he twentieth century has witnessed a number of dangerous, destabilizing, and expensive arms races. The outbreak of World War I (1914–1918) climaxed a rapid buildup of armaments among rival European powers. There was a similar mutual accumulation of conventional arms in the years just prior to World War II (1939–1945). The United States and the Soviet Union engaged in a costly nuclear arms race during the forty years of the Cold War. Stockpiling of ever more deadly weapons is common today in many parts of the world, including the Middle East and the Balkans.

The British meteorologist and educator Lewis F. Richardson (1881–1953) developed a number of mathematical models to help analyze the dynamics of such arms races. Richardson's primary model was based on mutual fear: a nation is spurred to increase its arms stockpile at a rate proportional to the level of armament expenditures of its rival. Richardson’s model takes into account internal constraints within a nation that slow down arms buildups: the more a nation is spending on arms, the harder it is to make greater increases, because it becomes increasingly difficult to divert society's resources from basic needs such as food and housing to weapons. Richardson also built into his model other factors driving or slowing down an arms race that are independent of levels of arms expenditures.

The mathematical structure of this model is a linked system of two first-order differential equations. If \( x \) and \( y \) represent the amount of wealth being spent on arms by two nations at time \( t \), then the model has the form

\[
\frac{dx}{dt} = ay - mx + r
\]

\[
\frac{dy}{dt} = bx - ny + s,
\]

where \( a, b, m, \) and \( n \) are positive constants and \( r \) and \( s \) are constants that can be positive or negative. The constants \( a \) and \( b \) measure mutual fear; the constants \( m \) and \( n \) represent proportionality factors for the "internal brakes" to further arms increases. Positive values for \( r \) and \( s \) correspond to underlying factors of ill will.
or distrust that would persist even if arms expenditures dropped to zero. Negative values for \( r \) and \( s \) indicate a contribution based on good will.

The dynamic behavior of this system of differential equations depends on the relative sizes of \( ab \) and \( mn \) together with the signs of \( r \) and \( s \). Although the model is a relatively simple one, it allows us to consider several different long-term outcomes. It's possible that two nations might move simultaneously toward mutual disarmament, with \( x \) and \( y \) each approaching zero. A vicious cycle of unbounded increases in \( x \) and \( y \) is another possible scenario. A third eventuality is that the arms expenditures asymptotically approach a stable point \((x^*, y^*)\) regardless of the initial level of arms expenditures. In other cases, the eventual outcome is very dependent on the starting point. Figure 1 shows one possible situation with four different initial levels, each of which leads to a "stable outcome."

Richardson's pioneering work has led to many fruitful applications of models of differential equations to problems in international relations and political science.

References

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