I Will if You Will...
Individual Thresholds and Group Behavior

JoAnne S. Gowney
I WILL IF YOU WILL... INDIVIDUAL THRESHOLDS AND GROUP BEHAVIOR

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APPLICATIONS OF ALGEBRA TO GROUP BEHAVIOR

Students (college freshmen) in a general education survey mathematics course or in a course in the behavioral sciences

The critical mass model gives insight into situations in which group members must choose between two opposite behaviors and in which the choice of each member depends on how many others do likewise. The model is applied to examples in which individuals must decide to cheat or not to cheat, to attend or not to attend, to participate or not to participate. Students will see and participate in the process of traveling back and forth between a mathematical model and the real world as they solve problems. They will learn to analyze cumulative frequency distributions and apply their analysis to a variety of situations involving group behavior.

Some experience with graphing and the concept of slope.
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The goal of UMAP was to develop, through a community of users and developers, a system of instructional modules in undergraduate mathematics and its applications to be used to supplement existing courses and from which complete courses may eventually be built.

The Project was guided by a National Advisory Board of mathematicians, scientists, and educators. UMAP was funded by a grant from the National Science Foundation and is now supported by the Consortium for Mathematics and Its Applications (COMAP), Inc., a nonprofit corporation engaged in research and development in mathematics education.

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1. Introduction

Honor codes receive a great deal of attention at U.S. Military Academies and some degree of attention at other educational institutions. It is surely true that every educational institution is concerned about cheating and deals with the problem to some degree. Some prefer to let individual instructors deal with it case by case. Others have policies that list at least the ranges of punishable offenses and permitted punishments. Still others prefer that the problem receive as little recognition as possible, fearing that calling attention to cheating has a net effect of increasing it.

We are uncertain of the effects of crime publicity on the crime rate. We do believe, however, that many decisions made by individuals about how to behave are based on how each thinks that others will behave under similar circumstances. In this module we describe a method for analyzing the cumulative effect of individual attitudes on group behavior. We will introduce our method with an example about cheating.

Our presentation below illustrates the role of mathematics as "helper" in dealing with a situation that is primarily nonmathematical. As we investigate a problem of how to combat cheating, we demonstrate the interaction between mathematics and a real problem situation. While the solution to the problem lies outside mathematics, nevertheless, our mathematical analysis gives insight into how to solve it. Many applications of mathematics require frequent trips back and forth between a problem situation and mathematical processes, repeatedly asking new questions, experimenting with new procedures, and checking new answers. Our discussion of threshold analysis illustrates this modus operandi.

2. A Prototype Example

2.1 To Cheat or Not to Cheat?—That Is the Question

Professor C. F. Ogive, a statistics instructor at Cranwell State College, became concerned about an alarming rate of cheating in his Introductory Statistics course. To gain insight into what to do, Professor Ogive decided to gather some information about his students' attitudes toward this problem. He surveyed his class of 100
students and asked each to respond to the following questions:

1. Which of the following situations would you prefer (check one)?
   _____ (a) No student will cheat.
   _____ (b) Every student feels free to cheat.

2. Regardless of how you answered question 1, please respond to one of the following statements:
   _____ I will never cheat in this Introductory Statistics course.
   _____ I will cheat in this Introductory Statistics course when at least _____ of the class members choose to cheat.

Instructions for filling in the final blank

Use a number that designates the smallest group of cheaters (within the class size of 100) which you would be willing to be part of. For example, if you are determined not to cheat unless everyone else does, you would fill in 100; if you would be willing to cheat as part of a group of 20 cheaters (but no smaller size group), fill in 20. The value you supply is called your cheating threshold for this class situation. Round your cheating threshold to a multiple of 10; this will make the results easier to tabulate. For example, if you would be willing to cheat if only a small group of people are doing so, then fill in 10 as a value (rather than 4 or 7 or 12).

The results of the survey that Professor Ogive conducted are provided below:

1. (a) Number of students preferring no cheating: 98.
   (b) Number of students preferring universal cheating: 2.

As he examined the tabulated data, Professor Ogive began to list some observations:

Observation 1: 10 of the 100 students surveyed indicate that they will never cheat, no matter what the others may do.
Observation 2: 90 of the 100 students surveyed are willing to cheat under certain circumstances.
### Table 1.
Frequency Tabulation of Student Survey Responses

<table>
<thead>
<tr>
<th>Frequency Tabulation of Cheating Thresholds (group sizes)</th>
<th>Student Survey Responses: Number of Students with Given Threshold</th>
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</thead>
<tbody>
<tr>
<td>0</td>
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<td>100</td>
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<tr>
<td>No Cheating Threshold</td>
<td>10</td>
</tr>
</tbody>
</table>

**Exercise**

1. (a) Professor Ogive’s survey indicated that 98 students preferred a class situation with no cheaters, whereas only 10 students indicated that they would never cheat. Explain why these results are not necessarily contradictory. (b) The survey results state that 90 of the 100 students are willing to cheat if enough others do, yet only 2 students responded in favor of universal cheating. Are these results contradictory?

**Having watched his students for about half a semester, Professor Ogive was aware that cheating had increased. In the beginning it seemed as if only a few students cheated—probably no more than normal. The number engaged in cheating had increased until now, midway through the semester, surely half the class had become involved. He feared it would increase still further. He turned to his survey results to see what further insights they might provide.**

Professor Ogive wanted a way to assess the extent to which cheating by some students caused cheating by others. He extended Table 1 to obtain Table 2. Each entry in column 3 of Table 2 is the sum of the corresponding entry and all those above it from column 2.

