IN THEIR OWN WORDS

Jean-Philippe Rameau, *Treatise on Harmony* (1722)

Jean-Philippe Rameau (1684–1764) was an important composer and music theorist working in Dijon and Paris, France, at the beginning of what has come to be called the Enlightenment. Having a scientific bent to his thinking, Rameau was much interested in the recent writings of mathematicians and philosophers, such as René Descartes (see Chapter 34) and Joseph Sauveur (1656–1717). In 1701 Sauveur had expounded in print a principle hugely important to our understanding of music: the overtone series. In the course of his musical writings, Rameau only gradually came to recognize and acknowledge the importance of Sauveur's discovery—that there was a natural, or physical, basis for why Western music has developed over the centuries with primacy accorded to the intervals of the octave, fifth, and fourth, and major and minor thirds, and has made use of the triad as the fundamental building block of tonal music. In Sauveur's discovery Rameau came to find a purely scientific explanation for the theory that Rameau, ironically, had been heading toward from an entirely different direction—by means of the old math involved in Greek intervallic ratios (see Chapter 1) and in the division of the medieval monochord (see Chapter 2). There was more in the way of "natural principles" involved in Rameau's new theory of chord structure than even he at first suspected.

Book I of Rameau's *Treatise on Harmony* is entitled "On the Nature and Properties of Chords and on Everything That May Be Used to Make Music Perfect." In its initial chapters, Rameau uses an old division of the monochord (2:1, 3:2, 4:3, 5:4, 6:5) to generate the octave, fifth, fourth, major third, and minor third in a system akin to what we call just intonation (both Zarlino in the sixteenth century and Ptolemy back in the first century C.E. had advocated it). In Chapter 7, "On the Harmonic Division or the Origin of Chords," Rameau shows through simple arithmetic how the triad can be taken to be the primary building block of tonal harmony. In Chapter 8, "On the Inversion of Chords," he demonstrates how the triad can be inverted and establishes the terminology of "sixth" chord and "six-four" chord, as well as the concept of "perfect" and "imperfect" triads.

Chapter 7: On the Harmonic Division or the Origin of Chords

The harmonic division according to our system is nothing other than the arithmetic division, and it generates as a harmonic mean only the fifth and two thirds. That is because if the fourth and other intervals enter in, it is only by means of the octave. All other intervals that one might perceive are generated only by the sounds of the fifth and these thirds, so that whatever arbitrary combination of sounds that harmony might present us, in order to demonstrate the force of its perfection, we should not lose sight of the fact that a principle is always at work [the fundamental nature of the triad]. Now the fifth and the thirds not only divide all the principal chords but also create them, be it through the process of squaring [their ratios], or be it through the process of their addition. If we wish to apply the rules of multiplication and subtraction to these basic intervals, we can extract all harmonious chords. For example, the multiplication of the two thirds generates the fifth; and by means of their subtraction, we arrive at the harmonic mean of this fifth:

Major third	4:5		20
	×		
Minor third	5:6	Product of this multiplication	30 [= 3:2 given the fifth]
		Product of this subtraction 24:25	

Perfect chords: 20: 25: 30 [creating the major third] or 20: 24: 30 [creating the minor third] 20 and 30 divided at 25 give us the prefect chord that we call major. In this case the fifth is divided by the major third below. Those same numbers divided at 24 give us the perfect chord that we call minor, in that one the fifth is divided by the minor third below. Moreover, these numbers 24:25 give the ratio of the minor semitone [a very small one, compared to other tunings and temperaments], which is the difference between the major third and the minor third.

The square of the major third gives the augmented fifth and that of the minor third gives the diminished fifth. The subtraction of each square divides harmonically each of these intervals.

Major third	4:5	Minor third	5:6
	4:5		5:6
Product of multiplication	16:25		25:36
Product of subtraction	20		30

It is important to observe now that there is no chord that can be called perfect if it doesn't posses the fifth, nor consequently without the presence of the two thirds that comprise the fifth. This is true because this is the perfect chord that is formed from their union, and from this all chords have their origin. From this it follows that if the fifth cannot be heard in a chord, the root [*fondament*] is in an inverted position, implied or extracted, unless the chord is incomplete; failing this, the chord will not be valid. Thus we have not given the name "chord" to the diminished fifth and augmented fifth divided harmonically because the resultant chord is not complete [because it does not contain a pure fifth]. . . .

If there are harmonious chords in addition to the preceding perfect chords, they must have been formed by one perfect [chord] and one of its members, that is, one of its thirds. For example, the addition of a third to the fifth will give us the interval of the seventh, and the subtraction of it will yield the basic chord. [Adding now a minor and a major third above a fifth:]

Minor third 5:6
Fifth 2:3
Product of multiplication: 10:18 [ratio of minor seventh]
Product of subtraction: 12:15 [ratio of major third]
Seventh chord: 10:12:15:18 [ratios of a minor seventh chord]
Major third 4:5
Fifth 2:3
Product of multiplication: 8:15
Product of subtraction: 10:12
Seventh chord: 8:10:12:15 [ratios of major seventh chord] . . . [pp. 29ff]

Chapter 8: On the Inversion of Chords

If there are only three accordant numbers [2, 3 and 5] (as Descartes says), it is appropriate to notice that there are also only three principal consonances, those being the fifth and the two thirds, from which are derived the fourth and the sixths. Let us see how to differentiate these consonances among chords.

Article 1: On the Perfect Major Chord and Its Derivatives

Of the three first numbers 2, 3, and 5 we will substitute 4 and 6 so that the fifth is divided into two thirds, as it should be. Thus the perfect major chord is formed from these three numbers: 4:5:6, and if we transpose 4 [the root] up an octave, we get 5:6:8 [8, of course, being a doubling of 4]. This gives us what we call a "sixth" chord because in this case the interval of a sixth is to be understood between the two extremes. If we transpose 5 [the first third of the chord] up an octave, we get 6:8:10. This gives us another chord that we call "six-four" because the intervals of a sixth and a fourth are to be understood between the high and low sounds, to which all intervals of a chord should be compared. If we transpose 6 to its octave, we get 8:10:12, which is the same as our original proportion 4:5:6. It is for this reason that it is pointless to continue to transpose in this manner because the perfect chord, consisting of only three different pitches, can thus only produce in this way three different chords, of which it is the first and most basic.

Although the two derivative chords [the sixth and six-four chords] come from the perfect one and are consonant, they are called imperfect, not only to distinguish them from the principal one but also because their quality is different . . . [pp. 34ff]

Source: Extracts translated from the original French in *Traité de l'harmonie*.