A letter to students and instructors:

When I was first asked to teach microeconomics, I was surprised to learn that the course had been one of the least popular in my department. It was unclear what the goals of the course were – and without such clarity at the outset, students had come to view the course as a disjointed mess of graphs and math with little real world relevance and no sense of what value it could add. As I came to define what goals I would like my course to develop, I had trouble finding a text that would help my students aim toward these goals without over-emphasizing just one or two to the exclusion of others. So we largely de-emphasized textbooks – but something was working: the course had suddenly become one of the most popular in the department!

I have therefore attempted to build a framework around the 5 primary goals that I believe any microeconomics course should accomplish:

1. It should present microeconomics not as a collection of unrelated models but as a way of looking at the world. People respond to incentives because they try to do the best they can given their circumstances. That’s microeconomics in a nutshell – and everything – everything – flows from it.

2. It should persuade that microeconomics does not just change the way we think about the world – it also tells us a lot about how and why the world works (and sometimes doesn’t work).

3. It should not only get us to think more clearly about economics but also to think more clearly in general – without relying on memorization. Such conceptual thinking skills are the very skills that are most sought after and most rewarded in the modern world.

4. It should directly confront the fact that few of us can move from memorizing to conceptual thinking without applying concepts directly, but different students learn differently, and instructors need the flexibility to target material to their students’ needs.

5. Finally, it should provide students with a roadmap for further studies – a sense of what the most compelling next courses might be given their interests.

The structure of Microeconomics: An Intuitive Approach is flexible but keeps us rooted in a way of thinking while developing a coherent overview to help us better understand the world around us. Half the text builds up to the most fundamental result in all of economics – that self-interested individuals will – under certain
conditions and without intending to – give rise to a spontaneous order that has great benefits for society. But the second half probes these “certain conditions” and develops insights into how firms, governments and civil society can contribute to human welfare when markets by themselves “fail.”

This text offers a variety of flexible paths for a one-semester (or two-semester) course. In each chapter, instructors can emphasize an intuitive A-part or link it to a more mathematical B-part; in fact, a version of the text that includes only the A sections from each chapter is also available (and may be ideal for courses in which calculus is not required). Other potential paths through this book include one focused on policy and another focused on business, with all paths including core material as well as optional topics. Throughout, the models build in complexity with applications woven into the narrative (rather than being relegated to side-boxes). They are then further developed in an extensive array of exercises that allow students to apply concepts to Everyday Business and Policy settings for themselves.

Finally, the complete learning package for Microeconomics: An Intuitive Approach acknowledges the fact that students have different learning styles and few can develop conceptual thinking skills without continually applying them. Complex graphs are decomposed into panels – and each graph is accompanied by a unique online animation – LiveGraphs – that students and instructors can play backward and forward, as many times as they would like. In addition, supplemental technology modules allow students to explore economic relationships and develop links between math and intuition. Many of my students have found that they learn the material best through these tools, and I – as an instructor – can cover more material more clearly in less time when I use them in class.

To see an online demonstration of the LiveGraphs – and in particular if you are using the chapters from this preview guide in class – visit www.cengage.com/economics/nechybademo (live in January 2010). There you’ll find all of the animated LiveGraphs and activities that accompany the two chapters shown in this preview guide (chapters 6 & 7 in the full text).

I hope this preview guide gives you an idea of what the rest of the text has to offer and that you will take the time to check it out in January of 2010. In the meantime, please do not hesitate to e-mail me if you have comments or suggestions.

Sincerely,

Thomas J. Nechyba
Thomas J. Nechyba
nechyba@econ.duke.edu
BRIDGING THE GAP...
Between Required Course and Real-World Understanding

For too long, too many students have viewed the microeconomics course as a disconnected
collection of graphs, math, and rote memorization—instead of an invaluable roadmap to
exploring the world around them. Thomas J. Nechyba’s *Microeconomics: An Intuitive Approach*
bridges the gap—between information and knowledge, introduction and real understanding—as it empowers students
with the tools and insight to make sense of their world as students and as future professionals.

Empowers students with the tools and
insight to make sense of their world.

A gifted instructor at Stanford and then Duke University,
Professor Nechyba revolutionized the way microeconomics
was taught, and in the process elevated it from one of the
department’s least popular courses to one of the most popular.
In this exciting first edition, he applies his proven methods to a coherent, comprehensive, and
cutting-edge approach to microeconomics that is applauded by instructors and students alike.

BRIDGING THE GAP...
Between What Instructors Want and What Other Texts Provide

One size does not fit all -- or even most. *Microeconomics: An Intuitive Approach* offers un-
precedented flexibility in terms of topical coverage, graphical analysis, and mathematical
level. No longer do instructors have to waste time trying to wedge their course goals into a
text’s rigid structure. *Microeconomics: An Intuitive Approach* delivers the utmost flexibility,
allowing instructors to easily tailor the text to their students’ distinctive needs.
BRIDGING THE GAP...
Between Conventional Graphs and Lively Animation

Each new copy of the text includes access to a premium website featuring Nechyba’s LiveGraphs—an exciting suite of interactive, animated graphs that give students hands-on experience working with content-filled graphs and functions found in the book as well. Students can play and replay full-color LiveGraphs while listening to a brief explanation of what they are viewing. Because many students learn best by doing and viewing at their own pace, LiveGraphs allow the learner to view and review graphs over and over, in some cases manipulating variables to watch how the results can change. Enjoying real hands-on learning, students can watch as lines are plotted, see curves move, and in some cases even change variables to see the resulting impact.

Coming in January 2010! See a demo at www.cengage.com/economics/nechybademo
BRIDGING THE GAP...
Between Majors and Nonmajors

- MICROECONOMICS: AN INTUITIVE APPROACH AND
- MICROECONOMICS: AN INTUITIVE APPROACH WITH CALCULUS

The text’s unique approach caters to students with a wide range of backgrounds. Each chapter of the book is divided into “A” and “B” sections. “A” sections introduce concepts using intuition, a conversational writing style, everyday examples, and graphs—requiring no background in economics or mathematics. “B” sections cover the same concepts with precise, accessible mathematical analysis that pre-supposes one semester of single-variable calculus. Correlating intuition with mathematics, “B” sections clearly demonstrate how mathematics is simply another way of analyzing the same concepts that have just been illustrated graphically.

Nechyba’s unique A & B approach promotes a more comprehensive and intuitive understanding of the way that graphical analysis is linked to underlying mathematical relationships. By exploring a narrative, intuitive, and graphical approach to the topics before linking the concepts, point by point, with mathematical analysis, students can build both their mathematical skill and the conceptual, abstract thinking that allows them to apply understanding beyond what is covered in the text. The result is better understanding of microeconomics foundations and better preparation for further course work in economics, business, or policy studies.

Adding even more convenience: Instructors can choose to adopt Microeconomics: An Intuitive Approach with both “A” and “B” sections or a version with only “A” sections—thus requiring no calculus prerequisite.

BRIDGING THE GAP...
Between “Principles” and “Intermediate” Microeconomics

Microeconomics: An Intuitive Approach eliminates the artificial distinction between principles of micro and intermediate microeconomics by building from the outset on the most basic foundations of modern economics—individual-level behavior. The text shows students the bigger picture of economic intuition while helping to train their conceptual thinking “muscle” with carefully written mathematical analysis. The first several chapters of the book meticulously build up a set of tools leading up to demand and supply and ultimately the First Welfare Theorem which in turn frames the second half of the book. While the text begins with tools, these are ultimately used in a bigger-picture narrative to illustrate the power and limits of economics.
BRIDGING THE GAP...

Between Book Knowledge and Real-World Experience

IN-CHAPTER EXERCISES: Each chapter is chunked into digestible segments punctuated by in-chapter learning exercises. Thoughtfully written by the author, these insightful exercises help students check their understanding at key points throughout the chapter. (Answers will be available for students to access in an online study guide.)

APPLICATIONS THAT WORK: Through practical end-of-chapter exercises, Prof. Nechyba encourages students to apply text concepts to everyday situations, current business scenarios, and policy applications.

• How does pricing of one product affect demand for another product produced by the same business?
• How might a firm most efficiently use cap-and-trade pollution vouchers?
• What is the impact of a capital-gains-tax-induced increase in the rental price of capital on firms within an industry?
• How do private school vouchers affect tuition levels?
• How effective are anti-price gauging laws during times of supply disruption?
• How do governments consider policies for subsidizing saving vs. taxing borrowing?

By working through exercises that exemplify key applications, students learn by doing and can apply their knowledge to applications beyond what the text has specified. Each exercise is carefully considered, checked for accuracy, and written by Prof. Nechyba, with step-by-step solutions to all exercises available to instructors.

BRIDGING THE GAP...

Between Classroom and Careers

With its bold approach, flexibility, and emphasis on conceptual skills, Microeconomics: An Intuitive Approach bridges the gap between textbook knowledge and real-world application, understanding, and—ultimately—success. The result is a text that not only gives students a solid understanding of key microeconomics concepts but also a new way to view the world. It provides a strong foundation that offers students excellent preparation for upper-level studies and their future careers.
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To Students

As a student, I often felt both alienated and insulted by textbooks: alienated because they seemed to make no attempt to speak to rather than at me, insulted because they seemed to talk down to me by giving me lots of "visuals" (like pictures of monkeys—seriously) to keep me awake and by feeding me endless definitions to memorize—all while never acknowledging the obvious conceptual limits of what was being presented.

I have therefore tried to write a book that is a little different and that I think I might have liked to use when I was a student. Some have commented that you might not like it because it doesn’t lend itself to memorizing definitions for exams. Others find it strange that I address you so directly throughout much of the book and that I occasionally even admit that this or that assumption we make is in many ways “silly.”

I don’t actually have anything against monkeys or definitions or assumptions that seem “silly,” but my experience with students over the years tells me that you do not mind being challenged a bit and actually enjoy being part of a conversation rather than committing one to memory. The modern world has few rewards for people who are really good at memorizing but offers much to those who can conceptualize ideas and integrate them with one another. Economics offers a path to practice this—and it does so in a way that can be exciting and interesting, interesting enough to not actually require monkey pictures even if it is sometimes frustrating to get through some of the details.

I will say more about much of this in Chapter 1—so I’ll try to avoid repeating myself here and instead just offer a few points on how best to use this text:

1. You may want to review parts of Chapter 0 (on the accompanying product support web site www.cengage.com/economics/nechyba) to review some basics before proceeding to Chapter 2.
2. Attempt the within-chapter exercises as you read—and check your answers with those in the accompanying (web-based) Study Guide that contains answers to all within-chapter-exercises. (My students who have used drafts of this text have done considerably better on exams when using within-chapter exercises and solutions.)
3. Before you read each chapter, particularly as the book progresses, print out the Print Graphics from the accompanying product support web site (www.cengage.com/economics/nechyba). This will reduce frustrations as the discussion of graphs in the text often extends across multiple pages—requiring you to flip back and forth unless you also have the print graphics with you. The print graphics might also prove handy in class as you can take notes directly on them. (And if you really want pictures of monkeys to stay awake, just keep them with your print graphs, or let me know and we’ll put some monkey pictures on the web site.)
4. Use the LiveGraphs feature of the web site, particularly if the discussion of graphs in the text leaves you with questions. These animated versions of the text graphs come with visual and audio explanations (by yours truly) that you can rewind and fast forward at your own pace. (Some chapters also have additional animated graphs that are not directly related to the print graphs in the text, and you may also access the Print Graphics from the LiveGraphs site.)

¹The full site will not go live until Summer of 2010.
5. Look for interesting applications in **end-of-chapter exercises**, but know that some of these are designed to be challenging. Don’t get frustrated if they don’t make sense at first. It helps to work with others to solve these (assuming your instructor allows this). The * symbol denotes exercises with solutions provided in the Study Guide, with solutions to the remainder of the exercises provided to instructors. (The symbol ** denotes conceptually more challenging exercises, and the symbol *** denotes computationally more intensive exercises.)

6. While you will often feel like you are getting lost in details within chapters, the **Introductions** (to the Parts as well as the Chapters) and the **Conclusions** (in each chapter) attempt to keep an eye on the big picture. Don’t skip them!

7. The book has an extensive **Glossary** and **Index** but develops definitions within a narrative rather than pulling them out within the text. Use the Glossary to remind yourself of the meaning of terms and the Index to find where the associated concepts are discussed in detail. But resist the temptation to memorize too much. The terms aren’t as important as the concepts.

8. No textbook is without **errors**, and this is particularly true for first editions. In anticipation of this, we have provided a place on the accompanying web site for reporting all errors in real time as they are identified. So if you think there might be an error, check the site and if it is not yet reported, let your instructor know so that it can be passed along to me.

**To Instructors**

When I was first asked to teach microeconomics, I was surprised to learn that the course had been one of the least popular in my department. It was unclear what the goals of the course were—and without such clarity at the outset, students had come to view the course as a disjointed mess of graphs and math with little real-world relevance and no sense of what value it could add. As I came to define what goals I would like my course to develop, I had trouble finding a text that would help my students aim toward these goals without over-emphasizing just one or two to the exclusion of others. So we largely de-emphasized textbooks—but something was working: the course had suddenly become one of the most popular in the department!

I have therefore attempted to build a framework around the five primary goals that I believe any microeconomics course should accomplish:

1. **It should present microeconomics not as a collection of unrelated models but as a way of looking at the world.** People respond to incentives because they try to do the best they can given their circumstances. That’s microeconomics in a nutshell—and everything—**everything**—flows from it.

2. **It should persuade that microeconomics does not just change the way we think about the world—it also tells us a lot about how and why the world works** (and sometimes doesn’t work).

3. **It should not only get us to think more clearly about economics but also to think more clearly in general**—without relying on memorization. Such **conceptual thinking skills** are the very skills that are most sought after and most rewarded in the modern world.

4. **It should directly confront the fact that few of us can move from memorizing to conceptual thinking without applying concepts directly,** but different students learn differently, and instructors need the **flexibility** to target material to their students’ needs.

5. **Finally, it should provide students with a roadmap for further studies**—a sense of what the most compelling next courses might be given their interests.

I am thus trying to provide a flexible framework that keeps us rooted in a **way of thinking** while developing a **coherent overview** to help us better understand the world around us. Half the
text builds up to the most fundamental result in all of economics—that self-interested individuals will—under certain conditions and without intending to—give rise to a spontaneous order that has great benefits for society. But the second half probes these “certain conditions” and develops insights into how firms, governments, and civil society can contribute to human welfare when markets by themselves “fail.” Future courses can then be seen as sub-fields that come to terms with these “certain conditions.”

While the material in the full text is more than enough for a two-semester sequence, the text offers a variety of flexible paths for a one-semester course. In each chapter, you can emphasize an intuitive A-part or link it to a more mathematical B-part; and, while the last part of the text relies heavily on game theory, the underlying narrative can also be developed through a non-game theoretic approach. Substantive paths include some focused on theory, others focused on policy, and yet others focused on business, with all paths including core material as well as optional topics. Throughout, the models build in complexity, with applications woven into the narrative (rather than being relegated to side-boxes). They are then further developed in an extensive array of exercises that get students—not me or you—to apply concepts to Everyday, Business, and Policy settings.

For more details on how you might use the various parts of the text and its accompanying tools, I hope you will have a look at the Instructor’s Manual that I have written to go along with the text. Here are just a few examples of how you might weave through the book depending on your focus:

1. Traditional Theory Emphasis:
   Ch. 1–23 (with Ch. 3, 8, the latter sections of 9 and 13 optional) plus
   Ch. 29–30 optional

2. Theory Emphasis with Game Theory:
   Ch. 1–18 (with 3, 8, the latter sections of 9, 13, and 18 optional) plus
   Ch. 23–27 (with 28 through 30 optional)

3. Business Focus:
   Ch. 1–18 (with Ch. 3, 8, 16, the latter sections of 9, 13, and 18 optional) plus
   Ch. 23–26

4. Policy Focus:
   Ch. 1–15 (with Ch. 3, 8, and the latter sections of 9 and 13 optional), plus
   Ch. 18–23, 28–30 (with Ch. 24–27 optional depending on level of game theory usage)

Finally, I would like to invite you to be a partner in shaping the future of this textbook. No text is perfect the first time around, and this one is no exception. But to achieve serious improvements with future editions, I need feedback on what is working and what is not, what is too much and what is missing. For this reason, we have created a place on the text web site where I can engage with instructors directly, where we can give one another feedback and where I can learn about how things are working out in your classroom. I hope you will make use of this and we will meet on the web site.

Acknowledgments

I never intended to write this textbook and, had it not been for the persuasive pressures applied by Mike Worls (who is formally the executive editor for the project), the book would in fact not have been written. It is for this reason that, during the more trying times of getting this project finished (when, as my kids put it, dad was “grumpy”), Mike became know as “that bad man”
in my household. Still, I tell my children that even “bad” people sometimes give rise to good things—and I hope this is the case here. Regardless, I am grateful for Mike’s insistence that I give this a shot. The project has taken longer and become more comprehensive than either of us envisioned at first, but Mike continued to believe in it throughout. Jennifer Thomas (the development editor for the project) has been with us since the beginning, and I often wondered whether her children, not yet born when we started, might be in college before the text gets finished. (I think we made it in time.)

Just before launching this project, I had the good fortune to meet John Gross and Deborah Antkoviak of EconWeb. As Director of Undergraduate Studies at Duke, I was interested in their ideas of how to bring graphical analysis alive in animations, and I began to collaborate with them to produce animations that could assist in teaching economics. So it became only natural to extend our partnership to this textbook, with John and Deb producing not only the (299!) graphics that appear in the text but also the LiveGraphs (and related material) that appear on the accompanying web site. I do not believe that the approach taken in this text would be possible without the quality of these graphics, and I know that there would have been no way to produce these without the partnership and friendship that we have developed. I cannot overstate the extent to which I am in their debt.

I am not sure I fully appreciated the challenge of compiling an internally consistent and partially mathematical text when I started typing away in Word, and so I thank my brother (Mike Nechyba) for introducing me to the wonders of LaTeX (while occasionally tutoring me on basics in Mathematica). I also thank Stas Kolenikov for arranging to have the initial Word chapters transferred into LaTeX. Although, much to my regret, the text in your hands was not ultimately laid out using LaTeX, we would not have been able to use pdf drafts of chapters in classrooms over the past few years had it not been for Mike’s and Stan’s early intervention, nor would I have had the patience to see the process through without the benefit of this approach. I am also grateful to the production team at Cengage. Tamborah Moore diligently kept the content moving throughout, and Michelle Kunkler kept an artist’s eye on it as I, lacking any artistic talent whatsoever, was too busy checking subscripts. My thanks to both of them and the entire team.

When I started seriously working on this text, I had just taken over as the new chair of the economics department at Duke. I have had some good ideas in my life, but I suspect that becoming department chair and textbook author at the same time was not one of these. So it seems only appropriate to thank all of my colleagues for their patience when time spent on the book came at the cost of time spent on the department. (Life would be so much better without the need for trade-offs, but then again, that would leave no place for the fascinating area of economics.) I particularly want to thank the department’s wonderful staff that helped maintain the illusion of my presence when I was hiding to work on this project, leaving some with the impression that I was actually a competent chair. Above all, I owe them my thanks for keeping me laughing even during the most trying times. Jim Speckart should be particularly acknowledged for processing the many hand-drawn graphs for the solution sets to the exercises in the textbook (while offering his persistently irreverent but unfailingly entertaining “feedback”).

Early versions of this text have been patiently endured by many. First and foremost among these, I thank the hundreds of Duke students that have taken microeconomics with me and quasi-voluntarily served as guinea pigs for this text. Their feedback and diligent reporting of errors, sometimes for extra credit, have made this a better book, and it is to all of those students that I therefore dedicate this first edition. Other instructors have also used early drafts of this text, and I thank them and their students as well. And I want to acknowledge in particular the students in the American Economic Association’s (AEA) Summer Program who were the first to live through the B-portions of the text when the AEA program was housed at Duke. I am perpetually amazed at the generosity with which all of these students allowed me to teach them from flawed and incomplete pdf documents.