**Table 2 confirmed Professor Ogive’s worries. He examined its values and noted the following additional observations.**

**Observation 3:** If class members are aware of each others’ thresholds and if a stressful situation suggests the need to cheat, the 15 students whose cheating thresholds are 10 are likely to cheat. This will occur because that group of 15 is large enough to satisfy all of its members’ thresholds.
Table 2.
Frequencies and Cumulative Frequencies
of Student Survey Responses

<table>
<thead>
<tr>
<th>Cheating Thresholds (group sizes)</th>
<th>Number of Students with Given Threshold</th>
<th>Number of Students with Thresholds Not More than Given Threshold</th>
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</thead>
<tbody>
<tr>
<td>0</td>
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<td>No Cheating Threshold</td>
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</tbody>
</table>

Observation 4: If class members are aware of each others' thresholds and if a stressful situation suggests the need to cheat, the 15 students whose cheating thresholds are 20 are likely to cheat. This will occur because they, together with those who will cheat with less provocation, form a group (of 30) large enough to satisfy all of its members' thresholds.

Exercise
2. Following the pattern of Observations 3 and 4, develop a rationale for each of the following observations:

Observation 5: Those 10 students whose threshold is 30 are likely to cheat if they know the thresholds of the other class members and if the need to cheat arises.

Observation 6: Those 30 students whose threshold is 90 are likely to cheat if they know the thresholds of the other class members and if the need to cheat arises.

Professor Ogive debated, "What shall I do with these survey results?" One view said that sharing the results with his class could be damaging since each would then know the position of other class members and many students would then be able to justify their own cheating. However, further thought suggested that the students already had this information and were acting on it.
Believing the latter, Professor Ogive decided to discuss the survey results with his class. Persuaded that "a picture is worth a thousand words," he constructed a graphical display (Figure 1) of the data collected.

Threshold values are located along the horizontal axis in Figure 1. The vertical axis indicates the number of students who will cheat when a given threshold is satisfied. A dotted line, called the equilibrium line, indicates points with equal numerical coordinates. Data points lie on the equilibrium line when the threshold and the number who are willing to cheat are equal.

Data points are obtained directly from columns 1 and 3 of Table 2. The segments joining the points are drawn solely to aid the eye in viewing the graph. In general, no meaning should be assigned to graphed values between the original data pairs. (For an exception to this, see Exercise 12.)

As we move along our graph from left to right, we observe that beyond (0, 0), until we reach the point (90, 90), the first component of each pair is greater than the second. For each such point, the number of students who will cheat is greater than the threshold. If the survey incorporates honest information from the students, we would expect that, as time passes and the students get to know each other, the class behavior will tend toward a situation in which 90 of the 100 students cheat.
Exercises
3. Reexamine Table 2 and Figure 1 and develop an argument in support of the last sentence above, "The class behavior will tend toward a situation in which 90 of the 100 students cheat."

4. In Professor Ogive's statistics class, if cheating activity were highly secretive and no class member knew of the opinions or of the cheating activity of others, what class behavior would you expect to result?

2.2 A Second Survey

As we might reasonably suppose, the statistics class was dismayed and embarrassed by the results of the cheating survey. In the survey, 98 of the 100 class members had indicated that they would prefer "no cheating" to "universal cheating," and yet their combined willingness to cheat, if others also do so, had led to a situation in which almost everyone would cheat. After discussion of the problem among themselves, many agreed that they should and would like to use more restraint in their own willingness to cheat. That is, each should raise his cheating threshold.

To evaluate the results of their discussion, the students requested that the survey be conducted again. This was done. Responses to survey item 1 remained the same. Responses to item 2 are summarized in Table 3.

<table>
<thead>
<tr>
<th>Cheating Thresholds (group sizes)</th>
<th>Number of Students with Given Threshold</th>
<th>Number of Students with Thresholds Not More than Given Threshold</th>
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<tr>
<td>No Cheating Threshold</td>
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Table 3.
Frequencies and Cumulative Frequencies of Second Survey Responses
These tabulated values show increased restraint on the part of many students. Now, for example, only 50 of the students (as compared with 80 on the first survey) will cheat if half of their classmates engage in the practice. As with the first set of data, we can use a graph to aid in analysis. Figure 2 summarizes the results of the second survey.

Inspection of Figure 2 identifies four data points that lie on the equilibrium line: (0, 0), (10, 10), (50, 50), and (90, 90). Because our analysis of the first survey concluded that class behavior would tend toward a situation with 90 cheaters, a situation identified in Figure 1 by the equilibrium point (90, 90), we again turn to examination of equilibrium points for insight into the class behavior that will emerge.

Each equilibrium point designates a situation in which attitudes (expressed in terms of thresholds) and behaviors of group members coincide. When a graph of threshold data shows more than one equilibrium point, environmental conditions will influence the group in its selection of an equilibrium. Here are some possible scenarios:

50 Students Cheat:
Professor Ogive perceives this as the present situation. Because this state of affairs is one in which attitudes and behavior match, it could continue.

