Module 232

Kinetics of Single Reactant Reactions

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Applications of Calculus to Chemistry
KINETICS OF SINGLE REACTANT REACTIONS

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Intermodule Description Sheet: UMAP Unit 282

Title: KINETICS OF SINGLE REACTANT REACTIONS

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Classification: APPLIC CALC CHEM/KINETICS

Suggested Support Material:

References:

Prerequisite Skills:
1. Be familiar with the Cartesian coordinate system.
2. Understand that \( a'(t) \) describes the rate of change of \( a(t) \).
3. Be able to integrate \( \int_0^t a'(t) \, dt \) for \( n = 0, 1, 2, 3 \).
4. Be able to solve an exponential equation.

This unit is intended for Calculus students with an active interest in and some background knowledge of chemistry. This background may be represented by concurrent registration in a college level chemistry course.

Output Skills:
1. Be able to describe single reactant irreversible reactions, including definitions of rate constant, reaction order, and half-life.
2. Be able to find explicit formulas for \( a(t) \) and for the half-life for a reaction of order \( n \).
3. Be able to determine the reaction order and rate constant of a reaction, given data on \( a(t) \), provided the reaction is of order 0, 1, or 2.

Other Related Units:
1. SINGLE REACTANT IRREVERSIBLE REACTIONS

1.1 Definition and Some Examples

Suppose we have a chemical reaction of a particularly simple sort, one which involves only one substance (let's call it A) as a reactant, and which is irreversible, therefore going to completion. It may be represented by writing:

$$A \rightarrow B_1 + B_2 + \ldots + B_n,$$

where $B_1, B_2, \ldots B_n$ are the products. Suppose at time $t = 0$ we have a certain concentration $a_0$ of A (measured, for example, in moles per liter). It is possible to observe and record the concentration $a(t)$ of A at various later times $t$.

### TABLE I
Experimental Data from Three Single Reactant Irreversible Reactions.

<table>
<thead>
<tr>
<th>$t$ (seconds)</th>
<th>0</th>
<th>51</th>
<th>206</th>
<th>454</th>
<th>751</th>
<th>1132</th>
<th>1575</th>
<th>2215</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a(t)$ (mm Hg)</td>
<td>15.03</td>
<td>14.58</td>
<td>13.32</td>
<td>11.49</td>
<td>9.73</td>
<td>7.79</td>
<td>6.08</td>
<td>4.17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$t$ (minutes)</th>
<th>0</th>
<th>1</th>
<th>4</th>
<th>10</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a(t)$ (mm Hg)</td>
<td>55</td>
<td>50</td>
<td>38</td>
<td>21</td>
<td>3</td>
<td>1.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$t$ (seconds)</th>
<th>0</th>
<th>120</th>
<th>180</th>
<th>240</th>
<th>330</th>
<th>530</th>
<th>600</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a(t)/a_0$</td>
<td>1</td>
<td>.5705</td>
<td>.5825</td>
<td>.512</td>
<td>.4195</td>
<td>.310</td>
<td>.2965</td>
</tr>
</tbody>
</table>
Table I gives three sets of such observations. Part (a) is for an experiment conducted at 280°C involving the decomposition of trichloromethyl chloroformate into phosgene:

\[ \text{Cl-C-O-CCl}_3 \rightarrow 2\text{Cl-C-Cl}. \]

Part (b) is for the decomposition at 500°C of ethylamine into ethylene and ammonia:

\[ \text{C}_2\text{H}_5\text{NH}_2 \rightarrow \text{C}_2\text{H}_4 + \text{NH}_3. \]

Part (c) is for alkaline hydrolysis of ethyl nitrobenzoate at an initial concentration of 0.05 moles per liter.

The reactants in parts (a) and (b) are gaseous. At constant temperature and volume, \( a(t) \) is proportional to its partial pressure, and it is this figure, in millimeters of mercury (mm Hg) that appears in Table I. In part (c), \( a(t) \) is given as a fraction of \( a_0 \).

In the conversion of trichloromethyl chloroformate to phosgene, both the reactant and the product are gaseous, and the total pressure actually increases as the reaction proceeds because each trichloromethyl chloroformate molecule gives rise to two phosgene molecules. The partial pressure of the trichloromethyl chloroformate is deduced from the total pressure by taking the reaction equation and the original pressure into account. In many reactions, however, the amount of the reactant is determined by techniques based on its absorption of light.