Isaac Linnartz, Jesse Patrone-Werdiger, Suzy Silk, Wendy Wang, and Sejal Shah, all Duke undergraduates some years ago, read patiently through early chapters and gave feedback that helped
shape the rest of the book. I should thank my many TAs individually for all their patience as we
taught from drafts, but space (and memory problems) keep me from doing so here. I do, however,
want to highlight Bethany Peterson, Terry Yang, and Liad Wagman for their comments, encouragement,
and contributions. Toward the completion of the text, proof reading and feedback from Chase
Wilson, Christina Shin, and Chen Xiaoyan kept many errors from making it into the final text.
Naturally, the errors that remain are almost entirely my fault (and, of course, my wife’s since she
insisted that I actually spend time away from the textbook doing “my share” around the house).

Anyone that has ever tried to undertake a project like this in the midst of a busy family life
knows how difficult the burden on family can be at those times when the frustrations of the
project seem to outweigh the bright prospect of future royalties. For living through this with me,
my thanks to the family that sustains and completes me—Stacy, Ellie, Jenny, Katie, and, most
recently, Blake. We will celebrate on our vacation in the Cayman Islands soon.

Thomas J. Nechyba
Durham, NC

FURTHER ACKNOWLEDGMENTS

The author and editorial team at South-Western Cengage Learning would also like to acknowledge
the feedback and assistance of many instructors who reviewed this text in various drafts and par-
ticipated in several focus groups concerning its development and LiveGraphs. Their time and
comments have been invaluable in crafting the final product.

J. Ulyses Balderas, Sam Houston State
University
Klaus G. Becker, Texas Tech University
Allen Bellas, Metropolitan State College
of Denver
Tibor Besedes, Louisiana State University
Maharukh Bhiladwalla, Rutgers University
Volodymyr Bilotkach, University
of California Irvine
Benjamin F. Blair, Mississippi State
University
Victor Brajer, California State University
Fullerton
Nancy Brooks, University of Vermont
James Cardon, Brigham Young University
Kalyan Chakraborty, Emporia State
University
Basant Chaudhuri, Rutgers University
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Mary Deily, Lehigh University
Wayne Edwards, University of Alaska
Adem Y. Elveren, University of Utah
Robert M. Feinberg, American University
Rhona Free, Eastern Connecticut State
University
Jaqueline Geoghegan, Clark University
Dipak Ghosh, Emporia State University
Rajeev K. Goel, Illinois State University
Tiffani A. Gottschall, Washington
and Jefferson College
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Ross LaRoe, *Denison University*
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Ranganath Murthy, *Bucknell University*
Kathryn Nantz, *Fairfield University*
Tara Natarjan, *Saint Michael’s College*
Ronald Nate, *Brigham Young University*
Catherine Norman, *Johns Hopkins University*
Terry Olson, *Truman State University*
Mete Ozcan, *Brooklyn College CUNY*
Ebru Isil Ozturk, *University of Wisconsin*
Silve Parviainen, *University of Illinois at Urbana-Champaign*
Brian Peterson, *Central College*
Jonas Prager, *New York University*
James Prieger, *Pepperdine University School of Public Policy*
Salim Rashid, *University of Illinois at Urbana-Champaign*
Tyler R. Ross, *University of Washington Seattle*
Jeremy Sandford, *University of Wisconsin*
Jonathan Sandy, *University of San Diego*
Mustafa Sawani, *Truman State University*
Kwang Soo Cheong, *Johns Hopkins University*
Charles Steele, *Hillsdale College*
Vasant Sukhatme, *Macalester College*
Jeffrey Sundberg, *Lake Forest College*
Jose Vasquez-Cognet, *University of Illinois at Urbana-Champaign*
Richard Vogel, *Farmingdale State College*
Eleanor von Ende, *Texas Tech University*
Rob Wassmer, *California State University Sacramento*
Tetsuji Yamada, *Rutgers University*
Ben Young, *University of Missouri Kansas City*
Sourushe Zandvakili, *University of Cincinnati*
We began our introduction of microeconomics with the simple premise that economic agents try to do the best they can given their circumstances. For three types of economic agents—consumers, workers, and individuals planning for the future—we showed in Chapters 2 and 3 how choice sets can be used to illustrate the circumstances these economic agents face when making choices. We then illustrated in Chapters 4 and 5 how we can model individual tastes, giving us a way of now addressing how individuals will judge which of their available choices is indeed the “best.” Chapters 2 through 5 therefore developed our basic model of individual choice sets and tastes, the first step in our economic analysis of choice. We now begin the second step, the analysis of how individuals in our basic model optimize; i.e., how they would behave if they are indeed doing the best they can.

6A Choice: Combining Economic Circumstances with Tastes

We begin by building some intuition about how tastes and choice sets interact to determine optimal choices. This means that we will essentially combine the graphs of Chapters 2 and 3 with those of Chapters 4 and 5 as we return to some of the examples we raised in those chapters. In the process, we’ll begin to get our first glimpse at the important role market prices play in helping us exploit all the potential gains from trade that would be difficult to realize in the absence of such prices. Then, in Section 6A.2, we consider scenarios under which individuals may choose not to purchase any quantity of a particular good, scenarios we will refer to as corner solutions. And, in Section 6A.3, we will uncover scenarios under which individuals may discover that more than one choice is optimal for them, scenarios that arise when either choice sets or tastes exhibit non-convexities.

6A.1 The “Best” Bundle of Shirts and Pants

Suppose we return to my story of me going to Wal-Mart with $200 to spend on shirts and pants, with shirts costing $10 each and pants costing $20 per pair. We know from our work in Chapter 2 that in a graph with pants on the horizontal axis and shirts on the vertical, my budget constraint

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1Chapters 2, 4, and 5 are required as reading for this chapter. Chapter 3 is not necessary.
intersects at 20 on the vertical and at 10 on the horizontal. Its slope, which gives expression to the opportunity cost of one more pair of pants in terms of how many shirts I have to give up, is $-2$. Suppose further that the marginal rate of substitution is equal to $-2$ at all bundles where I have twice as many shirts as pants, that it is equal to $-1$ at bundles where I have an equal number of shirts and pants, and that it is equal to $-1/2$ at bundles where I have twice as many pants as shirts. (This is an example of what we called “homothetic” tastes in Chapter 5.) My budget constraint and choice set are then graphed in Graph 6.1a, and some of the indifference curves from the indifference map that represents my tastes are graphed in Graph 6.1b. To determine which of the available choices is “best” given my circumstances, we now have to combine the information from Graphs 6.1a and 6.1b. This is done in Graph 6.1c where panel (b) is simply laid on top of panel (a). Of the three indifference curves that are graphed, the green curve contains only bundles that are in fact not available to me given my circumstances because the entire curve lies outside my choice set. The magenta indifference curve has many bundles that fall within my choice set, but none of these is “best” for me because there are bundles in the shaded area to the northeast that all lie within my choice set and above this indifference curve, bundles that are “better” for someone with my tastes. We could now imagine me starting at some low indifference curve like this one and pushing northeast to get to higher and higher indifference curves without leaving the choice set. This process would end at the blue indifference curve in Graph 6.1c, an indifference curve that contains 1 bundle that lies in the choice set (bundle $A$) with no bundles above the indifference curve that also lie in the choice set. Bundle $A$, then, is the bundle I would choose if indeed I am trying to do the best I can given my circumstances. More precisely, I would consume 5 pair of pants and 10 shirts at my optimal bundle $A$.  

Exercise 6A.1 In Chapter 2, we discussed a scenario under which my wife gives me a coupon that reduces the effective price of pants to $10 per pair. Assuming the same tastes, what would be my best bundle?

\[2\text{This optimal bundle lies at the intersection of the budget line } (x_2 = 20 - 2x_1) \text{ and the ray } x_2 = 2x_1 \text{ representing all the points with } MRS \text{ of } -2. \text{ Solving these by substituting the second equation into the first gives us the answer that } x_1 = 5, \text{ and putting that into either of the two equations gives us that } x_2 = 10.\]
6A.1.1 Opportunity Cost = Marginal Rate of Substitution  At bundle \(A\) in Graph 6.1c, a very particular relationship exists between the slope of the budget constraint and the slope of the indifference curve that contains bundle \(A\): the two slopes are equal. This is no accident, and it should make intuitive sense why this is true. The slope of the budget constraint represents the opportunity cost of pants in terms of shirts, which is the number of shirts I have to give up to get one more pair of pants (given the prices Wal-Mart charges for pants and shirts). Put differently, the slope of the budget constraint represents the rate at which Wal-Mart is allowing me to change pants into shirts. The slope of the indifference curve, in contrast, represents the marginal rate of substitution, which is the number of shirts I am willing to give up to get one more pair of pants. If I have a bundle in my shopping basket at which the value I place on pants (in terms of shirts) differs from the rate at which Wal-Mart is allowing me to change pants into shirts, I can make myself better off by choosing a different bundle. Thus, at the optimal bundle, the rate at which I am willing to trade pants for shirt and the rate at which I have to trade them must be equal.

Suppose, for instance, that I have \(B\) from Graph 6.1c (8 pants, 4 shirts) in my shopping basket. The marginal rate of substitution at \(B\) is \(-1/2\). This means that I am willing to trade 1 pair of pants for half a shirt, but Wal-Mart will give me 2 shirts for every pair of pants that I put back on the rack. If I am willing to trade a pair of pants for just half a shirt and Wal-Mart will give me 2 shirts for a pair of pants, then I can clearly make myself better off by trading pants for more shirts. Put differently, when I have \(B\) in my basket, the marginal value I place on pants is lower than the marginal value Wal-Mart is placing on those pants, and Wal-Mart is therefore willing to give me more for pants (in terms of shirts) than I think they are worth. Therefore cannot possibly be a “best” bundle because I can make myself better off by exchanging pants for shirts.

Suppose you and I each have a bundle of 6 pants and 6 shirts, and suppose that my MRS of shirts for pants is \(-1\) and yours is \(-2\). Suppose further that neither one of us has access to Wal-Mart. Propose a trade that would make both of us better off.

6A.1.2 How Wal-Mart Makes Us All the Same at the Margin  I am not the only one who rushes to buy shirts and pants right before the school year starts; lots of others do the same. Some of those consumers have tastes very different than mine, so their indifference maps look very different. Others will have more generous wives (and thus more generous budgets); yet others may be poorer and may only be able to spend a fraction of what my wife is permitting me to spend. Imagine all of us—rich and poor, some in more need of pants and some in more need of shirts—all coming to Wal-Mart to do the best we can. Coming into Wal-Mart, we will be very different; but coming out of Wal-Mart, it turns out that we will be quite the same in one important respect: our marginal rates of substitution of pants for shirts given what we have just purchased will all be the same.

Consider, for instance, the two consumers whose choice sets and tastes are graphed in Graph 6.2a and 6.2b. Consumer 1 is rich (and thus has a large choice set) whereas consumer 2 is poor (and thus has a small choice set). Consumer 1 and consumer 2 also have very different indifference maps. In the end, however, they both choose an optimal bundle of shirts and pants at which their marginal rate of substitution is equal to the slope of their budget constraint. Since the slope of each consumer’s budget constraint is determined by the ratio of prices for shirts and pants at Wal-Mart, and since Wal-Mart charges the same prices to anyone who enters the store, the marginal rates of substitution for both people is thus equal once they have chosen their best bundle. Put differently, while the two consumers enter the store with very different incomes and tastes, they leave the store with the same tastes for pants and shirts at the margin (i.e., around the bundle they purchase).

6A.1.3 How Wal-Mart Eliminates Any Need for Us to Trade  An important and unintended side effect of Wal-Mart’s policy to charge everyone the same price is that all gains from trade in pants
and shirts occur inside Wal-Mart, eliminating any need for us to trade with one another once we leave the store. As we all enter the store, we may have different quantities of pants and shirts at home, and we could probably benefit from trading shirts and pants among us given that some of us might be willing to trade shirts for pants more easily than others. But once we leave Wal-Mart, we value pants and shirts exactly the same at the margin; i.e., we all have the same marginal rate of substitution of pants for shirts. There is therefore no more possibility for us to trade and become better off because we became as well off as we could by simply doing the best we can inside Wal-Mart.

This is an important initial insight into a more general result we will develop later on in this book. Whenever two people have bundles of goods at which they value the goods in the bundle differently on the margin, there is the potential for gains from trade, the potential for trade to make both people better off. We already illustrated this in the end-of-chapter exercise 4.5 in Chapter 4 as well as in within-chapter exercise 6A.2, but here is another example. Suppose I am willing to trade 1 can of Coke for 1 can of Pepsi (i.e., my marginal rate of substitution is 1) but my wife is willing to trade 1 can of Coke for 2 cans of Pepsi (i.e., her marginal rate of substitution is 2). Then we can gain from trading with one another so long as we each have both Coke and Pepsi in our bundles. In particular, I could offer my wife 2 Cokes for 3 Pepsis. This will make me better off because I would have been willing to take only 2 Pepsis for 2 Cokes, and it will make my wife better off because she would have been willing to give me as many as 4 Pepsis for 2 Cokes. The fact that our marginal rates of substitution are different, the fact that we value goods differently at the margin, makes it possible for us to trade in a way that makes both of us better off.

**Economists say that a situation is efficient if there is no way to change the situation so as to make some people better off without making anyone worse off.** A situation is therefore inefficient

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3Sometimes economists refer to this as Pareto efficient or Pareto optimal after Vilfredo Pareto (1848–1923). Pareto was among the first economists in the late 19th century to realize that economic analysis did not require utility to be objectively measurable, that all that was required was for individuals to be able rank different alternatives. This led him to his definition of efficiency, which stands in contrast to earlier “utilitarian” theories that relied on adding up people’s “utils.” We will return to some of this in Chapter 29.
if we can think of a way to change the situation and make some people better off without making anyone else worse off. If we find ourselves in a situation where people value goods that they possess differently at the margin, we know there is a way to make everyone better off through trade. Thus, situations where people have different marginal rates of substitution for goods that they possess are inefficient. Since Wal-Mart’s policy of charging the same prices to everyone results in a situation where everyone leaves the store with marginal rates of substitution between goods in their baskets identical, *Wal-Mart ensures that the distribution of pants and shirts is efficient among those that purchase pants and shirts at Wal-Mart.*

We keep using the phrase “at the margin” as, for example, when we say that tastes for those leaving Wal-Mart will be the “same at the margin.” What do economists mean by this “at the margin” phrase?

I doubt you have ever thought of approaching someone in the Wal-Mart parking lot to propose a trade of goods in your shopping basket with goods you see in his or her basket. It turns out, there is a very good reason for this: It would be an exercise in futility because all gains from trade have been exhausted within Wal-Mart, and the distribution of goods is already efficient. Put differently, once we leave Wal-Mart, any trade that I propose to you will either leave us just as well off as we would be without trading or would make one of us worse off. So we don’t need to bother trying.

### 6A.2 To Buy or Not to Buy

With the indifference maps and budget sets used above, “doing the best I can” led me to purchase both pants and shirts at Wal-Mart. But sometimes our tastes and circumstances are such that doing the best we can implies we will choose not to consume any of a particular good. This certainly happens for goods that we consider “bads,” goods of which we would prefer less rather than more. Peanut butter is such a good for me. I simply cannot imagine why anyone would ever consume any unless there was an immediate need to induce vomiting. Ketchup is another such good for me. I will never buy peanut butter or ketchup. But there are also goods that I like of which I will consume none. For instance, I like both Coke and Pepsi equally (and in fact cannot tell the difference between the two), but whenever Pepsi is more expensive than Coke, I will buy no Pepsi. My tastes for goods that I like combine, in this case, with my economic circumstances to lead to my “best” choice at a “corner” of my budget constraint.

#### 6A.2.1 Corner Solutions

Let’s consider the case of me choosing between Coke and Pepsi in the context of our model of tastes and circumstances. Suppose that I get sent to the store with $15 to spend on soft drinks, and suppose that the store sells only Coke and Pepsi. Suppose further that the price of Coke is $1 per can and the price of Pepsi is $1.50 per can. Graph 6.3a then illustrates my choice set and budget constraint. In Chapter 5, we further illustrated my tastes for Coke and Pepsi with an indifference map containing indifference curves that all have a marginal rate of substitution equal to $-1$ everywhere. Such indifference curves, illustrated again in Graph 6.3b, give expression to the fact that I cannot tell the difference between Coke and Pepsi and therefore am always willing to trade them one for one.

In panel (c) of Graph 6.3, we again overlay my choice set (from panel (a)) and my indifference map (from panel (b)). My goal is to reach the highest indifference curve that contains at least one bundle in the choice set. I could start with the lowest (magenta) indifference curve, note that all bundles on that indifference curve lie in my choice set, then move to the northeast to higher indifference curves. Eventually, I will reach the blue indifference curve in Graph 6.3c, which contains one bundle (bundle A) that lies both on the indifference curve and within my choice set.
Since any bundle on an indifference curve higher than this lies outside my choice set, bundle A is my “best” bundle. It contains 15 Cokes and no Pepsi and is called a “corner solution” because it lies on one corner of my choice set.

In the previous section, we argued that Wal-Mart’s policy of charging the same price to all consumers ensures that there are no further gains from trade for goods contained in the shopping baskets of individuals who leave Wal-Mart. The argument assumed that all consumers end up at an interior solution, not a corner solution. Can you see why the conclusion still stands when some people optimize at corner solutions where their MRS may be quite different from the MRS’s of those who optimize at interior solutions?

Suppose the prices of Coke and Pepsi were the same. Illustrate that now there are many optimal bundles for someone with my kind of tastes. What would be my “best” bundle if Pepsi is cheaper than Coke?

Of course, tastes do not have to be as extreme as those for perfect substitutes in order for corner solutions to arise. Panels (d), (e), and (f) of Graph 6.3, for instance, illustrate a less
extreme set of indifference curves that nevertheless results in corner solutions for certain economic circumstances.

**6A.2.2 Ruling Out Corner Solutions** In Chapter 5, we discussed how a good is “essential” if indifference curves do not intersect the axes on which the other good is measured, essential in the sense that no utility above that of consuming at the origin of the graph can be attained without at least some consumption of such “essential” goods. If all goods in a particular model of a consumer’s tastes are “essential,” then corner solutions are not possible; it can never be optimal to choose a bundle with zero quantity of one of the goods because that would be the same as choosing zero quantity of all goods. *Whenever indifference curves intersect an axis, however, some goods are not essential, and there is thus a potential for a corner solution to be the optimal choice under some economic circumstances.*

Consider, for instance, my wife’s tastes for iced tea and sugar as described in Chapter 5. Suppose that sugar costs $0.25 per packet and iced tea costs $0.50 per glass, and suppose that my wife has budgeted $15 for her weekly iced tea drinking. Her weekly choice set is illustrated in Graph 6.4a, and her tastes for iced tea and sugar packets are illustrated with three indifference curves in Graph 6.4b (given that these are perfect complements for her). Panel (c) of Graph 6.4 then illustrates her optimal choice as bundle A, with equal numbers of glasses of iced tea and sugar packets.

**Graph 6.4: Ruling Out Corner Solutions**

![Graph 6.4: Ruling Out Corner Solutions](image)
We could now think of changing the prices of iced tea and sugar packets, of making sugar packets really cheap and making iced tea really expensive, for instance. While the total quantity of iced tea and sugar packets that is optimal will be different, it will always be true that my wife will consume equal numbers of iced tea glasses and sugar packets, and never a corner solution.

The case of perfect complements is an extreme case that ensures that no corner solutions will ever be optimal. But the same logic holds for any map of indifference curves that do not intersect either axis, or, put differently, for any set of goods that are all essential. Panels (d) through (f) of Graph 6.4, for instance, model my wife’s tastes for iced tea and sugar as less extreme, with some willingness to trade off some sugar for more iced tea and vice versa. Still, the indifference map in panel (e) of the graph is such that no indifference curve ever intersects either axis, ensuring an interior solution where the marginal rate of substitution is exactly equal to the slope of the budget constraint.

