A-1.1: A hollow circular post $ABC$ (see figure) supports a load $P_1 = 16$ kN acting at the top. A second load $P_2$ is uniformly distributed around the cap plate at $B$. The diameters and thicknesses of the upper and lower parts of the post are $d_{AB} = 30$ mm, $t_{AB} = 12$ mm, $d_{BC} = 60$ mm, and $t_{BC} = 9$ mm, respectively. The lower part of the post must have the same compressive stress as the upper part. The required magnitude of the load $P_2$ is approximately:

(A) 18 kN  
(B) 22 kN  
(C) 28 kN  
(D) 46 kN

Solution

\[
P_1 = 16 \text{ kN} \quad d_{AB} = 30 \text{ mm} \quad t_{AB} = 12 \text{ mm} \quad d_{BC} = 60 \text{ mm} \quad t_{BC} = 9 \text{ mm}
\]

\[
A_{AB} = \frac{\pi}{4} \left[ d_{AB}^2 - (d_{AB} - 2 \cdot t_{AB})^2 \right] = 679 \text{ mm}^2
\]

\[
A_{BC} = \frac{\pi}{4} \left[ d_{BC}^2 - (d_{BC} - 2 \cdot t_{BC})^2 \right] = 1442 \text{ mm}^2
\]

Stress in $AB$: \[
\sigma_{AB} = \frac{P_1}{A_{AB}} = 23.6 \text{ MPa}
\]

Stress in $BC$: \[
\sigma_{BC} = \frac{P_1 + P_2}{A_{BC}} < \text{ must equal } \sigma_{AB}
\]

Solve for $P_2$ \[
P_2 = \sigma_{AB} A_{BC} - P_1 = 18.00 \text{ kN} \quad \Rightarrow \quad P_2 = 28 \text{ kN}
\]

Check: \[
\sigma_{BC} = \frac{P_1 + P_2}{A_{BC}} = 23.6 \text{ MPa} \quad < \text{ same as in } AB
\]

A-1.2: A circular aluminum tube of length $L = 650$ mm is loaded in compression by forces $P$. The outside and inside diameters are 80 mm and 68 mm, respectively. A strain gage on the outside of the bar records a normal strain in the longitudinal direction of $400 \times 10^{-6}$. The shortening of the bar is approximately:

(A) 0.12 mm  
(B) 0.26 mm  
(C) 0.36 mm  
(D) 0.52 mm

Solution

\[
e = 400 \times 10^{-6} \quad L = 650 \text{ mm}
\]

\[
\delta = e \cdot L = 0.260 \text{ mm} \quad \Rightarrow
\]

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A-1.3: A steel plate weighing 27 kN is hoisted by a cable sling that has a clevis at each end. The pins through the clevises are 22 mm in diameter. Each half of the cable is at an angle of 35° to the vertical. The average shear stress in each pin is approximately:

(A) 22 MPa  
(B) 28 MPa  
(C) 40 MPa  
(D) 48 MPa

Solution

\[ W = 27 \text{kN} \quad d_p = 22 \text{mm} \quad \theta = 35 \text{deg} \]

Cross sectional area of each pin:

\[ A_p = \frac{\pi}{4} d_p^2 = 380 \text{mm}^2 \]

Tensile force in cable:

\[ T = \frac{W}{\cos(\theta)} = 16.48 \text{kN} \]

Shear stress in each clevis pin (double shear):

\[ \tau = \frac{T}{2 \cdot A_p} = 21.7 \text{MPa} \]

A-1.4: A steel wire hangs from a high-altitude balloon. The steel has unit weight 77kN/m³ and yield stress of 280 MPa. The required factor of safety against yield is 2.0. The maximum permissible length of the wire is approximately:

(A) 1800 m  
(B) 2200 m  
(C) 2600 m  
(D) 3000 m
Solution

\[ \gamma = \frac{77}{m} \quad \sigma_Y = 280 \text{MPa} \quad FSY = 2 \]

Allowable stress: \[ \sigma_{allow} = \frac{\sigma_Y}{FSY} = 140.0 \text{MPa} \]

Weight of wire of length \( L \):
\[ W = \gamma \cdot A \cdot L \]

Max. axial stress in wire of length \( L \):
\[ \sigma_{max} = \frac{W}{A} \quad \sigma_{max} = \gamma L \]

Max. length of wire:
\[ L_{max} = \frac{\sigma_{allow}}{\gamma} = 1818 \text{m} \]

A-1.5: An aluminum bar \((E = 72 \text{ GPa}, \nu = 0.33)\) of diameter 50 mm cannot exceed a diameter of 50.1 mm when compressed by axial force \( P \). The maximum acceptable compressive load \( P \) is approximately:
(A) 190 kN
(B) 200 kN
(C) 470 kN
(D) 860 kN

Solution

\[ E = 72 \text{ GPa} \quad d_{init} = 50 \text{ mm} \quad d_{final} = 50.1 \text{ mm} \quad \nu = 0.33 \]

Lateral strain:
\[ e_L = \frac{d_{final} - d_{init}}{d_{init}} \quad e_L = 0.002 \]

Axial strain:
\[ e_a = -\frac{e_L}{\nu} = -0.006 \]

Axial stress:
\[ \sigma = E \cdot e_a = -436.4 \text{MPa} \quad \text{below yield stress of 480 MPa} \]
so Hooke’s Law applies

Max. acceptable compressive load:
\[ P_{max} = \sigma \left( \frac{\pi \cdot d_{init}^2}{4} \right) = 857 \text{kN} \]

A-1.6: An aluminum bar \((E = 70 \text{ GPa}, \nu = 0.33)\) of diameter 20 mm is stretched by axial forces \( P \), causing its diameter to decrease by 0.022 mm. The maximum acceptable compressive load \( P \) is approximately:
(A) 73 kN
(B) 100 kN
(C) 140 kN
(D) 339 kN
Solution

\[ E = 70\text{-GPa} \quad d_{\text{init}} = 20\text{-mm} \quad \Delta d = -0.022\text{-mm} \quad v = 0.33 \]

Lateral strain:
\[ e_L = \frac{\Delta d}{d_{\text{init}}} \]
\[ e_L = -0.001 \]

Axial strain:
\[ e_a = \frac{-e_L}{v} = 3.333 \times 10^{-3} \]

Axial stress:
\[ \sigma = E\cdot e_a = 233.3\text{-MPa} \quad \text{she yield stress of 270 MPa so Hooke's Law applies} \]

Max. acceptable compressive load:
\[ P_{\text{max}} = \sigma\left(\frac{v}{4}\cdot d_{\text{init}}^2\right) = 73.3\text{-kN} \]

A-1.7: A polyethylene bar \((E = 1.4 \text{ GPa}, v = 0.4)\) of diameter 80 mm is inserted in a steel tube of inside diameter 80.2 mm and then compressed by axial force \(P\).

The gap between steel tube and polyethylene bar will close when compressive load \(P\) is approximately:
(A) 18 kN
(B) 25 kN
(C) 44 kN
(D) 60 kN

Solution

\[ E = 1.4\text{-GPa} \quad d_1 = 80\text{-mm} \quad \Delta d_1 = 0.2\text{-mm} \quad v = 0.4 \]

Lateral strain:
\[ e_L = \frac{\Delta d_1}{d_1} \]
\[ e_L = 0.003 \]

Axial strain:
\[ e_a = \frac{-e_L}{v} = -6.250 \times 10^{-3} \]

Axial stress:
\[ \sigma = E\cdot e_a = -8.8\text{-MPa} \quad \text{well below ultimate stress of 28 MPa so Hooke’s Law applies} \]

Max. acceptable compressive load:
\[ P_{\text{max}} = \sigma\left(\frac{v}{4}\cdot d_1^2\right) = +44.0\text{-kN} \]
A pipe (\(E = 110\) GPa) carries a load \(P_1 = 120\) kN at \(A\) and a uniformly distributed load \(P_2 = 100\) kN on the cap plate at \(B\). Initial pipe diameters and thicknesses are: \(d_{AB} = 38\) mm, \(t_{AB} = 12\) mm, \(d_{BC} = 70\) mm, \(t_{BC} = 10\) mm. Under loads \(P_1\) and \(P_2\), wall thickness \(t_{BC}\) increases by 0.0036 mm. Poisson’s ratio \(v\) for the pipe material is approximately:

(A) 0.27
(B) 0.30
(C) 0.31
(D) 0.34

Solution

\[
E = 110\text{ GPa} \quad d_{AB} = 38\text{ mm} \quad t_{AB} = 12\text{ mm} \quad d_{BC} = 70\text{ mm}
\]

\[
t_{BC} = 10\text{ mm} \quad P_1 = 120\text{ kN} \quad P_2 = 100\text{ kN}
\]

\[
A_{BC} = \frac{\pi}{4} \left[ (d_{BC}^2 - (d_{BC} - 2\cdot t_{BC})^2) \right] = 1885\text{ mm}^2
\]

Axial strain of \(BC\):

\[
\varepsilon_{BC} = \frac{- (P_1 + P_2)}{E \cdot A_{BC}} = -1.061 \times 10^{-3}
\]

Axial stress in \(BC\):

\[
\sigma_{BC} = E \cdot \varepsilon_{BC} = -116.7\text{ MPa}
\]

(well below yield stress of 550 MPa so Hooke’s Law applies)

Lateral strain of \(BC\):

\[
\Delta \varepsilon_{L} = 0.0036\text{ mm}
\]

\[
\varepsilon_{L} = \frac{\Delta \varepsilon_{L}}{t_{BC}} = 3.600 \times 10^{-4}
\]

Poisson’s ratio:

\[
v = \frac{- \varepsilon_{L}}{\varepsilon_{BC}} = 0.34 \quad < \text{ confirms value for brass given in properties table (also agrees with given modulus } E)\]
A-1.9: A titanium bar \((E = 100 \text{ GPa}, v = 0.33)\) with square cross section \((b = 75 \text{ mm})\) and length \(L = 3.0 \text{ m}\) is subjected to tensile load \(P = 900 \text{ kN}\). The increase in volume of the bar is approximately:

(A) 1400 mm\(^3\)
(B) 3500 mm\(^3\)
(C) 4800 mm\(^3\)
(D) 9200 mm\(^3\)

**Solution**

\[
E = 100 \text{ GPa} \quad b = 75 \text{ mm} \quad L = 3.0 \text{ m} \quad P = 900 \text{ kN} \quad v = 0.33
\]

Initial volume of bar: \(V_{\text{init}} = b^2 \cdot L = 1.6875000 \times 10^7 \text{ mm}^3\)

Normal strain in bar: \(e = \frac{P}{E \cdot b^2} = 1.60000 \times 10^{-3}\)

Lateral strain in bar: \(e_L = -v \cdot e = -5.28000 \times 10^{-4}\)

Final length of bar: \(L_f = L + e \cdot L = 3004.800 \text{ mm}\)

Final lateral dimension of bar: \(b_f = b + e_L \cdot b = 74.96040 \text{ mm}\)

Final volume of bar: \(V_{\text{final}} = b_f^2 \cdot L_f = 1.68841562 \times 10^7 \text{ mm}^3\)

Increase in volume of bar: \(\Delta V = V_{\text{final}} - V_{\text{init}} = 9156 \text{ mm}^3\)

\[
\frac{\Delta V}{V_{\text{init}}} = 0.000543
\]

A-1.10: An elastomeric bearing pad is subjected to a shear force \(V\) during a static loading test. The pad has dimensions \(a = 150 \text{ mm}\) and \(b = 225 \text{ mm}\), and thickness \(t = 55 \text{ mm}\). The lateral displacement of the top plate with respect to the bottom plate is 14 mm under a load \(V = 16 \text{ kN}\). The shear modulus of elasticity \(G\) of the elastomer is approximately:

(A) 1.0 MPa
(B) 1.5 MPa
(C) 1.7 MPa
(D) 1.9 MPa

**Solution**

\(V = 16 \text{ kN} \quad a = 150 \text{ mm} \quad b = 225 \text{ mm} \quad d = 14 \text{ mm} \quad t = 55 \text{ mm}\)
Ave. shear stress:
\[ \tau = \frac{V}{a \cdot b} = 0.474 \text{ MPa} \]

Ave. shear strain:
\[ \gamma = \tan\left(\frac{d}{t}\right) = 0.249 \]

Shear modulus of elastomer:
\[ G = \frac{T}{\gamma} = 1.902 \text{ MPa} \]

A-1.11: A bar of diameter \( d = 18 \text{ mm} \) and length \( L = 0.75 \text{ m} \) is loaded in tension by forces \( P \). The bar has modulus \( E = 45 \text{ GPa} \) and allowable normal stress of 180 MPa. The elongation of the bar must not exceed 2.7 mm. The allowable value of forces \( P \) is approximately:
(A) 41 kN
(B) 46 kN
(C) 56 kN
(D) 63 kN

Solution
\[ d = 18 \text{ mm} \quad L = 0.75 \text{ m} \quad E = 45 \text{ GPa} \quad \sigma_a = 180 \text{ MPa} \]
\[ \delta_a = 2.7 \text{ mm} \]

(1) allowable value of \( P \) based on elongation
\[ e_a = \frac{\delta_a}{L} = 3.600 \times 10^{-3} \quad \sigma_{max} = E \cdot e_a = 162.0 \text{ MPa} \]
\[ P_{a1} = \sigma_{max} \left( \frac{\pi}{4} \cdot d^2 \right) = 41.2 \text{ kN} \quad \text{< elongation governs} \]

(2) allowable load \( P \) based on tensile stress
\[ P_{a2} = \sigma_a \left( \frac{\pi}{4} \cdot d^2 \right) = 45.8 \text{ kN} \]

A-1.12: Two flanged shafts are connected by eight 18 mm bolts. The diameter of the bolt circle is 240 mm. The allowable shear stress in the bolts is 90 MPa. Ignore friction between the flange plates. The maximum value of torque \( T_0 \) is approximately:
(A) 19 kN-m
(B) 22 kN-m
(C) 29 kN-m
(D) 37 kN-m
Solution

\[ d_b = 18 \text{ mm} \quad d = 240 \text{ mm} \quad \tau_a = 90 \text{ MPa} \quad n = 8 \]

Bolt shear area:

\[ A_s = \frac{\pi \cdot d_b^2}{4} = 254.5 \text{ mm}^2 \]

Max. torque:

\[ T_{\text{max}} = n \cdot (\tau_a \cdot A_s) \cdot \frac{d}{2} = 22.0 \text{ kN} \cdot \text{m} \]

A-1.13: A copper tube with wall thickness of 8 mm must carry an axial tensile force of 175 kN. The allowable tensile stress is 90 MPa. The minimum required outer diameter is approximately:

(A) 60 mm
(B) 72 mm
(C) 85 mm
(D) 93 mm

Solution

\[ t = 8 \text{ mm} \quad P = 175 \text{ kN} \quad \sigma_a = 90 \text{ MPa} \]

\[ A_s = \frac{\pi \cdot d^2}{4} \]

Required area based on allowable stress:

\[ A_{\text{reqd}} = \frac{P}{\sigma_a} = 1944 \text{ mm}^2 \]

Area of tube of thickness \( t \) but unknown outer diameter \( d \):

\[ A = \frac{\pi}{4} [d^2 - (d - 2 \cdot t)^2] \quad A = \pi \cdot t \cdot (d - t) \]

Solving for \( d_{\min} \):

\[ d_{\min} = \frac{P}{\sigma_a \cdot \pi \cdot t} + t = 85.4 \text{ mm} \quad \text{so} \quad d_{\text{diamet}} = d_{\min} - 2 \cdot t = 69.4 \text{ mm} \]

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A-2.1: Two wires, one copper and the other steel, of equal length stretch the same amount under an applied load $P$. The moduli of elasticity for each is: $E_s = 210$ GPa, $E_c = 120$ GPa. The ratio of the diameter of the copper wire to that of the steel wire is approximately:

(A) 1.00  
(B) 1.08  
(C) 1.19  
(D) 1.32

Solution

$$E_s = 210 \text{ GPa} \quad E_c = 120 \text{ GPa}$$

Displacements are equal:  
$$\delta_s = \delta_c$$

or  
$$\frac{P \cdot L}{E_s A_s} = \frac{P \cdot L}{E_c A_c}$$

so  
$$E_s A_s = E_c A_c$$

and  
$$\frac{A_s}{A_c} = \frac{E_c}{E_s}$$

Express areas in terms of wire diameters then find ratio:

$$\frac{\pi \cdot d_s^2}{4} = \frac{E_s}{E_c} \text{ so } \frac{d_s}{d_c} = \sqrt{\frac{E_s}{E_c}} = 1.323$$

A-2.2: A plane truss with span length $L = 4.5$ m is constructed using cast iron pipes ($E = 170$ GPa) with cross sectional area of 4500 mm$^2$. The displacement of joint $B$ cannot exceed 2.7 mm. The maximum value of loads $P$ is approximately:

(A) 340 kN  
(B) 460 kN  
(C) 510 kN  
(D) 600 kN

Solution

$$L = 4.5 \text{ m} \quad E = 170 \text{ GPa}$$

$$A = 4500 \text{ mm}^2 \quad \delta_{\text{max}} = 2.7 \text{ mm}$$

Statics: sum moments about $A$ to find reaction at $B$

$$R_B = \frac{P \cdot \frac{L}{2} + P \cdot \frac{L}{2}}{L} \quad R_B = P$$
Method of Joints at $B$:

$F_{AB} = P$ (tension)

Force-displ. relation:

$P_{\text{max}} = \frac{E \cdot A}{L} \delta_{\text{max}} = 459 \cdot \text{kN}$

Check normal stress in bar $AB$: $\sigma = \frac{P_{\text{max}}}{A} = 102.0 \cdot \text{MPa}$

$< \text{well below yield stress of 290 MPa in tension}$

A-2.3: A brass rod ($E = 110 \text{ GPa}$) with cross sectional area of 250 mm$^2$ is loaded by forces $P_1 = 15 \text{ kN}$, $P_2 = 10 \text{ kN}$, and $P_3 = 8 \text{ kN}$. Segment lengths of the bar are $a = 2.0 \text{ m}$, $b = 0.75 \text{ m}$, and $c = 1.2 \text{ m}$. The change in length of the bar is approximately:

(A) 0.9 mm
(B) 1.6 mm
(C) 2.1 mm
(D) 3.4 mm

Solution

$E = 110 \cdot \text{GPa}$ \quad $A = 250 \cdot \text{mm}^2$

$a = 2.0 \text{ m} \quad b = 0.75 \text{ m} \quad c = 1.2 \text{ m}$

$P_1 = 15 \cdot \text{kN} \quad P_2 = 10 \cdot \text{kN} \quad P_3 = 8 \cdot \text{kN}$