1.2 Graphs of the Results

We have plotted these results in Figures 1, 2, and 3. In that all of the curves decrease as \( t \) increases, these curves look very similar. But there is at least one significant difference (aside from the differences of scales). In each figure we have selected various concentrations of \( A \) and determined graphically approximately how long it takes for \( a(t) \) to decrease from the selected concentration to half of it. For example, Figure 1 shows us
Figure 1. Decomposition of Trichloromethyl Chloroformate (from Table 1(a)).

Figure 2. Decomposition of Ethylamine (from Table 1(b)).
that it takes approximately 1170 seconds for $a(t)$ to decrease from 14 to 7 mm Hg, or 1220 seconds for it to decrease from 10 to 5 mm Hg. In each of the first two figures the measured time intervals are approximately equal, but in Figure 3 they are not.

1.3 Questions

Can we explain this difference in terms of the reactions? Or, turning the question around, can we draw any conclusions based on these observations, about the nature of the reactions?

1.4 Chemical Kinetics

Questions such as these are part of a branch of chemistry known as chemical kinetics. Chemical kinetics is concerned with the rates and mechanisms of chemical
reactions. The name reflects the fact that "kinetics" is concerned with the changing aspects of systems, as distinguished from "statics" which concerns systems at equilibrium. We should also point out here that the rate at which a chemical process takes place and the mechanism of the process (i.e., what exactly happens during the transformation of A into $B_1 + B_2 + \ldots B_n$) are two different things. The study of reaction mechanisms lies at a higher theoretical level than the study of reaction rates. In general, experimentally determined reaction rates can be used to rule out a proposed mechanism if they are inconsistent with it. But experimental data that are consistent with a proposed mechanism can only serve as supporting evidence for it; they cannot be used directly to prove its correctness.

2. REACTION ORDER

2.1 Definitions

To make the question in Section 1.3 more specific, we shall summarize some background information about the reaction rates in reactions of this type. If substance A (in gas or liquid form) is uniformly distributed, and if the temperature and volume are kept constant, then it usually turns out that the rate $a'(t)$ at which A decomposes is proportional to a non-negative integer power $(0,1,2,\ldots)$ of the concentration $a(t)$. In other words

\begin{equation}
    a'(t) = -k[a(t)]^n
\end{equation}

where $k$ is a positive constant and $n$ is a non-negative integer. We call $k$ the rate constant and $n$ the order of the reaction. Equation (1) with $n$ established is called the rate law for the reaction.

We shall consider reaction orders 0, 1 and 2 in detail. Higher reaction orders for reactions of the type we are discussing are considerably more rare.
2.2 Zero-order Reactions

Setting \( n = 0 \) in Equation (1) gives

\[
\text{(2)} \quad \frac{dA}{dt} = -k_0
\]

where we have introduced the subscript to denote the reaction order. The rate is independent of the concentration of \( A \). It is determined by other factors such as temperature, the intensity of light in light-induced reactions, the surface area available in surface-catalyzed reactions, or the amount of catalyst in homogeneous catalysis. (A catalyst is a chemical substance that controls the rate of a reaction without undergoing any net change in itself over the course of the reaction.)

2.3 First-order Reactions

In this case we have

\[
\text{(3)} \quad \frac{dA}{dt} = -k_1A(t).
\]

Most simple decomposition reactions involving a single reactant are of first-order. This is not surprising if we imagine the reaction process to consist of molecules of \( A \) decomposing randomly. If, for example, each molecule has 1 chance in 10 of decomposing in the next second, then about \( \frac{1}{10} \) th of those present will in fact decompose in that second. In other words, the change in \( A(t) \) in that second is about \( \frac{1}{10} A(t) \). We describe this by writing

\[
\frac{dA}{dt} = \frac{1}{10} A(t).
\]

2.4 Second-order Reactions

The rate law for second-order reactions is:

\[
\text{(4)} \quad \frac{dA}{dt} = -k_{II}A^2(t).
\]

In general, elementary reactions which require the collision of two molecules are good candidates for this category.