6A.2.3 Is it Realistic to Rule Out Corner Solutions? In many of our applications throughout this book, we will assume tastes with indifference maps that rule out corner solutions by assuming that all goods are essential. Our first reaction to this might be that this is highly unrealistic. After all, we are all at corner solutions because there are many goods at Wal-Mart that never end up in our shopping baskets. This is certainly true, but remember that we are not trying to model everything that happens in the world when we write down an economic model. Rather, we try to isolate the aspects of the world that are essential for a proper analysis of particular questions, and so it may often make sense simply to abstract away from the existence of all those goods that we never purchase.

For instance, I might be interested in analyzing how your housing choices change as your circumstances change. I might therefore abstract away from your tastes over Coke and Pepsi and pants and shirts, and simply model your tastes for square feet of housing and “other consumption.” In that case, of course, it makes perfect sense to assume indifference maps that exclude the possibility of corner solutions because you will almost certainly choose to consume some housing and some other goods regardless of how much your circumstances change. Similarly, when I am interested in analyzing your choice of leisure and consumption, it is likely that you will always choose some leisure and some consumption. The same is probably the case when I model your choice of how much to consume this year versus next year: Few people will consciously plan to consume only today or only next year regardless of how much individual circumstances change. Thus, while we certainly are at corner solutions almost all the time in the sense that we do not consume many types of goods, economic modeling of the relevant choices often makes it quite reasonable to assume tastes that prohibit corner solutions by assuming that the goods relevant to our analysis are all essential.

6A.3 More than One “Best” Bundle? Non-Convexities of Choice Sets and Tastes

Thus far, almost all our examples have made it appear as if a consumer will always be able to reach a unique optimal decision. It turns out that this “uniqueness” occurs in most of our models because of two assumptions that have held throughout the earlier portions of this chapter: First, all budget constraints were lines, and second, all tastes were assumed to satisfy the “averages are better than extremes” assumption. More generally, we will find next that the “uniqueness” of the “best” choice may disappear as “non-convexities” in choice sets or tastes enter the problem we are modeling.

4The one exception to this has been the case of indifference curves with linear components such as those for perfect substitutes, where a whole set of bundles may be optimal when the ratio of prices is exactly equal to the slope of the linear component of the budget line (see the within-chapter exercise 6A.5 in Section 6A.2.1.)
6A.3.1 Optimizing with Kinked Budgets  

As we illustrated in Chapters 2 and 3, there are two basic types of kinks in budget constraints that may arise under various circumstances: those that point “outward” and those that point “inward.” We introduced these in Chapter 2 with two types of coupons for pants. First we considered a coupon that gave a consumer 50% off for the first 6 pairs of pants (Graph 2.4a) and then turned toward thinking about a coupon that gave 50% off for any pair of pants a consumer purchases after buying 6 at regular price. We will demonstrate now that multiple “best” bundles may arise only in the second case but not in the first (assuming for now that our tastes satisfy the basic five assumptions laid out in Chapter 4).

Graph 6.5 considers how three different types of tastes may result in three different optimal bundles on the same “outwardly” kinked budget constraint derived from the first type of coupon (see Section 2A.2). In each case, the general shape of our standard indifference curves guarantees only a single “best” choice because there is no way to draw our usual shapes for indifference curves and get more than one tangency to the outwardly kinked budget constraint.

Graph 6.6, in contrast, considers the “inwardly” kinked budget that arises under the second type of coupon (see also Section 2A.2) and particularly models tastes that lead to two “best” bundles: bundles A and B. You can immediately see how this is possible: Since indifference curves begin steep and become shallower as we move toward the right in the graph, the only way we can have two bundles at which the budget constraint has the same slope at the best indifference curve is for the budget constraint itself also to become shallower as we move to the right. This can happen with an “inward” kink in the budget, but it cannot happen with an “outward” kink such as that in Graph 6.5.

6A.3.2 Non-Convexities in Choice Sets  

In fact, a “kink” in the budget is, strictly speaking, not necessary for the possibility of multiple “best” bundles when indifference maps satisfy the “averages better than extreme” assumption. Rather, what is necessary is a property known as “non-convexity” of the choice set.

A set of points is said to be convex whenever the line connecting any two points in the set is itself contained within the set. Conversely, a set of points is said to be non-convex whenever some part of a line connecting two points in the set lies outside the set. No such non-convexity exists in the choice set of Graph 6.5. Regardless of which two points in the set we pick, the line connecting them always also lies within the set. But in the choice set of Graph 6.6, it is easy to...
find pairs of points where the line connecting those points lies outside the set. For instance, both points $A$ and $B$ in Graph 6.6 lie in the choice set, but the line connecting the two points lies outside the set. Thus, the choice set in Graph 6.6 is non-convex.

**Exercise 6A.6**  
Consider a set of points that compose a solid sphere. Is this set convex? What about the set of points contained in a donut?

**Exercise 6A.7**  
We have just defined what it means for a set of points to be convex—it must be the case that any line connecting two points in the set is fully contained in the set as well. In Chapter 4, we defined tastes to be convex when “averages are better than (or at least as good as) extremes.” The reason such tastes are called “convex” is because the set of bundles that is better than any given bundle is a convex set. Illustrate that this is the case with an indifference curve from an indifference map of convex tastes.

Now, notice that a regularly shaped indifference curve can be tangent to the boundary of a choice set more than once only if the choice set is non-convex. The series of graphs in Graph 6.7 attempts to show this intuitively by beginning with a convex choice set (in panel (a)), continuing with a linear budget that is still convex (in panel (b)), and then proceeding to two non-convex choice sets in panels (c) and (d). **The important characteristic of a choice set to produce multiple “best bundles” is therefore not the existence of a kink but rather the existence of a non-convexity (which may or may not involve a kink).** While we can think of examples of non-convex choice sets, we will see that convex choice sets are most common in most of the economic applications we will discuss in the remainder of this book.

**Exercise 6A.8**  
*True/False:* If a choice set is non-convex, there are definitely multiple “best” bundles for a consumer whose tastes satisfy the usual assumptions.

**Exercise 6A.9**  
*True/False:* If a choice set is convex, then there will be a unique “best” bundle, assuming consumer tastes satisfy our usual assumptions and averages are strictly better than extremes.
6A.3.3 Non-Convexities in Tastes  Suppose next that an indifference map had indifference curves that looked like those graphed in Graph 6.8a. You can demonstrate that such indifference curves violate the “averages are better than extremes” (or convexity) assumption by considering bundles $A$ and $B$ together with the average between those bundles, labeled $C$ in the graph. Since $C$ falls below the indifference curve that contains $A$ and $B$, it is worse than $A$ and $B$; thus the average bundle is not as good as the more extreme bundles. As already suggested in exercise 6A.7, the reason we call such tastes non-convex is that the set of bundles that is better than a given bundle is a non-convex set. In our example, bundle $C$ lies on the line connecting bundles $A$ and $B$ but is worse, not better, than bundles $A$ and $B$. Thus, the set of bundles that are better than those on the indifference curve containing bundle $A$ (the shaded area in Graph 6.8a) is non-convex.

Graph 6.8: Example of 2 Optimal Bundles when Tastes are Non-Convex
Now suppose we consider an individual with tastes that can be represented by the indifference map in Graph 6.8a trying to do the best he or she can on the linear (and thus convex) budget in Graph 6.8b. This can then result in both A and B in Graph 6.8c being optimal. Our “averages are better than extremes” assumption rules this scenario out by explicitly ruling out non-convexities in tastes. We have argued in Chapter 4 that assuming “averages are better than extremes” is reasonable for most economic models. It makes sense that people are more willing to trade shirts for pants if they have lots of shirts and relatively few pants. In most economic models, we therefore feel comfortable ruling out “non-convex” tastes, and thus ruling out multiple optimal bundles due to non-convexities in tastes.

Suppose that the choice set is defined by linear budget constraint and tastes satisfy the usual assumptions but contain indifference curves with linear components (or “flat spots”). True/False: There might then be multiple “best” bundles, but we can be sure that the set of “best” bundles is a convex set.

Exercise 6A.10*

There are instances, however, when we might think that tastes should be modeled as non-convex, and should thus permit multiple optimal solutions. Suppose, for instance, we modeled our tastes for steak dinners versus chicken dinners, and suppose we considered a model in which we are trying to predict whether someone will choose a steak or a chicken dinner, or some combination of the two. It may well be reasonable for someone to have non-convex tastes that allow for both a steak dinner and a chicken dinner to be optimal, with a half steak and half chicken dinner being worse. At the same time, if we instead modeled someone’s weekly tastes for steak and chicken dinners (rather than just his or her tastes at a single meal), the non-convexity is less reasonable because, over the course of a week, someone is much more likely to be willing to have some steak and some chicken dinners.

Putting the insights from this and the previous section together, we can conclude that we can be sure that an individual has a single, unique “best” choice given a particular set of economic circumstances only if neither his or her choice set nor his or her tastes exhibit non-convexities. More precisely, we need tastes to be strictly convex—averages to be strictly better than (and not just as good as) extremes, because, as we saw in exercise 6A.10, multiple optimal bundles (forming a convex set) are possible when indifference curves contain linear segments or “flat spots.”

6A.4 Learning about Tastes by Observing Choices in Supermarkets or Laboratories

It is impossible for you to look at me and know whether or not I like Coke and Pepsi, whether I enjoy peanut butter or would rather have more shirts than pants or the other way around. We do not carry our tastes around on our sleeves for all the world to see. Thus, you may think all this “theory” about tastes is a little pie in the sky, that it wreaks of the cluttered mind of an academic who has lost his marbles and his connection to the real world. Not so! Despite the fact that tastes are not directly observable, we are able to observe people’s choices under different economic circumstances, and from those choices we can conclude something about their tastes. In fact, if we observe enough real-world choices under enough different economic circumstances, we can pretty much determine what a person’s indifference map looks like. Economists and neuroscientists are also beginning to map tastes directly to features of our brain through the use of sophisticated brain scanning equipment in laboratories.
6A.4.1 Estimating Tastes from Real-World Choices

It is not difficult to see how we can estimate tastes by observing people’s choices in the real world (even though the statistical methods required for an economist actually to determine a consumer’s underlying tastes are quite sophisticated and beyond the scope of this text). Take our example of me shopping for pants and shirts at Wal-Mart, for instance, and suppose that you observe that I purchase 10 shirts and 5 pants with my $200 budget when the prices of shirts and pants are $10 and $20 respectively. This tells you that my \( MRS \) at the bundle (5, 10) is equal to the slope of my budget (−2). Then suppose that my economic circumstances change because Wal-Mart changes the price of pants to $10 and the price of shirts to $20, and suppose you now see me purchasing 10 pants and 5 shirts. You now know that my \( MRS \) at the bundle (10, 5) is −1/2. If you continue to see changes in my economic circumstances and my response to those changes in terms of my choices, you can keep collecting information about the \( MRS \) at each of the bundles that I purchase under each scenario. The more such choices you observe, the easier it is for you to estimate what my underlying indifference map must look like.

Thus, economists have developed ways to estimate underlying tastes by observing choices under different economic circumstances. Many supermarkets, for instance, provide consumers with cards that can be scanned at the check-out counter and that give consumers some discounts on certain products. Every time I shop in our local supermarket, I give the check-out clerk my card so that I get the discounts on advertised items. The supermarket then automatically collects data on my consumption patterns. It knows what I buy when I shop and how my consumption patterns change with the supermarket’s discounts and price changes. Economists can then analyze such data to recover underlying tastes for particular consumers or the “average consumer.”

6A.4.2 Learning about the Link from the Brain to Tastes

Over the last few years, a new area has emerged within economics known as neuroeconomics. Many neuroeconomists are actually neuroscientists who specialized in understanding how our brain makes decisions, and a small but increasing number have been trained as economists who collaborate with neuroscientists. Their aim is, in part, to unravel the “black box” of tastes: to understand what determines our tastes and how they change over time, to what extent tastes are “hard-wired” into our brain, and how our brain uses tastes to make decisions. In doing their work, neuroeconomists rely on both the economic theory of choice as well as experimental evidence gathered from observing individuals make choices within a laboratory where various aspects of their physiology can be closely monitored. Neuroeconomists can, for instance, see which parts of the brain are active—and how active they are—when individuals confront a variety of choices, and through this they are beginning to be able to infer something about the mapping of features of tastes (such as marginal rates of substitution) to the structure of the brain. They are also able to see how the decision-making process is altered when the brain is altered by such factors as substance abuse. This is fascinating research, but it is beyond the scope of this book. However, within a relatively short period, it is likely that you will be able to take course work in neuroeconomics and should consider doing so if the intersection between economics and neuroscience seems interesting to you.

6B Optimizing within the Mathematical Model

In part 6A, we found ways of depicting mathematical optimization problems in intuitive graphs, and we now turn toward an exposition of the mathematics that underlies this intuition. Specifically, we will see that consumers face what mathematicians call a constrained optimization problem, a problem where some variables (the goods in the consumption bundle) are chosen so as to optimize a function (the utility function), subject to the fact that there are constraints (the choice set).
6B.1 Optimizing by Choosing Pants and Shirts

Letting \( x_1 \) and \( x_2 \) denote pants and shirts, consider once again the example of me choosing a consumption bundle \((x_1, x_2)\) in Wal-Mart given that the price for a pair of pants is $20 and the price for a shirt is $10, and given that my wife gave me a total of $200 to spend. Suppose further that my tastes can be represented by the Cobb–Douglas utility function \( u(x_1, x_2) = x_1^{1/2}x_2^{1/2} \), which gives rise to the indifference curves drawn in Graph 6.1 of Section 2A.1. Then the mathematical problem I face is that I would like to choose the quantities of \( x_1 \) and \( x_2 \) so that they are affordable (i.e., they lie within the choice set) and so that they attain for me the highest possible utility as evaluated by the utility function \( u \). That is, of course, exactly the same problem we were solving graphically in Graph 6.1, where we were finding the “best” bundle by finding the highest indifference curve (and thus the highest level of utility) that contains at least one point in the budget set.

Put differently, I would like to choose \((x_1, x_2)\) so as to maximize the function \( u(x_1, x_2) \), subject to the constraint that my expenditures on good \( x_1 \) plus my expenditures on good \( x_2 \) are no larger than $200. Formally, we write this as

\[
\max_{x_1, x_2} u(x_1, x_2) = x_1^{1/2}x_2^{1/2} \quad \text{subject to} \quad 20x_1 + 10x_2 \leq 200. \tag{6.1}
\]

The “max” notation at the beginning of the expression signifies that we are attempting to maximize or “get to the highest possible value” of a function. The variables that appear immediately below the “max” notation as subscripts signify those variables that we are choosing, or the choice variables in the optimization problem. I am able to choose the quantities of the two goods, but I am not able to choose the prices at which I purchase them or, since my wife determined it, my money budget. Thus, \( x_1 \) and \( x_2 \) are the only choice variables in this optimization problem. This is then followed by the function that we are maximizing, called the objective function of the optimization problem. Finally, if there is a constraint to the optimization problem, it appears as the last item of the formal statement of the problem following the words “subject to.” We will follow this general format for stating optimization problems throughout this text.

Since we know that Cobb–Douglas utility functions represent tastes that satisfy our “more is better” assumption, we can furthermore rewrite expression (6.1) with the certainty that the bundle \((x_1, x_2)\) that solves the optimization problem is one that lies on the budget line, not inside the choice set. When such an inequality constraint holds with equality in an optimization problem, we say that the constraint is binding. In other words, we know that I will end up spending all of my allocated money budget, so we might as well write that constraint as an equality rather than as an inequality. Expression (6.1) then becomes

\[
\max_{x_1, x_2} u(x_1, x_2) = x_1^{1/2}x_2^{1/2} \quad \text{subject to} \quad 20x_1 + 10x_2 = 200. \tag{6.2}
\]

6B.1.1 Two Ways of Approaching the Problem Mathematically

We begin by viewing the problem strictly through the eyes of a mathematician, and we illustrate two equivalent methods to solving the problem defined in equation (6.2).

Method 1: Converting the Constrained Optimization Problem into an Unconstrained Optimization Problem

One way is to turn the problem from a constrained optimization to an unconstrained optimization problem by inserting the constraint into the objective function. For example, we can solve the constraint for \( x_2 \) by subtracting \( 20x_1 \) from both sides and dividing both sides by 10 to get \( x_2 = 20 - 2x_1 \). When we insert this into the utility function for \( x_2 \), we get a new function that is simply a function of the variable \( x_1 \). We can call this function \( f(x_1) \) and rewrite the problem defined in (6.2) as

\[
\max_{x_1} f(x_1) = x_1^{1/2}(20 - 2x_1)^{1/2}. \tag{6.3}
\]
Chapter 6. Doing the “Best” We Can

Graph 6.9 plots this function, and this graph illustrates that the function $f$ attains a maximum at $x_1 = 5$, which is exactly the same answer we derived graphically in Graph 6.1. Furthermore, the $f$ function attains a value of zero at $x_1 = 10$. Thinking back to the underlying economics, when $x_1$ (the number of pants) is 10, I have no money left over for shirts. Since the tastes are such that both shirts and pants are “essential,” it makes sense that the function returns back to zero when I purchase no shirts.

Rather than plotting the whole function and finding the maximum graphically, we can of course use calculus to find the maximum. More precisely, since the function has a slope of zero when it attains its maximum, all we have to do to find this maximum mathematically is find where the slope (or derivative) of the function is zero. Taking the derivative of $f$ with respect to $x_1$, we get

$$\frac{df}{dx_1} = \frac{1}{2} x_1^{1/2} (20 - 2x_1)^{1/2} - x_1^{1/2} (20 - 2x_1)^{-1/2}. \quad (6.4)$$

When we then set this expression to zero and solve for $x_1$, we get $x_1 = 5$ as the maximum of the function, just as Graph 6.9 illustrates. Thus, we know that I will purchase 5 pairs of pants (costing a total of $100), leaving $100 to purchase 10 shirts (at a price of $10 each). We have found mathematically what we found graphically in Graph 6.1: the “best” choice for me “given my circumstances.”

**Method 2:** The Lagrange Method for Solving the Constrained Optimization Problem

A second (and more general) way to solve problems of the type expressed in (6.2) is to use a method that is known as the *Lagrange Method*. If you have taken a full calculus sequence, you have probably covered this in your last calculus course, but the method is not very complicated and does not require all the material usually covered in the entire calculus sequence. The method does essentially what we did in Method 1: It defines a new function and sets derivatives equal to zero in order to find the maximum of that new function. The function that we define is called the *Lagrange function*, and it is always constructed as a combination of the objective function in the optimization problem plus a term $\lambda$ multiplied by the constraint (where the terms in the constraint are all collected to one side, with the other side equal to zero). For instance, expression (6.2) results in the *Lagrange function* $\mathcal{L}$ given by

$$\mathcal{L}(x_1, x_2, \lambda) = x_1^{1/2} x_2^{1/2} + \lambda (200 - 20x_1 - 10x_2). \quad (6.5)$$
Notice that the function $L$ is a function of three variables: the two choice variables $(x_1, x_2)$ and $\lambda$, which is called the Lagrange multiplier. Without explaining exactly why the following solution method works, Lagrange problems of this type are solved by solving the system of three equations that arises when we take the partial derivatives of $L$ with respect to each of the three variables and set these derivatives to zero; i.e., we solve the following system of equations known jointly as the first order conditions of the constrained optimization problem:

$$\frac{\partial L}{\partial x_1} = \frac{1}{2} x_1^{1/2} x_2^{1/2} - 20\lambda = 0,$$

$$\frac{\partial L}{\partial x_2} = \frac{1}{2} x_1^{1/2} x_2^{1/2} - 10\lambda = 0,$$

$$\frac{\partial L}{\partial \lambda} = 200 - 20x_1 - 10x_2 = 0. \quad (6.6)$$

One easy way to solve this system of equations is to rewrite the first two by adding the $\lambda$ terms to both sides, thus getting

$$\frac{1}{2} x_1^{1/2} x_2^{1/2} = 20\lambda \quad (6.7)$$

and then dividing these two equations by each other to get

$$\frac{x_2}{x_1} = 2. \quad (6.8)$$

Multiplying both sides of (6.8) by $x_1$ then gives us

$$x_2 = 2x_1, \quad (6.9)$$

which we can insert into the third equation in expression (6.6) to get

$$200 - 20x_1 - 10(2x_1) = 0. \quad (6.10)$$

Solving this expression for $x_1$ then gives the same answer we calculated using our first method: $x_1 = 5$, and substituting that into expression (6.9) gives us $x_2 = 10$. Doing the “best” I can “given my circumstances” in Wal-Mart again means that I will purchase 5 pants and 10 shirts. Intuitively, condition (6.9) tells us that, for the type of tastes we are modeling and the prices that we are facing at Wal-Mart (20 and 10), it will be optimal for me to consume twice as many shirts ($x_2$) as pants ($x_1$); i.e., it will be optimal for me to consume on the ray emanating from the origin that contains bundles with twice as many shirts as pants. That is exactly the ray containing point A in Graph 6.1c, where we modeled the same homothetic tastes graphically. In fact, steps (6.9) and (6.10) above are exactly the same as the steps we used to solve for the optimal solutions when all we had to go on was the graphical information in Section 6A.1!