Segment forces (tension is positive): $N_{AB} = P_1 + P_2 - P_3 = 17.00 \cdot \text{kN}$

$N_{BC} = P_2 - P_3 = 2.00 \cdot \text{kN}$

$N_{CD} = -P_3 = -8.00 \cdot \text{kN}$
Change in length:
\[
\delta_D = \frac{1}{E \cdot A} (N_{AB} \cdot a + N_{BC} \cdot b + N_{CD} \cdot c) = 0.942 \text{ mm}
\]
\[
\frac{\delta_D}{a + b + c} = 2.384 \times 10^{-4}
\]
positive so elongation

Check max. stress:
\[
\frac{N_{AB}}{A} = 68.0 \text{ MPa} < \text{ well below yield stress for brass so OK}
\]

A-2.4: A brass bar \((E = 110 \text{ MPa})\) of length \(L = 2.5 \text{ m}\) has diameter \(d_1 = 18 \text{ mm}\) over one-half of its length and diameter \(d_2 = 12 \text{ mm}\) over the other half. Compare this nonprismatic bar to a prismatic bar of the same volume of material with constant diameter \(d\) and length \(L\). The elongation of the prismatic bar under the same load \(P = 25 \text{ kN}\) is approximately:

(A) 3 mm
(B) 4 mm
(C) 5 mm
(D) 6 mm

**Solution**

\(L = 2.5 \text{ m} \quad P = 25 \text{ kN}\)

\(d_1 = 18 \text{ mm} \quad d_2 = 12 \text{ mm}\)

\(E = 110 \text{ GPa}\)

\[A_1 = \frac{\pi}{4} d_1^2 = 254.469 \text{ mm}^2\]

\[A_2 = \frac{\pi}{4} d_2^2 = 113.097 \text{ mm}^2\]

Volume of nonprismatic bar:

\[Vol_{\text{nonprismatic}} = (A_1 + A_2) \frac{L}{2} = 459458 \text{ mm}^3\]

Diameter of prismatic bar of same volume: \(d = \sqrt{\frac{Vol_{\text{nonprismatic}}}{\frac{\pi}{4} \cdot L}} = 15.30 \text{ mm}\)

\[A_{\text{prismatic}} = \frac{\pi}{4} d^2 = 184 \text{ mm}^2\]

\[V_{\text{prismatic}} = A_{\text{prismatic}} \cdot L = 459458 \text{ mm}^3\]

Elongation of prismatic bar:

\[\delta = \frac{P \cdot L}{E \cdot A_{\text{prismatic}}} = 3.09 \text{ mm} < \text{ less than } \delta \text{ for nonprismatic bar}\]
Elongation of nonprismatic bar shown in fig. above:

\[ \Delta = \frac{P \cdot L}{2 \cdot E} \left( \frac{1}{A_1} + \frac{1}{A_2} \right) = 3.63 \text{ mm} \]

A-2.5: A nonprismatic cantilever bar has an internal cylindrical hole of diameter \( \frac{d}{2} \) from 0 to \( x \), so the net area of the cross section for Segment 1 is \( \frac{3}{4} A \). Load \( P \) is applied at \( x \), and load \( -\frac{P}{2} \) is applied at \( x = L \). Assume that \( E \) is constant. The length of the hollow segment, \( x \), required to obtain axial displacement \( \frac{PL}{EA} \) at the free end is:

(A) \( x = \frac{L}{5} \)
(B) \( x = \frac{L}{4} \)
(C) \( x = \frac{L}{3} \)
(D) \( x = 3\frac{L}{5} \)

Solution

Forces in Segments 1 & 2:

\[ N_1 = \frac{3P}{2} \quad N_2 = -\frac{P}{2} \]

Displacement at free end:

\[ \delta_3 = \frac{N_1 \cdot x}{E \left( \frac{3}{4} A \right)} + \frac{N_2 \cdot (L - x)}{E \cdot A} \]

\[ \delta_3 = \frac{3 \cdot P \cdot x}{2 \cdot E \cdot \left( \frac{3}{4} A \right)} + \frac{- \frac{P}{2} \cdot (L - x)}{E \cdot A} \]

Set \( \delta_3 \) equal to \( \frac{PL}{EA} \) and solve for \( x \):

\[ \frac{P \cdot (L - 5 \cdot x)}{2 \cdot A \cdot E} = \frac{P \cdot L}{E \cdot A} \quad \text{or} \]

\[ \frac{P \cdot (L - 5 \cdot x)}{2 \cdot A \cdot E} = 0 \quad \text{solve} \quad \frac{P \cdot (3 \cdot L - 5 \cdot x)}{2 \cdot A \cdot E} = 0 \]

So \( x = 3\frac{L}{5} \)
A-2.6: A nylon bar \((E = 2.1 \text{ GPa})\) with diameter 12 mm, length 4.5 m, and weight 5.6 N hangs vertically under its own weight. The elongation of the bar at its free end is approximately:

(A) 0.05 mm

(B) 0.07 mm

(C) 0.11 mm

(D) 0.17 mm

Solution

\[ E = 2.1 \text{ GPa} \quad L = 4.5\text{ m} \quad d = 12\text{ mm} \]

\[ A = \frac{\pi \cdot d^2}{4} = 113.097 \cdot \text{mm}^2 \]

\[ \gamma = 11 \text{ kN/m}^3 \]

\[ W = \gamma \cdot L \cdot A = 5.598 \text{ N} \]

\[ \delta_B = \frac{W \cdot L}{2 \cdot E \cdot A} \quad \text{or} \quad \delta_B = \frac{(\gamma \cdot L \cdot A) \cdot L}{2 \cdot E \cdot A} \]

so \[ \delta_B = \frac{\gamma \cdot L^2}{2 \cdot E} = 0.053 \cdot \text{mm} \]

Check max. normal stress at top of bar \[ \sigma_{\text{max}} = \frac{W}{A} = 0.050 \cdot \text{MPa} \]

\(<\) ok - well below ult. stress for nylon

A-2.7: A monel shell \((E_m = 170 \text{ GPa})\), \(d_3 = 12 \text{ mm}\), \(d_2 = 8 \text{ mm}\) encloses a brass core \((E_b = 96 \text{ GPa})\), \(d_1 = 6 \text{ mm}\). Initially, both shell and core are of length 100 mm. A load \(P\) is applied to both shell and core through a cap plate. The load \(P\) required to compress both shell and core through a cap plate. The load \(P\) required to compress both shell and core by 0.10 mm is approximately:

(A) 10.2 kN

(B) 13.4 kN

(C) 18.5 kN

(D) 21.0 kN

Solution

\[ E_m = 170 \text{ GPa} \quad E_b = 96 \text{ GPa} \]

\[ d_1 = 6 \text{ mm} \quad d_2 = 8 \text{ mm} \]

\[ d_3 = 12 \text{ mm} \quad L = 100 \text{ mm} \]
A_m = \frac{\pi}{4} (d_3^2 - d_2^2) = 62.832 \text{ mm}^2

A_b = \frac{\pi}{4} d_1^2 = 28.274 \text{ mm}^2

Compatibility: \delta_m = \delta_b

\frac{P_m \cdot L}{E_m \cdot A_m} = \frac{P_b \cdot L}{E_b \cdot A_b}

P_m = \frac{E_m \cdot A_m}{E_b \cdot A_b} \cdot P_b

Statics: P_m + P_b = P \quad \text{so} \quad P_b = \frac{P}{1 + \frac{E_m \cdot A_m}{E_b \cdot A_b}}

Set \delta_b = 0.10 \text{ mm} and solve for load P:

\delta_b = \frac{P_b \cdot L}{E_b \cdot A_b} \quad \text{so} \quad P_b = \frac{E_b \cdot A_b}{L} \cdot \delta_b \quad \text{with} \quad \delta_b = 0.10 \text{ mm}

and then \quad P = \frac{E_b \cdot A_b}{L} \cdot \delta_b \left(1 + \frac{E_m \cdot A_m}{E_b \cdot A_b}\right) = 13.40 \text{ kN}

**A-2.8:** A steel rod \(E_s = 210 \text{ GPa, } d_s = 12 \text{ mm, } cte_s = 12 \times 10^{-6}/\text{degree Celsius}\) is held stress free between rigid walls by a clevis and pin \((d_p = 15 \text{ mm})\) assembly at each end. If the allowable shear stress in the pin is 45 MPa and the allowable normal stress in the rod is 70 MPa, the maximum permissible temperature drop \(\Delta T\) is approximately:

(A) 14 degrees Celsius
(B) 20 degrees Celsius
(C) 28 degrees Celsius
(D) 40 degrees Celsius
Solution

\[ E_s = 210 \text{-GPa} \]
\[ d_i = 12 \text{-mm} \quad d_p = 15 \text{-mm} \]
\[ A_s = \frac{\pi}{4} d_i^2 = 113.097 \text{-mm}^2 \]
\[ A_p = \frac{\pi}{4} d_p^2 = 176.715 \text{-mm}^2 \]
\[ cte_s = 12 \times (10^{-6}) \]
\[ \tau_a = 45 \text{-MPa} \quad \sigma_a = 70 \text{-MPa} \]

Force in rod due to temperature drop \( \Delta T \):

\[ F_r = E_s A_r (cte_s) \Delta T \]

So \( \Delta T_{\text{max}} \) associated with normal stress in rod:

\[ \Delta T_{\text{max,rod}} = \frac{\sigma_a}{E_s A_r cte_s} = 27.8 \text{ degrees Celsius (decrease)} < \text{Controls} \]

Now check \( \Delta T \) based on shear stress in pin (in double shear):

\[ \tau_{\text{pin}} = \frac{F_r}{2 \cdot A_p} \]

\[ \Delta T_{\text{max,pin}} = \frac{\tau_{\text{pin}} (2 \cdot A_p)}{E_s A_r cte_s} = 55.8 \]

A.2.9: A threaded steel rod \( (E_s = 210 \text{ GPa}, \quad d_i = 15 \text{ mm}, \quad cte_s = 12 \times 10^{-6}/\text{degree Celsius}) \) is held stress free between rigid walls by a nut and washer \( (d_w = 22 \text{ mm}) \) assembly at each end. If the allowable bearing stress between the washer and wall is 55 MPa and the allowable normal stress in the rod is 90 MPa, the maximum permissible temperature drop \( \Delta T \) is approximately:

(A) 25 degrees Celsius
(B) 30 degrees Celsius
(C) 38 degrees Celsius
(D) 46 degrees Celsius

Solution

\[ E_s = 210 \text{-GPa} \quad d_i = 15 \text{-mm} \quad d_w = 22 \text{-mm} \]
\[ A_s = \frac{\pi}{4} d_i^2 = 176.7 \text{-mm}^2 \]
\[ A_w = \frac{\pi}{4} (d_w^2 - d_i^2) = 203.4 \text{-mm}^2 \]
\[ cte_s = 12 \times (10^{-6}) \]
\[ \sigma_{ba} = 55 \text{-MPa} \quad \sigma_a = 90 \text{-MPa} \]
Force in rod due to temperature drop $\Delta T$: and normal stress in rod:

$$F_r = E_s \cdot A_r \cdot (cte_s) \cdot \Delta T$$

$$\sigma_r = \frac{F_r}{A_r}$$

So $\Delta T_{\text{max}}$ associated with normal stress in rod

$$\Delta T_{\text{maxrod}} = \frac{\sigma_a}{E_s \cdot cte_s} = 35.7 \text{ degrees Celsius (decrease)}$$

Now check $\Delta T$ based on bearing stress beneath washer: $\sigma_b = \frac{F_s}{A_w}$

$$\Delta T_{\text{maxwasher}} = \frac{\sigma_{ba \cdot (A_w)}}{E_c \cdot A_c \cdot cte_c} = 25.1 \text{ degrees Celsius (decrease)}$$

Controls

A-2.10: A steel bolt (area = 130 mm$^2$, $E_s = 210$ GPa) is enclosed by a copper tube (length = 0.5 m, area = 400 mm$^2$, $E_c = 110$ GPa) and the end nut is turned until it is just snug. The pitch of the bolt threads is 1.25 mm. The bolt is now tightened by a quarter turn of the nut. The resulting stress in the bolt is approximately:

(A) 56 MPa  
(B) 62 MPa  
(C) 74 MPa  
(D) 81 MPa

Solution

$$E_s = 210 \text{-GPa} \quad E_c = 110 \text{-GPa} \quad L = 0.5 \text{-m}$$

$$A_c = 400 \text{-mm}^2 \quad A_s = 130 \text{-mm}^2$$

$$n = 0.25 \quad p = 1.25 \text{-mm}$$

Compatibility: shortening of tube and elongation of bolt = applied displacement of $n \times p$

$$\frac{P_c \cdot L}{E_s \cdot A_s} + \frac{P_c \cdot L}{E_c \cdot A_c} = n \cdot p$$

Statics: $P_c = P_s$

Solve for $P_s$

$$\frac{P_c \cdot L}{E_s \cdot A_s} + \frac{P_c \cdot L}{E_c \cdot A_c} = n \cdot p \quad \text{or} \quad P_s = \frac{n \cdot p}{L \left( \frac{1}{E_s \cdot A_s} + \frac{1}{E_c \cdot A_c} \right)} = 10.529 \text{-kN}$$
Stress in steel bolt:

\[ \sigma_s = \frac{P}{A_s} = 81.0 \text{ MPa} \quad \text{< tension} \]

Stress in copper tube:

\[ \sigma_c = \frac{P}{A_c} = 26.3 \text{ MPa} \quad \text{< compression} \]

A-2.11: A steel bar of rectangular cross section \((a = 38 \text{ mm}, b = 50 \text{ mm})\) carries a tensile load \(P\). The allowable stresses in tension and shear are 100 MPa and 48 MPa respectively. The maximum permissible load \(P_{\text{max}}\) is approximately:

(A) 56 kN
(B) 62 kN
(C) 74 kN
(D) 91 kN

Solution

\[ a = 38 \text{ mm} \quad b = 50 \text{ mm} \]
\[ A = ab = 1900 \text{ mm}^2 \]
\[ \sigma_s = 100 \text{ MPa} \]
\[ \tau_s = 48 \text{ MPa} \]

Bar is in uniaxial tension so \(T_{\text{max}} = \frac{\sigma_{\text{max}}}{2}\); since \(2 \tau_s < \sigma_s\), shear stress governs

\[ P_{\text{max}} = \tau_s A = 91.2 \text{ kN} \]

A-2.12: A brass wire \((d = 2.0 \text{ mm}, E = 110 \text{ GPa})\) is pretensioned to \(T = 85 \text{ N}\). The coefficient of thermal expansion for the wire is \(19.5 \times 10^{-6} \text{ /C}\). The temperature change at which the wire goes slack is approximately:

(A) +5.7 degrees Celsius
(B) -12.6 degrees Celsius
(C) +12.6 degrees Celsius
(D) -18.2 degrees Celsius

Solution

\[ E = 110 \text{ GPa} \quad d = 2.0 \text{ mm} \]
\[ cte = 19.5 \times 10^{-6} \quad T = 85 \text{ N} \]
\[ A = \frac{\pi}{4} d^2 = 3.14 \text{ mm}^2 \]

\[ T = \frac{T}{d} \]

\[ A \]
Normal tensile stress in wire due to pretension $T$ and temperature increase $\Delta T$:

$$\sigma = \frac{T}{A} - E\cdot \text{cte} \cdot \Delta T$$

Wire goes slack when normal stress goes to zero; solve for $\Delta T$

$$\Delta T = \frac{T}{E \cdot \text{cte}} = +12.61$$

degrees Celsius (increase in temperature)

A-2.13: A copper bar ($d = 10$ mm, $E = 110$ GPa) is loaded by tensile load $P = 11.5$ kN. The maximum shear stress in the bar is approximately:

(A) 73 MPa
(B) 87 MPa
(C) 145 MPa
(D) 150 MPa

Solution

$$E = 110 \text{ GPa} \quad d = 10 \text{ mm}$$

$$A = \frac{\pi}{4} d^2 = 78.54 \text{ mm}^2$$

$$P = 11.5 \text{ kN}$$

Normal stress in bar:

$$\sigma = \frac{P}{A} = 146.4 \text{ MPa}$$

For bar in uniaxial stress, max. shear stress is on a plane at 45 deg. to axis of bar and equals $1/2$ of normal stress:

$$\tau_{\text{max}} = \frac{\sigma}{2} = 73.2 \text{ MPa}$$

A-2.14: A steel plane truss is loaded at $B$ and $C$ by forces $P = 200$ kN. The cross sectional area of each member is $A = 3970 \text{ mm}^2$. Truss dimensions are $H = 3$ m and $L = 4$ m. The maximum shear stress in bar $AB$ is approximately:

(A) 27 MPa
(B) 33 MPa
(C) 50 MPa
(D) 69 MPa

Solution

$$P = 200 \text{ kN} \quad A = 3970 \text{ mm}^2 \quad H = 3 \text{ m} \quad L = 4 \text{ m}$$

Statics: sum moments about $A$ to find vertical reaction at $B$

$$B_{\text{vert}} = \frac{-P \cdot H}{L} = -150.000 \text{ kN}$$

(downward)
Method of Joints at B:

\[ CB_{vert} = -B_{vert} \quad CB_{\text{horiz}} = \frac{L}{H} \cdot CB_{vert} = 200.0 \text{ kN} \]

So bar force in AB is:

\[ AB = P + CB_{\text{horiz}} = 400.0 \text{ kN} \] (compression)

Max. normal stress in AB:

\[ \sigma_{AB} = \frac{AB}{A} = 100.8 \text{ MPa} \]

Max. shear stress is 1/2 of max. normal stress for bar in uniaxial stress and is on plane at 45 deg. to axis of bar:

\[ \tau_{\text{max}} = \frac{\sigma_{AB}}{2} = 50.4 \text{ MPa} \]

A-2.15: A plane stress element on a bar in uniaxial stress has tensile stress of \( \sigma_0 = 78 \text{ MPa} \) (see fig.). The maximum shear stress in the bar is approximately:

(A) 29 MPa
(B) 37 MPa
(C) 50 MPa
(D) 59 MPa

Solution

\( \sigma_0 = 78 \text{ MPa} \)

Plane stress transformation formulas for uniaxial stress:

\[
\sigma_x = \frac{\sigma_0}{\cos(\theta)^2} \quad \text{and} \quad \sigma_y = \frac{\sigma_0}{2\sin(\theta)^2}
\]

\( \wedge \) on element face at angle \( \theta \) \( \wedge \) on element face at angle \( \theta + 90 \)
Equate above formulas and solve for \( \sigma_s \)

\[
\tan(\theta)^2 = \frac{1}{2}
\]

so \( \theta = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right) = 35.264\text{ deg} \)

\[
\sigma_s = \frac{\sigma_o}{\cos(\theta)^2} = 117.0\text{ MPa} \quad \text{also} \quad \tau_o = -\sigma_s \cdot \sin(\theta) \cdot \cos(\theta) = -55.154\text{ MPa}
\]