2.5 Statement of the Problem

Equation (1) has been confirmed for many reactions by numerous experiments, and also explained theoretically.
We shall not get into the theoretical explanation except to say (as has already been indicated in Sections 2.2, 2.3 and 2.4) that different reaction orders are the result of different underlying reaction mechanisms. So if we have a reaction and want to know more about its mechanism a very useful first step is to determine its reaction order experimentally.

Can we use data such as that given in Table 1 to determine whether a reaction has one of the orders we have discussed, and if so, which one?

3. DETERMINING THE REACTION ORDER

3.1 Solving for \( a(t) \)

To begin with, we can use Equations (2), (3), and (4) to obtain explicit formulas for \( a(t) \) in the three cases.

(a) Zero-order reactions: If \( a'(t) = -k_0 \) then \( a(t) = -k_0 t + C \) where \( C \) is a constant of integration. Using the fact that \( a(0) = a_0 \) we see that \( C = a_0 \) and

\[
(5) \quad a(t) = a_0 - k_0 t.
\]

(b) First-order reactions: Starting with Equation (3), divide both sides by \( a(t) \) (which is never zero):

\[
\frac{a'(t)}{a(t)} = -k_1
\]

\[
\int_0^t \frac{a'(t)}{a(t)} \, dt = \int_0^t -k_1 \, dt
\]

\[
\ln a(t) \bigg|_0^t = -k_1 t \bigg|_0^t
\]

\[
\ln a(t) = -k_1 t + \ln a_0
\]

\[
(6) \quad a(t) = a_0 e^{-k_1 t}.
\]
(c) Second-order reactions: In Equation (4) we divide each side by \( a^2(t) \), and conclude that

\[
\frac{a'(t)}{a^2(t)} = -k_{II}
\]

\[
\int_0^t \frac{a'(t)}{a^2(t)} \, dt = \int_0^t k_{II} \, dt
\]

\[
-\frac{1}{a(t)} \bigg|_0^t = -k_{II} t \bigg|_0^t
\]

\[
\frac{1}{a(t)} = k_{II} t + \frac{1}{a_0} = \frac{a_0 k_{II} t + 1}{a_0}
\]

(7)

\[
a(t) = \frac{a_0}{a_0 k_{II} t + 1}
\]

Exercise 1

Find \( a(t) \) explicitly for a third-order reaction.

Exercise 2

Assume two reactions are of first and second order respectively:

\[
a'(t) = -k_1 a(t)
\]

\[
b'(t) = -k_2 b^2(t)
\]

Assume they begin with the same amount of reactant \( a_0 = b_0 \), and their initial rates are the same \([a'(0) = b'(0)]\). Prove that \( a(t) < b(t) \) for all \( t > 0 \).

(Hint: Note that \( \frac{a(t)}{b(t)} = 1 \) when \( t = 0 \) and show that it is strictly decreasing for \( t > 0 \).)

3.2 The Difficulty

The rate constant \((k_0, k_1, k_{II})\) is of course not known, so we cannot get away with anything so naive as plugging our data into Equations (5), (6), and (7) to see which one checks out. It is true that the graphs
of these equations have three distinctive "shapes", whatever the constants are (for example, Equation (5) is a straight line). So we could consider graphing our experimental data and trying to determine which "shape" curve fits it best. In this unit however, we present a method of determining the reaction order that does not depend on graphing, and which also gives us the rate constant at no extra cost.

3.3 Solving the Difficulty

The method starts with solving Equations (5), (6), and (7) for $k_0$, $k_1$, and $k_{II}$:

\begin{align}
(8) \quad k_0 &= \frac{a_0 - a(t)}{t}, \quad t > 0 \\
(9) \quad k_1 &= \frac{1}{t} \ln \frac{a_0}{a(t)}, \quad t > 0 \\
(10) \quad k_{II} &= \frac{1}{t} \left( \frac{1}{a(t)} - \frac{1}{a_0} \right), \quad t > 0.
\end{align}

Now if, for example, the reaction order is zero, then all the data points should satisfy Equation (5) for some constant $k_0$. Thus whenever we substitute any data point $(t, a(t))$ to the right side of Equation (8) we should get more or less the same value (namely $k_0$). Naturally there will be small variations due to experimental error. Similar comments apply to Equation (9) if the reaction order is one, and Equation (10) if the reaction order is two.