The Lagrange Method of solving constrained optimization problems is the preferred method for economists because it generalizes most easily to cases where we are choosing more than two goods. For instance, suppose that I was at Wal-Mart choosing bundles of pants ($x_1$), shirts ($x_2$), and socks ($x_3$) with the price of socks being equal to 5 (and all other prices the same as before), and suppose one utility function that can represent my tastes is the Cobb–Douglas function $u(x_1, x_2, x_3) = x_1^{1/2} x_2^{1/2} x_3^{1/2}$. Then my constrained optimization problem would be written as

$$\max_{x_1, x_2, x_3} u(x_1, x_2, x_3) = x_1^{1/2} x_2^{1/2} x_3^{1/2} \text{ subject to } 20x_1 + 10x_2 + 5x_3 = 200, \quad (6.11)$$
and the Lagrange function would be written as

\[ L(x_1, x_2, x_3, \lambda) = x_1^{1/2}x_2^{1/2}x_3^{1/2} + \lambda(200 - 20x_1 - 10x_2 - 5x_3). \quad (6.12) \]

We would then solve a system of 4 equations made up of the partial derivatives of \( L \) with respect to each of the choice variables \( (x_1, x_2, x_3) \) and \( \lambda \).

Solve for the optimal quantities of \( x_1, x_2, \) and \( x_3 \) in the problem defined in equation 6.11. (Hint: The problem will be considerably easier to solve if you take the logarithm of the utility function (which you can do since logarithms are order preserving transformations that do not alter the shapes of indifference curves).)

\[ \text{Exercise 6B.1} \]

### 6B.1.2 Opportunity Cost = Marginal Rate of Substitution: Solving the Problem by Combining Intuition and Math

When we solved my Wal-Mart consumer problem graphically in Graph 6.1, we discovered that once I made my “best” choice “given my circumstances,” my MRS of shirts for pants (the slope of my indifference curve at the optimal bundle) was exactly equal to the opportunity cost of pants (given by the slope of the budget constraint), at least as long as my tastes are such that I end up buying at least some of each good. The Lagrange Method we have just learned implicitly confirms this.

Specifically, suppose we just write the general constrained optimization problem for a consumer who chooses a bundle \( (x_1, x_2) \) given prices \( (p_1, p_2) \), an exogenous income \( I \) and tastes that can be summarized by a utility function \( u(x_1, x_2) \):

\[ \max_{x_1, x_2} u(x_1, x_2) \text{ subject to } p_1 x_1 + p_2 x_2 = I. \quad (6.13) \]

We then write the Lagrange function \( L(x_1, x_2, \lambda) \) as

\[ L(x_1, x_2, \lambda) = u(x_1, x_2) + \lambda(I - p_1 x_1 - p_2 x_2), \quad (6.14) \]

and we know that, at the optimal bundle, the partial derivatives of \( L \) with respect to each of the three variables is equal to zero. Thus,

\[ \frac{\partial L}{\partial x_1} = \frac{\partial u(x_1, x_2)}{\partial x_1} - \lambda p_1 = 0, \]

\[ \frac{\partial L}{\partial x_2} = \frac{\partial u(x_1, x_2)}{\partial x_2} - \lambda p_2 = 0. \quad (6.15) \]

These first order conditions can then be rewritten as

\[ \frac{\partial u(x_1, x_2)}{\partial x_1} = \lambda p_1, \]

\[ \frac{\partial u(x_1, x_2)}{\partial x_2} = \lambda p_2. \quad (6.16) \]

and the two equations can be divided by one another and multiplied by \(-1\) to give us

\[ -\left( \frac{\partial u(x_1, x_2)/\partial x_1}{\partial u(x_1, x_2)/\partial x_2} \right) = -\frac{p_1}{p_2}. \quad (6.17) \]

Notice that the left-hand side of equation (6.17) is the definition of the MRS whereas the right-hand side is the definition of the slope of the budget line. Thus, at the optimal bundle,

\[ MRS = -\frac{p_1}{p_2} = \text{opportunity cost of } x_1 \text{ (in terms of } x_2). \quad (6.18) \]
Knowing that this condition has to hold at the optimum, we can now illustrate a third method for solving the constrained optimization problem defined in (6.2):

**Method 3: Using $MRS = -\frac{p_1}{p_2}$ to Solve the Constrained Optimization Problem**

Returning to the case of my Wal-Mart problem, we arrived in the previous section at two equivalent methods of solving for my “best” bundle (as evaluated by the utility function $u(x_1, x_2) = x_1^{1/2} x_2^{1/2}$) given my circumstances of facing prices of $20 for pants and $10 for shirts as well as a budget of $200. In each case, the best option for me was to purchase 5 pants and 10 shirts. We could also, however, simply use the fact that we know expression (6.17) must hold at the optimum to get the same solution.

In particular, the left-hand side of equation (6.17) for the utility function $u(x_1, x_2) = x_1^{1/2} x_2^{1/2}$ is simply equal to $-x_2/x_1$ (which we previously derived in Chapter 4 when we derived the $MRS$ for such a function). Thus, the full equation (6.17) reduces to

$$-\frac{x_2}{x_1} = -\frac{p_1}{p_2} = -2,$$

which can also be written as

$$x_2 = 2x_1. \quad (6.20)$$

The budget constraint must also hold at the optimum, so we can plug (6.20) into the budget constraint $20x_1 + 10x_2 = 200$ to get

$$20x_1 + 10(2x_1) = 200. \quad (6.21)$$

Solving for $x_1$, we then get $x_1 = 5$, and plugging this back into (6.20) we get $x_2 = 10$; i.e., 5 pants and 10 shirts are once again optimal.

Notice that expressions (6.9) and (6.10) are exactly equivalent to equations (6.20) and (6.21). This is no accident. Method 3 of solving the constrained optimization problem simply substitutes some of our intuition (i.e., $MRS = -p_1/p_2$) to take a shortcut that is implicitly a part of the Lagrange Method (Method 2). Put differently, the two methods are rooted in the same underlying logic, with one using only mathematics and the other using the intuition that $MRS = -p_1/p_2$, an intuition that is based on the graphical logic of Graph 6.1.

This also confirms our intuition from Section 6A.1.2 that when all consumers face the same prices (as they do at Wal-Mart), their tastes are the same at the margin after they optimize. This is because the equality $MRS = -p_1/p_2$ holds for all consumers who consume both goods, regardless of how different their underlying tastes or money budgets are. Thus, tastes can differ even if tastes at the margin are the same after consumers choose their optimal bundles. Our discussion of gains from trade and efficiency in Section 6A.1.3 then follows from this.

### 6B.2 To Buy or Not to Buy: How to Find Corner Solutions

Although we have assumed throughout our mathematical discussion in this chapter that optimal choices always involve consumption of each of the goods, we had demonstrated in Section 6A.2 that, for certain types of tastes and certain economic circumstances, it is optimal to choose zero consumption of some goods, or, put differently, to choose a corner solution. This is important for the three mathematical optimization approaches we have discussed so far because each of them assumes an interior, not a corner, solution. We will see in this section what goes wrong with the mathematical approach when there are corner solutions and what assumptions we can make in order to be certain that the mathematical approach in Section 6B.1 does not run into problems due to the possible existence of corner solutions.
6B.2.1 Corner Solutions and First Order Conditions Consider, for instance, our example of me shopping in Wal-Mart for pants \((x_1)\) and shirts \((x_2)\) when the prices are $20 and $10 and my money budget is $200. Now, however, suppose that my tastes are properly summarized by the quasilinear utility function

\[
\ln u(x_1, x_2) = \alpha \ln x_1 + x_2, \tag{6.22}
\]

where “\(\ln\)” stands for the natural logarithm. Notice that tastes that can be represented by this utility function are such that \(\alpha\) is not essential and the indifference curves thus cross the \(x_1\) axis. The \(MRS\) of good \(x_1\) for \(x_2\) for this function is \(-\alpha/x_1\). Using our optimization Method 3, this implies that the optimal bundle must be such that \(-\alpha x_1 = -p_1/p_2 = -2\), which implies \(x_1 = \alpha/2\). Plugging this into the budget constraint and solving for \(x_2\), we get

\[
x_2 = \frac{(200 - 10\alpha)}{10}. \tag{6.23}
\]

Set up the Lagrange function for this problem and solve it to see whether you get the same solution.

Now suppose that \(\alpha = 25\) in the utility function (6.22). Then our solution for how much of \(x_2\) is “best” in equation (6.23) would suggest that I should consume a negative quantity of shirts \((x_2)\), negative 5 shirts to be specific! This is of course nonsense, and we can see what went wrong with the mathematics by illustrating the problem graphically.

More specifically, in Graph 6.10a we illustrate the shape of the optimal indifference curve derived from the utility function (6.22) (when \(\alpha = 25\)) as well as the budget constraint. The optimal bundle, bundle \(A\), contains no shirts and 10 pants. Our mathematical optimization missed this point because we did not explicitly add the constraint that consumption of neither good can be negative and simply assumed an interior solution where \(MRS = -p_1/p_2\). At the actual optimum \(A\), however, \(MRS \neq -p_1/p_2\).

Our mathematical solution method (without the constraint that consumption cannot be negative) pictured the problem as extending into a quadrant of the graph that we usually do not picture, the quadrant in which consumption of \(x_2\) is negative. This is illustrated in panel (b) of Graph 6.10, where indifference curves represented by the utility function (6.22) are allowed to cross into this new quadrant of the graph, as is the budget constraint. The “solution” found by solving first order conditions is illustrated as the tangency of the higher (magenta) indifference curve with the extended budget line, where \(MRS = -p_1/p_2\) as would be the case if the optimum was an interior solution.

The bottom line you should take from this example is that the mathematical methods of optimization we introduced in this chapter assume that the actual optimum is an interior solution and thus involves a positive level of consumption of all goods. When this is not the case, the math will give us the nonsensical answer unless we employ a more complicated method that explicitly introduces nonnegativity constraints for all consumption goods.\(^5\) Instead of resorting to more complex methods, however, we can just use common sense to conclude that the true optimum is a corner solution whenever our solution method suggests a negative level of consumption as optimal.

Demonstrate how the Lagrange Method (or one of the related methods we introduced earlier in this chapter) fails even more dramatically in the case of perfect substitutes. Can you explain what the Lagrange Method is doing in this case?

\(^5\)This more complicated method is a generalization of the Lagrange Method known as the “Kuhn Tucker method,” but it goes beyond the scope of this chapter. You can find it developed in graduate texts such as that by Mas-Colell, et al. (1992).
6B.2.2 Ruling Out Corner Solutions  We have already concluded intuitively in Section 6A.2.2 what assumptions on tastes are required in order for us to be sure that the optimum is an interior rather than a corner solution. Specifically, we argued that all goods that are modeled must be “essential” in the sense we defined in Chapter 5; i.e., indifference curves can converge to each axis but can never cross any axis. This should be even clearer now that we have seen how the mathematics of the Lagrange or related methods fails when indifference curves do cross an axis. Since our mathematical solution methods are guaranteed to work only in cases when we assume utility functions that represent tastes for goods that are all essential, the easiest way to model economic circumstances and use only the solution methods we have introduced is to assume only such utility functions. This does, however, rule out the important class of quasilinear tastes unless we simply modify our solution to be zero whenever the Lagrange (or a related) Method indicates a negative optimal consumption level.

The good news is that we will certainly know when we use the Lagrange (or a related) Method and we miss a corner solution because we will get the nonsensical solution of a negative optimal consumption level. But if we use these methods in models where not all goods are essential and we obtain solutions in which all consumption levels are positive, the methods are still giving us the correct answer. For instance, if \( \alpha \) in equation (6.22) is 10 instead of 25, the answer from equation (6.23) is that I should optimally consume 10 shirts (and five pants with the remainder of my budget). This solution is illustrated graphically in Graph 6.11 where, despite the fact that pants are not essential (and thus my indifference curves cross the shirt axis), my optimal choice is to purchase both shirts and pants under the economic circumstances I am facing at Wal-Mart.
Chapter 6. Doing the “Best” We Can

6B.3 Non-Convexities and First Order Conditions

When all goods in our optimization problem are essential—i.e., when indifference curves do not cross the axes—we have shown that any optimum of the problem must satisfy the first order conditions of the Lagrange problem. In other words, when all goods are essential, the first order conditions are necessary conditions for a point to be optimal. Unless non-convexities are absent from the optimization problem, however, the system of first order conditions may have multiple “solutions” (as we demonstrated in Section 6A.3 of the chapter), and not all of these are true optima (as we will show later). Put differently, in the presence of non-convexities, the first order conditions of the constrained optimization problem are necessary but not sufficient for a point to be a true optimum.

For this reason, we can simply solve for the solution of the first order condition equations and know for sure that the solution will be optimal only if we know that the problem has an interior solution and that the model has no non-convexities in choice sets or tastes. In the following section, we briefly explore the intuition of how such non-convexities can in fact result in nonoptimal solutions to the first order conditions of the Lagrange problem.

In the previous section, we concluded that the first order conditions of the Lagrange problem may be misleading when goods are not essential. Are these conditions either necessary or sufficient in that case?

6B.3.1 Non-Convexities in Choice Sets

In Section 6A.3 of the chapter, we motivated the potential for non-convex choice sets by appealing to one of our coupon examples from an earlier chapter, an example in which a kink in the budget constraint emerges. Solving optimizations problems with kinked budgets is a little involved, and so we leave it to be explored in the appendix to this chapter where a problem with an “outward” kink is solved. The same logic can be used to solve a problem with a non-convex kinked budget, one with an “inward kink.”

Exercise 6B.4

At what value for \( \alpha \) will the Lagrange Method correctly indicate an optimal consumption of zero shirts? Which of the panels of Graph 6.10 illustrates this?

Graph 6.11: The Presence of Nonessential Goods Does not Have to Result in a Corner Solution

Exercise 6B.5

In the previous section, we concluded that the first order conditions of the Lagrange problem may be misleading when goods are not essential. Are these conditions either necessary or sufficient in that case?
The mathematics of solving for the optimum when a budget is non-convex without the presence of a kink is somewhat different. We rarely encounter such budget constraints in micro-economic analysis, so we will not spend much time discussing them here. A problem of this type could be formally written as

$$\max_{x_1, x_2} u(x_1, x_2) \text{ subject to } f(x_1, x_2) = 0$$

(6.24)

where the function $f$ represents the nonlinear budget constraint. Such a problem could be set up exactly as we set up problems with linear budget constraints using a Lagrange function. The intuition of how just using first order conditions might yield misleading answers is seen relatively clearly with graphical examples. Consider, for instance, the shaded choice set in Graph 6.12 and the indifference curves that are tangent at points $A$ and $B$. At both points, the MRS is equal to the slope of the budget constraint, and thus both points would be solutions to the system of first derivative equations of the Lagrange function. But it is clear from the picture that only point $A$ is truly optimal since it lies on a higher indifference curve than point $B$. Whenever we solve a problem of this kind, we would therefore have to be careful to identify the true optimum from the possible optima that are produced through the Lagrange Method. Put differently, first order conditions are now necessary but not sufficient for identifying an optimal bundle.\(^6\)

6B.3.2 Non-Convexities in Tastes

In Section 6A.3.3, we discussed an example in which non-convex tastes result in multiple optimal solutions to an optimization problem (Graph 6.8). In the presence of such non-convexities in tastes, the Lagrange Method will still identify these optimal bundles, but it will once again also identify nonoptimal bundles. This is again because when non-convexities appear in constrained optimization problems, the first order conditions we use to solve for optimal solutions are necessary but not sufficient.

Graph 6.13 expands Graph 6.8 by adding another indifference curve to the picture, thus giving three points at which the MRS is equal to the ratio of prices. We can see immediately in this picture, however, that, while bundles $A$ and $B$ are optimal, bundle $C$ is not (since it lies on

\(^6\)You may have learned in your calculus classes about second order conditions. These conditions, involving second derivatives, ensure that points identified by first order conditions are indeed optimal. For an exploration of the mathematics of second order conditions, the reader is referred to E. Silberberg and W. Suen, *The Structure of Economics: A Mathematical Analysis*, 3rd ed. (Boston: McGraw-Hill, 2001) or other mathematical economics texts.
an indifference curve below that which contains bundles A and B.) The Lagrange Method will offer all three of these points as solutions to the system of first order conditions, which implies that, when we know that the underlying tastes are non-convex, we must check to see which of the points the Lagrange Method suggests are actually optimal. One way to do this is simply to plug the bundles the Lagrange method identifies back into the utility function to see which gives the highest utility. In the example of Graph 6.13, bundles A and B will give the same utility, but bundle C will give less. Thus, we could immediately conclude that only A and B are optimal.

While this method of plugging in the “candidate” optimal points (identified by the first order conditions) back into the utility function works, there exists a more general method by which to ensure that the Lagrange Method only yields truly optimal points. This method involves checking second derivative conditions, known in mathematics as second order conditions. Since we will rarely find a need to model tastes as non-convex, we will not focus on developing this method here. In general, you should simply be aware that we introduce greater complexity to the mathematical approach when we model situations in which non-convexities are important, complexities we do not need to worry about when the optimization problem is convex.

6B.4 Estimating Tastes from Observed Choices

In Section 6A.4, we acknowledged explicitly that tastes in themselves are not observable but also suggested that economists have developed ways of estimating the underlying tastes that are implied by choice behavior that we can observe. Essentially, we saw that the more choices we observe under different economic circumstances, the more information we can gain regarding the marginal rates of substitutions at different bundles that individuals are choosing. One interesting implication of this, however, is that the tastes that choice behavior implies are always going to satisfy our convexity assumption even when the true underlying tastes of a consumer are non-convex.

To see the intuition behind this, consider the case of a consumer whose indifference map contains the indifference curves drawn in Graph 6.13. We may observe such a consumer choosing bundles A and B, but we will never observe her choosing a bundle that lies on the non-convex portion of the indifference curve between A and B (unless the budget sets take on very odd shapes).
The reason for this is that tangencies with budget lines that lie on the non-convex portion of an indifference curve are not true optimal choices because they are like the bundle C in Graph 6.13. Thus, since we never observe choice behavior on non-convex portions of indifference maps, we can rarely infer the existence of non-convexities in tastes from choice behavior. An economist who observes the types of choices an individual makes with indifference curves like the ones in Graph 6.13 could simply conclude that there might be a “flat spot” in the indifference curve between A and B, but such an indifference curve would not contain the underlying non-convexity. The economist might suspect that there is a non-convexity in the indifference curve, but there is no way to identify it from observing consumption behavior easily.