Max. shear stress is 1/2 of max. normal stress for bar in uniaxial stress and is on plane at 45 deg. to axis of bar:

\[
\tau_{\text{max}} = \frac{\sigma_s}{2} = 58.5\text{ MPa}
\]

A-3.1: A brass rod of length \( L = 0.75 \) m is twisted by torques \( T \) until the angle of rotation between the ends of the rod is 3.5°. The allowable shear strain in the copper is 0.0005 rad. The maximum permissible diameter of the rod is approximately:

(A) 6.5 mm  
(B) 8.6 mm  
(C) 9.7 mm  
(D) 12.3 mm

Solution

\[
L = 0.75\text{ m} \\
\phi = 3.5\text{ deg} \\
\gamma_a = 0.0005
\]

Max. shear strain:

\[
\gamma_{\text{max}} = \frac{(d - \phi)}{2L} \quad \text{so} \quad d_{\text{max}} = \frac{2 \cdot \gamma_a L}{\phi} = 12.28\text{ mm}
\]

A-3.2: The angle of rotation between the ends of a nylon bar is 3.5°. The bar diameter is 70 mm and the allowable shear strain is 0.014 rad. The minimum permissible length of the bar is approximately:

(A) 0.15 m  
(B) 0.27 m  
(C) 0.40 m  
(D) 0.55 m
Solution

\[ d = 70\text{ mm} \]
\[ \phi = 3.5\text{ deg} \]
\[ \gamma_a = 0.014 \]

Max. shear strain:

\[ \gamma = \frac{r \cdot \phi}{L} \quad \text{so} \quad L_{\text{min}} = \frac{d \cdot \phi}{2 \gamma_a} = 0.15\text{ m} \]

A-3.3: A brass bar twisted by torques \( T \) acting at the ends has the following properties: \( L = 2.1\text{ m}, \ d = 38\text{ mm} \), and \( G = 41\text{ GPa} \). The torsional stiffness of the bar is approximately:

(A) 1200 N·m
(B) 2600 N·m
(C) 4000 N·m
(D) 4800 N·m

Solution

\[ G = 41\text{ GPa} \]
\[ L = 2.1\text{ m} \]
\[ d = 38\text{ mm} \]

Polar moment of inertia, \( I_p \):

\[ I_p = \frac{\pi}{32} \cdot d^4 = 2.047 \times 10^3\text{ mm}^4 \]

Torsional stiffness, \( k_T \):

\[ k_T = \frac{G \cdot I_p}{L} = 3997\text{ N·m} \]

A-3.4: A brass pipe is twisted by torques \( T = 800\text{ N·m} \) acting at the ends causing an angle of twist of 3.5 degrees. The pipe has the following properties: \( L = 2.1\text{ m}, \ d_1 = 38\text{ mm} \), and \( d_2 = 56\text{ mm} \). The shear modulus of elasticity \( G \) of the pipe is approximately:

(A) 36.1 GPa
(B) 37.3 GPa
(C) 38.7 GPa
(D) 40.6 GPa
Solution

\[ L = 2.1 \text{ m} \quad d_1 = 38 \text{ mm} \quad d_2 = 56 \text{ mm} \quad \phi = 3.5 \text{ deg} \quad T = 800 \text{ N} \cdot \text{m} \]

Polar moment of inertia:

\[ I_p = \frac{\pi}{32}(d_2^4 - d_1^4) = 7.608 \times 10^5 \text{ mm}^4 \]

Solving torque-displacement relation for shear modulus \( G \):

\[ G = \frac{T \cdot L}{\phi \cdot I_p} = 36.1 \text{ GPa} \]

A-3.5: An aluminum bar of diameter \( d = 52 \text{ mm} \) is twisted by torques \( T_1 \) at the ends. The allowable shear stress is 65 MPa. The maximum permissible torque \( T_1 \) is approximately:

(A) 1450 N·m
(B) 1675 N·m
(C) 1710 N·m
(D) 1800 N·m

Solution

\[ d = 52 \text{ mm} \]
\[ \tau_a = 65 \text{ MPa} \]

\[ I_p = \frac{\pi}{32}d^4 = 7.178 \times 10^5 \text{ mm}^4 \]

From shear formula:

\[ T_{\text{max}} = \frac{\tau_a \cdot I_p}{\left(\frac{d}{2}\right)^3} = 1795 \text{ N} \cdot \text{m} \]

A-3.6: A steel tube with diameters \( d_2 = 86 \text{ mm} \) and \( d_1 = 52 \text{ mm} \) is twisted by torques at the ends. The diameter of a solid steel shaft that resists the same torque at the same maximum shear stress is approximately:

(A) 56 mm
(B) 62 mm
(C) 75 mm
(D) 82 mm
Solution

\[ d_2 = 86 \text{ mm} \quad d_1 = 52 \text{ mm} \]

\[ I_{re} = \frac{\pi}{32}(d_2^4 - d_1^4) = 4.652 \times 10^6 \text{ mm}^4 \]

Shear formula for hollow pipe:

\[ \tau_{\text{max}} = \frac{T\left(\frac{d_2}{2}\right)}{I_{re}} \]

Shear formula for solid shaft:

\[ \tau_{\text{max}} = \frac{T\left(\frac{d}{2}\right)}{\frac{\pi}{32}d^4} \quad \text{simplify} \rightarrow \frac{16 \cdot T}{\pi d^3} \]

Equate and solve for \( d \) of solid shaft:

\[ d = \left(\frac{32 \cdot I_{re}}{\pi d_2^4}\right)^{\frac{1}{3}} = 82.0 \text{ mm} \]

A-3.7: A stepped steel shaft with diameters \( d_1 = 56 \text{ mm} \) and \( d_2 = 52 \text{ mm} \) is twisted by torques \( T_1 = 3.5 \text{ kN} \cdot \text{m} \) and \( T_2 = 1.5 \text{ kN} \cdot \text{m} \) acting in opposite directions. The maximum shear stress is approximately:

(A) 54 MPa
(B) 58 MPa
(C) 62 MPa
(D) 79 MPa

Solution

\[ d_1 = 56 \text{ mm} \quad d_2 = 52 \text{ mm} \]

\[ T_1 = 3.5 \text{ kN} \cdot \text{m} \quad T_2 = 1.5 \text{ kN} \cdot \text{m} \]
Polar moments of inertia:

\[ I_{p1} = \frac{\pi}{32} d_1^4 = 9.655 \times 10^3 \text{mm}^4 \]

\[ I_{p2} = \frac{\pi}{32} d_2^4 = 7.178 \times 10^3 \text{mm}^4 \]

Shear formula - max. shear stresses in segments 1 & 2:

\[
\tau_{\text{max}1} = \frac{(T_1 - T_2) \frac{d_1}{2}}{I_{p1}} = 58.0 \text{MPa} \quad \tau_{\text{max}2} = \frac{T_2 \left( \frac{d_2}{2} \right)}{I_{p2}} = 54.3 \text{MPa}
\]

A-3.8: A stepped steel shaft \((G = 75 \text{ GPa})\) with diameters \(d_1 = 36 \text{ mm}\) and \(d_2 = 32 \text{ mm}\) is twisted by torques \(T\) at each end. Segment lengths are \(L_1 = 0.9 \text{ m}\) and \(L_2 = 0.75 \text{ m}\). If the allowable shear stress is 28 MPa and maximum allowable twist is 1.8 degrees, the maximum permissible torque is approximately:

(A) 142 N·m  
(B) 180 N·m  
(C) 185 N·m  
(D) 257 N·m

### Solution

**d_1 = 36 \text{ mm}**  
**d_2 = 32 \text{ mm}**  
**G = 75 \text{ GPa}**  
**\(\tau_a = 28 \text{ MPa}\)**  
**L_1 = 0.9 \text{ m} \quad L_2 = 0.75 \text{ m}**  
**\(\phi_a = 1.8 \text{ deg}\)**

Polar moments of inertia:

\[ I_{p1} = \frac{\pi}{32} d_1^4 = 1.649 \times 10^3 \text{mm}^4 \]

\[ I_{p2} = \frac{\pi}{32} d_2^4 = 1.029 \times 10^3 \text{mm}^4 \]

Max torque based on allowable shear stress - use shear formula:

\[ T_{\text{max}1} = \tau_a \left( \frac{2 \cdot I_{p1}}{d_1} \right) = 257 \text{ N·m} \]

\[ T_{\text{max}2} = \tau_a \left( \frac{2 \cdot I_{p2}}{d_2} \right) = 180 \text{ N·m < controls} \]
Max. torque based on max. rotation & torque-displacement relation:
\[
\phi = \frac{T}{G \left( \frac{L_1}{I_{pl1}} + \frac{L_2}{I_{pl2}} \right)}
\]
\[
T_{\text{max}} = \frac{G \cdot \phi_a}{\left( \frac{L_1}{I_{pl1}} + \frac{L_2}{I_{pl2}} \right)} = 185 \cdot \text{N} \cdot \text{m}
\]

A-3.9: A gear shaft transmits torques \( T_A = 975 \text{ N} \cdot \text{m}, \ T_B = 1500 \text{ N} \cdot \text{m}, \ T_C = 650 \text{ N} \cdot \text{m} \) and \( T_D = 825 \text{ N} \cdot \text{m} \). If the allowable shear stress is 50 MPa, the required shaft diameter is approximately:
(A) 38 mm
(B) 44 mm
(C) 46 mm
(D) 48 mm

Solution
\[\tau_a = 50 \cdot \text{MPa}\]
\[T_A = 975 \cdot \text{N} \cdot \text{m}\]
\[T_B = 1500 \cdot \text{N} \cdot \text{m}\]
\[T_C = 650 \cdot \text{N} \cdot \text{m}\]
\[T_D = 825 \cdot \text{N} \cdot \text{m}\]

Find torque in each segment of shaft:
\[T_{AB} = T_A = 975.0 \cdot \text{N} \cdot \text{m}\]
\[T_{BC} = T_A - T_B = -525.0 \cdot \text{N} \cdot \text{m}\]
\[T_{CD} = T_D = 825.0 \cdot \text{N} \cdot \text{m}\]

Shear formula:
\[\tau = \frac{T}{\frac{\pi}{32} d^4} \text{ simplify } \rightarrow \frac{16 \cdot T}{\pi \cdot d^4}\]

Set \( \tau \) to \( \tau_{\text{allowable}} \) and \( T \) to torque in each segment; solve for required diameter \( d \) (largest controls)

Segment AB: \[d = \left( \frac{16 \cdot |T_{\text{allowable}}|}{\pi \cdot \tau_a} \right)^{\frac{1}{3}} = 46.3 \cdot \text{mm}
\]

Segment BC: \[d = \left( \frac{16 \cdot |T_{BC}|}{\pi \cdot \tau_a} \right)^{\frac{1}{3}} = 37.7 \cdot \text{mm}
\]

Segment CD: \[d = \left( \frac{16 \cdot |T_{CD}|}{\pi \cdot \tau_a} \right)^{\frac{1}{3}} = 43.8 \cdot \text{mm}
\]
A-3.10: A hollow aluminum shaft ($G = 27$ GPa, $d_2 = 96$ mm, $d_1 = 52$ mm) has an angle of twist per unit length of $1.8/\text{m}$ due to torques $T$. The resulting maximum tensile stress in the shaft is approximately:

(A) 38 MPa  
(B) 41 MPa  
(C) 49 MPa  
(D) 58 MPa

Solution

\[ G = 27 \text{ GPa} \]
\[ d_2 = 96 \text{ mm} \]
\[ d_1 = 52 \text{ mm} \]
\[ \theta = 1.8 \text{ deg/m} \]

Max. shear strain due to twist per unit length:

\[ \gamma_{\text{max}} = \left( \frac{d_2}{2} \right) \cdot \theta = 1.508 \times 10^{-3} \text{ radians} \]

Max. shear stress: \[ \tau_{\text{max}} = G \cdot \gamma_{\text{max}} = 40.7 \text{ MPa} \]

Max. tensile stress on plane at 45 degrees & equal to max. shear stress:

\[ \sigma_{\text{max}} = \tau_{\text{max}} = 40.7 \text{ MPa} \]

A-3.11: Torques $T = 5.7 \text{ kN-m}$ are applied to a hollow aluminum shaft ($G = 27$ GPa, $d_1 = 52$ mm). The allowable shear stress is 45 MPa and the allowable normal strain is $8.0 \times 10^{-4}$. The required outside diameter $d_2$ of the shaft is approximately:

(A) 38 mm  
(B) 56 mm  
(C) 87 mm  
(D) 91 mm

Solution

\[ T = 5.7 \text{ kN-m} \quad G = 27 \text{ GPa} \quad d_1 = 52 \text{ mm} \]
\[ \tau_{a1} = 45 \text{ MPa} \quad e_a = 8.0 \times 10^{-4} \]

Allowable shear strain based on allowable normal strain for pure shear

\[ \gamma_a = 2 \cdot e_a = 1.600 \times 10^{-3} \quad \text{so resulting allow. shear stress is:} \]
\[ \tau_{a2} = G \cdot \gamma_a = 43.2 \text{ MPa} \]
So allowable shear stress based on normal strain governs $\tau_a = \tau_{a2}$.

Use torsion formula to relate required $d_2$ to allowable shear stress:

$$\tau_{\text{max}} = \frac{T}{\pi d_2^2} \frac{d_2}{2}$$

so rearrange equation to get

$$d_2^4 - d_1^4 = \frac{16}{\pi \tau_a} T d_2$$

Solve resulting 4th order equation numerically, or use a calculator and trial & error

$T = 5700000$ N-mm $d_1 = 52$ $\tau_a = 43.2$

$f(d_2) = d_2^4 - \left(\frac{16 T}{\pi \tau_a}\right) d_2 - d_1^4$ gives $d_2 = 91$ mm

A-3.12: A motor drives a shaft with diameter $d = 46$ mm at $f = 5.25$ Hz and delivers $P = 25$ kW of power. The maximum shear stress in the shaft is approximately:

(A) 32 MPa
(B) 40 MPa
(C) 83 MPa
(D) 91 MPa

Solution

$$f = 5.25 \text{ Hz} \quad d = 46 \text{ mm}$$

$$P = 25 \text{ kW}$$

$$I_p = \frac{\pi}{32} d^4 = 4.396 \times 10^5 \text{ mm}^4$$

Power in terms of torque $T$:

$$P = \frac{\pi f}{2} T$$

Solve for torque $T$:

$$T = \frac{P}{2 \pi f} = 757.9 \text{ N-m}$$

Max. shear stress using torsion formula:

$$\tau_{\text{max}} = \frac{T d_2}{I_p} = 39.7 \text{ MPa}$$

A-3.13: A motor drives a shaft at $f = 10$ Hz and delivers $P = 35$ kW of power. The allowable shear stress in the shaft is 45 MPa. The minimum diameter of the shaft is approximately:

(A) 35 mm
(B) 40 mm
(C) 47 mm
(D) 61 mm
Solution

\[ f = 10\text{ Hz} \quad P = 35\text{ kW} \]
\[ \tau_a = 45\text{ MPa} \]

Power in terms of torque \( T \):
\[ P = 2\pi f T \]

Solve for torque \( T \):
\[ T = \frac{P}{2\pi f} = 557.0\text{ N\cdot m} \]

Shear formula:
\[ \tau = \frac{T \left( \frac{d}{2} \right)}{\pi d^4} \quad \text{or} \quad \tau = \frac{16T}{\pi d^2} \]

Solve for diameter \( d \):
\[ d = \left( \frac{16T}{\pi \tau_a} \right)^{\frac{1}{3}} = 39.8\text{ mm} \]

A-3.14: A drive shaft running at 2500 rpm has outer diameter 60 mm and inner diameter 40 mm. The allowable shear stress in the shaft is 35 MPa. The maximum power that can be transmitted is approximately:
(A) 220 kW
(B) 240 kW
(C) 288 kW
(D) 312 kW

Solution

\[ n = 2500 \text{ rpm} \]
\[ \tau_a = 35\text{ MPa} = 35 \times 10^6 \text{ N/m}^2 \]
\[ d_2 = 0.060 \text{ m} \]
\[ d_1 = 0.040 \text{ m} \]
\[ I_p = \frac{\pi}{32} (d_2^4 - d_1^4) = 1.021 \times 10^{-6} \text{ m}^4 \]

Shear formula:
\[ \tau = \frac{T \left( \frac{d_2}{2} \right)}{I_p} \quad \text{or} \quad T_{\text{max}} = \frac{2\tau_a I_p}{d_2} = 1191.2 \text{ N\cdot m} \]

Power in terms of torque \( T \):
\[ P = 2\pi f T = 2\pi (n/60) T = (\pi n/30) T \]
\[ P_{\text{max}} = \frac{2\pi n}{60} T_{\text{max}} = 3.119 \times 10^5 \text{ W} \quad P_{\text{max}} = 312 \text{ kW} \]
A-4.1: A simply supported beam with proportional loading \( P = 4.1 \text{kN} \) has span length \( L = 5 \text{ m} \). Load \( P \) is 1.2 m from support \( A \) and load \( 2P \) is 1.5 m from support \( B \). The bending moment just left of load \( 2P \) is approximately:

(A) 5.7 kN·m  
(B) 6.2 kN·m  
(C) 9.1 kN·m  
(D) 10.1 kN·m

Solution

\[ a = 1.2 \text{ m} \quad b = 2.3 \text{ m} \quad c = 1.5 \text{ m} \]

\[ L = a + b + c = 5.00 \text{ m} \]

\[ P = 4.1 \text{ kN} \]

Statics to find reaction force at \( B \):

\[ R_B = \frac{1}{L} \left[ P \cdot a + 2 \cdot P \cdot (a + b) \right] = 6.724 \text{ kN} \]

Moment just left of load \( 2P \):

\[ M = R_Bc = 10.1 \text{ kN·m} \quad \text{< compression on top of beam} \]

A-4.2: A simply-supported beam is loaded as shown in the figure. The bending moment at point \( C \) is approximately:

(A) 5.7 kN·m  
(B) 6.1 kN·m  
(C) 6.8 kN·m  
(D) 9.7 kN·m

Solution

Statics to find reaction force at \( A \):

\[ R_A = \frac{1}{5 \cdot \text{m}} \left[ 1.8 \cdot \text{kN/m} \cdot \frac{(3 \text{ m} - 0.5 \text{ m})^2}{2} + 7.5 \text{kN} \cdot (3 \text{ m} + 1 \text{ m}) \right] = 7.125 \text{kN} \]