10 Students Cheat:
The statistics class is concerned about cheating. This concern may lead to a cooperative effort to reduce the amount of cheating. (Perhaps Professor Ogive will conduct review sessions or take
other steps to reduce student worries that lead to consideration of cheating as a possible strategy; perhaps the students will establish group study sessions to help each other.) Within this environment, a class in which many are cheating may be able to reduce cheating behavior if each member acts on the belief that others are cooperating. If, for example, class members believe that the number of cheaters is no more than 40, then only the 35 students with thresholds of 40 or less will be willing to cheat. Once this is known, since only 25 students have thresholds less than 40, the number of cheaters will drop again, to 25. Following this chain of reasoning, we can deduce that the number of cheaters will continue to drop, until only 10 are willing to cheat. At this point, attitudes and behaviors match, and the situation may be expected to persist.

No Students Cheat:

The equilibrium point (0, 0) is a by-product of the way we formulated the survey. Without additional information we cannot be sure that a situation with no cheaters can be achieved.

90 Students Cheat:

If, as Professor Ogive suspects, the number of cheaters is about 50, and if their cheating activity is very public—perhaps they even brag about it—the non-cheaters may become convinced that at least 60 of the class members are cheating. If this is perceived then, since 90 of the class members have thresholds of 60 or less, there may be a big jump in cheating activity to the level with 90 students cheating. If an equilibrium of 90 is achieved, it will be difficult to change.

Exercise
5. (a) Just above, the following statement is made:

"If an equilibrium of 90 is achieved, it will be difficult to change."

Do you agree with the statement? Supply reasons for your view.

(b) Suppose that the statistics class contains no more than 40 students who are cheating. In your own words, and with more detail than given above, explain how the number of cheaters can decline to 10.

3. Formal Characteristics of Threshold Analysis

Threshold analysis, the process that we have used in describing the cheating situation in Professor Ogive's statistics class, can be
applied in a variety of situations. The characteristics necessary for a situation to be amenable to threshold analysis are these:

(1) There is a group of \( N \) individuals, each of whom has a choice between two opposite behaviors, one of which may be called \( B \); each member of the group may choose either to engage in Behavior \( B \) or to refrain from Behavior \( B \).

(2) Each individual either asserts, "I will never choose Behavior \( B \)," or is able to supply a threshold value \( x \) (\( x \) is greater than 0 but not greater than \( N \)) to complete the following statement:

"I will choose Behavior \( B \) when at least \( x \) members of the group choose \( B \). Otherwise I will not engage in Behavior \( B \)."

In some cases (as with the cheating survey), it is convenient to require threshold values to be rounded to multiples of 5 or 10 or some other suitable value. When \( N \) is large, it may be convenient to express threshold values as percentages.

Once individual responses have been collected and the frequencies and cumulative frequencies have been tabulated, a graph can be constructed. The graph is the keystone of threshold analysis.

Certain characteristics are always present in graphs of threshold data. Two of these are:

(1) If \( A \) is a data point to the right of data point \( B \), then \( A \) cannot be lower than \( B \). (That is, for data points \((x_1, y_1)\) and \((x_2, y_2)\), if \( x_2 > x_1 \), then \( y_2 \geq y_1 \).)

(2) A graph of threshold data will have at least one equilibrium point (i.e., a point \((x, y)\) for which \( x = y \)).

**Exercises**

6. Explain why a graph of threshold data must have characteristic (1).

7. Explain why a graph of threshold data must have characteristic (2).

On a graph of threshold data, points \((x, y)\) for which \( x = y \) are called equilibrium points. Equilibrium points are identified as stable or unstable depending on the slope of the graph at the point of intersection. (Recall that the slope of a (non-vertical) line segment joining points \((x_1, y_1)\) and \((x_2, y_2)\) is equal to the ratio \( (y_2 - y_1)/(x_2 - x_1) \).)

If the segment to the left of an equilibrium point has slope less than 1, the equilibrium is identified as stable against increases. If the segment to the right of the equilibrium point has slope less than 1, the equilibrium is stable against decreases. An equilibrium that is stable
against decreases and stable against increases is stable. Equilibria that have no type of stability are unstable.

Let us examine the stability of the four equilibrium points of Figure 2:

Equilibrium point $(0,0)$ is unstable. The slope of the segment to the right of $(0,0)$ is 1.

Equilibrium point $(10,10)$ is stable against increases. The slope of the segment to the left of $(10,10)$ is 1, and the slope of the segment to the right of $(10,10)$ is $1/2$.

Equilibrium point $(50,50)$ is unstable. The slope of the segment to the left of $(50,50)$ is 1.5, and the slope of the segment to the right of $(50,50)$ is 4.

Equilibrium point $(90,90)$ is stable. The slopes of the segments on both sides of the segments of $(90,90)$ are 0.

Assessment of the stability of various equilibria can be far more complex than our simple classification scheme. The magnitudes of the slopes and the widths of the intervals over which a level or steep slope persists are factors that can be utilized in a more discriminating assessment of degrees of stability.

Important factors in determining the group behavior that will result from the aggregation of individual thresholds include how members of the group learn and how much they know about what other members are really doing. Some, for example, may observe only a few close friends rather than considering the group as a whole; if half of a particular student's friends are cheating, that student may suppose that this proportion is true of the entire class and may act on the basis of this incorrect supposition.

**Exercises**

8. Answer questions (a) and (b) for each of the non-zero equilibrium points $(x, y)$ of Figure 2.

(a) If class behavior has stabilized with $x$ students cheating and if external conditions cause $x$ to decrease by 10, what can be expected to follow this change?

(b) If class behavior has stabilized with $x$ students cheating and if external conditions cause $x$ to increase by 10, what can be expected to follow this change?