So all we need to do is compute three rows of figures -- the right sides of Equations (8), (9), and (10) -- for our data points, and see if any row remains more or less constant. If so, that row gives us the reaction order, and its constant value is the rate constant ($k_0$, $k_1$, or $k_{II}$).
3.4. An Example

As an example, let's go back to part (a) of Table I. In Table II, we have repeated the data and also tabulated the right sides of Equations (8), (9), and (10). The figures in the row corresponding to Equation (9) are nearly constant ($= 5.8 \times 10^{-4}\text{sec}^{-1}$) while those in the other rows are not. So this reaction is apparently a first order reaction with $k_1 \approx 5.8 \times 10^{-4}\text{sec}^{-1}$.

<table>
<thead>
<tr>
<th>$t$</th>
<th>sec</th>
<th>0</th>
<th>51</th>
<th>206</th>
<th>454</th>
<th>751</th>
<th>1132</th>
<th>1575</th>
<th>2215</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a(t)$</td>
<td>mm Hg</td>
<td>15.01</td>
<td>14.58</td>
<td>13.32</td>
<td>11.49</td>
<td>9.73</td>
<td>7.79</td>
<td>6.08</td>
<td>4.17</td>
</tr>
<tr>
<td>$\frac{a_a - a(t)}{t} \times 10^3$</td>
<td>mm Hg</td>
<td>8.82</td>
<td>8.30</td>
<td>7.80</td>
<td>7.06</td>
<td>6.40</td>
<td>5.68</td>
<td>4.90</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{t} \ln \frac{a_a}{a(t)} \times 10^3$</td>
<td>$\text{sec}^{-1}$</td>
<td>5.96</td>
<td>5.89</td>
<td>5.92</td>
<td>5.79</td>
<td>5.81</td>
<td>5.75</td>
<td>5.79</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{t(a(t) - a_a)} \times 10^3$</td>
<td>mm Hg</td>
<td>4.03</td>
<td>4.15</td>
<td>4.52</td>
<td>4.83</td>
<td>5.46</td>
<td>6.22</td>
<td>7.82</td>
<td></td>
</tr>
</tbody>
</table>

Exercise 3

Determine the reaction order and rate constant from the data given in part (b) of Table I.

Exercise 4

Determine the reaction order and rate constant from the data given in part (c) of Table I.
4. HALF-LIFE

4.1 Definition

The half-life $t_{1/2}$ of a certain amount of a reactant is the length of time required for exactly half of it to be used up. In other words, if the amount of reactant is $a_0$ at time $t = 0$, and if $a(t)$ is the amount at a later time $t$, then $t_{1/2}$ is the solution of the equation

$$a(t) = \frac{1}{2} a_0.$$

In Section 1.2 we determined graphically the half-lives of various amounts of three reactants, and discovered that for two of the reactants $t_{1/2}$ did not seem to depend upon the initial amount, but for the third reactant it did. Let us see if this phenomenon can shed a little more light on the concept of reaction order.

4.2 Formulas for Half-Life

To start with, let us compute $t_{1/2}$ for each of the three reaction orders we are considering. All we need to do is set $a(t) = \frac{1}{2} a_0$ in each of Equations (5), (6), and (7) and solve for $t$:

$$t_{1/2} = \frac{1}{2k_0} a_0 \quad \text{(Zero-order)}$$

$$t_{1/2} = \frac{\ln 2}{k_1} \quad \text{(First-order)}$$

$$t_{1/2} = \frac{1}{k_{II}} a_0 \quad \text{(Second-order)}.$$

Exercise 5

Find $t_{1/2}$ as a function of $a_0$ for a third-order reaction.

Exercise 6

We define $t_{3/4}$ as the time required for $\frac{3}{4}$ of a reactant to be used up. That is, $a(t_{3/4}) = \frac{1}{4} a_0$. Find $t_{3/4}$ as a function of $a_0$ for reactions of zero, first, and second-order.
Exercise 7

Find the ratio \( \frac{t_{\frac{1}{4}}}{t_{\frac{1}{2}}} \) for reactions of zero, first, and second-order.