Moment at point \( C \), 2 m from \( A \):

\[ M = R_A \cdot (2 \text{ m}) - 7.5 \text{kN} \cdot (1.0 \text{ m}) = 6.75 \text{ kN·m} \quad \text{< compression on top of beam} \]
A-4.3: A cantilever beam is loaded as shown in the figure. The bending moment at 0.5 m from the support is approximately:

(A) 12.7 kN·m
(B) 14.2 kN·m
(C) 16.1 kN·m
(D) 18.5 kN·m

Solution

Cut beam at 0.5 m from support; use statics and right-hand FBD to find internal moment at that point

\[ M = 0.5 \cdot m \cdot (4.5 \text{ kN}) + \left( 0.5 \cdot m + 1.0 \cdot m + \frac{3.0 \cdot m}{2} \right) \cdot 1.8 \text{ kN/m} \cdot (3.0 \cdot m) \]

\[ = 18.5 \text{ kN·m} \quad \text{(tension on top of beam)} \]

A-4.4: An L-shaped beam is loaded as shown in the figure. The bending moment at the midpoint of span AB is approximately:

(A) 6.8 kN·m
(B) 10.1 kN·m
(C) 12.3 kN·m
(D) 15.5 kN·m

Solution

Use statics to find reaction at B; sum moments about A

\[ R_B = \frac{1}{5 \cdot m} \left[ 9 \text{ kN} \cdot (6 \cdot m) - 4.5 \text{ kN} \cdot (1 \cdot m) \right] = 9.90 \text{ kN} \]
Cut beam at midpoint of $AB$; use right hand FBD, sum moments

$$M = R_b \left( \frac{5 \cdot m}{2} \right) - 9 \cdot kN \left( \frac{5 \cdot m}{2} + 1 \cdot m \right) = 6.75 \cdot kN \cdot m \quad \text{< tension on top of beam}
$$

**A-4.5:** A T-shaped simple beam has a cable with force $P$ anchored at $B$ and passing over a pulley at $E$ as shown in the figure. The bending moment just left of $C$ is $1.25 \; kN \cdot m$. The cable force $P$ is approximately:

(A) 2.7 kN  
(B) 3.9 kN  
(C) 4.5 kN  
(D) 6.2 kN

**Solution**

$$M_C = 1.25 \; kN \cdot m$$

Sum moments about $D$ to find vertical reaction at $A$:

$$V_A = -\frac{1}{7} \cdot m \cdot [P \cdot (4 \cdot m)]$$

$$V_A = -\frac{4}{7} \cdot P \quad \text{(downward)}$$

Now cut beam & cable just left of $CE$ & use left FBD; show $V_A$ downward & show vertical cable force component of $(4/5)P$ upward at $B$; sum moments at $C$ to get $M_C$ and equate to given numerical value of $M_C$ to find $P$:

$$M_C = \frac{4}{5} \cdot P \cdot (3) + V_A \cdot (2 + 3)$$

$$M_C = \frac{4}{5} \cdot P \cdot (3) + \left( -\frac{4}{7} \cdot P \right) \cdot (2 + 3)$$

Simplify:

$$M_C = \frac{16 \cdot P}{35}$$

Solve for $P$:

$$P = \frac{35}{16} (1.25) = 2.73 \; kN$$

**A-4.6:** A simple beam ($L = 9 \; m$) with attached bracket $BDE$ has force $P = 5 \; kN$ applied downward at $E$. The bending moment just right of $B$ is approximately:

(A) 6 $kN \cdot m$  
(B) 10 $kN \cdot m$  
(C) 19 $kN \cdot m$  
(D) 22 $kN \cdot m$
Solution

Sum moments about A to find reaction at C:

\[ R_C = \frac{1}{L} \left[ p \left( \frac{L}{6} + \frac{L}{3} \right) \right] \rightarrow \frac{P}{2} \]

Cut through beam just right of B, then use FBD of BC to find moment at B:

\[ M_B = R_C \left( \frac{L}{2} \right) \left( \frac{L}{3} \right) \rightarrow \frac{5 \cdot L \cdot P}{12} \]

Substitute numbers for L and P:

\[ L = 9 \text{ m} \quad P = 5 \text{ kN} \]

\[ M_B \rightarrow \frac{5 \cdot L \cdot P}{12} = 18.8 \text{ kN\cdot m} \]

A-4.7: A simple beam AB with an overhang BC is loaded as shown in the figure. The bending moment at the midspan of AB is approximately:

(A) 8 kN\cdot m
(B) 12 kN\cdot m
(C) 17 kN\cdot m
(D) 21 kN\cdot m

Solution

Sum moments about B to get reaction at A:

\[ R_A = \frac{1}{3.2} \left[ 15 \cdot (1.6) \left( 1.6 + \frac{1.6}{2} \right) + 4.5 \right] \rightarrow 19.40625 \text{ kN} \]

Cut beam at midspan, use left FBD & sum moments to find moment at midspan:

\[ M_{\text{midspan}} = R_A \cdot (1.6) - 15 \cdot (1.6) \left( \frac{1.6}{2} \right) \rightarrow 11.85 \text{ kN}\cdot \text{m} \]
A-5.1: A copper wire \((d = 1.5\, \text{mm})\) is bent around a tube of radius \(R = 0.6\, \text{m}\). The maximum normal strain in the wire is approximately:

(A) \(1.25 \times 10^{-3}\)

(B) \(1.55 \times 10^{-3}\)

(C) \(1.76 \times 10^{-3}\)

(D) \(1.92 \times 10^{-3}\)

**Solution**

\[
e_{\text{max}} = \frac{d}{2R + \frac{d}{2}} \rightarrow \frac{d}{2\left(R + \frac{d}{2}\right)}
\]

\(d = 1.5\, \text{mm} \quad R = 0.6\, \text{m}\)

\[e_{\text{max}} = \frac{d}{2\left(R + \frac{d}{2}\right)} = 1.248 \times 10^{-3}\]

A-5.2: A simply supported wood beam \((L = 5\, \text{m})\) with rectangular cross section \((b = 200\, \text{mm}, h = 280\, \text{mm})\) carries uniform load \(q = 6.5\, \text{kN/m}\) which includes the weight of the beam. The maximum flexural stress is approximately:

(A) 8.7 MPa

(B) 10.1 MPa

(C) 11.4 MPa

(D) 14.3 MPa

**Solution**

\(L = 5\, \text{m} \quad b = 200\, \text{mm} \quad h = 280\, \text{mm}\)

\(q = 9.5\, \frac{\text{kN}}{\text{m}}\)

Section modulus:

\[S = \frac{bh^2}{6} = 2.613 \times 10^6\, \text{m}^3\]

Max. moment at midspan:

\[M_{\text{max}} = \frac{qL^2}{8} = 29.7\, \text{kN} \cdot \text{m}\]

Max. flexural stress at midspan:

\[\sigma_{\text{max}} = \frac{M_{\text{max}}}{S} = 11.4\, \text{MPa}\]
A-5.3: A cast iron pipe \((L = 12 \text{ m}, \text{ weight density} = 72 \text{ kN/m}^3, d_2 = 100 \text{ mm}, d_1 = 75 \text{ mm})\) is lifted by a hoist. The lift points are 6 m apart. The maximum bending stress in the pipe is approximately:

(A) 28 MPa  
(B) 33 MPa  
(C) 47 MPa  
(D) 59 MPa

Solution

\[ L = 12 \cdot \text{m} \quad s = 4 \cdot \text{m} \quad d_2 = 100 \cdot \text{mm} \quad d_1 = 75 \cdot \text{mm} \quad \gamma_{CT} = 72 \frac{\text{kN}}{\text{m}^3} \]

Pipe cross sectional properties:

\[ A = \frac{\pi}{4}(d_2^2 - d_1^2) = 3436 \cdot \text{mm}^2 \quad I = \frac{\pi}{64}(d_2^4 - d_1^4) = 3.356 \times 10^6 \cdot \text{mm}^4 \]

Uniformly distributed weight of pipe, \(q\):

\[ q = \gamma_{CT} \cdot A = 0.247 \frac{\text{kN}}{\text{m}} \]

Vertical force at each lift point:

\[ F = \frac{q \cdot L}{2} = 1.484 \cdot \text{kN} \]

Max. moment is either at lift points \((M_1)\) or at midspan \((M_2)\):

\[ M_1 = -q \left( \frac{L - s}{2} \right) \left( \frac{L - s}{2} \right) = -3.958 \cdot \text{kN} \cdot \text{m} \quad < \text{controls, tension on top} \]

\[ M_2 = F \cdot \frac{s}{2} - q \cdot \frac{L}{2} \left( \frac{L}{4} \right) = -1.484 \cdot \text{kN} \cdot \text{m} \quad < \text{tension on top} \]

Max. bending stress at lift point:

\[ \sigma_{\text{max}} = \frac{|M_1| \cdot \left( \frac{d_2}{2} \right)}{I} = 59.0 \cdot \text{MPa} \]

A-5.4: A beam with an overhang is loaded by a uniform load of 3 kN/m over its entire length. Moment of inertia \(I_1 = 3.36 \times 10^6 \text{ mm}^4\) and distances to top and bottom of the beam cross section are 20 mm and 66.4 mm, respectively. It is
known that reactions at $A$ and $B$ are 4.5 kN and 13.5 kN, respectively. The maximum bending stress in the beam is approximately:

(A) 36 MPa  
(B) 67 MPa  
(C) 102 MPa  
(D) 119 MPa

Solution

![Beam diagram](image)

$R_A = 4.5\text{ kN}$  
$I_c = 3.36\times10^6\text{ mm}^4$  
$q = 3\text{ kN/m}$

Location of max. positive moment in $AB$ (cut beam at location of zero shear & use left FBD):

$x_{\text{max}} = \frac{R_A}{q} \times 1.5\cdot\text{m}$  
$M_{\text{pos}} = R_A \cdot x_{\text{max}} - 3\frac{\text{kN} \cdot x_{\text{max}}^2}{\text{m}} = 3.375\times\text{kN}\cdot\text{m}$

< compression on top of beam

Compressive stress on top of beam at $x_{\text{max}}$:

$\sigma_{c1} = \frac{M_{\text{pos}} \cdot (20\cdot\text{mm})}{I_c} = 20.1\text{ MPa}$

Tensile stress at bottom of beam at $x_{\text{max}}$:

$\sigma_{t1} = \frac{M_{\text{pos}} \cdot (66.4\cdot\text{mm})}{I_c} = 66.696\text{ MPa}$

Max. negative moment at $B$ (use FBD of $BC$ to find moment; compression on bottom of beam):

$M_{\text{neg}} = \left(3\frac{\text{kN}}{\text{m}}\right) \times \frac{(2\cdot\text{m})^2}{2} = 6.000\times\text{kN}\cdot\text{m}$

$\sigma_{c2} = \frac{M_{\text{neg}} \cdot (66.4\cdot\text{mm})}{I_c} = 118.6\text{ MPa}$

$\sigma_{t2} = \frac{M_{\text{neg}} \cdot (20\cdot\text{mm})}{I_c} = 35.7\text{ MPa}$
A-5.5: A steel hanger with solid cross section has horizontal force $P = 5.5\, \text{kN}$ applied at free end $D$. Dimension variable $b = 175\, \text{mm}$ and allowable normal stress is $150\, \text{MPa}$. Neglect self weight of the hanger. The required diameter of the hanger is approximately:

(A) 5 cm  
(B) 7 cm  
(C) 10 cm  
(D) 13 cm

Solution

$P = 5.5\, \text{kN} \quad b = 175\, \text{mm} \quad \sigma_a = 150\, \text{MPa}$

Reactions at support:

$N_A = P$ (leftward)  
$M_A = P\cdot(2-b) = 1.9\, \text{kN}\cdot\text{m}$ (tension on bottom)

Max. normal stress at bottom of cross section at $A$:

$$\sigma_{\text{max}} = \frac{P}{\left(\frac{\pi\cdot d^2}{4}\right)} + \frac{(2\cdot P\cdot b)\left(\frac{d}{2}\right)}{\left(\frac{\pi\cdot d^4}{64}\right)} \quad \sigma_{\text{max}} = \frac{4\cdot P\cdot(16\cdot b + d)}{\pi\cdot d^3}$$

Set $\sigma_{\text{max}} = \sigma_a$ and solve for required diameter $d$:

$$(\pi\cdot \sigma_a)\cdot d^3 - (4\cdot P)\cdot d - 64\cdot P\cdot b = 0 \quad \text{solve numerically or by trial & error to find} \quad d_{\text{reqd}} = 5.11\, \text{cm}$$

A-5.6: A cantilever wood pole carries force $P = 300\, \text{N}$ applied at its free end, as well as its own weight (weight density $= 6\, \text{kN/m}^3$). The length of the pole is $L = 0.75\, \text{m}$ and the allowable bending stress is $14\, \text{MPa}$. The required diameter of the pole is approximately:

(A) 4.2 cm  
(B) 5.5 cm  
(C) 6.1 cm  
(D) 8.5 cm

Solution

$P = 300\, \text{N} \quad L = 0.75\, \text{m} \quad \sigma_a = 14\, \text{MPa} \quad \gamma_w = 6\, \frac{\text{kN}}{\text{m}}$
Uniformly distributed weight of pole:
\[ w = \gamma_w \left( \frac{\pi \cdot d^4}{4} \right) \]

Max. moment at support:
\[ M_{\text{max}} = P \cdot L + w \cdot L \cdot \frac{L}{2} \]

Section modulus of pole cross section:
\[ S = \frac{I}{(d/2)^2} \quad S = \frac{64}{(d/2)^2} \rightarrow \frac{\pi \cdot d^4}{32} \]

Set \( M_{\text{max}} \) equal to \( \sigma_a \times S \) and solve for required min. diameter \( d \):
\[ P \cdot L + \left[ \gamma_w \left( \frac{\pi \cdot d^4}{4} \right) \right] \cdot L \cdot \frac{L}{2} - \sigma_a \left( \frac{\pi \cdot d^4}{32} \right) = 0 \]

Or
\[ \left( \frac{\pi \cdot \sigma_a}{32} \right) \cdot d^4 - \left( \frac{\pi \cdot \gamma_w \cdot L^2}{8} \right) \cdot d^2 - P \cdot L = 0 \quad < \text{solve numerically or by trial} \]

\( d_{\text{reqd}} = 5.50 \text{ cm} \)

Since wood pole is light, try simpler solution which ignores self weight:
\[ P \cdot L = \sigma_a \cdot S \quad \text{Or} \quad \left( \frac{\pi \cdot \sigma_a}{32} \right) \cdot d^4 = P \cdot L \]
\[ d_{\text{reqd}} = \left[ P \cdot L \cdot \left( \frac{32}{\pi \cdot \sigma_a} \right) \right]^{1/4} = 5.47 \cdot \text{cm} \]

A-5.7: A simply supported steel beam of length \( L = 1.5 \text{ m} \) and rectangular cross section \((h = 75 \text{ mm}, b = 20 \text{ mm})\) carries a uniform load of \( q = 48 \text{ kN/m} \), which includes its own weight. The maximum transverse shear stress on the cross section at 0.25 m from the left support is approximately:

(A) 20 MPa
(B) 24 MPa
(C) 30 MPa
(D) 36 MPa

Solution
\[ L = 1.5 \cdot \text{m} \quad q = 48 \frac{\text{kN}}{\text{m}} \]
\[ h = 75 \cdot \text{mm} \quad b = 20 \cdot \text{mm} \]
Cross section properties:
\[ A = b \cdot h = 1500 \text{ mm}^2 \]
\[ Q = \left( b \cdot \frac{h}{2} \right) \frac{h}{4} = 14062 \text{ mm}^3 \]
\[ I = \frac{b \cdot h^3}{12} = 7.031 \times 10^5 \text{ mm}^4 \]

Support reactions:
\[ R = \frac{q \cdot L}{2} = 36.0 \text{ kN} \]

Transverse shear force at 0.25 m from support:
\[ V_{0.25} = R - q \cdot (0.25 \text{ m}) = 24.0 \text{ kN} \]

Max. shear stress at NA at 0.25 m from support:
\[ \tau_{\text{max}} = \frac{V_{0.25} \cdot Q}{I \cdot b} = 24.0 \cdot \text{MPa} \]
Or more simply . . .
\[ \tau_{\text{max}} = \frac{3 \cdot V_{0.25}}{2 \cdot A} = 24.0 \cdot \text{MPa} \]

A-5.8: A simply supported laminated beam of length \( L = 0.5 \text{ m} \) and square cross section weighs 4.8 N. Three strips are glued together to form the beam, with the allowable shear stress in the glued joint equal to 0.3 MPa. Considering also the weight of the beam, the maximum load \( P \) that can be applied at \( L/3 \) from the left support is approximately:
(A) 240 N
(B) 360 N
(C) 434 N
(D) 510 N

Solution

\[ L = 0.5 \text{ m} \quad W = 4.8 \text{ N} \quad q = \frac{W}{L} = 9.60 \frac{\text{N}}{\text{m}} \]
\[ h = 36 \text{ mm} \quad b = 36 \text{ mm} \quad \tau_a = 0.3 \text{ MPa} \]
Cross section properties:

\[ A = b \cdot h = 1296 \text{ mm}^2 \]
\[ I = \frac{b \cdot h^3}{12} = 1.400 \times 10^6 \text{ mm}^4 \]

Max. shear force at left support:

\[ V_{max} = \frac{q \cdot L}{2} + P \left( \frac{2}{3} \right) \]

Shear stress on glued joint at left support; set \( \tau = \tau_a \) then solve for \( P_{max} \):

\[ \tau = \frac{V_{max} \cdot Q_{joint}}{I \cdot b} \quad \text{Or} \quad \tau = \frac{V_{max} (b \cdot h^2)}{9} \cdot b \quad \text{Or} \quad \tau_a = \frac{4 \cdot V_{max}}{3 \cdot b \cdot h} \]

\[ \tau_a = \frac{4}{3 \cdot b \cdot h} \left[ \frac{q \cdot L}{2} + P \left( \frac{2}{3} \right) \right] \quad \text{so for } \tau_a = 0.3 \text{ MPa} \]

\[ P_{max} = \frac{3}{2} \left( \frac{3 \cdot b \cdot h \cdot \tau_a}{4} - \frac{q \cdot L}{2} \right) = 434 \text{ N} \]

**A-5.9:** An aluminum cantilever beam of length \( L = 0.65 \text{ m} \) carries a distributed load, which includes its own weight, of intensity \( q/2 \) at \( A \) and \( q \) at \( B \). The beam cross section has width 50 mm and height 170 mm. Allowable bending stress is 95 MPa and allowable shear stress is 12 MPa. The permissible value of load intensity \( q \) is approximately:

(A) 110 kN/m
(B) 122 kN/m
(C) 130 kN/m
(D) 139 kN/m

**Solution**

\[ L = 0.65 \text{ m} \quad b = 50 \text{ mm} \quad h = 170 \text{ mm} \quad \frac{q}{2} \]

\( \sigma_a = 95 \text{ MPa} \quad \tau_a = 12 \text{ MPa} \)

Cross section properties:

\[ A = b \cdot h = 8500 \text{ mm}^2 \]
\[ I = \frac{b \cdot h^3}{12} = 2.047 \times 10^6 \text{ mm}^4 \quad S = \frac{b \cdot h^2}{6} = 2.408 \times 10^5 \text{ mm}^3 \]

Reaction force and moment at \( A \):

\[ R_A = \frac{1}{2} \left( \frac{q}{2} + q \right) \cdot L \quad R_A = \frac{3}{4} q \cdot L \quad M_A = \frac{q}{2} \cdot L \cdot \frac{L}{2} + \frac{1}{2} \cdot \frac{q}{2} \cdot L \cdot \frac{2 \cdot L}{3} \]
\[ M_A = \frac{5}{12} q \cdot L^2 \]
Compare max. permissible values of \( q \) based on shear and moment allowable stresses; smaller value controls

\[
\tau_{\text{max}} = \frac{3}{2} \frac{R_A}{A} \quad \tau_a = \frac{3}{2} \left( \frac{3}{4} \cdot \frac{q \cdot L}{A} \right) \quad \text{So, since } \tau_a = 12 \text{ MPa}
\]

\[
q_{\text{max}1} = \frac{8}{9} \frac{\tau_a \cdot A}{L} = 139 \text{ kN/m}
\]

\[
\sigma_{\text{max}} = \frac{M_A}{S} \quad \sigma_a = \frac{5}{12} \frac{q \cdot L^2}{S} \quad \text{So, since } \sigma_a = 95 \text{ MPa}
\]

\[
q_{\text{max}2} = \frac{12}{5} \frac{\sigma_a \cdot S}{L^2} = 130.0 \text{ kN/m}
\]

A-5.10: An aluminum light pole weighs 4300 N and supports an arm of weight 700 N, with arm center of gravity at 1.2 m left of the centroidal axis of the pole. A wind force of 1500 N acts to the right at 7.5 m above the base. The pole cross section at the base has outside diameter 235 mm and thickness 20 mm. The maximum compressive stress at the base is approximately:

(A) 16 MPa
(B) 18 MPa
(C) 21 MPa
(D) 24 MPa

Solution

\[ H = 7.5 \text{ m} \quad B = 1.2 \text{ m} \]

\[ W_1 = 4300 \text{ N} \quad W_2 = 700 \text{ N} \]

\[ P_1 = 1500 \text{ N} \]

\[ d_2 = 235 \text{ mm} \quad t = 20 \text{ mm} \]

\[ d_1 = d_2 - 2 \cdot t = 195 \text{ mm} \]

Pole cross sectional properties at base:

\[ A = \frac{\pi}{4} (d_2^2 - d_1^2) = 13509 \text{ mm}^2 \]

\[ I = \frac{\pi}{64} (d_2^4 - d_1^4) = 7.873 \times 10^7 \text{ mm}^4 \]

Compressive (downward) force at base of pole:

\[ N = W_1 + W_2 = 5.0 \text{ kN} \]

Bending moment at base of pole:

\[ M = W_2 \cdot B - P_1 \cdot H = -10.410 \text{ kN.m} \quad \text{< results in compression at right} \]
Compressive stress at right side at base of pole:

\[
\sigma_c = \frac{N}{A} + \frac{|M| \left( \frac{d^2}{2} \right)}{I} = 15.9 \text{ MPa}
\]

**A-5.11:** Two thin cables, each having diameter \( d = \frac{t}{6} \) and carrying tensile loads \( P \), are bolted to the top of a rectangular steel block with cross section dimensions \( b \times t \). The ratio of the maximum tensile to compressive stress in the block due to loads \( P \) is:

(A) 1.5  
(B) 1.8  
(C) 2.0  
(D) 2.5

**Solution**

Cross section properties of block:

\[
A = b \times t \\ I = \frac{b \times t^3}{12} \\ d = \frac{t}{6}
\]

Tensile stress at top of block:

\[
\sigma_t = \frac{P}{A} + \frac{P \left( \frac{d}{2} + \frac{t}{2} \right) \left( \frac{t}{2} \right)}{I} = \frac{9 \cdot P}{2 \cdot b \cdot t}
\]

Compressive stress at bottom of block:

\[
\sigma_c = \frac{P}{A} - \frac{P \left( \frac{d}{2} + \frac{t}{2} \right) \left( \frac{t}{2} \right)}{I} = -\frac{5 \cdot P}{2 \cdot b \cdot t}
\]

Ratio of max. tensile to compressive stress in block:

\[
\text{ratio} = \left| \frac{\sigma_t}{\sigma_c} \right| = \frac{9}{5} = 1.8
\]

**A-5.12:** A composite beam is made up of a 200 mm \( \times \) 300 mm core \((E_c = 14 \text{ GPa})\) and an exterior cover sheet \((300 \text{ mm} \times 12 \text{ mm}, E_e = 100 \text{ GPa})\) on each side. Allowable stresses in core and exterior sheets are 9.5 MPa and 140 MPa, respectively. The ratio of the maximum permissible bending moment about the \( z \)-axis to that about the \( y \)-axis is most nearly:

(A) 0.5  
(B) 0.7  
(C) 1.2  
(D) 1.5
Solution

\[ b = 200\text{-mm} \quad t = 12\text{-mm} \]

\[ h = 300\text{-mm} \]

\[ E_c = 14\text{-GPa} \quad E_e = 100\text{-GPa} \]

\[ \sigma_{ac} = 9.5\text{-MPa} \]

\[ \sigma_{ae} = 140\text{-MPa} \]

Composite beam is symmetric about both axes so each NA is an axis of symmetry.

Moments of inertia of cross section about \( z \) and \( y \) axes:

\[ I_{cz} = \frac{b \cdot h^3}{12} = 4.500 \times 10^8\text{mm}^4 \]

\[ I_{cy} = \frac{h \cdot b^3}{12} = 2.000 \times 10^8\text{mm}^4 \]

\[ I_{ez} = \frac{2 \cdot t \cdot h^3}{12} = 5.400 \times 10^7\text{mm}^4 \]

\[ I_{ey} = \frac{2 \cdot h \cdot t^3}{12} = 2 \cdot (t \cdot h) \left( \frac{b}{2} + \frac{t}{2} \right)^2 = 8.099 \times 10^7\text{mm}^4 \]

**Bending about \( z \) axis based on allowable stress in each material (lesser value controls)**

\[ M_{\text{max},cz} = \sigma_{ac} \frac{E_c \cdot I_{cz} + E_e \cdot I_{cz}}{b \cdot \frac{h}{2} \cdot E_c} = 52.9\text{-kN} \cdot \text{m} \]

\[ M_{\text{max},cz} = \sigma_{ae} \frac{E_c \cdot I_{cz} + E_e \cdot I_{cz}}{b \cdot \frac{h}{2} \cdot E_e} = 109.2\text{-kN} \cdot \text{m} \]

**Bending about \( y \) axis based on allowable stress in each material (lesser value controls)**

\[ M_{\text{max},cy} = \sigma_{ac} \frac{E_e \cdot I_{ey} + E_c \cdot I_{ey}}{b \cdot \frac{h}{2} \cdot E_e} = 360\text{-kN} \cdot \text{m} \]

\[ M_{\text{max},cy} = \sigma_{ae} \frac{E_e \cdot I_{ey} + E_c \cdot I_{ey}}{b \cdot \frac{h}{2} \cdot E_e} = 136.2\text{-kN} \cdot \text{m} \]

\[ \text{ratio}_{z\rightarrow y} = \frac{M_{\text{max},cz}}{M_{\text{max},cy}} = 0.72 \quad \rightarrow \text{ allowable stress in the core, not exterior cover sheet, controls moments about both axes} \]
A-5.13: A composite beam is made up of a 90 mm × 160 mm wood beam (\(E_w = 11\) GPa) and a steel bottom cover plate (90 mm × 8 mm, \(E_s = 190\) GPa). Allowable stresses in wood and steel are 6.5 MPa and 110 MPa, respectively. The allowable bending moment about the \(z\)-axis of the composite beam is most nearly:

(A) 2.9 kN·m  
(B) 3.5 kN·m  
(C) 4.3 kN·m  
(D) 9.9 kN·m

Solution

\[ b = 90\text{\,mm} \quad t = 8\text{\,mm} \]
\[ h = 160\text{\,mm} \]
\[ E_w = 11\text{\,GPa} \quad E_s = 190\text{\,GPa} \]
\[ \sigma_{aw} = 6.5\text{\,MPa} \]
\[ \sigma_{as} = 110\text{\,MPa} \]
\[ A_w = b\cdot h = 14400\text{\,mm}^2 \]
\[ A_s = b\cdot t = 720\text{\,mm}^2 \]

Locate NA (distance \(h_2\) above base) by summing 1st moments of EA about base of beam; then find \(h_1 = \) dist. from NA to top of beam:

\[
\begin{align*}
\sum x = 0 & \quad \frac{E_s \cdot A_s \cdot \frac{t}{2} + E_w \cdot A_w \cdot \left(t + \frac{h}{2}\right)}{E_s \cdot A_s + E_w \cdot A_w} = 49.07\text{\,mm} \\
h_2 & = h + t - h_2 = 118.93\text{\,mm} \\
h_1 & = h + t - h_2 = 118.93\text{\,mm}
\end{align*}
\]

Moments of inertia of wood and steel about NA:

\[
\begin{align*}
I_w & = \frac{b\cdot t^3}{12} + A_w \left(h_2 - \frac{t}{2}\right)^2 = 1.467 \times 10^6\text{\,mm}^4 \\
I_s & = \frac{b\cdot t^3}{12} + A_s \left(h_1 - \frac{h}{2}\right)^2 = 5.254 \times 10^7\text{\,mm}^4
\end{align*}
\]

Allowable moment about \(z\) axis based on allowable stress in each material (lesser value controls)

\[
\begin{align*}
M_{\text{max,w}} & = \frac{\sigma_{aw} \cdot (E_w \cdot I_w + E_s \cdot I_s)}{h_1 \cdot E_w} = 4.26\text{\,kN·m} \\
M_{\text{max,s}} & = \frac{\sigma_{as} \cdot (E_w \cdot I_w + E_s \cdot I_s)}{h_2 \cdot E_s} = 10.11\text{\,kN·m}
\end{align*}
\]
A-5.14: A steel pipe \((d_3 = 104 \text{ mm}, d_2 = 96 \text{ mm})\) has a plastic liner with inner diameter \(d_1 = 82 \text{ mm}\). The modulus of elasticity of the steel is 75 times that of the modulus of the plastic. Allowable stresses in steel and plastic are 40 MPa and 550 kPa, respectively. The allowable bending moment for the composite pipe is approximately:

(A) 1100 N·m  
(B) 1230 N·m  
(C) 1370 N·m  
(D) 1460 N·m

Solution

\[ d_3 = 104 \text{ mm} \quad d_2 = 96 \text{ mm} \quad d_1 = 82 \text{ mm} \]
\[ \sigma_{as} = 40 \text{ MPa} \]
\[ \sigma_{ap} = 550 \text{ kPa} \]

Cross section properties:

\[ A_s = \frac{\pi}{4}(d_3^2 - d_2^2) = 1256.6 \text{ mm}^2 \]
\[ A_p = \frac{\pi}{4}(d_3^2 - d_1^2) = 1957.2 \text{ mm}^2 \]
\[ I_s = \frac{\pi}{64}(d_3^4 - d_2^4) = 1.573 \times 10^6 \text{ mm}^4 \]
\[ I_p = \frac{\pi}{64}(d_3^4 - d_1^4) = 1.950 \times 10^6 \text{ mm}^4 \]

Due to symmetry, NA of composite beam is the \(z\) axis

Allowable moment about \(z\) axis based on allowable stress in each material (lesser value controls)

\[ M_{\text{max}, s} = \sigma_{as} \left( \frac{E_p}{E_s} \cdot I_p + \frac{E_s}{E_s} \cdot I_s \right) \left( \frac{d_1}{2} \right) \cdot E_s \]
\[ M_{\text{max}, p} = \sigma_{ap} \left( \frac{E_p}{E_p} \cdot I_p + \frac{E_s}{E_s} \cdot I_s \right) \left( \frac{d_2}{2} \right) \cdot E_p \]

Modular ratio: \( n = \frac{E_s}{E_p} \quad n = 75 \)

Divide through by \(E_p\) in moment expressions above

\[ M_{\text{max}, s} = \sigma_{as} \left( \frac{I_p + n \cdot I_s}{\left( \frac{d_1}{2} \right) \cdot n} \right) = 1230 \text{ N·m} \]
\[ M_{\text{max}, p} = \sigma_{ap} \left( \frac{I_p + n \cdot I_s}{\left( \frac{d_2}{2} \right)} \right) = 1370 \text{ N·m} \]
A-5.15: A bimetallic beam of aluminum \((E_a = 70 \text{ GPa})\) and copper \((E_c = 110 \text{ GPa})\) strips has width \(b = 25 \text{ mm}\); each strip has thickness \(t = 1.5 \text{ mm}\). A bending moment of \(1.75 \text{ N}\cdot\text{m}\) is applied about the \(z\) axis. The ratio of the maximum stress in the aluminum to that in the copper is approximately:

(A) 0.6  
(B) 0.8  
(C) 1.0  
(D) 1.5

Solution:

\[
\begin{align*}
A_a &= b \cdot t = 37.5 \text{ mm}^2 \\
A_c &= A_a = 37.500 \text{ mm}^2 \\
E_a &= 70 \text{ GPa} \\
E_c &= 110 \text{ GPa} \\
M &= 1.75 \text{ N}\cdot\text{m} \\
I_a &= b \cdot t^3/12 + A_a \left( h_2 - \frac{t}{2} \right)^2 = 38.542 \text{ mm}^4 \\
I_c &= b \cdot t^3/12 + A_c \left( h_1 - \frac{t}{2} \right)^2 = 19.792 \text{ mm}^4
\end{align*}
\]

Bending stresses in aluminum and copper:

\[
\sigma_a = \frac{M \cdot h_1 \cdot E_a}{E_c \cdot I_c + E_a \cdot I_a} = 41.9 \cdot \text{MPa} \quad \sigma_c = \frac{M \cdot h_2 \cdot E_c}{E_c \cdot I_c + E_a \cdot I_a} = 52.6 \cdot \text{MPa}
\]

Ratio of the stress in the aluminum to that of the copper: \(\sigma_a / \sigma_c = 0.795\)

A-5.16: A composite beam of aluminum \((E_a = 72 \text{ GPa})\) and steel \((E_s = 190 \text{ GPa})\) has width \(b = 25 \text{ mm}\) and heights \(h_a = 42 \text{ mm}\), \(h_s = 68 \text{ mm}\). A bending moment is applied about the \(z\) axis resulting in a maximum stress in the aluminum of \(55 \text{ MPa}\). The maximum stress in the steel is approximately:

(A) 86 MPa  
(B) 90 MPa  
(C) 94 MPa  
(D) 98 MPa

\[
\frac{h_2}{h_1} = 0.795
\]

\[
\frac{h_1}{h_2} = 1.333 \quad 2t = 3.000 \text{ mm}
\]
Solution

\[ b = 25\text{·mm} \quad h_a = 42\text{·mm} \quad h_s = 68\text{·mm} \]

\[ E_a = 72\text{·GPa} \quad E_s = 190\text{·GPa} \quad \sigma_a = 55\text{·MPa} \]

\[ A_a = b \cdot h_a = 1050.0\text{·mm}^2 \]

\[ A_s = b \cdot h_s = 1700.0\text{·mm}^2 \]

Locate NA (distance \( h_2 \) above base) by summing 1st moments of EA about base of beam; then find \( h_1 \) = dist. from NA to top of beam:

\[ h_2 = \frac{E_s \cdot A_s \cdot \left( h_a + \frac{h_s}{2} \right)}{E_a \cdot A_a + E_s \cdot A_s} = 44.43\text{·mm} \]

\[ h_1 = h_a + h_s - h_2 = 65.57\text{·mm} \]

\[ h_1 + h_2 = 110.00\text{·mm} \]

Moments of inertia of aluminum and steel parts about NA:

\[ I_a = \frac{b \cdot h_a^3}{12} + A_a \left( h_a - \frac{h_s}{2} \right) = 8.401 \times 10^5\text{·mm}^4 \]

\[ I_s = \frac{b \cdot h_s^3}{12} + A_s \left( h_1 - \frac{h_a}{2} \right) = 2.240 \times 10^6\text{·mm}^4 \]

Set max. bending stress in aluminum to given value then solve for moment \( M \):

\[ M = \frac{\sigma_a (E_s \cdot I_a + E_a \cdot I_s)}{h_1 \cdot E_a} = 3.738\text{·kN} \cdot \text{m} \]

Use \( M \) to find max. bending stress in steel: \( \sigma_s = \frac{M \cdot h_2 \cdot E_s}{E_s \cdot I_a + E_a \cdot I_s} = 98.4\text{·MPa} \)

A-6.1: A rectangular plate \((a = 120\text{ mm}, \ b = 160\text{ mm})\) is subjected to compressive stress \( \sigma_y = -4.5\text{ MPa} \) and tensile stress \( \sigma_x = 15\text{ MPa} \). The ratio of the normal stress acting perpendicular to the weld to the shear stress acting along the weld is approximately:

(A) 0.27
(B) 0.54
(C) 0.85
(D) 1.22

Solution

\[ a = 120\text{·mm} \quad b = 160\text{·mm} \]

\[ \theta = \tan^{-1} \left( \frac{a}{b} \right) = 36.87\text{·deg} \]
\[ \sigma_x = -4.5 \text{ MPa} \quad \sigma_y = 15 \text{ MPa} \]
\[ \tau_{xy} = 0 \]

Plane stress transformation: normal and shear stresses on \( y \)-face of element rotated through angle \( \theta \) (perpendicular to & along weld seam):
\[ \sigma_y = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos\left(2\left(\theta + \frac{\pi}{2}\right)\right) + \tau_{xy} \sin\left(2\left(\theta + \frac{\pi}{2}\right)\right) = 7.98 \text{ MPa} \]
\[ \tau_y = -\frac{\sigma_x - \sigma_y}{2} \sin\left(2\left(\theta + \frac{\pi}{2}\right)\right) + \tau_{xy} \cos\left(2\left(\theta + \frac{\pi}{2}\right)\right) = -9.36 \text{ MPa} \]
\[ \left| \frac{\sigma_y}{\tau_y} \right| = 0.85 \]