(c) Suppose that some members of Professor Ogive's class are inaccurate in their assessments of what others are doing. How does this affect the overall group behavior?
9. In the Introduction we remarked that some institutions fear that calling attention to cheating may increase the amount of cheating. Consider this fear in connection with Professor Ogive's class: what was the effect of publicity on cheating in this case?

Give reasons that support the view that publicity helps deter cheating. Also give reasons that support the view that publicity encourages cheating.

10. Class participation is a behavior to which threshold analysis can be applied. Suppose a psychology class of 50 students was surveyed and gave the following responses.

A/R: Responses to question 1 do not form a part of threshold analysis. Instead they provide a context for interpreting the results of the analysis. Using the responses to question 1 we can determine whether group behavior coincides with what the members of the group actually prefer.

1. 5 I prefer a class in which no students participate.

50 I prefer a class in which all students participate.

2. Responses to the statement, "I will participate when at least \( x \) of the class members (including me) participate," are given in Table 4. The number of students responding, "I will never choose to participate," is shown in the last line of Table 4.

Complete Table 4, make a graph of the threshold data, and identify equilibrium points. What level of participation is likely to result in this psychology class? Explain how you determined this. What factors can cause it to change?

Table 4.

<table>
<thead>
<tr>
<th>Participation Thresholds</th>
<th>Number of Students with Given Threshold</th>
<th>Number of Students with Thresholds Not More than Given Threshold</th>
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4. Another Example—The Optional Problem Session

A situation familiar to many mathematics students is the “dying problem session.” In a course where a great deal of class time is devoted to the explanation of new material, students often request special sessions for discussion of assigned “homework” problems.

Typically, in such situations, the instructor inquires how many class members are interested, and most say they are. The instructor then schedules the sessions at a time convenient to as many as possible and the sessions begin. Since these sessions are extra, rather than part of the formal course requirements, attendance is optional.

More often than not, the problem sessions last only a few weeks. Attendance is good at the first session, but decreases steadily after that until finally no one shows up. Why does this occur? Do students really not want the extra sessions after all? Are the sessions poorly conducted and of little value? What factors lead to this inescapable death?

We can turn to threshold analysis for a possible explanation. For specificity, we consider a mathematics class with 50 members who are faced with the choice of problem session attendance. Within the class, 40 of the members want the problem sessions and attend the first session. However, each attendee is conscious of how many others attend. None wants to attend and be the center of attention, but each wishes to listen to the discussions of others. Student attitudes toward attendance are summarized in Table 5. As before, a graph will aid

<table>
<thead>
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<th>Attendance Thresholds</th>
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Table 5.
Frequencies and Cumulative Frequencies of Math Class Attitudes Toward Problem-Solving Attendance

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<thead>
<tr>
<th>Attendance Threshold</th>
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<tr>
<td>No Attendance</td>
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<tr>
<td>Threshold</td>
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our analysis. We construct Figure 3. The only equilibrium point is 
(0, 0), which is stable. This is consistent with our description of the 
problem session as a "dying" one.

Let us take time to trace through the dying process. It is reasonable to suppose that each person who attends the problem session on a given day uses that day’s attendance as an estimate for the number expected to attend the following session. As we have already indicated, 40 students attend the first session. However, there are only 30 students who are satisfied with a group size of 40, and so only those 30 attend the second session. Since there are only 20 students who are satisfied with a group size of 30, the third sessions attract only 20 students. Of these 20, only 10 are satisfied with that group size. The fourth session thus has only 10 students in attendance. These 10 students are unwilling to be part of such a small group, and no one shows up the following week. Although most of the class “wanted” the sessions, their attendance depended on how many others did so rather than being an unconditional preference. Thus the problem session “died.”

Exercise
11. Suppose the mathematics instructor, in the optional problem session example described above, was unable to find a room large enough to accommodate more than 30 students a a single session. He thus decided to schedule two sessions, each at a time suitable to half of his class of 50 students. Based on these new circumstances, the students revised their thresholds to those of Table 6(a) and Table 6(b). The two halves of the class are designated as Group I and Group II.

Complete Table 6(a) and Table 6(b), construct graphs of the threshold data, and identify equilibrium points. What is likely to happen to each problem session? Explain how you determined this. What factors could cause a change in the “death” or “continued life” of either problem session? Compare the results of this exercise with the whole-class situation analyzed previously.
5. Summary and References

Threshold analysis supplies a method for analyzing situations rather than a method for solving problems. In other words, it helps us to understand how things are, instead of telling us what to do. Despite this limitation, threshold analysis can still be useful in problem solving. For example, Professor Ogive's statistics class used threshold
analysis as a basis for making changes in a situation that they didn’t like.

The following paper is a valuable reference for individuals interested in learning more about threshold analysis:


Two items that Granovetter mentions that are pertinent to our discussion are:

1. Thresholds vary from situation to situation. (An individual’s cheating threshold, for example, is not a single number that he carries with him from one class to another, but instead results from the evaluation of costs and benefits to him of different behaviors in a particular situation.)

2. There are cases in which a small change in the distribution of thresholds can generate a large difference in group behavior. (For an illustration of this, see Exercise 13.)

