{10^2} = \frac{1}{100}$ True or false? _____
- Identify the exponent in 4^2 . _____
- Write 3,10,5 in descending order.

- Add: $(5x + 2) + (3x - 6)$.
- Simplify: $-(4x - 8)$.
- Subtract: $(2x + 3y) - (4x - 2y)$.
- Multiply. $\begin{array}{r} 425 \\ \times 36 \\ \hline \end{array}$
- Multiply: $(2x)(-5)$.
- If $M = 2AB$ and we insert $A = 4x$ and $B = -3$, what is the new formula for M ? _____
- Evaluate: $(2 + 5)^2$.
- Simplify: $x^2 - x - x + 1$.
- Divide. $12 \overline{)1632}$
- Evaluate: $\frac{12 + 36}{12}$.

Assessment 5A: Instructor Tally Sheet

Pretest and Diagnostic Tool: Exponents and Polynomials

Use this sheet to tally the *incorrect* responses to each question. Each group of questions correlates to specific skills that are needed in this chapter. From the tally sheet, you can see the areas where your students are likely to have more difficulty than others, and thus, where you may need to do additional review before beginning new topics.

Answers:

1. 27 2. 4 3. -16 4. $2 \cdot 2 \cdot 2$ 5. 4 6. 4^5 7. 1 8. $\frac{1}{3}, \frac{1}{9}$ 9. False 10. 7 11. False
12. False 13. True 14. 2 15. 10, 5, 3 16. $8x - 4$ 17. $-4x + 8$ 18. $-2x + 5y$
19. 15300 20. $-10x$ 21. $-24x$ 22. 49 23. $x^2 - 2x + 1$ 24. 136 25. 4

Skill: *Working with exponents.*

1.	2.	3.	4.
----	----	----	----

Skill: *Working with exponent rules (product, quotient, power).*

5.	6.
----	----

Skill: *Working with negative and zero exponents.*

7.	8.	9.	10.
----	----	----	-----

Skill: *Understanding scientific notation.*

11.	12.	13.
-----	-----	-----

Skill: *Understanding the language of polynomials.*

14.	15.
-----	-----

Skill: *Adding or subtracting polynomials.*

16.	17.	18.
-----	-----	-----

Skill: *Multiplying Polynomials.*

19.	20.
-----	-----

Skill: *Special Products.*

21.	22.	23.
-----	-----	-----

Skill: *Dividing polynomials.*

24.	25.
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Assessment 6D

Chapter 6: Metacognitive Skills Assessment

Metacognitive skills refer to the ability to judge how well you have learned something and to effectively direct your own learning and studying. This is a self evaluation tool designed to help you focus your studying and to improve your metacognitive skills with regards to this math class.

Fill the 1st column out before you begin studying.

Fill the 2nd column out after you study and before you take the test.

Go back to this page after your test and circle any of the ratings that you would now change – this identifies the “disconnects” between what you think you know well and what you actually know well.

Use the scale below to assign a number to each topic.

5 I am confident I can do any problems in this category correctly.

4 I am confident I can do most of the problems in this category correctly.

3 I understand how to do the problems in this category, but I still make a lot of mistakes.

2 I feel unsure about how to do these problems.

1 I know I don't understand how to do these problems.

Topic or Skill	Before Studying	After Studying
Finding the GCF for several terms.		
Factoring out the GCF.		
Factoring out a -1 . Why is it sometimes necessary to factor out a -1 ?		
Factoring by grouping and knowing when to employ this method.		
Distinguishing between trinomials of the form $x^2 + bx + c$ and the form $ax^2 + bx + c$.		
Factoring a trinomial of the form $x^2 + bx + c$.		
Factoring a trinomial of the form $ax^2 + bx + c$.		
Knowing the numbers that are perfect squares and perfect cubes.		
Recognizing expressions that are perfect squares and perfect cubes.		
Recognizing binomials that are either a sum or difference of squares.		
Factoring a difference of squares.		
Knowing about the factorability of a sum of squares.		
Recognizing binomials that are either a sum or difference of cubes.		
Factoring a sum or difference of cubes.		
Recognizing a perfect-square trinomial and properly factoring it.		
Using the general factoring strategy. What should you always do first?		
How can the number of terms in a polynomial help you to make decisions about how to factor it?		
Understanding when an expression is not yet completely factored, and how to complete it. Ex: Is $x(2x+4)(x^2-9)$ completely factored?		
Being able to check the answer to a factoring problem by multiplying out the answer.		

Use the scale below to assign a number to each topic.

- 5 *I am confident I can do any problems in this category correctly.*
- 4 *I am confident I can do most of the problems in this category correctly.*
- 3 *I understand how to do the problems in this category, but I still make a lot of mistakes.*
- 2 *I feel unsure about how to do these problems.*
- 1 *I know I don't understand how to do these problems.*

Topic or Skill	Before Studying	After Studying
Understanding why you must first get “=0” on one side of a quadratic equation before solving by factoring.		
Rewriting a quadratic equation in standard quadratic form.		
Solving a quadratic equation by factoring.		
Knowing how to properly write a solution set.		
Checking the solution to a quadratic equation.		
Distinguishing between the instructions: multiply, factor, or solve.		
Knowing basic geometry formulas for perimeter, area, and volume.		
Solving application problems involving geometry.		
Knowing the Pythagorean Theorem. What kind of triangles does the Pythagorean Theorem apply to?		
Solving application problems involving the Pythagorean Theorem.		
Knowing how to write consecutive integers algebraically.		
Solving application problems involving consecutive integers.		
Given a quadratic equation model, solve an application problem.		