**A-6.2:** A rectangular plate in plane stress is subjected to normal stresses \( \sigma_x \) and \( \sigma_y \) and shear stress \( \tau_{xy} \). Stress \( \sigma_x \) is known to be 15 MPa but \( \sigma_y \) and \( \tau_{xy} \) are unknown. However, the normal stress is known to be 33 MPa at counterclockwise angles of 35° and 75° from the \( x \) axis. Based on this, the normal stress \( \sigma_y \) on the element below is approximately:

(A) 14 MPa
(B) 21 MPa
(C) 26 MPa
(D) 43 MPa

**Solution**
\[ \sigma_x = 15 \quad \sigma_{35} = 33 \quad \sigma_{75} = \sigma_{35} \]

Plane stress transformations for 35 deg & 75 deg:
\[ \sigma_y = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos(2\cdot\theta) + \tau_{xy} \sin(2\cdot\theta) \]
For \( \theta = 35 \) deg: 
\[ \theta_{35} = 35 \cdot \frac{\pi}{180} \]
\[
\frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos[2 \cdot (\theta_{35})] + \tau_{xy} \sin[2 \cdot (\theta_{35})] = \sigma_{35}
\]
Or \( \sigma_x + 2.8563 \cdot \tau_{xy} = 69.713 \)

And for \( \theta = 75 \) deg: 
\[ \theta_{75} = 75 \cdot \frac{\pi}{180} \]
\[
\frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos[2 \cdot (\theta_{75})] + \tau_{xy} \sin[2 \cdot (\theta_{75})] = \sigma_{75}
\]
Or \( \sigma_x + 0.5359 \cdot \tau_{xy} = 34.292 \)

Solving above two equations for \( \sigma_y \) and \( \tau_{xy} \) gives:
\[
\begin{pmatrix} \sigma_y \\ \tau_{xy} \end{pmatrix} = \begin{pmatrix} 1 & 2.8563 \\ 1 & 0.5359 \end{pmatrix}^{-1} \begin{pmatrix} 69.713 \\ 34.292 \end{pmatrix} = \begin{pmatrix} 26.1 \\ 15.3 \end{pmatrix} \text{ MPa}
\]
so \( \sigma_y = 26.1 \) MPa

A-6.3: A rectangular plate in plane stress is subjected to normal stresses \( \sigma_x = 35 \) MPa, \( \sigma_y = 26 \) MPa, and shear stress \( \tau_{xy} = 14 \) MPa. The ratio of the magnitudes of the principal stresses \( (\sigma_1/\sigma_2) \) is approximately:

(A) 0.8 
(B) 1.5 
(C) 2.1 
(D) 2.9

Solution
\[
\sigma_x = 35 \cdot \text{MPa} \quad \sigma_y = 26 \cdot \text{MPa} \quad \tau_{xy} = 14 \cdot \text{MPa}
\]

Principal angles:
\[
\theta_{p1} = \frac{1}{2} \cdot \arctan \left( \frac{2 \cdot \tau_{xy}}{\sigma_x - \sigma_y} \right) = 36.091 \cdot \text{deg}
\]
\[
\theta_{p2} = \theta_{p1} + \frac{\pi}{2} = 126.091 \cdot \text{deg}
\]

Plane stress transformations:
\[
\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos(2 \cdot \theta_{p1}) + \tau_{xy} \sin(2 \cdot \theta_{p1}) = 45.21 \cdot \text{MPa}
\]
\[
\sigma_2 = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos(2 \cdot \theta_{p2}) + \tau_{xy} \sin(2 \cdot \theta_{p2}) = 15.79 \cdot \text{MPa}
\]
Ratio of principal stresses:
\[
\frac{\sigma_1}{\sigma_2} = 2.86
\]

A-6.4: A drive shaft resists torsional shear stress of 45 MPa and axial compressive stress of 100 MPa. The ratio of the magnitudes of the principal stresses \((\sigma_1/\sigma_2)\) is approximately:
(A) 0.15
(B) 0.55
(C) 1.2
(D) 1.9

Solution
\[
\sigma_x = -100 \text{ MPa} \quad \sigma_y = 0
\]
\[
\tau_{xy} = -45 \text{ MPa}
\]
Principal angles:
\[
\theta_{p1} = \frac{1}{2} \arctan \left( \frac{2 \cdot \tau_{xy}}{\sigma_x - \sigma_y} \right) = 20.994^\circ
\]
\[
\theta_{p2} = \theta_{p1} + \frac{\pi}{2} = 110.994^\circ
\]
Plane stress transformations:
\[
\sigma_{sp1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cdot \cos(2 \cdot \theta_{p1}) + \tau_{xy} \cdot \sin(2 \cdot \theta_{p1}) = -117.27 \text{ MPa}
\]
< actually \(\sigma_2\)
\[
\sigma_{sp2} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cdot \cos(2 \cdot \theta_{p2}) + \tau_{xy} \cdot \sin(2 \cdot \theta_{p2}) = 17.27 \text{ MPa}
\]
< this is \(\sigma_1\)

So
\[
\sigma_1 = \max(\sigma_{sp1}, \sigma_{sp2}) = 17.268 \text{ MPa} \quad \sigma_2 = \min(\sigma_{sp1}, \sigma_{sp2}) = -117.268 \text{ MPa}
\]

Ratio of principal stresses:
\[
\left| \frac{\sigma_1}{\sigma_2} \right| = 0.15
\]

A-6.5: A drive shaft resists torsional shear stress of 45 MPa and axial compressive stress of 100 MPa. The maximum shear stress is approximately:
(A) 42 MPa
(B) 67 MPa
(C) 71 MPa
(D) 93 MPa
Max. shear stress:

\[
\tau_{\text{max}} = \sqrt{\left(\frac{\tau_x - \tau_y}{2}\right)^2 + \tau_y^2} = 67.3\,\text{MPa}
\]

A-6.6: A drive shaft resists torsional shear stress of \(\tau_{xy} = 40\,\text{MPa}\) and axial compressive stress \(\sigma_x = -70\,\text{MPa}\). One principal normal stress is known to be 38 MPa (tensile). The stress \(\sigma_y\) is approximately:

(A) 23 MPa
(B) 35 MPa
(C) 62 MPa
(D) 75 MPa

Solution

\[
\sigma_x = -70\,\text{MPa} \quad \sigma_y < \text{unknown} \quad \sigma_{\text{prin}} = 38\,\text{MPa}
\]

Stresses \(\sigma_x\) and \(\sigma_y\) must be smaller than the given principal stress so:

\[
\sigma_x = \sigma_{\text{prin}}
\]

Substitute into stress transformation equation and solve for \(\sigma_y\):

\[
\frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_{\text{prin}} \Rightarrow \sigma_y = \frac{626\,\text{MPa}}{27} = 23.2\,\text{MPa}
\]

A-6.7: A cantilever beam with rectangular cross section \((b = 95\,\text{mm}, h = 300\,\text{mm})\) supports load \(P = 160\,\text{kN}\) at its free end. The ratio of the magnitudes of the principal stresses \((\sigma_1/\sigma_2)\) at point \(A\) (at distance \(c = 0.8\,\text{m}\) from the free end and distance \(d = 200\,\text{mm}\) up from the bottom) is approximately:

(A) 5
(B) 12
(C) 18
(D) 25
Solution

\[ P = 160 \cdot \text{kN} \quad b = 95 \cdot \text{mm} \quad h = 300 \cdot \text{mm} \]
\[ c = 0.8 \cdot \text{m} \quad d = 200 \cdot \text{mm} \quad \frac{d}{h} = 0.667 \]

Cross section properties:
\[ A = b \cdot h = 28500 \cdot \text{mm}^2 \]
\[ I = \frac{b \cdot h^3}{12} = 2.138 \times 10^6 \cdot \text{mm}^4 \]
\[ Q_A = [b \cdot (h - d)] \left( \frac{h}{2} - \frac{(h - d)}{2} \right) = 9.500 \times 10^5 \cdot \text{mm}^3 \]

Moment, shear force and normal and shear stresses at \( A \):
\[ M_A = -P \cdot c = -1.280 \times 10^5 \cdot \text{kN} \cdot \text{mm} \quad V_A = P \]
\[ \tau_A = \frac{V_A \cdot Q_A}{I \cdot b} = 7.485 \cdot \text{MPa} \quad \sigma_A = \frac{-M_A \left( \frac{d}{2} - \frac{h}{2} \right)}{I} = 29.942 \cdot \text{MPa} \]

Plane stress state at \( A \):
\[ \sigma_x = \sigma_A \quad \tau_{xy} = \tau_A \quad \sigma_y = 0 \]

Principal stresses:
\[ \theta_p = \frac{1}{2} \tan \left( \frac{2 \cdot \tau_{xy}}{\sigma_x - \sigma_y} \right) = 13.283 \cdot \text{deg} \]
\[ \sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2} = 31.709 \cdot \text{MPa} \]
\[ \sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2} = -1.767 \cdot \text{MPa} \]

Ratio of principal stresses (\( \sigma_1 / \sigma_2 \)):
\[ \left| \frac{\sigma_1}{\sigma_2} \right| = 17.9 \]

A-6.8: A simply supported beam (\( L = 4.5 \) m) with rectangular cross section \( (b = 95 \text{ mm}, h = 280 \text{ mm}) \) supports uniform load \( q = 25 \text{ kN/m} \). The ratio of the magnitudes of the principal stresses \( (\sigma_1/\sigma_2) \) at a point \( a = 1.0 \) m from the left support and distance \( d = 100 \text{ mm} \) up from the bottom of the beam is approximately:
(A) 9  
(B) 17  
(C) 31  
(D) 41
Solution

\[ q = 25 \text{ kN/m} \quad L = 4.5 \text{ m} \]

\[ b = 95 \text{ mm} \quad h = 280 \text{ mm} \]
\[ a = 1.0 \text{ m} \quad d = 100 \text{ mm} \]

Cross section properties:
\[ A = b \cdot h = 26600 \text{ mm}^2 \]
\[ I = \frac{b \cdot h^3}{12} = 1.738 \times 10^6 \text{ mm}^4 \]
\[ Q = [b \cdot (h - d)] \left[ \frac{h}{2} - \frac{(h - d)}{2} \right] = 8.550 \times 10^3 \text{ mm}^3 \]

Moment, shear force and normal and shear stresses at distance \( a \) from left support:
\[ V_a = \frac{q \cdot L}{2} - q \cdot a = 31.250 \text{ kN} \quad M_a = \frac{q \cdot L}{2} \cdot a - \frac{q \cdot a^2}{2} = 4.375 \times 10^4 \text{ kN} \cdot \text{mm} \]
\[ \tau = \frac{V_a \cdot Q}{I \cdot b} = 1.618 \text{ MPa} \quad \sigma = \frac{-M_a \left( \frac{d - h}{2} \right)}{I} = 10.070 \text{ MPa} \]

Plane stress state: \( \sigma_x = \sigma \quad \tau_{xy} = \tau \quad \sigma_y = 0 \text{ MPa} \)

Principal stresses:
\[ \theta_p = \frac{1}{2} \cdot \tan \left( \frac{2 \cdot \tau_{xy}}{\sigma_x - \sigma_y} \right) = 8.909 \text{ deg} \]
\[ \sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2} = 10.324 \text{ MPa} \]
\[ \sigma_2 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2} = -0.254 \text{ MPa} \]

Ratio of principal stresses \( (\sigma_1 / \sigma_2) \):
\[ \frac{\sigma_1}{\sigma_2} = 40.7 \]
A-7.1: A thin wall spherical tank of diameter 1.5 m and wall thickness 65 mm has internal pressure of 20 MPa. The maximum shear stress in the wall of the tank is approximately:

(A) 58 MPa  
(B) 67 MPa  
(C) 115 MPa  
(D) 127 MPa

Solution

\[ d = 1.5 \text{ m} \quad t = 65 \text{ mm} \quad p = 20 \text{ MPa} \]

Thin wall tank since: \( \frac{t}{d/2} = 0.087 \)

Biaxial stress:

\[ \sigma = \frac{p\left(\frac{d}{2}\right)}{2\cdot t} \quad \sigma = 115.4 \text{ MPa} \]

Max. shear stress at 45 deg. rotation is 1/2 of \( \sigma \)

\[ \tau_{\text{max}} = \frac{\sigma}{2} = 57.7 \text{ MPa} \]

A-7.2: A thin wall spherical tank of diameter 0.75 m has internal pressure of 20 MPa. The yield stress in tension is 920 MPa, the yield stress in shear is 475 MPa, and the factor of safety is 2.5. The modulus of elasticity is 210 GPa, Poisson’s ratio is 0.28, and maximum normal strain is \( 1220 \times 10^{-6} \). The minimum permissible thickness of the tank is approximately:

(A) 8.6 mm  
(B) 9.9 mm  
(C) 10.5 mm  
(D) 11.1 mm

Solution

\[ d = 0.75 \text{ m} \quad p = 20 \text{ MPa} \quad E = 210 \text{ GPa} \]

\[ \sigma_y = 920 \text{ MPa} \quad \tau_y = 475 \text{ MPa} \quad FS_y = 2.5 \]

\[ v = 0.28 \quad \varepsilon_n = 1220 \times 10^{-6} \]

Thickness based on tensile stress:

\[ t_t = \frac{p\left(\frac{d}{2}\right)}{2\left(\frac{\sigma_y}{FS_y}\right)} = 10.190 \text{ mm} \]

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Thickness based on shear stress:

\[ t_2 = \frac{p \left( \frac{d}{2} \right)}{4 \left( \frac{\tau_y}{FS_y} \right)} = 9.868 \text{ mm} \]

Thickness based on normal strain:

\[ t_3 = \frac{p \left( \frac{d}{2} \right) (1 - v)}{2 \cdot E \cdot (1 - v)} = 10.54 \text{ mm} \]

\[ t_3 \text{ is largest value controls} \]

A-7.3: A thin wall cylindrical tank of diameter 200 mm has internal pressure of 11 MPa. The yield stress in tension is 250 MPa, the yield stress in shear is 140 MPa, and the factor of safety is 2.5. The minimum permissible thickness of the tank is approximately:

(A) 8.2 mm
(B) 9.1 mm
(C) 9.8 mm
(D) 11.0 mm

Solution

\[ d = 200 \text{ mm} \quad p = 11 \text{ MPa} \]
\[ \sigma_y = 250 \text{ MPa} \quad \tau_y = 140 \text{ MPa} \quad FS_y = 2.5 \]

Wall thickness based on tensile stress:

\[ t_1 = \frac{p \left( \frac{d}{2} \right)}{\frac{\sigma_y}{FS_y}} = 11.00 \text{ mm} \]

\[ \frac{t_1}{\left( \frac{d}{2} \right)} = 0.110 \]

Wall thickness based on shear stress:

\[ t_2 = \frac{p \left( \frac{d}{2} \right)}{2 \cdot \frac{\tau_y}{FS_y}} = 9.821 \text{ mm} \]

\[ \frac{t_2}{\left( \frac{d}{2} \right)} = 0.098 \]

A-7.4: A thin wall cylindrical tank of diameter 2.0 m and wall thickness 18 mm is open at the top. The height \( h \) of water (weight density = 9.81 kN/m\(^3\)) in the tank at which the circumferential stress reaches 10 MPa in the tank wall is approximately:

(A) 14 m
(B) 18 m
(C) 20 m
(D) 24 m

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Solution

\[ d = 2 \text{ m} \quad t = 18 \text{ mm} \quad \sigma_a = 10 \text{ MPa} \quad \gamma_c = 9.81 \text{ kN/m} \]

Pressure at height \( h \):

\[ p_h = \gamma_c \cdot h \]

Circumferential stress:

\[ \sigma_c = \frac{p_h \left( \frac{d}{2} \right)}{t} \quad \sigma_c = \frac{(\gamma_c \cdot h) \left( \frac{d}{2} \right)}{t} \]

Set \( \sigma_c \) equal to \( \sigma_a \) and solve for \( h \):

\[ h = \frac{\sigma_a \cdot t}{(\gamma_c) \left( \frac{d}{2} \right)} = 18.3 \text{ m} \]

A-7.5: The pressure relief valve is opened on a thin wall cylindrical tank, with radius to wall thickness ratio of 128, thereby decreasing the longitudinal strain by \( 150 \times 10^{-6} \). Assume \( E = 73 \text{ GPa} \) and \( v = 0.33 \). The original internal pressure in the tank was approximately:

(A) 370 kPa
(B) 450 kPa
(C) 500 kPa
(D) 590 kPa

Solution

\[ r_i = \frac{r}{t} \quad r_t = 128 \]

\[ \varepsilon_L = 148 \times (10^{-6}) \]

\[ E = 73 \text{ GPa} \quad v = 0.33 \]

Longitudinal strain:

\[ \varepsilon = \frac{p \cdot \left( \frac{r}{t} \right)}{2 \cdot E} \cdot (1 - 2 \cdot v) \]

Set \( \varepsilon \) to \( \varepsilon_L \) and solve for pressure \( p \):

\[ p = \frac{2 \cdot E \cdot \varepsilon_L}{r_t \cdot (1 - 2 \cdot v)} = 497 \text{ kPa} \]

A-7.6: A cylindrical tank is assembled by welding steel sections circumferentially. Tank diameter is 1.5 m, thickness is 20 mm, and internal pressure is 2.0 MPa. The maximum stress in the heads of the tank is approximately:

(A) 38 MPa
(B) 45 MPa
(C) 50 MPa
(D) 59 MPa
A-7.7: A cylindrical tank is assembled by welding steel sections circumferentially. Tank diameter is 1.5 m, thickness is 20 mm, and internal pressure is 2.0 MPa. The maximum tensile stress in the cylindrical part of the tank is approximately:

(A) 45 MPa  
(B) 57 MPa  
(C) 62 MPa  
(D) 75 MPa

Solution
\[ \sigma_t = \frac{p}{2} \left( \frac{d}{2} \right) = 37.5 \text{ MPa} \]

A-7.8: A cylindrical tank is assembled by welding steel sections circumferentially. Tank diameter is 1.5 m, thickness is 20 mm, and internal pressure is 2.0 MPa. The maximum tensile stress perpendicular to the welds is approximately:

(A) 22 MPa  
(B) 29 MPa  
(C) 33 MPa  
(D) 37 MPa

Solution
\[ \sigma_c = \frac{p}{t} \left( \frac{d}{2} \right) = 75.0 \text{ MPa} \]

A-7.9: A cylindrical tank is assembled by welding steel sections circumferentially. Tank diameter is 1.5 m, thickness is 20 mm, and internal pressure is 2.0 MPa. The maximum shear stress in the heads is approximately:

(A) 19 MPa  
(B) 23 MPa  
(C) 33 MPa  
(D) 35 MPa

Solution
\[ \sigma_s = \frac{p}{2} \left( \frac{d}{2} \right) = 37.5 \text{ MPa} \]
A-7.10: A cylindrical tank is assembled by welding steel sections circumferentially. Tank diameter is 1.5 m, thickness is 20 mm, and internal pressure is 2.0 MPa. The maximum shear stress in the cylindrical part of the tank is approximately:

(A) 17 MPa
(B) 26 MPa
(C) 34 MPa
(D) 38 MPa

Solution
\[ \tau_{\text{max}} = \frac{p \left( \frac{d}{2} \right)}{4 \cdot t} = 18.8 \cdot \text{MPa} \]

\[ \tau_{\text{max}} = \frac{2.0}{4 \cdot 0.02} = 18.8 \cdot \text{MPa} \]

A-7.11: A cylindrical tank is assembled by welding steel sections in a helical pattern with angle \( \alpha = 50 \) degrees. Tank diameter is 1.6 m, thickness is 20 mm, and internal pressure is 2.75 MPa. Modulus \( E = 210 \) GPa and Poisson’s ratio \( \nu = 0.28 \). The circumferential strain in the wall of the tank is approximately:

(A) 1.9 \( \times \) 10\(^{-4} \)
(B) 3.2 \( \times \) 10\(^{-4} \)
(C) 3.9 \( \times \) 10\(^{-4} \)
(D) 4.5 \( \times \) 10\(^{-4} \)

Solution
\[ d = 1.6 \text{ m} \quad t = 20 \text{ mm} \quad p = 2.75 \text{ MPa} \]

\[ \tau_{\text{max}} = \frac{p \left( \frac{d}{2} \right)}{2 \cdot t} = 37.5 \cdot \text{MPa} \]

\[ \tau_{\text{max}} = \frac{2.75}{2 \cdot 0.02} = 37.5 \cdot \text{MPa} \]

Circumferential stress:
\[ \sigma_c = \frac{p \left( \frac{d}{2} \right)}{t} = 110.00 \cdot \text{MPa} \]

Circumferential strain:
\[ e_c = \frac{\sigma_c}{2 \cdot E} \cdot (2 - \nu) = 4.50 \times 10^{-4} \]
A-7.12: A cylindrical tank is assembled by welding steel sections in a helical pattern with angle $\alpha = 50$ degrees. Tank diameter is 1.6 m, thickness is 20 mm, and internal pressure is 2.75 MPa. Modulus $E = 210$ GPa and Poisson’s ratio $\nu = 0.28$. The longitudinal strain in the wall of the tank is approximately:

(A) $1.2 \times 10^{-4}$
(B) $2.4 \times 10^{-4}$
(C) $3.1 \times 10^{-4}$
(D) $4.3 \times 10^{-4}$

Solution

$d = 1.6 \text{ m} \quad t = 20 \text{ mm} \quad p = 2.75 \text{ MPa}$
$E = 210 \text{ GPa} \quad \nu = 0.28 \quad \alpha = 50 \text{ deg}$

Longitudinal stress:

$$\sigma_L = \frac{p \left( \frac{d}{2} \right)}{2 \cdot t} = 55,000 \text{ MPa}$$

Longitudinal strain:

$$\varepsilon_L = \frac{\sigma_L}{E} \cdot (1 - 2 \cdot \nu) = 1.15 \times 10^{-4}$$

A-7.13: A cylindrical tank is assembled by welding steel sections in a helical pattern with angle $\alpha = 50$ degrees. Tank diameter is 1.6 m, thickness is 20 mm, and internal pressure is 2.75 MPa. Modulus $E = 210$ GPa and Poisson’s ratio $\nu = 0.28$. The normal stress acting perpendicular to the weld is approximately:

(A) 39 MPa
(B) 48 MPa
(C) 78 MPa
(D) 84 MPa

Solution

$d = 1.6 \text{ m} \quad t = 20 \text{ mm} \quad p = 2.75 \text{ MPa} \quad E = 210 \text{ GPa}$
$\nu = 0.28 \quad \alpha = 50 \text{ deg}$

Longitudinal stress:

$$\sigma_L = \frac{p \left( \frac{d}{2} \right)}{2 \cdot t} = 55,000 \text{ MPa} \quad \text{So} \quad \sigma_s = \sigma_L$$
Circumferential stress:

\[ \sigma_c = \frac{P \left( \frac{d_2}{2} \right)}{t} = 110.000 \text{ MPa} \quad \text{So} \quad \sigma_y = \sigma_c \]

Angle perpendicular to the weld: \( \theta = 90\degree - \alpha = 40.000\degree \)

Normal stress perpendicular to the weld:

\[ \sigma_{40} = \frac{\sigma_c + \sigma_y}{2} + \frac{\sigma_c - \sigma_y}{2} \cdot \cos (2 \cdot \theta) = 77.7 \text{ MPa} \]

A-7.14: A segment of a drive shaft \((d_2 = 200 \text{ mm}, \; d_1 = 160 \text{ mm})\) is subjected to a torque \(T = 30 \text{ kN} \cdot \text{m}\). The allowable shear stress in the shaft is 45 MPa. The maximum permissible compressive load \(P\) is approximately:

(A) 200 kN
(B) 286 kN
(C) 328 kN
(D) 442 kN

Solution

\[ d_2 = 200 \text{ mm} \quad d_1 = 160 \text{ mm} \quad \tau_a = 45 \text{ MPa} \]

\[ T = 30 \text{ kN} \cdot \text{m} \]

Cross section properties:

\[ A = \frac{\pi}{4} (d_2^2 - d_1^2) = 11310 \text{ mm}^2 \]

\[ I_p = \frac{\pi}{32} (d_2^4 - d_1^4) = 9.274 \times 10^7 \text{ mm}^4 \]

Normal and in-plane shear stresses:

\[ \sigma_x = 0 \quad \sigma_y = \frac{-P}{A} \quad \tau_{sy} = \frac{T \cdot \left( \frac{d_2}{2} \right)}{I_p} = 32.349 \text{ MPa} \]

Maximum in-plane shear stress: set \(\tau_{\text{max}} = \tau_{\text{allow}}\) then solve for \(\sigma_y\)

\[ \tau_{\text{max}} = \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{sy}^2} \quad \text{So} \quad \sigma_y = \sqrt{4 \cdot (\tau_a^2 - \tau_{\text{max}}^2)} = 25.303 \text{ MPa} \]

Finally solve for \(P = \sigma_y \cdot A\):

\[ P_{\text{max}} = \sigma_y \cdot A = 286 \text{ kN} \]

A-7.15: A thin walled cylindrical tank, under internal pressure \(p\), is compressed by a force \(F = 75 \text{ kN}\). Cylinder diameter is \(d = 90 \text{ mm}\) and wall thickness \(t = 5.5 \text{ mm}\). Allowable normal stress is 110 MPa and allowable shear stress is 60 MPa. The maximum allowable internal pressure \(p_{\text{max}}\) is approximately:

(A) 5 MPa
(B) 10 MPa
(C) 13 MPa
(D) 17 MPa
Solution

d = 90·mm  
\( t = 5.5\cdot\text{mm} \)  \( \sigma_a = 110\cdot\text{MPa} \)

\( F = 75\cdot\text{kN} \)  \( A = 2\cdot\pi\cdot\frac{d}{2}\cdot t = 1555\cdot\text{mm}^2 \)

Circumferential normal stress:

\[
\sigma_c = \frac{p_{\max}\left(\frac{d}{2}\right)}{t} \quad \text{and setting } \sigma_c = \sigma_a \text{ and solving for } p_{\max}:
\]

\[
p_{\max} = \sigma_a\left(\frac{2\cdot t}{d}\right) = 13.4\cdot\text{MPa} \quad < \text{controls}
\]

Longitudinal normal stress:

\[
\sigma_L = \frac{p_{\max}\left(\frac{d}{2}\right)}{2\cdot t} = \frac{F}{A} \quad \text{Or} \quad \sigma_L = \frac{p_{\max}\cdot d}{4\cdot t} = \frac{F}{A}
\]

So set \( \sigma_L = \sigma_a \) and solve for \( p_{\max} \):

\[
p_{\max} = \left(\sigma_a + \frac{F}{A}\right)\frac{4\cdot t}{d} = 38.7\cdot\text{MPa}
\]

Check also in-plane & out-of-plane shear stresses: all are below allowable shear stress so circumferential normal stress controls as noted above.

A-8.1: An aluminum beam (\( E = 72\ \text{GPa} \)) with a square cross section and span length \( L = 2.5\ \text{m} \) is subjected to uniform load \( q = 1.5\ \text{kN/m} \). The allowable bending stress is 60 MPa. The maximum deflection of the beam is approximately:

(A) 10 mm
(B) 16 mm
(C) 22 mm
(D) 26 mm

Solution

\( E = 72\cdot(10^3)\text{MPa} \)  \( \sigma_a = 60\\text{MPa} \)

\( q = 1.5\ \frac{\text{N}}{\text{mm}} \)  \( \text{MPa} = \frac{\text{N}}{\text{mm}^2} \)

\( L = 2500\ \text{mm} \)

Max. moment and deflection at \( L/2 \):

\[
M_{\max} = \frac{q\cdot L^2}{8} \quad \delta_{\max} = \frac{5\cdot q\cdot L^4}{384\cdot E\cdot I}
\]
Moment of inertia and section modulus for square cross section (height = width = $b$)

\[ I = \frac{b^4}{12} \quad S = \frac{1}{\left( \frac{b}{2} \right)} \rightarrow \frac{b^3}{6} \]

**Flexure formula**

\[ \sigma_{\text{max}} = \frac{M_{\text{max}}}{S} \quad \sigma_{\text{max}} = \frac{qL^2}{8\left( \frac{b^3}{6} \right)} \quad \text{so} \quad b^3 = \frac{3qL^2}{4\sigma_{\text{max}}} \]

**Max. deflection formula**

\[ \delta_{\text{max}} = \frac{5qL^4}{384E\left( \frac{b^3}{12} \right)} \quad \text{so solve for} \quad \delta_{\text{max}} = a \quad \sigma_{\text{max}} = \sigma_a \]

\[ \delta_{\text{max}} = \frac{5qL^4}{384E \left[ \sqrt{\left( \frac{3qL^2}{4\sigma_a} \right)} \right]^3} = 22.2 \text{ mm} \]

**A-8.2:** An aluminum cantilever beam ($E = 72$ GPa) with a square cross section and span length $L = 2.5$ m is subjected to uniform load $q = 1.5$ kN/m. The allowable bending stress is $55$ MPa. The maximum deflection of the beam is approximately:

(A) $10$ mm  
(B) $20$ mm  
(C) $30$ mm  
(D) $40$ mm

**Solution**

\[ E = 72 \times (10^3) \text{ MPa} \quad \sigma_a = 55 \text{ MPa} \]

\[ q = 1.5 \frac{N}{\text{mm}} \quad \text{MPa} = \frac{N}{\text{mm}^2} \]

\[ L = 2500 \text{ mm} \]

Max. moment at support & max. deflection at $L$:

\[ M_{\text{max}} = \frac{qL^2}{2} \quad \delta_{\text{max}} = \frac{qL^4}{8E \cdot I} \]

Moment of inertia and section modulus for square cross section (height = width = $b$)

\[ I = \frac{b^4}{12} \quad S = \frac{1}{\left( \frac{b}{2} \right)} \rightarrow \frac{b^3}{6} \]
Flexure formula

\[ \sigma_{\text{max}} = \frac{M_{\text{max}}}{S} \]
\[ \sigma_{\text{max}} = \frac{q \cdot L^2}{2 \left( \frac{b^3}{6} \right)} \quad \text{so} \quad b^3 = \frac{3 \cdot q \cdot L^2}{\sigma_{\text{max}}} \]

Max. deflection formula

\[ \delta_{\text{max}} = \frac{q \cdot L^4}{8 \cdot E \cdot \left( \frac{b^4}{12} \right)} \quad \text{so solve for} \ \delta_{\text{max}} \ \text{if} \ \sigma_{\text{max}} = \sigma_a \quad \delta_{\text{max}} = \frac{q \cdot L^4}{8 \cdot E \cdot \left( \frac{3 \cdot q \cdot L^2}{\sigma_{\text{max}}} \right)^{\frac{1}{3}}} = 29.9 \text{mm} \]

A-8.3: A steel beam \((E = 210 \text{ GPa})\) with \(I = 119 \times 10^6 \text{ mm}^4\) and span length \(L = 3.5 \text{ m}\) is subjected to uniform load \(q = 9.5 \text{ kN/m}\). The maximum deflection of the beam is approximately:

(A) 10 mm  
(B) 13 mm  
(C) 17 mm  
(D) 19 mm

Solution

\(E = 210 \cdot (10^3) \text{ MPa} \quad I = 119 \cdot (10^6) \text{ mm}^4 < \text{ strong axis } I \text{ for W310×52} \)
\(q = 9.5 \frac{N}{\text{mm}} \quad MPa = \frac{N}{\text{mm}^2} \)
\(L = 3500 \text{ mm} \)

Max. deflection at \(A\) by superposition of SS beam mid-span deflection & \(R_B/k\):

\[ \delta_{\text{max}} = \frac{5 \cdot q \cdot (2 \cdot L^4)}{384 \cdot E \cdot I} + \frac{(q \cdot L)}{\left( \frac{48 \cdot E \cdot I}{L^3} \right)} = 13.07 \text{mm} \]
A-8.4: A steel bracket $ABC (EI = 4.2 \times 10^6 \text{ N}\cdot\text{m}^2)$ with span length $L = 4.5 \text{ m}$ and height $H = 2 \text{ m}$ is subjected to load $P = 15 \text{ kN}$ at $C$. The maximum rotation of joint $B$ is approximately:
(A) 0.1 degrees 
(B) 0.3 degrees 
(C) 0.6 degrees 
(D) 0.9 degrees

Solution

\[ E = 210 \text{ GPa} \quad I = 20\cdot10^6 \text{ mm}^4 \quad \text{< strong axis } I \text{ for W200}\times22.5 \]
\[ EI = 4.20 \times 10^6 \text{ N}\cdot\text{m}^2 \]
\[ P = 15 \text{ kN} \]
\[ L = 4.5 \text{ m} \quad H = 2 \text{ m} \]

Max. rotation at $B$: apply statically-equivalent moment $P\times H$ at $B$ on SS beam

\[ \theta_{\text{Bmax}} = \frac{(P\cdot H)\cdot L}{3\cdot E\cdot I} = 0.614 \text{ deg} \quad \theta_{\text{Bmax}} = 0.011 \text{ rad} \]

A-8.5: A steel bracket $ABC (EI = 4.2 \times 10^6 \text{ N}\cdot\text{m}^2)$ with span length $L = 4.5 \text{ m}$ and height $H = 2 \text{ m}$ is subjected to load $P = 15 \text{ kN}$ at $C$. The maximum horizontal displacement of joint $C$ is approximately:
(A) 22 mm 
(B) 31 mm 
(C) 38 mm 
(D) 40 mm

Solution

\[ E = 210 \text{ GPa} \quad I = 20\cdot10^6 \text{ mm}^4 \quad \text{< strong axis } I \text{ for W200}\times22.5 \]
\[ EI = 4.20 \times 10^6 \text{ N}\cdot\text{m}^2 \]
\[ P = 15 \text{ kN} \]
\[ L = 4.5 \text{ m} \quad H = 2 \text{ m} \]
Max. rotation at $B$: apply statically-equivalent moment $P \times H$ at $B$ on SS beam

$$\theta_{B_{\text{max}}} = \frac{(P \cdot H) \cdot L}{3 \cdot E \cdot I} = 0.614 \cdot \text{deg} \quad \theta_{B_{\text{max}}} = 0.011 \cdot \text{rad}$$

Horizontal deflection of vertical cantilever $BC$:

$$\delta_{BC} = \frac{P \cdot H^3}{3 \cdot E \cdot I} = 9.524 \cdot \text{mm}$$

Finally, superpose $\theta_B \times H$ and $\delta_{BC}$

$$\delta_C = \theta_{B_{\text{max}}} \cdot H + \delta_{BC} = 31.0 \cdot \text{mm}$$

A-8.6: A nonprismatic cantilever beam of one material is subjected to load $P$ at its free end. Moment of inertia $I_2$.