A second highly readable source of further information about threshold analysis is the following book:


In Chapter 3 of this book, Schelling discusses critical mass models and incorporates much of what we have called threshold analysis. The term “critical mass” has been adopted from nuclear engineering, where it refers to the amount of a radioactive substance necessary to sustain a chain reaction. In Professor Ogive’s statistics class, the situation portrayed by the first survey can be described in these terms. At the beginning of the semester the existence of a few cheaters provided a “critical mass” that satisfied the cheating thresholds of others, and a “chain reaction” took place. In like manner, the outcome of the mathematics problem session can be translated as: the problem session died because there was not a “critical mass” to sustain it.

As we end our introduction to threshold analysis, a word of caution is in order. When you apply threshold analysis to study new situations—real situations that you seek to understand or improve—you may run into difficulties not encountered in the exercises of this module. Survey data may be hard to get, hard to summarize neatly, and hard to interpret with any but very tentative predictions about group behavior. Such difficulties are not unusual; seldom do real problems fit exactly into the patterns we have
learned. However, a variety of approaches, tried with patient persistence, ultimately will lead to some insights into confounding situations.

6. Additional Exercises and Project

Exercises

12. Although graphs of threshold data must intersect the equilibrium line, these intersections need not occur at data points. (See Figures 4(a) and 4(b).) Since we have stipulated that no meaning is assigned to points on the segments between data points, we face the dilemma of how to interpret these crossings.

(a) Explain why the behavior of the group whose threshold data is summarized in Figure 4(a) can be expected to stabilize with 45 members choosing Behavior B.

(b) Explain why the behavior of the group whose threshold data is summarized in Figure 4(b) will stabilize with either 0 members or 100 members choosing Behavior B.

(c) Generalize from your analysis of Figures 4(a) and 4(b) to complete the following statements:

1) If a graph of threshold data has a slope less than 1 when it crosses the equilibrium line at a non-data point, then ____________.

2) If a graph of threshold data has a slope greater than 1 when it crosses the equilibrium line at a non-data point, then ____________.


Near Bordentown and Grand Forks are sections of land being considered as sites for proposed nuclear power plants. Within each of these cities 100 individuals have formed a group called Nuclear Concern. These people are determined to educate themselves about the merits and dangers of nuclear plants—and then to take appropriate action. The degree of concern among group members varies
from individual to individual. Each group met recently to decide whether to stage a protest in front of the executive offices of the electric power company that has proposed nuclear construction. The attitudes of group members and the outcomes of their meetings are given below.

**Bordentown Nuclear Concern member attitudes:**
- one member favors protest even if no one else does;
- one member favors protest if at least one other agrees;
- one member favors protest if at least two others agree;
- one member favors protest if at least three others agree;
  ... one member favors protest if at least 99 others agree.

**Result of recent meeting:** the group decided to stage a protest.

**Grand Forks Nuclear Concern member attitudes:**
- one member favors protest if at least one other agrees;
- one member favors protest if at least two others agree;
- one member favors protest if at least three others agree;
  ... one member favors protest if at least 99 others agree;
- one member does not favor protesting no matter how many others do.

**Result of recent meeting:** The group showed insufficient interest to stage a protest.

Examination of the attitudes of the members of the two Nuclear Concern groups reveal that they are almost identical. Threshold analysis can provide a possible explanation of why their recent meetings led to different outcomes.

(a) Invent a brief description of how the meeting of the Bordentown Nuclear Concern group might have proceeded toward the decision to stage a protest.
(b) Invent a description of how things may have gone at the Grand Forks Nuclear Concern meeting in which the group did not decide to protest.
(c) Suppose that Mike Norad—who is committed to protesting nuclear power plants no matter what others do or don’t do—moves to Grand Forks and joins the Nuclear Concern group there. Describe what can be expected to happen.
(d) Observers of Nuclear Concern’s activity after Mike Norad’s arrival in Grand Forks might conclude that he changed a lot of people’s minds about protesting. Develop an explanation that would show these observers how the protest could have evolved without anyone changing his earlier attitude.

14. A certain Introduction to Sociology class was studying how a fad catches on. One student, who had spent her summer at a Jersey Shore resort, mentioned a fad that had swept the beach community—males shaving their heads. As a class experiment, the sociology class collected head-shaving threshold data from the 100 males enrolled in that course. The results are given in Table 7.

(a) Complete Table 7. Graph the threshold data and identify equilibrium points.
(b) If, at the time that the class was surveyed to obtain threshold information, none of the 100 males had shaved their heads, would you expect any changes to result? Explain why or why not.
(c) Suppose that 30 of the males in the sociology class have their heads shaved as a fraternity initiation requirement, explain how this could result in 70 of the males eventually sporting shaved heads.
Table 7.

<table>
<thead>
<tr>
<th>Head Shaving Threshold</th>
<th>Number of Males with Given Threshold</th>
<th>Number of Males with Threshold Not More than Given Threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>15</td>
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<td>30</td>
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<td>40</td>
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<td>60</td>
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<td></td>
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<td>70</td>
<td>0</td>
<td></td>
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<tr>
<td>80</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Number of males in Intro. to Sociology Who Will Never Shave Their Heads: 20

15. (This exercise has been adapted from an example suggested by Thomas C. Schelling of Harvard University.) Consider Bowie, a hypothetical “western” town in which the typical person’s decision on whether or not to carry a gun depends on how many others in the town wear guns. In general, the more guns that are seen on others, the more people that will be motivated to carry guns themselves. Some may wear guns regardless of what others do; some may refuse to wear guns no matter how many wear them; but most people will wear guns if enough others do. The meaning of “enough” varies from person to person.

We can apply threshold analysis to the situation. Suppose that the 5617 adult residents of Bowie have been surveyed with the following results. (For convenience, percents have been used instead of numbers.)