CONCLUSION

We have now begun our analysis of optimizing behavior, of economic agents “doing the best they can” given their economic circumstances. In the end, all we are doing is combining our model of economic circumstances (budget constraints and choice sets from Chapters 2 and 3) with our model of tastes (indifference curves and utility functions from Chapters 4 and 5). But, even though we are just at the beginning of exploring all the implications of optimizing behavior, we are already gaining some insights relevant to the real world. We have defined in this chapter what it means for a situation to be economically efficient and have shown that optimizing consumer behavior in markets leads to an efficient allocation of goods across consumers. Put differently, market prices organize optimizing consumers so as to ensure that, once they have optimized in the market, they all have the same tastes on the margin for the goods that they have purchased. And with the same tastes on the margin, there is no way for consumers to find trades among each other that would make both parties better off; there are no gains from trade that have not already occurred in the market.

Along the way, we have also explored some technical details of optimization. Interior solutions are guaranteed only when tastes are defined such that all goods are “essential,” and corner solutions may arise when some goods are not essential. The consumer optimization problem will furthermore have a single unique solution if the optimization problem is in every way convex, with convex choice sets and (strictly) convex tastes (where averages are strictly better than extremes). This “uniqueness” of the solution may disappear, however, when tastes are defined such that averages can be just as good as extremes, or when tastes are non-convex. In the former case, a convex set of bundles may emerge as the solution (tangent to a “flat spot” on an indifference curve), whereas in the latter case a non-convex set of multiple solutions may emerge. Furthermore, when non-convexities in budgets or tastes are part of the consumer choice problem, the Lagrange Method (or derivatives of it) will identify as solutions bundles that are in fact not optimal.

We are not, however, done with our building of conceptual tools in our optimization model. Rather, we now move to Chapter 7 in which we begin to explore how optimizing behavior changes as economic circumstances (income and prices) in the economy change. Chapter 8 will extend this analysis to labor and financial markets, and Chapter 9 will demonstrate how the individual optimizing behavior results in demand curves for goods and supply curves for labor and capital. Finally, we will conclude our analysis of consumer optimization in Chapter 10, where we explore the concept of consumer surplus.

APPENDIX: OPTIMIZATION PROBLEMS WITH KINKED BUDGETS

In Section 6A.3.2, we introduced non-convexities in choice sets by considering budget constraints that have “inward” kinks, budget constraints like that graphed in Graph 6.6. We then discovered that non-convexities in choice sets can also arise without kinks, as in the budget constraint graphed in Graph 6.7. The mathematics of solving for optimal bundles is now complicated in two ways: First, in budget constraints that have kinks, the optimization problem contains a constraint that cannot be captured in a single equation; and second, in non-convex budgets without kinks, the first order conditions are not sufficient for us to identify optimal bundles.
Consider first the shaded kinked (but convex) choice set in Graph 6.14a, which replicates the coupon example graphed initially in Graph 6.5. The budget constraint of this choice set consists of two line segments, with the dotted extension of each line segment indicating the intercepts. The constrained optimization problem can now be written in two parts as

\[
\begin{align*}
\max_{x_1,x_2} u(x_1,x_2) \text{ subject to } & x_2 = 20 - x_1 \text{ for } 0 \leq x_1 \leq 6 \\
\max_{x_1,x_2} u(x_1,x_2) \text{ subject to } & x_2 = 26 - 2x_1 \text{ for } 6 \leq x_1
\end{align*}
\]

(6.25)

with the true optimum represented by the solution that achieved greater utility. The easiest way to solve such a problem is to solve two separate optimization problems with the extended line segments in Graph 6.14a representing the budget constraints in those problems; i.e.,

\[
\begin{align*}
\max_{x_1,x_2} u(x_1,x_2) \text{ subject to } & x_2 = 20 - x_1 \text{ for } 0 \leq x_1 \leq 6 \\
\max_{x_1,x_2} u(x_1,x_2) \text{ subject to } & x_2 = 26 - 2x_1 \text{ for } 6 \leq x_1
\end{align*}
\]

(6.26)
For the convex budget in Graph 6.14a, the true optimal point will occur either to the left of the kink (as in Graph 6.5b), to the right of the kink (as in Graph 6.5c), or on the kink (as in Graph 6.5d). When solving the two separate optimization problems in expression (6.26), we may get one of several corresponding sets of solutions. First, both optimization problems could result in an optimum with $x_1 < 6$, in which case the true optimum is the one resulting from the first optimization problem that is relevant for $x_1 < 6$ represented by $A$ in Graph 6.14b. Second, both optimization problems could result in a solution with $x_1 > 6$, in which case the true optimum is the one resulting from the second optimization problem that is relevant when $x_1 > 6$ represented by point $B$ in Graph 6.14c. Third, the first optimization problem could result in $x_1 > 6$ while the second optimization problem results in $x_1 < 6$, as represented in Graph 6.14d. In this case, both problems give a solution on the dotted extensions of the linear segments of the true budget constraint, with both $A$ and $B$ lying outside the shaded choice set. In this case, the true optimal point is the kink point (on the green indifference curve). Finally, both optimization problems could result in $x_1 = 6$, thus again indicating that the kink point is optimal (as depicted in Graph 6.14e).

**Exercise 6B.6** Is it necessary for the indifference curve at the kink of the budget constraint to have a kink in order for both problems in (6.26) to result in $x_1 = 6$?

When solving mathematically for optimal bundles when budget constraints are kinked, it is then best to combine the mathematics described with the intuition we gain from the graphical analysis. While we have illustrated this here with an “outwardly” kinked budget, the same is true for “inwardly” kinked (and thus non-convex) budgets, which we leave here to the following exercise.

**Exercise 6B.7** Using the intuitions from graphical analysis similar to that in Graph 6.14, illustrate how you might go about solving for the true optimum when a choice set is non-convex due to an “inward” kink.

**END-OF-CHAPTER EXERCISES**

6.1 I have two 3-year-old girls, Ellie and Jenny, at home. Suppose I begin the day by giving each girl 10 toy cars and 10 princess toys. I then ask them to plot their indifference curves that contain these endowment bundles on a graph with cars on the horizontal and princess toys on the vertical axis.

A. Ellie’s indifference curve appears to have a marginal rate of substitution of $-1$ at her endowment bundle, whereas Jenny’s appears to have a marginal rate of substitution of $-2$ at the same bundle.

a. Can you propose a trade that would make both girls better off?

b. Suppose the girls cannot figure out a trade on their own. So I open a store where they can buy and sell any toy for $1. Illustrate the budget constraint for each girl.

c. Will either of the girls shop at my store? If so, what will they buy?

d. Suppose I do not actually have any toys in my store and simply want my store to help the girls make trades between themselves. Suppose I fix the price at which princess toys are bought and sold to $1$. Without being specific about what the price of toy cars would have to be, illustrate, using final indifference curves for both girls on the same graph, a situation where the prices in my store result in an efficient allocation of toys.

e. What values might the price for toy cars take to achieve the efficient trades you described in your answer to (d)?
B. Now suppose that my girls’ tastes could be described by the utility function 
\[ u(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}, \]
where \( x_1 \) represents toy cars, \( x_2 \) represents princess toys, and \( 0 < \alpha < 1 \).

a. What must be the value of \( \alpha \) for Ellie (given the information in part A)? What must the value be for Jenny?

b. When I set all toy prices to $1, what exactly will Ellie do? What will Jenny do?

c. Given that I am fixing the price of princess toys at $1, do I have to raise or lower the price of car toys in order for me to operate a store in which I don’t keep inventory but simply facilitate trades between the girls?

d. Suppose I raise the price of car toys to $1.40, and assume that it is possible to sell fractions of toys. Have I found a set of prices that allow me to keep no inventory?

6.2 Suppose Coke and Pepsi are perfect substitutes for me, and right and left shoes are perfect complements.

A. Suppose my income allocated to Coke/Pepsi consumption is $100 per month, and my income allocated to right/left shoe consumption is similarly $100 per month.

a. Suppose Coke currently costs $0.50 per can and Pepsi costs $0.75 per can. Then the price of Coke goes up to $1 per can. Illustrate my original and my new optimal bundle with Coke on the horizontal and Pepsi on the vertical axis.

b. Suppose right and left shoes are sold separately. If right and left shoes are originally both priced at $1, illustrate (on a graph with right shoes on the horizontal and left shoes on the vertical) my original and my new optimal bundle when the price of left shoes increases to $2.

c. True or False: Perfect complements represent a unique special case of homothetic tastes in the following sense: Whether income goes up or whether the price of one of the goods falls, the optimal bundle will always lie on a the same ray emerging from the origin.

B. Continue with the assumptions about tastes from part A.

a. Write down two utility functions: one representing my tastes over Coke and Pepsi, another representing my tastes over right and left shoes.

b. Using the appropriate equation derived in B(a), label the two indifference curves you drew in A(a).

c. Using the appropriate equation derived in B(a), label the two indifference curves you drew in A(b).

d. Consider two different equations representing indifference curves for perfect complements: 
\[ u^1(x_1, x_2) = \min\{x_1, x_2\} \] and \[ u^2(x_1, x_2) = \min\{x_1, 2x_2\}. \] By inspecting two of the indifference curves for each of these utility functions, determine the equation for the ray along which all optimal bundles will lie for individuals whose tastes these equations can represent.

e. Explain why the Lagrange Method does not seem to work for calculating the optimal consumption bundle when the goods are perfect substitutes.

f. Explain why the Lagrange Method cannot be applied to calculate the optimal bundle when the goods are perfect complements.

6.3 Pizza and Beer: Sometimes we can infer something about tastes from observing only two choices under two different economic circumstances.

A. Suppose we consume only beer and pizza (sold at prices \( p_1 \) and \( p_2 \) respectively) with an exogenously set income \( I \).

a. With the number of beers on the horizontal axis and the number of pizzas on the vertical, illustrate a budget constraint (clearly labeling intercepts and the slope) and some initial optimal (interior) bundle \( A \).

b. When your income goes up, I notice that you consume more beer and the same amount of pizza. Can you tell whether my tastes might be homothetic? Can you tell whether they might be quasilinear in either pizza or beer?

c. How would your answers change if I had observed you decreasing your beer consumption when income goes up?
d. How would your answers change if both beer and pizza consumption increased by the same proportion as income?

B. Suppose your tastes over beer \((x_1)\) and pizza \((x_2)\) can be summarize by the utility function 
\(u(x_1, x_2) = x_1^2x_2\) and that \(p_1 = 2, p_2 = 10\) and weekly income \(I = 180\).

a. Calculate your optimal bundle \(A\) of weekly beer and pizza consumption by simply using the fact that, at any interior solution, 
\[ MRS = -\frac{p_1}{p_2}. \]

b. What numerical label does this utility function assign to the indifference curve that contains your optimal bundle?

c. Set up the more general optimization problem where, instead of using the prices and income given earlier, you simply use \(p_1, p_2\) and \(I\). Then, derive your optimal consumption of \(x_1\) and \(x_2\) as a function of \(p_1, p_2\) and \(I\).

d. Plug the values \(p_1 = 2, p_2 = 10,\) and \(I = 180\) into your answer to B(c) and verify that you get the same result you originally calculated in B(a).

e. Using your answer to part B(c), verify that your tastes are homothetic.

f. Which of the scenarios in A(b) through (d) could be generated by the utility function 
\(u(x_1, x_2) = x_1^2x_2\)?

6.4† \(\text{Inferring Tastes for Roses (and Love) from Behavior}:\) I express my undying love for my wife through weekly purchases of roses that cost $5 each.

A. Suppose you have known me for a long time and you have seen my economic circumstances change with time. For instance, you knew me in graduate school when I managed to have $125 per week in disposable income that I could choose to allocate between purchases of roses and “other consumption” denominated in dollars. Every week, I brought 25 roses home to my wife.

a. Illustrate my budget as a graduate student, with roses on the horizontal and “dollars of other consumption” on the vertical axis. Indicate my optimal bundle on that budget as \(A\). Can you conclude whether either good is not “essential”?

b. When I became an assistant professor, my disposable income rose to $500 per week, and the roses I bought for my wife continued to sell for $5 each. You observed that I still bought 25 roses each week. Illustrate my new budget constraint and optimal bundle on your graph. From this information, can you conclude whether my tastes might be quasilinear in roses? Might they not be quasilinear?

c. Suppose for the rest of the problem that my tastes in fact are quasilinear in roses. One day while I was an assistant professor, the price of roses suddenly dropped to $2.50. Can you predict whether I then purchased more or fewer roses?

d. Suppose I had not gotten tenure, and the best I could do was rely on a weekly allowance of $50 from my wife. Suppose further that the price of roses goes back up to $5. How many roses will I buy for my wife per week?

e. \text{True or False}: Consumption of quasilinear goods always stays the same as income changes.

f. \text{True or False}: Over the range of prices and incomes where corner solutions are not involved, a decrease in price will result in increased consumption of quasilinear goods but an increase in income will not.

B. Suppose my tastes for roses \((x_1)\) and other goods \((x_2)\) can be represented by utility function 
\(u(x_1, x_2) = \beta x_1^2 + x_2\).

a. Letting the price of roses be denoted by \(p_1\), the price of other goods by \(I\), and my weekly income by \(I\), determine my optimal weekly consumption of roses and other goods as a function of \(p_1\) and \(I\).

b. Suppose \(\beta = 50\) and \(\alpha = 0.5\). How many roses do I purchase when \(I = 125\) and \(p_1 = 5\)? What if my income rises to $500?

c. Comparing your answers with your graph from part A, could the actions observed in part A(b) be rationalized by tastes represented by the utility function \(u(x_1, x_2)\)? Give an example of another utility function that can rationalize the behavior described in part A(b).
d. What happens when the price of roses falls to $2.50? Is this consistent with your answer to part A(c)?

e. What happens when my income falls to $50 and the price of roses increases back to $5? Is this consistent with your answer to part A(d)? Can you illustrate in a graph how the math is giving an answer that is incorrect?

6.5 Assume you have an income of $100 to spend on goods \( x_1 \) and \( x_2 \).

**A.** Suppose that you have homothetic tastes that happen to have the special property that indifference curves on one side of the 45-degree line are mirror images of indifference curves on the other side of the 45-degree line.

a. Illustrate your optimal consumption bundle graphically when \( p_1 = 1 = p_2 \).

b. Now suppose the price of the first 75 units of \( x_1 \) you buy is 1/3 while the price for any additional units beyond that is 3. The price of \( x_2 \) remains at 1 throughout. Illustrate your new budget and optimal bundle.

c. Suppose instead that the price for the first 25 units of \( x_1 \) is 3 but then falls to 1/3 for all units beyond 25 (with the price of \( x_2 \) still at 1). Illustrate this budget constraint and indicate what would be optimal.

d. If the homothetic tastes did not have the symmetry property, which of your answers might not change?

**B.** Suppose that your tastes can be summarized by the Cobb–Douglas utility function \( u(x_1, x_2) = x_1^{1/2} x_2^{1/2} \).

a. Does this utility function represent tastes that have the symmetry property described in part A?

b. Calculate the optimal consumption bundle when \( p_1 = 1 = p_2 \).

c. Derive the two equations that make up the budget constraint you drew in part A(b) and use the method described in the appendix to this chapter to calculate the optimal bundle under that budget constraint.

d. Repeat for the budget constraint you drew in A(c).

e. Repeat (b) through (d) assuming instead \( u(x_1, x_2) = x_1^{3/4} x_2^{1/4} \) and illustrate your answers in graphs.

6.6* **Coffee, Coke, and Pepsi:** Suppose there are three different goods: cans of Coke (\( x_1 \)), cups of coffee (\( x_2 \)), and cans of Pepsi (\( x_3 \)).

**A.** Suppose each of these goods costs the same price, \( p \), and you have an exogenous income, \( I \).

a. Illustrate your budget constraint in three dimensions and carefully label all intercepts and slopes.

b. Suppose each of the three drinks has the same caffeine content, and suppose caffeine is the only characteristic of a drink you care about. What do “indifference curves” look like?

c. What bundles on your budget constraint would be optimal?

d. Suppose that Coke and Pepsi become more expensive. How does your answer change? Are you now better or worse off than you were before the price change?

**B.** Assume again that the three goods cost the same price, \( p \).

a. Write down the equation of the budget constraint you drew in part A(a).

b. Write down a utility function that represents the tastes described in A(b).

c. Can you extend our notion of homotheticity to tastes over three goods? Are the tastes represented by the utility function you derived in (b) homothetic?

6.7* **Coffee, Milk, and Sugar:** Suppose there are three different goods: cups of coffee (\( x_1 \)), ounces of milk (\( x_2 \)), and packets of sugar (\( x_3 \)).

**A.** Suppose each of these goods costs $0.25 and you have an exogenous income of $15.

a. Illustrate your budget constraint in three dimensions and carefully label all intercepts.
b. Suppose that the only way you get enjoyment from a cup of coffee is to have at least 1 ounce of milk and 1 packet of sugar in the coffee, the only way you get enjoyment from an ounce of milk is to have at least 1 cup of coffee and 1 packet of sugar, and the only way you get enjoyment from a packet of sugar is to have at least 1 cup of coffee and 1 ounce of milk. What is the optimal consumption bundle on your budget constraint?

c. What does your optimal indifference curve look like?

d. If your income falls to $10, what will be your optimal consumption bundle?

e. If instead of a drop in income the price of coffee goes to $0.50, how does your optimal bundle change?

f. Suppose your tastes are less extreme and you are willing to substitute some coffee for milk, some milk for sugar, and some sugar for coffee. Suppose that the optimal consumption bundle you identified in (b) is still optimal under these less extreme tastes. Can you picture what the optimal indifference curve might look like in your picture of the budget constraint?

g. If tastes are still homothetic (but of the less extreme variety discussed in (f)), would your answers to (d) or (e) change?

B. Continue with the assumption of an income of $15 and prices for coffee, milk, and sugar of $0.25 each.

a. Write down the budget constraint.

b. Write down a utility function that represents the tastes described in A(b).

c. Suppose that instead your tastes are less extreme and can be represented by the utility function \( u(x_1, x_2, x_3) = x_1^a x_2^b x_3^c \). Calculate your optimal consumption of \( x_1, x_2, \) and \( x_3 \) when your economic circumstances are described by the prices \( p_1, p_2, \) and \( p_3 \) and income is given by \( I \).

d. What values must \( a \) and \( b \) take in order for the optimum you identified in A(b) to remain the optimum under these less extreme tastes?

e. Suppose \( a \) and \( b \) are as you concluded in part B(d). How does your optimal consumption bundle under these less extreme tastes change if income falls to $10 or if the price of coffee increases to $0.50? Compare your answers with your answer for the more extreme tastes in A(d) and (e).

f. True or False: Just as the usual shapes of indifference curves represent two-dimensional “slices” of a three-dimensional utility function, three-dimensional “indifference bowls” emerge when there are three goods, and these “bowls” represent slices of a four-dimensional utility function.

6.8 Grits and Cereal: In end-of-chapter exercise 4.1, I described my dislike for grits and my fondness for Coco Puffs Cereal.

A. In part A of exercise 4.1, you were asked to assume that my tastes satisfy convexity and continuity and then to illustrate indifference curves on a graph with grits on the horizontal axis and cereal on the vertical.

a. Now add a budget constraint (with some positive prices for grits and cereal and some exogenous income, \( I \), for me). Illustrate my optimal choice given my tastes.

b. Does your answer change if my tastes are non-convex (as in part (b) of exercise 4.1A)?

c. In part (c) of exercise 4.1A, you were asked to imagine that I hate cereal as well and that my tastes are again convex. Illustrate my optimal choice under this assumption.

d. Does your answer change when my tastes are not convex (as in part (d) of exercise 4.1A)?

B. In part B of exercise 4.1, you derived a utility function that was consistent with my dislike for grits.

a. Can you explain why the Lagrange Method will not work if you used it to try to solve the optimization problem using this utility function?

b. What would the Lagrange Method offer as the optimal solution if you used a utility function that captured a dislike for both grits and cereal when tastes are non-convex? Illustrate your answer using \( u(x_1, x_2) = -x_1 x_2 \) and graph your insights.
c. What would the Lagrange Method offer as a solution if a utility function that captures a dislike for both grits and cereal represented convex tastes? Illustrate your answer using the function \( u(x_1, x_2) = -x_1^2 - x_2^2 \) and show what happens graphically.