The ratio $r$ of the deflection $\delta_2$ to the deflection $\delta_1$ at the free end of a prismatic cantilever with moment of inertia $I_1$ carrying the same load is approximately:

(A) 0.25
(B) 0.40
(C) 0.56
(D) 0.78

Solution

Max. deflection of prismatic cantilever (constant $I_1$)

$$\delta_1 = \frac{P \cdot L^3}{3 \cdot E \cdot I_1}$$

Rotation at $C$ due to both load $P$ & moment $PL/2$ at $C$ for nonprismatic beam:

$$\theta_C = \frac{P \left( \frac{L}{2} \right)^2}{2 \cdot E \cdot I_2} + \frac{P \cdot \left( \frac{L}{2} \right) \cdot L}{E \cdot I_2} \quad \text{simplify} \quad \frac{3 \cdot L^2 \cdot P}{8 \cdot E \cdot I_2}$$

Deflection at $C$ due to both load $P$ & moment $PL/2$ at $C$ for nonprismatic beam:

$$\delta_{C_{\text{total}}} = \frac{P \left( \frac{L}{2} \right)^3}{3 \cdot E \cdot I_2} + \frac{P \cdot \left( \frac{L}{2} \right) \cdot \left( \frac{L}{2} \right)^2}{2 \cdot E \cdot I_2} \quad \text{simplify} \quad \frac{5 \cdot L^3 \cdot P}{48 \cdot E \cdot I_2}$$

Total deflection at $B$:  $\delta_B = \delta_{C_{\text{total}}} + \theta_C \cdot \frac{L}{2} + \frac{P \left( \frac{L}{2} \right)^3}{3 \cdot E \cdot I_2}$

$$\delta_B = \frac{5 \cdot L^3 \cdot P}{48 \cdot E \cdot I_2} + \frac{3 \cdot L^2 \cdot P}{8 \cdot E \cdot I_2} \cdot \frac{L}{2} + \frac{P \cdot \left( \frac{L}{2} \right)^3}{3 \cdot E \cdot I_1} \quad \text{simplify} \quad \frac{L^3 \cdot P \cdot (7 \cdot I_1 + I_2)}{24 \cdot E \cdot I_1 \cdot I_2}$$
A-8.7: A steel bracket ABCD \((EI = 4.2 \times 10^6 \text{ N\cdot m}^2)\), with span length \(L = 4.5 \text{ m}\) and dimension \(a = 2 \text{ m}\), is subjected to load \(P = 10 \text{ kN}\) at \(D\). The maximum deflection at \(B\) is approximately:

(A) 10 mm  
(B) 14 mm  
(C) 19 mm  
(D) 24 mm

**Solution**

\[ E = 210 \text{ GPa} \quad I = 20 \times 10^6 \text{ mm}^4 < \text{ strong axis } I \text{ for W200} \times 22.5 \]

\[ EI = 4.2 \times 10^6 \text{ N\cdot m}^2 \]

\[ P = 10 \text{ kN} \]

\[ L = 4.5 \text{ \cdot m} \quad a = 2 \text{ \cdot m} \]

\[ h = 206 \text{ \cdot mm} \]

Statically-equivalent loads at end of cantilever \(AB\):

- downward load \(P\)
- CCW moment \(P \times a\)

Downward deflection at \(B\) by superposition:

\[ \delta_B = \frac{P \cdot L^3}{3EI} - \frac{(P \cdot a) \cdot L^2}{2EI} = 24.1 \text{ \cdot mm} \quad \Rightarrow \quad \frac{\delta_B}{L} = 0.005 \]

A-9.1: Beam \(ACB\) has a sliding support at \(A\) and is supported at \(C\) by a pinned end steel column with square cross section \((E = 200 \text{ GPa}, b = 40 \text{ mm})\) and height \(L = 3.75 \text{ m}\). The column must resist a load \(Q\) at \(B\) with a factor of safety 2.0 with respect to the critical load. The maximum permissible value of \(Q\) is approximately:

(A) 10.5 kN  
(B) 11.8 kN  
(C) 13.2 kN  
(D) 15.0 kN
Solution

\[ E = 200 \text{ GPa} \quad n = 2.0 \]
\[ b = 40 \text{ mm} \quad L = 3.75 \text{ m} \]
\[ I = \frac{b^4}{12} = 2.133 \times 10^5 \text{ mm}^4 \]

Statics: sum vertical forces to find reaction at \( D \):
\[ R_D = Q \]
So force in pin-pin column is \( Q \)
\[ P_{cr} = Q_{cr} \quad Q_{cr} = \frac{\pi^2 \cdot E \cdot I}{L^2} = 29.9 \text{ kN} \]
Allowable value of \( Q \):
\[ \frac{Q_{allow}}{n} = 15.0 \text{ kN} \]

A-9.2: Beam \( ACB \) has a pin support at \( A \) and is supported at \( C \) by a steel column with square cross section \( (E = 190 \text{ GPa}, b = 42 \text{ mm}) \) and height \( L = 5.25 \text{ m} \). The column is pinned at \( C \) and fixed at \( D \). The column must resist a load \( Q \) at \( B \) with a factor of safety 2.0 with respect to the critical load. The maximum permissible value of \( Q \) is approximately:
(A) 3.0 kN
(B) 6.0 kN
(C) 9.4 kN
(D) 10.1 kN

Solution

\[ E = 190 \text{ GPa} \quad n = 2.0 \]
\[ b = 42 \text{ mm} \quad L = 5.25 \text{ m} \]

Effective length of pinned-fixed column:
\[ L_c = 0.699 \cdot L = 3.670 \text{ m} \]
\[ I = \frac{b^4}{12} = 2.593 \times 10^5 \text{ mm}^4 \]

Statics: use FBD of \( ACB \) and sum moments about \( A \) to find force in column as a multiple of \( Q \):
\[ F_{CD} = \frac{Q \cdot (3\cdot d)}{d} \rightarrow 3 \cdot Q \]
So force in pin-fixed column is \( 3Q \)
\[ P_{cr} = 3 \cdot Q_{cr} \quad P_{cr} = \frac{\pi^2 \cdot E \cdot I}{L_c^2} = 36.1 \text{ kN} \quad \text{So} \quad Q_{cr} = \frac{P_{cr}}{3} = 12.0 \text{ kN} \]
Allowable value of $Q$:

$$Q_{allow} = \frac{Q_{cr}}{n} = 6.0 \cdot \text{kN}$$

A-9.3: A steel pipe column ($E = 190 \text{ GPa}$, $\alpha = 14 \times 10^{-6}$ per degree Celsius, $d_2 = 82 \text{ mm}$, $d_1 = 70 \text{ mm}$) of length $L = 4.25 \text{ m}$ is subjected to a temperature increase $\Delta T$. The column is pinned at the top and fixed at the bottom. The temperature increase at which the column will buckle is approximately:

(A) 36 °C  
(B) 42 °C  
(C) 54 °C  
(D) 58 °C

Solution

$$E = 190 \text{ GPa} \quad L = 4.25 \text{ m} \quad \alpha = [14 \cdot (10^{-6})]$$

$$d_2 = 82 \text{ mm} \quad d_1 = 70 \text{ mm} \quad A = \frac{\pi}{4} \cdot (d_2^4 - d_1^4) = 1432.57 \cdot \text{mm}^2$$

Effective length of pinned-fixed column:

$$L_e = 0.699 \cdot L = 3.0 \text{ m}$$

$$I = \frac{\pi}{64} \cdot (d_2^4 - d_1^4) = 1.04076 \times 10^6 \cdot \text{mm}^4$$

Axial compressive load in bar:  

$$P = EA \alpha (\Delta T)$$

Equate to Euler buckling load and solve for $\Delta T$:

$$\Delta T = \frac{\pi^2 \cdot E \cdot I}{L_e^2} \text{ Or } \Delta T = \frac{\pi^2 \cdot I}{\alpha \cdot A \cdot L_e} = 58.0 \text{°C}$$

A-9.4: A steel pipe ($E = 190 \text{ GPa}$, $\alpha = 14 \times 10^{-6}$ per degree Celsius, $d_2 = 82 \text{ mm}$, $d_1 = 70 \text{ mm}$) of length $L = 4.25 \text{ m}$ hangs from a rigid surface and is subjected to a temperature increase $\Delta T = 50 \text{ °C}$. The column is fixed at the top and has a small gap at the bottom. To avoid buckling, the minimum clearance at the bottom should be approximately:

(A) 2.55 mm  
(B) 3.24 mm  
(C) 4.17 mm  
(D) 5.23 mm

Solution

$$E = 190 \text{ GPa} \quad L = 4250 \text{ mm}$$

$$\alpha = [14 \cdot (10^{-6})] / \text{°C} \quad \Delta T = 50 \text{ °C}$$
\[ \begin{align*}
    d_2 &= 82 \text{ mm} & d_1 &= 70 \text{ mm} \\
    A &= \frac{\pi}{4}(d_2^2 - d_1^2) = 1433 \text{ mm}^2 \\
    I &= \frac{\pi}{64}(d_2^4 - d_1^4) = 1.041 \times 10^6 \text{ mm}^4
\end{align*} \]

Effective length of fixed-roller support column:
\[ L_e = 2.0 - L = 8500.0 \text{ mm} \]

Column elongation due to temperature increase:
\[ \delta_1 = \alpha \cdot \Delta T \cdot L = 2.975 \text{ mm} \]

Euler buckling load for fixed-roller column:
\[ P_{\text{cr}} = \frac{\pi^2 \cdot E \cdot I}{L_e^2} = 27.013 \text{ kN} \]

Column shortening under load of \( P = P_{\text{cr}} \):
\[ \delta_2 = \frac{P_{\text{cr}} \cdot L}{E \cdot A} = 0.422 \text{ mm} \]

Minimum required gap size to avoid buckling: \( \text{gap} = \delta_1 - \delta_2 = 2.55 \text{ mm} \)

**A-9.5:** A pinned-end copper strut \((E = 110 \text{ GPa})\) with length \( L = 1.6 \text{ m} \) is constructed of circular tubing with outside diameter \( d = 38 \text{ mm} \). The strut must resist an axial load \( P = 14 \text{ kN} \) with a factor of safety 2.0 with respect to the critical load. The required thickness \( t \) of the tube is:

(A) 2.75 mm  
(B) 3.15 mm  
(C) 3.89 mm  
(D) 4.33 mm

**Solution**

\[ E = 110 \cdot \text{GPa} \quad L = 1.6 \cdot \text{m} \quad d = 38 \cdot \text{mm} \quad n = 2.0 \quad P = 14 \cdot \text{kN} \]

\[ P_{\text{cr}} = n \cdot P \quad P_{\text{cr}} = 28.0 \cdot \text{kN} \]

Solve for required moment of inertia \( I \) in terms of \( P_{\text{cr}} \) then find tube thickness

\[ P_{\text{cr}} = \frac{\pi^2 \cdot E \cdot I}{L_e^2} \quad I = \frac{P_{\text{cr}} \cdot L^2}{\pi^2 \cdot E} \]

\[ I = 66025 \cdot \text{mm}^4 \]

Moment of inertia  
Solve numerically for min. thickness \( t \):

\[ I = \frac{\pi}{64} \left[ d^4 - (d - 2 \cdot t)^4 \right] \quad d^4 - (d - 2 \cdot t)^4 = I \cdot \frac{64}{\pi} \]

\[ t_{\text{min}} = 4.33 \cdot \text{mm} \quad d - 2 \cdot t_{\text{min}} = 29.3 \cdot \text{mm} = \text{inner diameter} \]
A-9.6: A plane truss composed of two steel pipes ($E = 210$ GPa, $d = 100$ mm, wall thickness = 6.5 mm) is subjected to vertical load $W$ at joint $B$. Joints $A$ and $C$ are $L = 7$ m apart. The critical value of load $W$ for buckling in the plane of the truss is nearly:

(A) 138 kN
(B) 146 kN
(C) 153 kN
(D) 164 kN

Solution

\[
E = 210 \text{ GPa} \quad L = 7 \text{ m}
\]
\[
d = 100 \text{ mm} \quad t = 6.5 \text{ mm}
\]

Moment of inertia

\[
I = \frac{\pi}{64} \left[ d^4 - (d - 2 \cdot t)^4 \right] = 2.097 \times 10^6 \text{ mm}^4
\]

Member lengths:

\[
L_{BA} = L \cdot \cos(40^\circ) = 5.362 \text{ m}
\]
\[
L_{BC} = L \cdot \cos(50^\circ) = 4.500 \text{ m}
\]

Statics at joint $B$ to find member forces $F_{BA}$ and $F_{BC}$:

- Sum horizontal forces at joint $B$:

\[
F_{BA} \cdot \cos(40^\circ) = F_{BC} \cdot \cos(50^\circ) \quad F_{BA} = F_{BC} \cdot \frac{\cos(50^\circ)}{\cos(40^\circ)}
\]

where \( \alpha = \frac{\cos(50^\circ)}{\cos(40^\circ)} = 0.839 \)

- Sum vertical forces at joint $B$:

\[
W = F_{BA} \cdot \sin(40^\circ) + F_{BC} \cdot \sin(50^\circ)
\]
\[
W = F_{BC} \cdot \frac{\cos(50^\circ)}{\cos(40^\circ)} \cdot \sin(40^\circ) + F_{BC} \cdot \sin(50^\circ)
\]
\[
F_{BC} = W \cdot \beta \quad \text{where} \quad \beta = \frac{1}{\left( \frac{\cos(50^\circ)}{\cos(40^\circ)} \cdot \sin(40^\circ) + \sin(50^\circ) \right)} = 0.766
\]

So member forces in terms of $W$ are:

\[
F_{BC} = W \cdot \beta \quad \text{and} \quad F_{BA} = F_{BC} \cdot \alpha \quad \text{or} \quad F_{BA} = W \cdot (\alpha' \beta)
\]

with \( \alpha' \beta = 0.643 \)

Euler buckling loads in $BA$ & $BC$:

\[
F_{BA,cr} = \frac{\pi^2 \cdot E \cdot I}{L_{BA}^2} = 151.118 \text{ kN}
\]

so \( W_{BA,cr} = \frac{\beta}{\alpha} \cdot F_{BA,cr} = 138 \text{ kN} \) \( < \) lower value controls

\[
F_{BC,cr} = \frac{\pi^2 \cdot E \cdot I}{L_{BC}^2} = 214.630 \text{ kN}
\]

so \( W_{BC,cr} = \beta \cdot F_{BC,cr} = 164 \text{ kN} \)

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A-9.7: A beam is pin-connected to the tops of two identical pipe columns, each of height \( h \), in a frame. The frame is restrained against sidesway at the top of column 1. Only buckling of columns 1 and 2 in the plane of the frame is of interest here. The ratio \((a/L)\) defining the placement of load \( Q_{cr} \), which causes both columns to buckle simultaneously, is approximately:

(A) 0.25  
(B) 0.33  
(C) 0.67  
(D) 0.75

Solution

Draw FBD of beam only; use statics to show that \( Q_{cr} \) causes forces \( P_1 \) and \( P_2 \) in columns 1 & 2 respectively:

\[
P_1 = \left( \frac{L - a}{L} \right) Q_{cr}
\]

\[
P_2 = \frac{a}{L} Q_{cr}
\]

Buckling loads for columns 1 & 2:

\[
P_{cr1} = \frac{\pi^2 \cdot El}{(0.699 \cdot h^2)} \left( \frac{L}{L - a} \right) Q_{cr}
\]

\[
P_{cr2} = \frac{\pi^2 \cdot El}{h^2} = \left( \frac{a}{L} \right) Q_{cr}
\]

Solve above expressions for \( Q_{cr} \), then solve for required \( a/L \) so that columns buckle at the same time:

\[
\frac{\pi^2 \cdot El}{(0.699 \cdot h^2)} \left( \frac{L}{L - a} \right) = \frac{\pi^2 \cdot El}{h^2} \left( \frac{L}{a} \right)
\]

Or

\[
\frac{\pi^2 \cdot El}{(0.699 \cdot h^2)} \left( \frac{L}{L - a} \right) = \frac{\pi^2 \cdot El}{h^2} \left( \frac{L}{a} \right) = 0
\]

Or

\[
\frac{L}{0.699^2 \cdot (L - a)} - \frac{L}{a} = 0
\]

Or

\[
\frac{a}{L} = 0.699^2
\]

So

\[
\frac{a}{L} = 0.699^2 = 0.328
\]

A-9.8: A steel pipe column \((E = 210 \text{ GPa})\) with length \( L = 4.25 \text{ m} \) is constructed of circular tubing with outside diameter \( d_2 = 90 \text{ mm} \) and inner diameter \( d_1 = 64 \text{ mm} \). The pipe column is fixed at the base and pinned at the top and may buckle in any direction. The Euler buckling load of the column is most nearly:

(A) 303 kN  
(B) 560 kN  
(C) 690 kN  
(D) 720 kN
Solution

\[ E = 210 \text{ GPa} \quad L = 4.25 \text{ mm} \]
\[ d_2 = 90 \text{ mm} \quad d_1 = 64 \text{ mm} \]

Moment of inertia
\[ I = \frac{\pi}{64}(d_2^4 - d_1^4) \quad I = 2.397 \times 10^6 \text{ mm}^4 \]

Effective length of column for fixed-pinned case:
\[ L_e = 0.699 \cdot L = 2.971 \text{ m} \]
\[ P_{cr} = \frac{\pi^2 \cdot E \cdot I}{L_e^2} = 563 \cdot \text{kN} \]

**A-9.9:** An aluminum tube \((E = 72 \text{ GPa})\) \(AB\) of circular cross section has a pinned support at the base and is pin-connected at the top to a horizontal beam supporting a load \(Q = 600 \text{ kN}\). The outside diameter of the tube is 200 mm and the desired factor of safety with respect to Euler buckling is 3.0. The required thickness \(t\) of the tube is most nearly:
(A) 8 mm
(B) 10 mm
(C) 12 mm
(D) 14 mm

Solution

\[ E = 72 \text{ GPa} \quad L = 2.5 \text{ m} \quad n = 3.0 \quad Q = 600 \text{ kN} \quad d = 200 \text{ mm} \]

\[ \Sigma M_e = 0 \quad P = \frac{2.5 \cdot Q}{1.5} \quad P = 1000 \text{-kN} \]

Find required \(I\) based on critical buckling load

Critical load
\[ P_{cr} = P_{N} \]
\[ P_{cr} = 3000 \text{-kN} \]
\[ P_{cr} = \frac{\pi^2 \cdot E \cdot I}{L_e^2} \]
\[ I = \frac{P_{cr} \cdot L_e^2}{\pi^2 \cdot E} \]
\[ I = 26.386 \times 10^6 \text{ mm}^4 \]

Moment of inertia
\[ I = \frac{\pi}{64} [d^4 - (d - 2 \cdot t)^4] \]
\[ t_{\text{min}} = \frac{d - \sqrt{d^4 - I \cdot \frac{64}{\pi}}}{2} \quad t_{\text{min}} = 9.73 \text{-mm} \]
A-9.10: Two pipe columns are required to have the same Euler buckling load $P_{cr}$. Column 1 has flexural rigidity $EI$ and height $L_1$; column 2 has flexural rigidity $(4/3)EI$ and height $L_2$. The ratio ($L_2/L_1$) at which both columns will buckle under the same load is approximately:

(A) 0.55
(B) 0.72
(C) 0.81
(D) 1.10

**Solution**

Equate Euler buckling load expressions for the two columns considering their different properties, base fixity conditions and lengths:

$$\frac{\pi^2 EI}{(0.699 L_1)^2} = \frac{\pi^2 \left(\frac{2}{3}E\right) (2I)}{L_2^2}$$

Simplify then solve for $L_2/L_1$:

$$\left(\frac{L_2}{0.699 L_1}\right)^2 = \frac{4}{3}$$

$$\frac{L_2}{L_1} = \sqrt{\frac{4}{3} \cdot 0.699^2} = 0.807$$

A-9.11: Two pipe columns are required to have the same Euler buckling load $P_{cr}$. Column 1 has flexural rigidity $EI$ and height $L_1$; column 2 has flexural rigidity $(2/3)EI$ and height $L$. The ratio ($I_2/I_1$) at which both columns will buckle under the same load is approximately:

(A) 0.8
(B) 1.0
(C) 2.2
(D) 3.1

**Solution**

Equate Euler buckling load expressions for the two columns considering their different properties, base fixity conditions and lengths:

$$\frac{\pi^2 EI_1}{(0.699 L_1)^2} = \frac{\pi^2 \left(\frac{2}{3}E\right) (I_2)}{L^2}$$

$$\frac{2E/3 I_2}{L}$$

$$\frac{I_2}{I_1} = \sqrt{\frac{4}{3} \cdot 0.699^2} = 0.807$$
Simplify then solve for $I_2/I_1$:

$$\frac{I_2}{I_1} = \frac{\frac{L^2}{(0.699-L)^2}}{\frac{E}{2}}$$

$$\frac{I_2}{I_1} = \frac{\frac{3}{2}}{(0.699)^2} = 3.07$$