Bowie gun-carrying survey results

1. 90% I prefer a situation in which no (adult) resident of Bowie carries a gun.
10% I prefer a situation in which all (adult) residents of Bowie carry guns.
2. Responses to the statement “I will carry a gun when at least x% of the (adult) residents of Bowie do so” have been summarized in Table 8. The percent of residents who said “I will never carry a gun” is recorded in the last line of Table 8.

Apply threshold analysis to the data of Table 8 and describe the group behavior that is likely to result in Bowie as a result of the individual attitudes expressed above.

16. (continuation of Exercise 15) A resident of Bowie, Citizen Disarm, who observed that most of the residents of Bowie prefer that no one carry a gun, became concerned about the way the situation had been described. Disarm thought that people’s thresholds for gun carrying had been affected by the way the problem was posed. He proposed a different point of view. Consider instead the behavior of refusing to carry a gun and determine people’s thresholds for this behavior. Disarm took the data of Table 8 and reinterpreted it to obtain Table 9.
Table 8.

<table>
<thead>
<tr>
<th>Gun-Carrying Thresholds (percents)</th>
<th>Percent of Bowie Adult Residents with Given Threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>25</td>
<td>35</td>
</tr>
<tr>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>75</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>15</td>
</tr>
</tbody>
</table>

Percent of Bowie Adults with No Gun-Carrying Threshold

10

Table 9.

<table>
<thead>
<tr>
<th>Gun-Refusal Thresholds (percents)</th>
<th>Percent of Bowie Adult Residents with Given Threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>75</td>
<td>40</td>
</tr>
<tr>
<td>100</td>
<td>35</td>
</tr>
</tbody>
</table>

Percent of Bowie Adults with No Gun-Refusal Threshold

0

Explain how Disarm could have obtained the values for Table 9. Apply threshold analysis and describe the group behavior that is likely to result in Bowie as a result of these attitudes. Compare and contrast this solution with that found in Exercise 15.

17. Citizen Disarm was chagrined by the results of his efforts at reformulating the gun-carrying problem. He had been so sure that his new way of looking at the problem would make a difference. On his way to the paper shredder to destroy the evidence of his failed attempt, he met Citizen Cooperation who inquired about his reason for dismay. Ms. Cooperation urged him not to give up yet. She reasoned thus: Because the original survey data had been collected in response to the statement,

“I will carry a gun when at least x% of the adult residents of Bowie do so,”

the survey had contained the subtle suggestion that carrying a gun was cooperative behavior. Even when Mr. Disarm had looked at the data from an alternative point of view, this built-in bias could not be removed.
Table 10.
New Data Collected by Cooperation and Disarm

<table>
<thead>
<tr>
<th>Gun-Refusal Thresholds</th>
<th>Percent of Bowie Adults with Given Threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>75</td>
<td>15</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>Percent of Bowie Adults with No Gun-Refusal Threshold</td>
<td>10</td>
</tr>
</tbody>
</table>

Together, Cooperation and Disarm decided to gather a new set of data. They asked the 5617 adult residents of Bowie to respond to:

"I will refuse to carry a gun when at least x% of the adult residents of Bowie refuse to carry a gun."

As all western tales must, this one has a happy ending. Discover that happy ending by analyzing the threshold data from the second survey, summarized in Table 10. In this case, what group behavior is likely to result?

18. As summer progresses toward autumn, many communities begin to worry about whether their water supply will last until it is replenished by generous rainfall. At such times, these localities may impose restrictions that limit water use. Such conservation restrictions are hard to enforce, however. Furthermore, if one person is seen watering his lawn, his neighbors who have been scrupulously limiting their water use may wonder, "What good does it do for me to conserve water when others don't?" and may violate the limits as well.

(a) Consider the hypothetical community of Watertown in which 1000 families live and share a common reservoir. Suppose that 900 of the 1000 families support water conservation if everyone else conserves. Invent data that illustrate that, despite a 90% support of conservation, a situation can result in which no one conserves.

(b) Suppose that you are the mayor of Watertown and believe that it is essential for the community members to obey the conservation restriction. You know that 90% of your constituents favor conservation but have observed that their threshold values have a cumulative effect that is preventing conservation. What steps would you take to help the community to achieve the conservation that most of them want?

19. PROJECT.

Analysis of threshold data is of most interest when we can apply it to gain insights into the behavior of groups to which we ourselves belong. You can gain—and share—insight into a group of which you are a concerned member by conducting your own threshold analysis.
Step 1. Choose a group to which you belong and a situation in which members face a choice of engaging (or not) in a certain behavior $B$. Sample behaviors include:
- attending (or not attending) a particular activity;
- conserving (or not conserving) a valuable resource;
- supporting (or not supporting) a certain political candidate or issue;
- wearing (or not wearing) a particular type of clothing—such as a scarf in cold weather or a helmet when cycling;
- conforming (or not conforming) to certain rules—such as liquor or drug laws, parking restrictions or speed limits;
- joining (or resisting) a particular fad.

Step 2. Survey the group members to learn
(a) the number of members favoring each behavior ($B$ or not $B$);
(b) the threshold values.

Step 3. Tabulate the survey results and construct a graph.

Step 4. Identify the equilibrium points. Describe external circumstances that could lead to each different equilibrium behavior. How likely is a particular equilibrium to be sustained?