6.9† Everyday Application: Price Fluctuations in the Housing Market: Suppose you have $400,000 to spend on a house and “other goods” (denominated in dollars).

A. The price of 1 square foot of housing is $100, and you choose to purchase your optimally sized house at 2,000 square feet. Assume throughout that you spend money on housing solely for its consumption value (and not as part of your investment strategy).
   a. On a graph with “square feet of housing” on the horizontal axis and “other goods” on the vertical, illustrate your budget constraint and your optimal bundle \( A \).
   b. After you bought the house, the price of housing falls to $50 per square foot. Given that you can sell your house from bundle \( A \) if you want to, are you better or worse off?
   c. Assuming you can easily buy and sell houses, will you now buy a different house? If so, is your new house smaller or larger than your initial house?
   d. Does your answer to (c) differ depending on whether you assume tastes are quasilinear in housing or homothetic?
   e. How does your answer to (c) change if the price of housing went up to $200 per square foot rather than down to $50.
   f. What form would tastes have to take in order for you not to sell your 2,000-square-foot house when the price per square foot goes up or down?
   g. True or False: So long as housing and other consumption is at least somewhat substitutable, any change in the price per square foot of housing makes homeowners better off (assuming it is easy to buy and sell houses.)
   h. True or False: Renters are always better off when the rental price of housing goes down and worse off when it goes up.

B. Suppose your tastes for “square feet of housing” \( (x_1) \) and “other goods” \( (x_2) \) can be represented by the utility function \( u(x_1, x_2) = x_1 x_2 \).
   a. Calculate your optimal housing consumption as a function of the price of housing \( (p_1) \) and your exogenous income \( I \) (assuming of course that \( p_2 \) is by definition equal to 1).
   b. Using your answer, verify that you will purchase a 2,000-square-foot house when your income is $400,000 and the price per square foot is $100.
   c. Now suppose the price of housing falls to $50 per square foot and you choose to sell your 2,000-square-foot house. How big a house would you now buy?
   d. Calculate your utility (as measured by your utility function) at your initial 2,000-square-foot house and your new utility after you bought your new house. Did the price decline make you better off?
   e. How would your answers to B(c) and B(d) change if, instead of falling, the price of housing had increased to $200 per square foot?

6.10 Everyday Application: Different Interest Rates for Borrowing and Lending: You first analyzed intertemporal budget constraints with different interest rates for borrowing and saving (or lending) in end-of-chapter exercise 3.8.

A. Suppose that you have an income of $100,000 now and you expect to have a $300,000 income 10 years from now, and suppose that the interest rate for borrowing from the bank is twice as high as the interest rate the bank offers for savings.
   a. Begin by drawing your budget constraint with “consumption now” and “consumption in 10 years” on the horizontal and vertical axes. (Assume for purposes of this problem that your consumption in the intervening years is covered and not part of the analysis.)
   b. Can you explain why, for a wide class of tastes, it is rational for someone in this position not to save or borrow?
c. Now suppose that the interest rate for borrowing was half the interest rate for saving. Draw this new budget constraint.

d. Illustrate a case where it might be rational for a consumer to flip a coin to determine whether to borrow a lot or to save a lot.

B. Suppose that your incomes are as described in part A and that the annual interest rate for borrowing is 20% and the annual interest rate for saving is 10%. Also, suppose that your tastes over current consumption, $c_1$, and consumption 10 years from now, $c_2$, can be captured by the utility function $u(c_1, c_2) = c_1^a c_2^{1-a}$.

a. Assuming that interest compounds annually, what are the slopes of the different segments of the budget constraint that you drew in A(a)? What are the intercepts?

b. For what ranges of $a$ is it rational to neither borrow nor save?

### 6.11* Business Application: Quantity Discounts and Optimal Choices

In end-of-chapter exercise 2.9, you illustrated my department’s budget constraint between “pages copied in units of 100” and “dollars spent on other goods” given the quantity discounts our local copy service gives the department. Assume the same budget constraint as the one described in 2.9A.

**A.** In this exercise, assume that my department’s tastes do not change with time (or with who happens to be department chair). When we ask whether someone is “respecting the department’s tastes,” we mean whether that person is using the department’s tastes to make optimal decisions for the department given the circumstances the department faces. Assume throughout that my department’s tastes are convex.

a. *True or False:* If copies and other expenditures are very substitutable for my department, then you should observe either very little or a great deal of photocopying by our department at the local copy shop.

b. Suppose that I was department chair last year and had approximately 5,000 copies per month made. This year, I am on leave and an interim chair has taken my place. He has chosen to make 150,000 copies per month. Given that our department’s tastes are not changing over time, can you say that either I or the current interim chair is not respecting the department’s tastes?

c. Now the interim chair has decided to go on vacation for a month, and an interim interim chair has been named for that month. He has decided to purchase 75,000 copies per month. If I was respecting the department’s tastes, is this interim interim chair necessarily violating them?

d. If both the initial interim chair and I were respecting the department’s tastes, is the new interim interim chair necessarily violating them?

**B.** Consider the decisions made by the three chairs as previously described.

a. If the second interim chair (i.e., the interim interim chair) and I both respected the department’s tastes, can you approximate the elasticity of substitution of the department’s tastes?

b. If the first and second interim chairs both respected the department’s tastes, can you approximate the elasticity of substitution for the department?

c. Could the underlying tastes under which all three chairs respect the department’s tastes be represented by a CES utility function?

### 6.12* Business Application: Retail Industry Lobbying for Daylight Savings Time

In 2005, the U.S. Congress passed a bill to extend daylight savings time earlier into the spring and later into the fall (beginning in 2007). The change was made as part of an Energy Bill, with some claiming that daylight savings time reduces energy use by extending sunlight to later in the day (which means fewer hours of artificial light). Among the biggest advocates for daylight savings time, however, was the retail and restaurant industry that believes consumers will spend more time shopping and eating in malls for reasons explored here.

**A.** Consider a consumer who returns home from work at 6 p.m. and goes to sleep at 10 p.m. In the month of March, the sun sets by 7 p.m. in the absence of daylight savings time, but with daylight savings time, the sun does not set until 8 p.m. When the consumer comes home from work, she can either spend time (1) at home eating food from her refrigerator while e-mailing friends and surfing/shopping on the Internet or (2) at the local mall meeting friends for a bite to eat and strolling through stores...
to shop. Suppose this consumer gets utility from (1) and (2) (as defined here) but she also cares about \( x_3 \), which is defined as the fraction of daylight hours after work.

a. On a graph with “weekly hours at the mall” on the horizontal axis and “weekly hours at home” on the vertical, illustrate this consumer’s typical weekly after-work time constraint (with a total of 20 hours per week available, 4 hours on each of the 5 workdays). (For purposes of this problem, assume the consumer gets as much enjoyment from driving to the mall as she does being at the mall.)

b. Consider first the scenario of no daylight savings time in March. This implies only 1 hour of daylight in the 4 hours after work and before going to sleep; i.e., the fraction \( x_3 \) of daylight hours after work is \( \frac{1}{4} \). Pick a bundle \( A \) on the budget constraint from (a) as the optimum for this consumer given this fraction of after-work of daylight hours.

c. Now suppose daylight savings time is moved into March, thus raising the number of after-work daylight hours to 2 per day. Suppose this changes the at every bundle. If the retail and restaurant industry is right, which way does it change the ?

d. Illustrate how if the retail and restaurant industry is right, this results in more shopping and eating at malls every week.

e. Explain the following statement: “While it appears in our two-dimensional indifference maps that tastes have changed as a result of a change in daylight savings time, tastes really haven’t changed at all because we are simply graphing two-dimensional slices of the same three-dimensional indifference surfaces.”

f. Businesses can lobby Congress to change the circumstances under which we make decisions, but Congress has no power to change our tastes. Explain how the change in daylight savings time illustrates this in light of your answer to (e).

g. Some have argued that consumers must be irrational for shopping more just because daylight savings is introduced. Do you agree?

h. If we consider not just energy required to produce light but also energy required to power cars that take people to shopping malls, is it still clear that the change in daylight savings time is necessarily energy saving?

B. Suppose a consumer’s tastes can be represented by the utility function \( u(x_1, x_2, x_3) = 12x_3 \ln x_1 + x_2 \), where \( x_1 \) represents weekly hours spent at the mall, \( x_2 \) represents weekly after-work hours spent at home (not sleeping), and \( x_3 \) represents the fraction of after-work (before-sleep) time that has daylight.

a. Calculate the \( MRS \) of \( x_2 \) for \( x_1 \) for this utility function and check to see whether it has the property that retail and restaurant owners hypothesize.

b. Which of the three things the consumer cares about—\( x_1 \), \( x_2 \), and \( x_3 \)—are choice variables for the consumer?

c. Given the overall number of weekly after-work hours our consumer has (i.e., 20), calculate the number of hours per week this consumer will spend in malls and restaurants as a function of \( x_3 \).

d. How much time per week will she spend in malls and restaurants in the absence of daily savings time? How does this change when daylight savings time is introduced?

6.13 Policy Application: Food Stamps versus Food Subsidies: In exercise 2.13, you considered the food stamp programs in the United States. Under this program, poor households receive a certain quantity of “food stamps,” stamps that contain a dollar value that is accepted like cash for food purchases at grocery stores.

A. Consider a household with monthly income of $1,500 and suppose that this household qualifies for food stamps in the amount of $500.

a. Illustrate this household’s budget, both with and without the food stamp program, with “dollars spent on food” (on the horizontal axis) and “dollars spent on other goods” on the vertical. What has to be true for the household to be just as well off under this food stamp program as it would be if the government simply gave $500 in cash to the household (instead of food stamps)?

b. Consider the following alternate policy: Instead of food stamps, the government tells this household that it will reimburse 50% of the household’s food bills. On a separate graph,
illustrate the household’s budget (in the absence of food stamps) with and without this alternate program.

c. Choose an optimal bundle $A$ on the alternate program budget line and determine how much the government is paying to this household (as a vertical distance in your graph). Call this amount $S$.

d. Now suppose the government decided to abolish the program and instead gives the same amount $S$ in food stamps. How does this change the household’s budget?

e. Will this household be happy about the change from the first alternate program to the food stamp program?

f. If some politicians want to increase food consumption by the poor and others just want to make the poor happier, will they differ on what policy is best?

g. **True or False:** The less substitutable food is for other goods, the greater the difference in food consumption between equally funded cash and food subsidy programs.

h. Consider a third possible alternative: giving cash instead of food stamps. **True or False:** As the food stamp program becomes more generous, the household will at some point prefer a pure cash transfer over an equally costly food stamp program.

B. **Suppose this household’s tastes for spending on food ($x_1$) and spending on other goods ($x_2$) can be characterized by the utility function $u(x_1, x_2) = a \ln x_1 + \ln x_2$.**

   a. Calculate the level of food and other good purchases as a function of $I$ and the price of food $p_1$ (leaving the price of dollars on other goods as just 1).

   b. For the household described in part A, what is the range of $a$ that makes the $500 food stamp program equivalent to a cash gift of $500$?

   c. Suppose for the remainder of the problem that $a = 0.5$. How much food will this household buy under the alternate policy described in A(b)?

   d. How much does this alternate policy cost the government for this household? Call this amount $S$.

   e. How much food will the household buy if the government gives $S$ as a cash payment and abolishes the alternate food subsidy program?

   f. Determine which policy—the price subsidy that leads to an amount $S$ being given to the household or the equally costly cash payment in part (e)—the household prefers.

   g. Now suppose the government considered subsidizing food more heavily. Calculate the utility that the household will receive from three equally funded policies: a 75% food price subsidy (i.e., a subsidy where the government pays 75% of food bills), a food stamp program, and a cash gift program.

6.14 **Policy Application:** *Gasoline Taxes and Tax Rebates:* Given the concerns about environmental damage from car pollution, many have proposed increasing the tax on gasoline. We will consider the social benefits of such legislation later on in the text when we introduce externalities. For now, however, we can look at the impact on a single consumer.

A. Suppose a consumer has annual income of $50,000 and suppose the price of a gallon of gasoline is currently $2.50.

   a. Illustrate the consumer’s budget constraint with “gallons of gasoline” per year on the horizontal axis and “dollars spent on other goods” on the vertical. Then illustrate how this changes if the government imposes a tax on gasoline that raises the price per gallon to $5.00.

   b. Pick some bundle $A$ on the after tax budget constraint and assume that bundle is the optimal bundle for our consumer. Illustrate in your graph how much in gasoline taxes this consumer is paying, and call this amount $T$.

   c. One of the concerns about using gasoline taxes to combat pollution is that it will impose hardship on consumers (and, perhaps more importantly, voters). Some have therefore suggested that the government simply rebate all revenues from a gasoline tax to taxpayers. Suppose that our consumer receives a rebate of exactly $T$. Illustrate how this alters the budget of our consumer.

   d. Suppose our consumer’s tastes are quasilinear in gasoline. How much gasoline will he consume after getting the rebate?
e. Can you tell whether the tax/rebate policy is successful at getting our consumer to consume less gasoline than he would were there neither the tax nor the rebate?

f. True or False: Since the government is giving back in the form of a rebate exactly the same amount as it collected in gasoline taxes from our consumer, the consumer is made no better or worse off from the tax/rebate policy.

B. Suppose our consumer’s tastes can be captured by the quasilinear utility function \( u(x_1, x_2) = 200x_1^{0.5} + x_2 \), where \( x_1 \) denotes gallons of gasoline and \( x_2 \) denotes dollars of other goods.

a. Calculate how much gasoline this consumer consumes as a function of the price of gasoline \( p_1 \) and income \( I \). Since other consumption is denominated in dollars, you can simply set its price \( p_2 \) to 1.

b. After the tax raises the price of gasoline to $5, how much gasoline does our consumer purchase this year?

c. How much of a tax does he pay?

d. Can you verify that his gasoline consumption will not change when the government sends him a rebate check equal to the tax payments he has made?

e. How does annual gasoline consumption for our consumer differ under the tax/rebate program from what it would be in the absence of either a tax or rebate?

f. Illustrate that our consumer would prefer no tax/rebate program but, if there is to be a tax on gasoline, he would prefer to have the rebate rather than no rebate.

6.15* Policy Application: AFDC and Work Disincentives: Consider the AFDC program for an individual as described in end-of-chapter exercise 3.18.

A. Consider again an individual who can work up to 8 hours per day at a wage of $5 per hour.

a. Replicate the budget constraint you were asked to illustrate in 3.18A.

b. True or False: If this person’s tastes are homothetic, then he/she will work no more than 1 hour per day.

c. For purposes of defining a 45-degree line for this part of the question, assume that you have drawn hours on the horizontal axis 10 times as large as dollars on the vertical. This implies that the 45-degree line contains bundles like (1, 10), (2, 20), etc. How much would this person work if his tastes are homothetic and symmetric across this 45-degree line? (By “symmetric across the 45-degree line,” I mean that the portions of the indifference curves to one side of the 45-degree line are mirror images to the portions of the indifference curves to the other side of the 45-degree line.)

d. Suppose you knew that the individual’s indifference curves were linear but you did not know the MRS. Which bundles on the budget constraint could in principle be optimal and for what ranges of the MRS?

e. Suppose you knew that, for a particular person facing this budget constraint, there are two optimal solutions. How much in AFDC payments does this person collect at each of these optimal bundles (assuming the person’s tastes satisfy our usual assumptions)?

B. Suppose this worker’s tastes can be summarized by the Cobb–Douglas utility function \( u(c, \ell) = c^a \ell^{1-a} \), where \( \ell \) stands for leisure and \( c \) for consumption.

a. Forget for a moment the AFDC program and suppose that the budget constraint for our worker could simply be written as \( c = I - 5\ell \). Calculate the optimal amount of consumption and leisure as a function of \( a \) and \( I \).

b. On your graph of the AFDC budget constraint for this worker, there are two line segments with slope \(-5\): one for 0–2 hours of leisure and another for 7–8 hours of leisure. Each of these lies on a line defined by \( c = I - 5\ell \) except that \( I \) is different for the two equations that contain these line segments. What are the relevant Js to identify the right equations on which these budget constraint segments lie?

c. Suppose \( a = 0.25 \). If this worker were to optimize using the two budget constraints you have identified with the two different Js, how much leisure would he choose under each constraint?
Can you illustrate what you find in a graph and tell from this where on the real AFDC budget constraint this worker will optimize?

d. As $\alpha$ increases, what happens to the $MRS$ at each bundle?

e. Repeat B(c) for $\alpha = 0.3846$ and for $\alpha = 0.4615$. What can you now say about this worker’s choice for any $0 < \alpha < 0.3846$? What can you say about this worker’s leisure choice if $0.3846 < \alpha < 0.4615$?

f. Repeat B(c) for $\alpha = 0.9214$ and calculate the utility associated with the resulting choice. Compare this to the utility of consuming at the kink point $(7, 30)$ and illustrate what you have found on a graph. What can you conclude about this worker’s choice if $0.4615 < \alpha < 0.9214$?

g. How much leisure will the worker take if $\alpha > 0.9214$?

h. Describe in words what this tells you about what it would take for a worker to overcome the work disincentives under the AFDC program.

6.16 Policy Application: Cost of Living Adjustments of Social Security Benefits: Social Security payments to the elderly are adjusted every year in the following way: The government has in the past determined some average bundle of goods consumed by an average elderly person. Each year, the government then takes a look at changes in the prices of all the goods in that bundle and raises Social Security payments by the percentage required to allow the hypothetical elderly person to continue consuming that same bundle. This is referred to as a cost of living adjustment or COLA.

A. Consider the impact on an average senior’s budget constraint as cost of living adjustments are put in place. Analyze this in a two-good model where the goods are simply $x_1$ and $x_2$.

a. Begin by drawing such a budget constraint in a graph where you indicate the “average bundle” the government has identified as $A$ and assume that initially this average bundle is indeed the one our average senior would have chosen from his budget.

b. Suppose the prices of both goods went up by exactly the same proportion. After the government implements the COLA, has anything changed for the average senior? Is behavior likely to change?

c. Now suppose that the price of $x_1$ went up but the price of $x_2$ stayed the same. Illustrate how the government will change the average senior’s budget constraint when it calculates and passes along the COLA. Will the senior alter his behavior? Is he better off, worse off, or unaffected?

d. How would your answers change if the price of $x_2$ increased and the price of $x_1$ stayed the same?

e. Suppose the government’s goal in paying COLAs to senior citizens is to insure that seniors become neither better nor worse off from price changes. Is the current policy successful if all price changes come in the form of general “inflation”; i.e., if all prices always change together by the same proportion? What if inflation hits some categories of goods more than others?

f. If you could “choose” your tastes under this system, would you choose tastes for which goods are highly substitutable, or would you choose tastes for which goods are highly complementary?

B. ** Suppose the average senior has tastes that can be captured by the utility function $u(x_1, x_2) = (x_1^\rho + x_2^\rho)^{-\frac{1}{\rho}}$.

a. Suppose the average senior has income from all sources equal to $40,000 per year, and suppose that prices are given by $p_1$ and $p_2$. How much will our senior consume of $x_1$ and $x_2$? (Hint: It may be easiest simply to use what you know about the $MRS$ of CES utility functions to solve this problem.)

b. If $p_1 = p_2 = 1$ initially, how much of each good will the senior consume? Does your answer depend on the elasticity of substitution?

c. Now suppose that the price of $x_1$ increases to $p_1 = 1.25$. How much does the government have to increase the senior’s Social Security payment in order for the senior still to be able to purchase the same bundle as he purchased prior to the price change?
d. Assuming the government adjusts the Social Security payment to allow the senior to continue to purchase the same bundle as before the price increase, how much $x_1$ and $x_2$ will the senior actually end up buying if $\rho = 0$?

e. How does your answer change if $\rho = -0.5$ and if $\rho = -0.95$? What happens as $\rho$ approaches $-1$?

f. How does your answer change when $\rho = 1$ and when $\rho = 10$? What happens as $\rho$ approaches infinity?

g. Can you come to a conclusion about the relationship between how much a senior benefits from the way the government calculates COLAs and the elasticity of substitution that the senior’s tastes exhibit? Can you explain intuitively how this makes sense, particularly in light of your answer to A(f)?

h. Finally, show how COLAs affect consumption decisions by seniors under general inflation that raises all prices simultaneously and in proportion to one another as, for instance, when both $p_1$ and $p_2$ increase from 1.00 to 1.25 simultaneously.
We have just demonstrated in Chapter 6 how we can use our model of choice sets and tastes to illustrate optimal decision making by individuals such as consumers or workers. We now turn to the question of how such optimal decisions change when economic circumstances change. Since economic circumstances in this model are fully captured by the choice set, we could put this differently by saying that we will now ask how optimal choices change when income, endowments, or prices change.