Exercise 8

Table III gives \( t_{\frac{1}{4}} \) and \( t_{\frac{1}{2}} \) for three initial amounts of the reactant in the reaction:

\[
\text{CH}_3\text{CHO} \rightarrow \text{CH}_4 + \text{CO}
\]

acetaldehyde methane carbon monoxide

Determine if possible whether the reaction has one of the three orders discussed in this unit and, if so, which one.

**TABLE III**

Half-life and \( \frac{1}{4} \)-Life Data for the Reaction \( \text{CH}_3\text{CHO} \rightarrow \text{CH}_4 + \text{CO} \) (Exercise 8)

<table>
<thead>
<tr>
<th>( a_0 ) (mm Hg)</th>
<th>420</th>
<th>225</th>
<th>184</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_{\frac{1}{2}} ) (seconds)</td>
<td>395</td>
<td>572</td>
<td>665</td>
</tr>
<tr>
<td>( t_{\frac{1}{4}} ) (seconds)</td>
<td>1135</td>
<td>1710</td>
<td>1920</td>
</tr>
</tbody>
</table>

Exercise 9

Suppose, for every \( x \) between 0 and 1, we write \( t_x \) for the time required for fraction \( x \) of a reactant to be used up. (In Exercise 6 \( t_{\frac{1}{4}} \) is an example of \( t_x \) with \( x = \frac{1}{4} \).) Show that in a first-order reaction \( t_x \) is independent of the initial amount no matter what \( x \) is.

4.3 Zero-order Reactions

For a zero-order reaction, half-life is proportional to initial amount. The greater the amount, the longer
the half-life. To help yourself understand and remember this, think of a very large number of marbles, from which we remove, say, 10 each second \((k = 10)\). The more there are originally, the longer it will take to remove half of them.

4.4 First-order Reactions

For a first-order reaction, half-life is independent of initial amount!! To help understand and remember this, think again of a very large number of marbles. This time remove one half of the pile in the first second, then one half of the remaining pile in the next second, etc. \((k = \frac{1}{2})\). No matter how many we start with, it will take one second to remove half of them. Also, at any later stage it will take one second to remove half of what remains.

4.5 Second-order Reactions

For a second-order reaction, half-life is proportional to the reciprocal of the initial amount. Another way of saying this is that \(a_t \sim \frac{1}{2} \) is a constant. The more of \(A\) there is, the less time it takes for one half of it to decompose! Although this may seem paradoxical we invite you to consider the fact that second-order reactions depend upon collisions of pairs of molecules. Equation (13) says that the more molecules there are, the more likely they will collide, and the faster the reaction will proceed.

Exercise 10

The following data were obtained by F. Daniels and E.H. Johnston (J. Am. Chem. Soc., 43, 53 (1921)) for the decomposition of nitrogen pentoxide \((N_2O_5)\) in solution in carbon tetrachloride \((CCl_4)\) at 450°C:

\[2N_2O_5 \rightarrow 2N_2O_4 + O_2.\]
<table>
<thead>
<tr>
<th>t (seconds)</th>
<th>0</th>
<th>184</th>
<th>319</th>
<th>526</th>
<th>867</th>
<th>1198</th>
<th>1877</th>
<th>2315</th>
<th>3144</th>
</tr>
</thead>
<tbody>
<tr>
<td>concentration of (H_2O_2) (mole/(\ell))</td>
<td>2.33</td>
<td>2.06</td>
<td>1.91</td>
<td>1.67</td>
<td>1.36</td>
<td>1.11</td>
<td>.72</td>
<td>.55</td>
<td>.34</td>
</tr>
</tbody>
</table>

Determine the reaction order and the rate constant, as well as the half-life \(t_\frac{1}{2}\). How long would it take for 87.5% of the reactant to be used up?
5. **MODEL EXAM**

1. For some reactions the reaction order is found to be fractional. Find \( a(t) \) explicitly (in terms of \( a_0 \)) for a reaction with reaction order \( n = \frac{1}{2} \).

2. Define \( t_x \) as that time for which \( a(t_x) = (1 - x)a_0 \). Find \( t_x \) for a second order reaction. Is this \( t_x \) independent of \( a_0 \)?