Step 5. Think about the results. Consider questions such as:
(a) Is the group's behavior in accord with what most members want? If not, how can this be changed?
(b) What would be the probable effects of more (or less) publicity concerning members' thresholds?
(c) Has the wording of the survey influenced the data gathered? What effects may this have?
(d) Could the group be divided into two or more subgroups for which the behavior of (at least) one of the subgroups would be markedly different than the behavior of the entire group.

Step 6. Evaluate your methods.
(a) Is threshold analysis well suited to the situation to which you applied it?
(b) If you were to start over, what would you do differently to obtain better results?

7. Solutions to Exercises

1. (a) Many individuals prefer a situation in which no one cheats, but their preference is conditional: They consider it fair to cheat if enough others do.
(b) No. Most of the 90 individuals seem to view cheating as a defensive reaction, rather than an unequivocal policy. Willingness to cheat when others do is not the same as endorsement of cheating as a universal practice.

2. The 10 students with threshold 30, together with the 30 students with thresholds less than 30, form a group that is more than large enough to satisfy the cheating thresholds of its members.

The 30 students with thresholds 50, together with the 50 students with thresholds less than 50, form a group that is large enough to satisfy the cheating thresholds of its members.

3. The sentence to be justified follows directly from a sequence of observations like Observations 3, 4, 5, and 6. If one such observation is supplied for each non-zero data point, moving from left to right across the graph, the given sentence is the inescapable conclusion reached.

4. It is hard to predict what may happen. Individuals will base their decisions on suspicions rather than on knowledge. If no cheating is suspected, then students who will cheat only when others do will refrain from it. If a significant amount of cheating is suspected, we would expect a chain reaction of the type we have already discussed.

5. (a) If 90 students are cheating, the number will remain at 90 unless group members believe that the actual number of cheaters is less than 90. This could occur if many cheaters pretend not to cheat or if a large group of students get together and agree to stop simultaneously.

(b) If the number of cheaters is 40 then, since only 35 students have thresholds of 40 or less, the number of cheaters will drop to 35.

However, only 25 students have thresholds less than 40, so the number of cheaters will drop again, this time to 25.

Since only 15 students have thresholds less than 30, the number of cheaters will drop to 15.

Since only 10 students have thresholds less than 20, the number of cheaters will drop to 10.

Since 10 students have threshold 10, the number of cheaters may stabilize at that value.

N.B. This sequence of changes depends on class members knowing the behavior of others.

6. If \( x_2 > x_1 \), then the number, \( y_2 \), of individuals with threshold values \( < x_2 \) includes those \( y_1 \) individuals with threshold values \( \leq x_1 \). Thus \( y_2 \geq y_1 \).

7. \((0,0)\) will always be a data point.

8. For equilibrium point \((10,10)\):

(a) We do not have sufficient information to know whether a situation with no cheaters could continue.

(b) If thresholds remain as given, the number of cheaters can be expected to drop back from 20 to 10.

For equilibrium point \((50,50)\):

(a) If the number of cheaters drops to 40, we can expect (based on our previous analysis) it to drop down to 10.
(b) If the number of cheaters increases to 60, we expect it to increase further to 90.

For equilibrium point (90, 90):

(a) We would expect a drop of 10 from 90 only to be temporary since 90 students are satisfied with a threshold of 80.

(b) Since the survey data identifies 10 students who will never cheat, this situation will not occur unless the data is false.

Remark: The preceding responses illustrate the different degrees of stability of the three non-zero equilibrium points. The point (90, 90) identifies an equilibrium that is more stable than (10, 10) which, in turn, identifies a more stable equilibrium than (50, 50).

(c) The answer to this depends on how much inaccuracy there is. Using the answers to (a) and (b), we could assert, for example, that a "misconception of size 10" will shift group behavior away from the unstable equilibrium of 50 but will not shift behavior away from the stable equilibrium of 90.

9. Even in the statistics class example, it is hard to be sure of the effects of publicity. Perhaps, early in the semester, the reason that cheating "caught on" as a class behavior was that a few students were non-secretive about their cheating behavior. Later, however, group discussion and analysis of the cheating situation provided an opportunity to reduce it.

Some individuals cheat on their income tax because of the publicity that lets them know how many others do. Some others prepare honest tax returns because of publicity about the penalties for tax fraud.

Publicity itself may not be the critical factor; instead, what may matter is the amount of approval or disapproval that will result, or the promise of reward or penalty for the behavior.

10. The last column of Table 4 should be completed with the following values, in the order given 0, 0, 5, 5, 20, 30, 40, 40, 40, 40, 40. The following graph can then be drawn. The equilibrium points are (20, 20), (0, 0), and (40, 40), and the last two of these are stable. Since class participation is a very public event, stabilization could occur with exactly 20 students participating. However, slight misconceptions about actual numbers participating will cause an increase or decrease in this participation level. Thus, one of the stable equilibria—either no students participating—is more likely to be maintained, if achieved. One would suppose that, since 45 of the 50 class members prefer a class
in which all students participate, that the equilibrium with 40 students participating is the one likely to be achieved. We can, however, invent scenarios that include a climate of fear, in which universal nonparticipation takes place. In such a case, the group would profit from examination of the difference between what its members prefer and what they are doing and to take steps (as in Professor Ogive's statistics class) to try to change the situation.