As we proceed, it is important for us to keep in mind the difference between tastes and behavior. Behavior, or what we have been calling choice, emerges when tastes confront circumstances as individuals try to do the “best” they can given those circumstances. If I buy less wine because the price of wine has increased, my behavior has changed but my tastes have not. Wine still tastes the same as it did before, it just costs more. In terms of the tools we have developed, my indifference map remains exactly as it was. I simply move to a different indifference curve as my circumstances (i.e., the price of wine) change.

In the process of thinking about how behavior changes with economic circumstances, we will identify two conceptually distinct causes, known as income and substitution effects. At first it will seem like the distinction between these effects is abstract and quite unrelated to real-world issues we care about. As you will see later, however, this could not be further from the truth. Deep questions related to the efficiency of tax policy, the effectiveness of Social Security and health policy, and the desirability of different types of antipoverty programs are fundamentally rooted in questions related to income and substitution effects. While we are still in the stage of building tools for economic analysis, I hope you will be patient and bear with me as we develop an understanding of these tools.

Still, it may be useful to at least give an initial example to motivate the effects we will develop in this chapter, an example that will already be familiar to you if you have done end-of-chapter exercise 6.14. As you know, there is increasing concern about carbon-based emissions from automobiles, and an increased desire by policy makers to find ways of reducing such emissions. Many economists have long recommended the simple policy of taxing gasoline heavily in order to encourage consumers to find ways of conserving gasoline (by driving less and buying more fuel-efficient cars). The obvious concern with such a policy is that it imposes substantial hardship on households that rely heavily on their cars, particularly poorer households that would be hit pretty hard by such a tax. Some

1Chapters 2 and 4 through 6 are required reading for this chapter. Chapter 3 is not necessary.
2This distinction was fully introduced into neoclassical economics by Sir John Hicks in his influential book, Value and Capital, originally published in 1939. We had previously mentioned him in part B of Chapter 5 as the economist who first derived a way to measure substitutability through “elasticities of substitution.” Hicks was awarded the Nobel Prize in Economics in 1972 (together with Ken Arrowi).
economists have therefore proposed simply sending all tax revenues from such a gasoline tax back to taxpayers in the form of a tax refund. This has led many editorial writers to conclude that economists must be nuts; after all, if we send the money back to the consumers, wouldn’t they then just buy the same amount of gasoline as before since (at least on average) they would still be able to afford it? Economists may be nuts, but our analysis will tell us that they are also almost certainly right, and editorial writers are almost certainly wrong, when it comes to the prediction of how this policy proposal would change behavior. And the explanation lies fully in an understanding of substitution effects that economists understand and most noneconomists don’t think about. We’ll return to this in the conclusion to the chapter.

7A Graphical Exposition of Income and Substitution Effects

There are two primary ways in which choice sets (and thus our economic circumstances) can change: First, a change in our income or wealth might shift our budget constraints without changing their slopes, and thus without changing the opportunity costs of the various goods we consume. Second, individual prices in the economy—whether in the form of prices of goods, wages, or interest rates—may change and thus alter the slopes of our budget constraints and the opportunity costs we face. These two types of changes in choice sets result in different types of effects on behavior, and we will discuss them separately in what follows. First, we will look only at what happens to economic choices when income or wealth changes without a change in opportunity costs (Section 7A.1). Next, we will investigate how decisions are impacted when only opportunity costs change without a change in real wealth (Section 7A.2). Finally, we will turn to an analysis of what happens when changes in income and opportunity costs occur at the same time, which, as it turns out, is typically the case when relative prices in the economy change.

7A.1 The Impact of Changing Income on Behavior

What happens to our consumption when our income increases because of a pay raise at work or when our wealth endowment increases because of an unexpected inheritance or when our leisure endowment rises due to the invention of some time-saving technology? Would we consume more shirts, pants, Coke, housing, and jewelry? Would we consume more of some goods and fewer of others, work more or less, save more or less? Would our consumption of all goods go up by the same proportion as our income or wealth?

The answer depends entirely on the nature of our tastes, and the indifference map that represents our tastes. For most of us, it is likely that our consumption of some goods will go up by a lot while our consumption of other goods will increase by less, stay the same, or even decline. The impact of changes in our income or wealth on our consumption decisions (in the absence of changes in opportunity costs) is known as the income or wealth effect.

The economics “lingo” is not entirely settled on whether to call this kind of an effect a “wealth” or an “income” effect, and we will use the two terms in the following way: Whenever we are analyzing a model where the size of the choice set is determined by exogenously given income, as in Chapter 2 and for the remainder of this chapter, we will refer to the impact of a change in income as an income effect. In models where the size of the choice set is determined by the value of an endowment, as in Chapter 3 and in the next chapter, we will refer to the impact of changes in that endowment as a wealth effect. What should be understood throughout, however, is that by both income and wealth effect we mean an impact on consumer decisions that arises from a parallel shift in the budget constraint, a shift that does not include a change in opportunity costs as captured by a change in the slope of the budget line.
7A.1.1 Normal and Inferior Goods  During my first few years in graduate school, my wife and I made relatively little money. Often, our budget would permit few extravagances, with dinners heavily tilted toward relatively cheap foods such as potatoes and pasta. When my wife’s business began to take off, our income increased considerably, and she observed one night over a nice steak dinner that we seemed to be eating a lot less pasta these days. Our consumption of pasta, it turned out, declined as our income went up, whereas our consumption of steak and other goods increased. How could this happen within the context of the general model that we have developed in the last few chapters?

Consider a simple model in which we put monthly consumption of boxes of pasta on the horizontal axis and the monthly consumption of pounds of steak on the vertical. My wife and I began with a relatively low income and experienced an increase in income as my wife’s business succeeded. This is illustrated by the outward shift in our budget constraint (from blue to magenta) in each of the panels of Graph 7.1. As we then add the indifference curves that contain our optimal choices under the two budget constraints, we get less pasta consumption at the higher income only if the tangency on the budget line occurs to the left of our tangency on the lower budget line. This is illustrated in panel (a) of Graph 7.1. Panel (b), on the other hand, illustrates the relationship between the two indifference curves if pasta consumption had remained unchanged with the increase in our income, while panel (c) illustrates the case had our pasta consumption increased with our income. This change in consumer behavior as exogenous income changes is called the income effect.

Since my wife observed that our consumption of pasta declined with an increase in our income, our preferences must look more like those in panel (a), where increased income has a negative impact on pasta consumption. We will then say that the income effect is negative whenever an increase in exogenous income (without a change in opportunity cost) results in less consumption, and goods whose consumption is characterized by negative income effects are called inferior goods. In contrast, we will say that the income effect is positive whenever an increase in exogenous income (without a change in opportunity cost) results in more consumption, and goods whose consumption is characterized by positive income effects are called normal goods. Panel
(c) of Graph 7.1 illustrates an example of what our preferences could look like if pasta were in fact a normal good for us. Finally, panel (b) of Graph 7.1 illustrates an indifference map that gives rise to no income effect on our pasta consumption. Notice the following defining characteristic of this indifference map: The marginal rate of substitution is constant along the vertical line that connects points A and B. In Chapter 5, we called tastes that are represented by indifference curves whose marginal rates of substitution are constant in this way quasilinear (in pasta). The sequence of panels in Graph 7.1 then illustrates how quasilinear tastes are the only kinds of tastes that do not give rise to income effects for some good, and as such they represent the borderline case between normal and inferior goods.

It is worthwhile noting that whenever we observe a negative income effect on our consumption of one good, there must be a positive income effect on our consumption of a different good. After all, the increased income must be going somewhere, whether it is increased consumption of some good today or increased savings for consumption in the future. In Graph 7.1a, for instance, we observe a negative income effect on our consumption of pasta on the horizontal axis. At the same time, on the vertical axis we observe a positive income effect on our consumption of steak.

7A.1.2 Luxuries and Necessities  As we have just seen, quasilinear tastes represent one special case that divides two types of goods: normal goods whose consumption increases with income and inferior goods whose consumption decreases with income. The defining difference between these two types of goods is how consumption changes in an absolute sense as our income changes. A different way of dividing goods into two sets is to ask how our relative consumption of different goods changes as income changes. Put differently, instead of asking whether total consumption of a particular good increases or decreases with an increase in income, we could ask whether the fraction of our income spent on a particular good increases or decreases as our income goes up; i.e., whether our consumption increases relative to our income.

Consider, for instance, our consumption of housing. In each panel of Graph 7.2, we model choices between square footage of housing and “dollars of other goods.” As in the previous graph, we consider how choices will change as income doubles, with bundle A representing the optimal choice at the lower income and bundle B representing the optimal choice at the higher income. Suppose that in each panel, the individual spends 25% of her income on housing at bundle A. If housing remains a constant fraction of consumption as income increases, then the optimal consumption bundle B when income doubles would simply involve twice as much housing and twice as much “other good” consumption. This bundle would then lie on a ray emanating from the origin and passing through point A, as pictured in Graph 7.2b. If, on the other hand, the fraction of income allocated to housing declines as income rises, B would lie to the left of this ray (as in Graph 7.2a), and if the fraction of income allocated to housing increases as income rises, B would lie to the right of the ray (as in Graph 7.2c). It turns out that on average, people spend approximately 25% of their income on housing regardless of how much they make, which implies that tastes for housing typically look most like those in Graph 7.2b.

Is it also the case that whenever there is a positive income effect on our consumption of one good, there must be a negative income effect on our consumption of a different good?  

Exercise 7A.1  

Can a good be an inferior good at all income levels? (Hint: Consider the bundle (0,0).)  

Exercise 7A.2
Economists have come to refer to goods whose consumption as a fraction of income declines with income as necessities while referring to goods whose consumption as a fraction of income increases with income as luxuries. The borderline tastes that divide these two classes of goods are tastes of the kind represented in Graph 7.2b, tastes that we defined as homothetic in Chapter 5. (Recall that we said tastes were homothetic if the marginal rates of substitution are constant along any ray emanating from the origin.) Thus, just as quasilinear tastes represent the borderline tastes between normal and inferior goods, homothetic tastes represent the borderline tastes between necessary and luxury goods.

### Exercise 7A.3
Are all inferior goods necessities? Are all necessities inferior goods? (Hint: The answer to the first is yes; the answer to the second is no.) Explain.

### Exercise 7A.4
At a particular consumption bundle, can both goods (in a two-good model) be luxuries? Can they both be necessities?

### 7A.2 The Impact of Changing Opportunity Costs on Behavior

Suppose my brother and I go off on a week-long vacation to the Cayman Islands during different weeks. He and I are identical in every way, same income, same tastes. Since there is no public transportation on the Cayman Islands, you only have two choices of what to do once you step off the airplane: you can either rent a car for the week, or you can take a taxi to your hotel and then rely on taxis for any additional transportation needs. After we returned home from our respective vacations, we compared notes and discovered that, although we had stayed at exactly the same hotel, I had rented a car whereas my brother had used only taxis.

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3This assumption is for illustration only. Both my brother and I are horrified at the idea of anyone thinking we are identical, and he asked for this clarification in this text.
Which one of us do you think went on more trips away from our hotel? The difference between the number of car rides he and I took is what we will call a substitution effect.

7A.2.1 Renting a Car versus Taking Taxis on Vacation  
The answer jumps out straight away if we model the relevant aspects of the choice problem that my brother and I were facing when we arrived at the airport in the Cayman Islands. Basically, we were choosing the best way to travel by car during our vacation. We can model this choice by putting “miles travelled” on the horizontal axis and “dollars of other consumption” on the vertical. Depending on whether I rent a car or rely on taxis, I will face different budget constraints. If I rent a car, I end up paying a weekly rental fee that is the same regardless of how many miles I actually drive. I then have to pay only for the gas I use as I drive to different parts of the island. If I rely on taxis, on the other hand, I pay only for the miles I travel, but of course I pay a per mile cost that is higher than just the cost of gas. Translated into budget constraints with “miles driven” on the horizontal axis and “dollars of other consumption” on the vertical, this implies that my budget will have a higher intercept on the vertical axis if I choose to use taxis because I do not have to pay the fixed rental fee. At the same time, the slope of the budget constraint would be steeper if I chose to use taxis because each mile I travel has a higher opportunity cost.

The choice my brother and I faced when we arrived in the Cayman Islands is thus a choice between two different budget constraints, one with a higher intercept and steeper slope than the other, as depicted in Graph 7.3a. (If this looks familiar, it is because you may have done this in end-of-chapter exercise 2.6.) Since my brother and I are identical in every way and faced exactly the same choice, you can reasonably conclude that we were indifferent between these two modes of transportation (and thus between the two budget constraints). After all, if one choice was clearly better than the other, we should have ended up making the same choice.

Thus, although we made different choices, we must have ended up on the same indifference curve. (This statement—that we ended up on the same indifference curve—makes sense only because we know that my brother and I have the same tastes and thus the same map of indifference curves, and we have the same exogenous income.) Graph 7.3b therefore fits a single indifference curve tangent to the two budget constraints, illustrating that our optimal
choices on the two different budget constraints result in the same level of satisfaction. My brother’s optimal choice A then indicates fewer miles traveled than my optimal choice B.

The intuition behind the model’s prediction is straightforward. Once I sped off to my hotel in my rented car, I had to pay the rental fee no matter what else I did for the week. So, the opportunity cost or price of driving a mile (once I decided to rent a car) was only the cost of gasoline. My brother, on the other hand, faced a much higher opportunity cost since he had to pay taxi prices for every mile he traveled. Even though our choices made us equally well off, it is clear that my lower opportunity cost of driving led me to travel more miles and consume less of other goods than my brother.

Economists will often say that the flat weekly rental fee becomes a sunk cost as soon as I have chosen to rent a car. Once I have rented the car, there is no way for me to get back the fixed rental fee that I have agreed to pay, and it stays the same no matter what I do once I leave the rental car lot. So, the rental fee is never an opportunity cost of anything I do once I have rented the car. Such sunk costs, once they have been incurred, therefore do not affect economic decisions because our economic decisions are shaped by the trade-offs inherent in opportunity costs. We will return to the concept of sunk costs more extensively when we discuss producer behavior, and we will note in Chapter 29 that some psychologists quarrel with the economist’s conclusion that such costs should have no impact on behavior.

7A.2.2 Substitution Effects

The difference in my brother’s and my behavior in our Cayman Island example is what is known as a substitution effect. Substitution effects arise whenever opportunity costs or prices change. In our example, for instance, we analyzed the difference in consumer behavior when the price of driving changes, but the general intuition behind the substitution effect will be important for many more general applications throughout this book.

We will define a substitution effect more precisely as follows: The substitution effect of a price change is the change in behavior that results purely from the change in opportunity costs and not from a change in real income. By real income, we mean real welfare, so “no change in real income” should be taken to mean “no change in satisfaction” or “no change in indifference curves.” The Cayman Island example was constructed so that we could isolate a substitution effect clearly by focusing our attention on a single indifference curve or a single level of “real income.”

The fact that bundle B must lie to the right of bundle A is a simple matter of geometry: A steeper budget line fit tangent to an indifference curve must lie to the left of a shallower budget line that is tangent to the same indifference curve. The direction of a substitution effect is therefore always toward more consumption of the good that has become relatively cheaper and away from the good that has become relatively more expensive. Note that this differs from what we concluded about income effects whose direction depends on whether a good is normal or inferior.

7A.2.3 How Large Are Substitution Effects?

While the direction of substitution effects is unambiguous, the size of the effect is dependent entirely on the kinds of underlying tastes a consumer has. The picture in Graph 7.3b suggests a pretty clear and sizable difference between

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4This definition of “real income” differs from another definition you may run into during your studies of economics (one that we also used in an earlier chapter on budget constraints). Macroeconomists who study inflation, or microeconomists who want to study behavior that is influenced by inflation, often define “real income” as “inflation adjusted income.” For instance, when comparing someone’s income in 1990 to his or her income in 2000, an economist might adjust the 2000 income by the amount of inflation that occurred between 1990 and 2000, thus reporting 2000 “real income” expressed in 1990 dollars.
the number of miles I drove and the number of miles my brother drove given that we faced
different opportunity costs for driving while having the same level of satisfaction or welfare.
But I could have equally well drawn the indifference curve with more curvature, and thus with
less substitutability between miles driven and other consumption. The less substitutability is built
into a consumer’s tastes, the smaller will be substitution effects arising from changes in opportu-
nity costs.

For instance, consider the indifference curve in Graph 7.4b, an indifference curve with more
curvature than that in Graph 7.4a and thus less built-in substitutability along the portion on which
my brother and I are making our choices. Notice that, although the substitution effect points in
the same direction as before, the effect is considerably smaller. Graph 7.4c illustrates the role
played by the level of substitutability between goods even more clearly by focusing on the
extreme case of perfect complements. Such tastes give rise to indifference curves that permit no
substitutability between goods, leading to bundles A and B overlapping and a consequent disap-
pearance of the substitution effect.

**Exercise 7A.6**

True or False: If you observed my brother and me consuming the same number of miles driven
during our vacations, then our tastes must be those of perfect complements between miles
driven and other consumption.

### 7A.2.4 “Hicks” versus “Slutsky” Substitution

We have now defined the substitution effect as the change in consumption that is due to a change in
opportunity cost without a change in “real income”; i.e., without a change in the indifference curve. This is sometimes called
Hicksian substitution. A slightly different concept of a substitution effect arises when we ask how
a change in opportunity costs alters a consumer’s behavior assuming that her ability to purchase
the original bundle remains intact. This is called Slutsky substitution. It operates very similarly to
Hicksian substitution, and we will therefore leave it to end-of-chapter exercise 7.11 to explore
this further. We are also using the idea in exercise 7.11 (and its previous companion exercise
6.16) and 7.6 (as well as its previous companion exercise 6.9).
7A.3 Price Changes: Income and Substitution Effects Combined

As you were reading through the Cayman Island example, you may have wondered why I chose such an admittedly contrived story. The reason is that I wanted to follow our discussion of pure income effects (which occur in the absence of changes in opportunity costs) in Section 7A.1 with a discussion of pure substitution effects (which occur in the absence of any changes in real income or wealth) in Section 7A.2. Most real-world changes in opportunity costs, however, implicitly also give rise to changes in real income, causing the simultaneous operation of both income and substitution effects.

Let’s forget the Cayman Islands, then, and consider what happens when the price of a good that most of us consume goes up, as, for instance, the price of gasoline. When this happens, I can no longer afford to reach the same indifference curve as before if my exogenous income remains the same. Thus, not only do I face a different opportunity cost for gasoline but I also have to face the prospect of ending up with less satisfaction—or what we have called less “real” income—because I am doomed to operate on a lower indifference curve than before the price increase. Similarly, if the price of gasoline declines, I not only face a different opportunity cost for gasoline but will also end up on a higher indifference curve, and thus experience an increase in real income. A price change therefore typically results in both an income effect and a substitution effect. These can be conceptually disentangled even though they occur simultaneously, and it will become quite important for many policy applications to know the relative sizes of these conceptually different effects. You will see how this is important more clearly in later chapters. For now, we will simply focus on conceptually disentangling the two effects of price changes.