3. Determine the reaction order and rate constant from the following data for a hypothetical reaction.

<table>
<thead>
<tr>
<th>( t ) (seconds)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a(t) ) (moles/1)</td>
<td>10.0</td>
<td>3.98</td>
<td>2.51</td>
<td>1.02</td>
<td>1.44</td>
<td>1.19</td>
</tr>
</tbody>
</table>
The Project would like to thank Scott C. Mohr of Boston University, Andrew Jorgensen of Indiana State University at Evansville, Bernice Kastner of Montgomery College, Rockville, Maryland, Barbara Juister of Elgin Community College, Elgin, Illinois for their reviews, and all others who assisted in the production of this unit.

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1. \[ a(t) = \frac{1}{2} \left( \frac{1}{\sqrt{2a^2 + k_{II} t + 1}} \right) ^2 \]

2. The information given to us is:
   1. \( a'(t) = -k_I a(t) \)
   2. \( b'(t) = -k_{II} b^2(t) \)
   3. \( a(0) = b(0) \)
   4. \( a'(0) = b'(0) \)

   To see that \( \frac{a(t)}{b(t)} \) is a decreasing function of \( t \), we show that the derivative of the quotient is negative.

   \[
   \frac{d}{dt} \left[ \frac{a(t)}{b(t)} \right] = \frac{b(t) a'(t) - a(t) b'(t)}{b^2(t)}
   \]

   \[
   = \frac{b(t)(-k_I a(t)) - a(t)(-k_{II} b^2(t))}{b^2(t)}
   \]

   \[
   = a(t) \left( -k_I + k_{II} b^2(t) \right) \frac{a(t)}{b(t)}
   \]

   \[
   (14) \quad a(t) \left( k_{II} - \frac{k_I}{b(t)} \right)
   \]

   Now, \( a'(0) = b'(0) \) means that

   \[ k_I a(0) = k_{II} b^2(0) \]

   and \( a(0) = b(0) \) means further that

   \[ k_I b(0) = k_{II} b^2(0) \]

   \[ k_I = k_{II} b(0). \]

   When we substitute this value of \( k_I \) in Equation (14) we obtain

   \[
   \frac{d}{dt} \left[ \frac{a(t)}{b(t)} \right] = a(t) \left( k_{II} - \frac{k_{II} b(0)}{b(t)} \right)
   \]

   \[
   = k_{II} a(t) \left( 1 - \frac{b(0)}{b(t)} \right)
   \]

   Since \( b(t) < b(0) \) for \( t > 0 \),

   \[ \frac{b(0)}{b(t)} > 1 \]

   and

   \[ \left( 1 - \frac{b(0)}{b(t)} \right) < 0. \]
3. Reaction order = 1
   \[ k = 9.4 \times 10^{-2} \text{ min}^{-1}. \]

4. Reaction order = 2
   \[ a_0k = 4.1 \times 10^{-3} \text{ sec}^{-1} \text{ or, since } a_0 = 0.05 \]
   \[ k = 8.2 \times 10^{-2} \text{ mole}^{-1} \text{ sec}^{-1}. \]

5. \[ t_\frac{1}{2} = \frac{3}{2a_0^{-\frac{1}{2}}k_1}. \]

6. Zero order: \[ t_{\frac{3}{4}} = \frac{3a_0}{5k_0} \]
   First-order: \[ t_{\frac{1}{2}} = \frac{1}{k_1} \ln 4. \]
   Second-order: \[ t_{\frac{3}{4}} = \frac{3}{a_0k_{II}} \]

7. Zero-order: \[ \frac{3}{2} \]
   First-order: 2
   Second-order: 3

8. Second-order.

10. First order, \[ k = 6.2 \times 10^{-9} \text{ sec}^{-1}. \]
    \[ t_\frac{1}{2} = 1120 \text{ sec}, \]
    \[ 3t_\frac{1}{2} = 3360 \text{ sec}. \]
7. ANSWERS TO MODEL EXAM

1. \( a(t) = a_0 - \eta_0^c \kappa t + \frac{k^2 t^2}{4} \)

2. \( t_x = \frac{1}{a_0 k \eta} \left( \frac{x}{1 - x} \right) \) ; No.

3. \( k = 7.5 \times 10^{-1} \text{ mole}^{-2} \text{ sec}^{-1} \)
   Order = 2.