11. The final of Table 6(a) should contain these entries: 0, 9, 0, 15, 25, 25. The final column of Table 6(b) should contain these entries: 0, 0, 5, 15, 15. The critical mass model graphs are:

Group I is likely to maintain a full problem session with 25 students attending. The number 25 is obtained from the stable equilibrium point (25, 25). If unforeseen circumstances change the number of attendees by a few, as long as 20 students attend, the problem session will continue.

The attitudes of members of Group II will result in a "dying" problem session. We might expect that 15 will show up at the first meeting, but of these 15 students, only five students have indicated that they would be satisfied with 15 attending. Thus only these five will show up at the second session. Since a group of size five is not sufficient to justify their attendance, they will not attend the third session. The Group II problem session will be "dead" after the second meeting.

The fact that Group I members can sustain a problem session while the whole group did not suggest a strategy to apply in other situations. Even if a certain group cannot maintain a behavior in the group as a whole, perhaps a separated subgroup can. Sometimes religious and political subgroups adopt this strategy; they separate themselves from a larger group and within the separated group members willingly adopt certain behaviors.

12. (a) If the group members become well informed about the attitudes of each other, then 45 members who are satisfied with a threshold of 40 will discover each other and join together in Behavior B.

(b) In Figure 4(b) neither of the situations identified by (40, 20) and (60, 80) are equilibrium situations. As publicity increased, a group whose behavior started at (40, 20) would eventually achieve an equilibrium with 40 members choosing L. A group whose behavior started at (60, 80) would eventually achieve an equilibrium with 60 group members choosing L. The graphs of Figure 4(a) and Figure 4(b) do support assertions (1) and (2).
(e) (1) ... the vertical coordinate of the data point to the left of the equilibrium line identifies a possible equilibrium situation for the group.
(2) ... the intersection of the graph and the equilibrium line does not identify an equilibrium situation for the group.

13. (a) This is the way it might have been at Bordenton. The person willing to protest alone spoke first and loudly. The person willing to protest along with one other heard and joined. Thus, a chain reaction began, and eventually it included the entire group.
(b) At the Grand Forks members talked about protesting—perhaps at length—but no one would say, "I will!"
(c) Mike is likely to start a chain reaction such as the one that occurred in Bordenton; see (a).
(d) The attitudes, listed above, for Grand Forks Nuclear Concern members are sufficient—without changes—to generate a protest if Mike's attitude is added at the head of the list.

Remark 1. A graph of the data for Bordenton Nuclear Concern reveals every data point to be an equilibrium point, thus any number of protesters is possible. If external conditions—say a broken leg—force member number 63 to drop out, this would cause the number of protesters to stabilize at 62.

Remark 2. The threshold graphs for the two Nuclear Concern groups are very similar in appearance, yet the Grand Forks graph has only one equilibrium point while the Bordenton graph has many. We thus can see that it is a mistake to suppose that two groups whose members hold similar attitudes will exhibit similar overall behaviors.

14. (a) The following values complete the third column of Table 7: 0, 5, 20, 35, 50, 65, 70, 70, 70, 80, 80. The following graph results:

![Graph showing equilibrium points](image)

The graph shows three equilibrium points: (20, 20), (0, 0), and (70, 70); the latter two are stable.
(b) No. (0, 0) is a stable equilibrium point.
(c) If 30 males have their heads shaved, the 35 males will have thresholds that are satisfied, and so the number of baldies will increase. Each increase will cause a further increase until a total of 70 males sport shaved heads. 

25
15. The threshold graph based on the Bowie survey:

If guns are carried openly, behavior in Bowie would be expected to stabilize with 75% of the adult residents carrying guns.

16. We first note that no one can have a threshold of zero since an individual includes himself in the threshold group. Samples of possible reasoning by Disarm include:

The 10% of individuals who would never carry a gun and the 15% who would carry a gun only if everyone was doing it would be surely satisfied not to carry a gun as part of a group of 25%.

Since 40% of Bowie residents had thresholds of 50% for gun carrying, then these same 40% would be unwilling to carry guns if only 25% carried them. Thus that same group of 40% surely would refuse to carry a gun if they would be part of a 75% majority.

The threshold graph:

If guns are carried openly, behavior in Bowie can be expected to stabilize with 25% of the adult residents refusing to carry guns. Despite Disarm’s efforts, he has done little more than turn the graph of Exercise 15 upside down.
17. The graph resulting from the new data of Table 9:

![Graph](image)

While an unstable equilibrium of 0 or 25 residents refraining from gun carrying could possibly be maintained, the likely group behavior (since the residents favor the absence of guns) is the stable equilibrium in which 90% of the residents refuse to carry weapons.

18 (a) One possible collection of threshold data for Watertown is:

<table>
<thead>
<tr>
<th>Non-Conservation Threshold (percents)</th>
<th>Percent of Community Members with Given Threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
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<td>30</td>
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<td>90</td>
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<td>100</td>
<td>5</td>
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</tbody>
</table>

A graph for this data has two equilibrium points: (0, 0) and (100, 100). Only the latter one is stable. Thus we have a situation in which the likely outcome is 100% nonconservation. We may suppose that while 90% of the Watertown residents support conservation, many find it to be a sacrifice, and their willingness to conserve readily disappears when others fail to conserve.

(b) One possible approach would be for the mayor to plead with the residents of Watertown to raise their thresholds. A difficulty with this approach, though it may work, is that it appeals only to the people with strong consciences and leaves the others free to waste.

Another approach would be for the mayor to urge the town to impose a fine for conservation violators. The fine serves as a way of enforcing the commitment that conservation is what everyone wants.