7A.3.1 An Increase in the Price of Gasoline

To model the impact of an increase in the price of gasoline on my behavior, we can once again put “miles driven” on the horizontal axis and “dollars of other consumption” on the vertical. An increase in the price of gasoline then causes an inward rotation of the budget line around the vertical intercept, as illustrated in Graph 7.5a. My optimal bundle prior to the price increase is illustrated by the tangency of the indifference curve at point A.

Graph 7.5: Income and Substitution Effects when Gasoline Is a Normal Good
We can now begin our disentangling of income and substitution effects by asking how my consumption bundle would have changed had I only experienced the change in opportunity costs without a change in my real income. Put differently, we can ask how my consumption decision would change if I faced a new budget that incorporated the steeper slope implied by the price change but was large enough to permit me to be as satisfied as I was before the price change, large enough to keep me on my original indifference curve. This budget is illustrated as the green budget tangent to the indifference curve containing bundle $A$ in Graph 7.5b and is called the compensated budget. A compensated budget for a price change is the budget that incorporates the new price but includes sufficient monetary compensation to make the consumer as well off as she was before the price change. If income is exogenous (as it is in our example), the compensated budget requires positive compensation when prices increase and negative compensation when prices decrease.

Graph 7.5b then looks very much like Graph 7.4b that illustrated a pure substitution effect for our Cayman Islands example. This is because we have imagined that I was provided sufficient compensation at the higher gasoline price to keep my real income constant in order to focus only on the change in my consumption that is due to the change in my opportunity costs along a single indifference curve. As in the Cayman example, we can then quickly see that consumption of gasoline is less at point $B$ than at point $A$. When real income is unchanged, the substitution effect tells us that I will consume less gasoline because gasoline has become more expensive relative to other goods.

Rarely, however, will someone come to me and offer me compensation for a price change in real life. Rather, I will have to settle for a decrease in my real income when prices go up. In Graph 7.5c, we thus start with the compensated budget and ask how my actual consumption decision will differ from the hypothetical outcome $B$. Before answering this question, notice that the compensated budget and the final budget in Graph 7.5c have the same slope and thus differ only by the hypothetical compensation we have assumed when plotting the compensated budget. Thus when going from the compensated (green) to the final (magenta) budget, we are simply analyzing the impact of a change in my exogenous money income, or what we called a pure income effect in Section 7A.1.

Whether my optimal consumption of gasoline on my final budget line is larger or smaller than at point $B$ then depends entirely on whether gasoline is a normal or an inferior good for me. We defined a normal good as one whose consumption moves in the same direction as changes in exogenous income, while we defined an inferior good as one whose consumption moved in the opposite direction of changes in exogenous income. Thus, the optimal bundle on the final budget might lie to the left of point $B$ if gasoline is a normal good, and it might lie to the right of $B$ if gasoline is an inferior good. In the latter case, it could lie in between $A$ and $B$ if the income effect is smaller than the substitution effect, or it might lie to the right of point $A$ if the income effect is larger than the substitution effect. In Graph 7.5c, we illustrate the case where gasoline is a normal good, and the optimal final bundle $C$ lies to the left of $B$. In this case, both income and substitution effects suggest that I will purchase less gasoline as the price of gasoline increases.

**7A.3.2 Regular Inferior and Giffen Goods** Notice that we can conclude unambiguously that my consumption of gasoline will decline if its price increases whenever gasoline is a normal good (as is the case if bundle $C$ in Graph 7.5c is my optimal final choice). This is because both the substitution and the income effect suggest declining consumption. If, on the other hand, gasoline is an inferior good for me, then my gasoline consumption could increase or decrease depending on whether my final consumption bundle lies between $A$ and $B$ as in Graph 7.6a or whether it lies to the right of $A$ as in Graph 7.6b. We can therefore divide inferior goods into two subcategories: those whose consumption decreases with an increase in price and those whose consumption increases with an increase in price (when exogenous income remains constant). We will call the former regular inferior goods and the latter Giffen goods.

When initially introduced to the possibility that a consumer might purchase more of a good when its price goes up, students often misinterpret what economists mean by this. A common example that students will think of is that of certain goods that carry a high level of prestige...
precisely because everyone knows they are expensive. For instance, it may be true that some consumers who care about the prestige value of a BMW will be more likely to purchase BMWs as the price (and thus the prestige value) increases. This is not, however, the kind of behavior we have in mind when we think of Giffen goods. The person who attaches a prestige value to the price of a BMW is really buying two different goods when he or she buys this car: the car itself and the prestige value of the car. As the price of the BMW goes up, the car remains the same but the quantity of prestige value rises. So, a consumer who is more likely to buy BMWs as the price increases is not buying more of a single good but is rather buying a different mix of goods when the price of the BMW goes up. When the same consumer’s income falls (and the price of BMWs remains the same), the consumer would almost certainly be less likely to buy BMWs, which indicates that the car itself (with the prestige value held constant) is a normal good.  

Real Giffen goods are quite different, and we rarely observe them in the real world. Economists have struggled for literally centuries to find examples; this is how rare they are. At the end of the 19th century, Alfred Marshall (1842–1924), one of the great economists of that century, included a hypothetical example in his economics textbook and attributed it to Robert Giffen, a contemporary of his. Over the years, a variety of attempts to find credible historical examples that are not hypothetical have been discredited, although a recent paper demonstrates that rice in poor areas of China may indeed be a Giffen good there.  

While an increase in the price still causes an increase in the consumption of the physical good we observe, such goods are examples of what is known as Veblen Goods after Thorstein Veblen (1857–1929) who hypothesized that preferences for certain goods intensify as price increases, which can cause what appear to be increases in consumption as price goes up. You can think through this more carefully in end-of-chapter exercise 7.9, where you are asked to explain an increase in the consumption of Gucci accessories when the price increases. In Chapter 21, we revisit Veblen goods in end-of-chapter exercise 21.5 in the context of network externalities.  

6To quote from his text: “As Mr. Giffen has pointed out, a rise in the price of bread makes so large a drain on the resources of the poorer labouring families that they are forced to curtail their consumption of meat and the more expensive farinaceous foods: and bread being still the cheapest food which they can get and will take, they consume more, and not less of it.” A. Marshall, Principles of Economics (MacMillan: London, 1895). While Robert Giffen (1837–1910) was a highly regarded economist and statistician, it appears no one has located a reference to the kinds of goods that are named after him in any of his own writings, only in Marshall’s.  

A friend of mine in graduate school once told me a story that is the closest example I have ever personally heard of a real Giffen good. He came from a relatively poor family in the Midwest where winters get bitterly cold and where they heated their home with a form of gasoline. Every winter, they would spend a month over Christmas with relatives in Florida. One year during the 1973 energy crisis, the price of gasoline went up so much that they decided they could not afford to go on their annual vacation in Florida. So, they stayed in the Midwest and had to heat their home for one additional month. While they tried to conserve on gasoline all winter, they ended up using more than usual because of that extra month. Thus, their consumption of gasoline went up precisely because the price of gasoline went up and the income effect outweighed the substitution effect. This example, as well as the recent research on rice in China, both illustrate that, in order to find the “Giffen behavior” of increasing consumption with an increase in price, it must be that the good in question represents a large portion of a person’s income to begin with, with a change in price therefore causing a large income effect. It furthermore must be the case that there are no very good substitutes for the good in order for the substitution effect to remain small. Given the variety of substitutable goods in the modern world and the historically high standard of living, it therefore seems very unlikely that we will find much “Giffen behavior” in the part of the world that has risen above subsistence income levels.

Can you re-tell the Heating Gasoline-in-Midwest story in terms of income and substitution effects in a graph with “yearly gallons of gasoline consumption” on the horizontal axis and “yearly time on vacation in Florida” on the vertical?

7A.3.3 Income and Substitution Effects for Pants and Shirts. Now let’s return to our example from Chapter 2: My wife sends me to Wal-Mart with a fixed budget to buy pants and shirts. Since I know how much Wal-Mart charges for pants and shirts, I enter the store already having solved for my optimal bundle. Now suppose that one of the greeters at Wal-Mart hands me a 50% off coupon for pants, effectively decreasing the price of pants I face. We already know that this will lead to an outward rotation of my budget as shown in Graph 7.7a. Armed with the new information presented in this chapter, however, we can now predict how my consumption of pants and shirts will change depending on whether pants and shirts are normal, regular inferior, or Giffen goods.

Graph 7.7: Inferring the Type of Good from Observed Choices
First, we isolate once again the substitution effect by drawing my (green) compensated budget under the new price in Graph 7.7b. Notice that the “compensation” in this case is negative: In order to keep my “real income” (i.e., my indifference curve) constant and concentrate only on the impact of the change in opportunity costs, you would have to take away some of the money my wife had given me. As always, the substitution effect, the shift from A to B, indicates that I will switch away from the good that has become relatively more expensive (shirts) and toward the good that has become relatively cheaper (pants).

In Graph 7.7c, we then focus on what happens when we switch from the hypothetical optimum on the compensated (green) budget to our new optimum on the final (magenta) budget. Since this involves no change in opportunity costs, we are left with a pure income effect as we jump from the optimal point B on the compensated budget line to the final optimum on the final budget constraint. Suppose we know that both shirts and pants are normal goods for me. This would tell me that, when I experience an increase in income from the compensated to the final budget, I will choose to consume more pants and fewer shirts than I did at point B. If shirts are inferior and pants are normal, I will consume more pants and fewer shirts than at B; and if pants are inferior and shirts are normal, I will consume fewer pants and more shirts. Given that I am restricted in this example to consuming only shirts and pants, it cannot be the case that both goods are inferior because this would imply that I consume fewer pants and fewer shirts on my final budget than I did at point B, which would put me at a bundle to the southwest of B. Since “more is better,” I would not be at an optimum given that I can move to a higher indifference curve from there.

Now suppose that you know not only that pants are an inferior good but also that pants are a Giffen good. The definition of a Giffen good implies that I will consume less of the good as its price decreases when exogenous income remains unchanged. Thus, I would end up consuming not just fewer pants than at point B but also fewer than at point A. Notice that this is the only scenario under which we would not even have to first find the substitution effect; if we know something is a Giffen good and we know its price has decreased, we immediately know that consumption will decrease as well. In each of the other scenarios, however, we needed to find the compensated optimum before being able to apply the definition of normal or inferior goods.

Finally, suppose you know that shirts rather than pants are a Giffen good. Remember that in order to observe a Giffen good, we must observe a price change for that good (with exogenous income constant) since Giffen goods are goods whose consumption moves in the same direction as price (when income is exogenous and unchanged). In this example, we did not observe a price change for shirts, which means that we cannot usefully apply the definition of a Giffen good to predict how consumption will change. Rather, we can simply note that, since all Giffen goods are also inferior goods, I will consume fewer shirts as my income increases from the compensated budget to the final budget. Thus, knowing that shirts are Giffen tells us nothing more in this example than knowing that shirts are inferior goods.

**Exercise 7A.8** Replicate Graph 7.7 for an increase in the price of pants (rather than a decrease).

**7B The Mathematics of Income and Substitution Effects**

In this section, we will now begin to explore income and substitution effects mathematically. I say that we will “begin” doing this because our exploration of these effects will become deeper as we move through the next few chapters. For now, we will try to illustrate how to relate the intuitions developed in part A of this chapter most directly to some specific mathematics, and in the process we will build
the tools for a more general treatment later on. As you read through this section, you will undoubtedly
get lost a bit unless you sit with pencil and paper and follow the calculations we undertake closely on
your own. As you do this, you will begin to get a feel for how we can use the various mathematical
concepts introduced thus far to identify precisely the points A, B, and C that appear in our graphs of
this chapter. It might help you even more to then reread the chapter and construct simple spreadsheets
in a program like Microsoft Excel, which is precisely how I kept track of the different numerical
answers that are presented in the text as I wrote this section. Setting up such spreadsheets will give
you a good feel for how the mathematics of consumer choice works for specific examples.

7B.1 The Impact of Changing Income on Behavior

In Chapter 6, we solved the consumer’s constrained optimization problem for specific economic cir-
cumstances; i.e., for specific prices and incomes. In Section 7A.1, we became interested in how con-
sumer behavior changes when exogenous income changes, and we discovered that the answer
depends on the nature of the underlying map of indifference curves. We will now translate some of this
analysis from Section 7A.1 into the mathematical optimization language we developed in Chapter 6.

7B.1.1 Inferior and Normal Goods  Consider, for instance, the example of pasta and steak
we introduced in Section 7A.1.1, and suppose my wife and I had discovered that our consump-
tion of pasta remained unchanged as our income increased (as depicted in Graph 7.1b). Suppose
that the price of a box of pasta is $2 and the price of a pound of steak is $10, and suppose we let
boxes of pasta be denoted by and pounds of steak by . We know from our discussion in
Section 7A.1.1 that pasta consumption can remain constant as income increases only if the
underlying tastes are quasilinear in pasta; i.e., when utility functions can be written as

\[ u(x_1, x_2) = v(x_1) + x_2 \]

For an income level and for tastes that can be described by a utility
function , the constrained optimization problem can then be written as

\[
\max_{x_1, x_2} u(x_1, x_2) = v(x_1) + x_2 \quad \text{subject to} \quad 2x_1 + 10x_2 = I, \tag{7.1}
\]

with a corresponding Lagrange function

\[ \mathcal{L}(x_1, x_2, \lambda) = v(x_1) + x_2 + \lambda(I - 2x_1 - 10x_2). \tag{7.2} \]

Taking the first two first order conditions, we get

\[
\frac{\partial \mathcal{L}}{\partial x_1} = \frac{dv(x_1)}{dx_1} - 2\lambda = 0, \tag{7.3}
\]

\[
\frac{\partial \mathcal{L}}{\partial x_2} = 1 - 10\lambda = 0.
\]

The second of the expressions in (7.3) can then be rewritten as \( \lambda = 1/10 \), which, when sub-
stituted into the first expression in (7.3), gives

\[ \frac{dv(x_1)}{dx_1} = \frac{1}{5}. \tag{7.4} \]

Notice that the left-hand side of (7.4) is just a function of \( x_1 \), whereas the right-hand side is just
a real number, which implies that, when we have a specific functional form for the function \( v \),
we can solve for \( x_1 \) as just a real number. For instance, if \( u(x_1, x_2) = \ln x_1 + x_2 \) (implying
\( v(x_1) = \ln x_1 \)), expression (7.4) becomes

\[ \frac{1}{x_1} = \frac{1}{5} \quad \text{or} \quad x_1 = 5. \tag{7.5} \]
When the underlying tastes are quasilinear, the optimal quantity of pasta \( x_1 \) is therefore 5 (when prices of pasta and steak are 2 and 10) and is thus always the same regardless of what value the exogenous income \( I \) takes in the optimization problem (7.1). Put differently, the variable \( I \) simply drops out of the analysis as we solve for \( x_1 \). Thus, borderline normal/inferior goods have no income effects.

This is not true, of course, for tastes that cannot be represented by quasilinear utility functions. Consider, for instance, the same problem but with underlying tastes that can be represented by the Cobb–Douglas utility function \( u(x_1, x_2) = x_1^a x_2^{1-a} \). The Lagrange function is then

\[
\mathcal{L}(x_1, x_2, \lambda) = x_1^a x_2^{1-a} + \lambda (I - 2x_1 - 10x_2),
\]

and the first order conditions for this problem are

\[
\frac{\partial \mathcal{L}}{\partial x_1} = a x_1^{(a-1)} x_2^{(1-a)} - 2\lambda = 0, \\
\frac{\partial \mathcal{L}}{\partial x_2} = (1 - a) x_1^a x_2^{-a} - 10\lambda = 0, \\
\frac{\partial \mathcal{L}}{\partial \lambda} = I - 2x_1 - 10x_2 = 0.
\]

Adding \( 2\lambda \) to both sides of the first equation and \( 10\lambda \) to both sides of the second equation, and then dividing these equations by each other, we get \( ax_2/(1 - \alpha)x_1 = 1/5 \) or \( x_2 = (1 - \alpha) x_1/5\alpha \). Substituting this into the third equation of expression (7.7) and solving for \( x_1 \), we get

\[
x_1 = \frac{\alpha I}{2}.
\]

Thus, for the underlying Cobb–Douglas tastes specified here, the optimal consumption of pasta \( x_1 \) depends on income, with higher income leading to greater consumption of pasta. Cobb–Douglas tastes (as well as all other homothetic tastes) therefore represent tastes for normal goods as depicted in Graph 7.1c.

Finally, none of the utility functions we have discussed thus far represent tastes for inferior goods. This is because such tastes are difficult to capture in simple mathematical functions, in part because there are no tastes such that a particular good is always an inferior good. To see this, imagine beginning with zero income, thus consuming the origin in our graphs. Now suppose I give you $10. Since we cannot consume negative amounts of goods, it is not possible for you to consume less pasta than you did before I gave you $10, and it is therefore not possible to have tastes that represent inferior goods around the origin of our graphs. All goods are therefore normal or borderline normal/inferior goods at least around the bundle (0,0). Goods can be inferior only for some portion of an indifference map, and this logical conclusion makes it difficult to represent such tastes in simple utility functions.

### 7B.1.2 Luxury Goods and Necessities

We also defined in Section 7A.1.2 the terms luxury goods and necessities, with borderline goods between the two represented by homothetic tastes. We know from our discussion of homothetic tastes in Chapter 5 that such tastes have the feature that the marginal rates of substitution stay constant along linear rays emanating from the origin, and it is this feature of such tastes that ensures that, when exogenous income is increased by \( x\% \) (without a change in opportunity costs), our consumption of each good also increases by \( x\% \), leaving the ratio of our consumption of one good relative to the other unchanged.

For instance, in equation (7.8), we discovered that my optimal consumption of pasta is equal to \( \alpha l/2 \) when my tastes are captured by the Cobb–Douglas function \( u(x_1, x_2) = x_1^a x_2^{1-a} \), when the price of pasta is $2 and the price of steak is $10 and when my income is given by \( I \). When plugging this value into the budget constraint for \( x_1 \) and solving for \( x_2 \), we can also determine that
my optimal consumption of steak is \((1 - \alpha)/10\). Thus, the ratio \((x_1/x_2)\) of my pasta consumption to my steak consumption under these economic circumstances is \(5\alpha/(1 - \alpha)\). Put differently, my consumption of pasta relative to steak is independent of income. Since we know that Cobb–Douglas utility functions represent homothetic tastes, this simply confirms what our intuition already tells us: both pasta and steak are borderline luxury/necessity goods when the underlying tastes can be represented by Cobb–Douglas utility functions.

Again, this is not true for all types of tastes. If my tastes could be represented by the quasilinear utility function \(u(x_1, x_2) = \ln x_1 + x_2\), we concluded in expression (7.5) that my optimal consumption of pasta would be equal to 5 boxes regardless of my income level (assuming, of course, that I had at least enough income to cover that much pasta consumption). Plugging this into the budget constraint for \(\alpha\) and solving for \(\beta\), we also get that my optimal steak consumption is \(\beta\); i.e., my optimal steak consumption is a function of my income whereas my optimal pasta consumption is not. Put differently, my consumption of pasta relative to my consumption of steak declines with income, making pasta a necessity (and steak a luxury good).

### 7B.2 The Impact of Changing Opportunity Costs on Behavior

We introduced the concept of a substitution effect in Section 7A.2 by focusing on a particular example in which my brother chose to use taxis for transportation on his Cayman Islands vacation whereas I rented a car. To really focus on the underlying ideas, we assumed that my brother and I were identical in every way, allowing us to infer from the fact that we made two different choices that he and I were indifferent between renting a car and using taxis when we arrived at the airport in Cayman. The choice we made was one of choosing one of two budget constraints between “miles driven” and “other consumption” on our vacation. Renting a car requires a large fixed payment (thus reducing the level of other consumption that is possible if little or no driving occurs) but has the advantage of making additional miles cheap. Using taxis, on the other hand, involves no fixed payment but makes additional miles more expensive. Graph 7.3a illustrated the resulting choice sets, and Graph 7.3b illustrated a substitution effect from the different opportunity costs arising from those choice sets.

#### 7B.2.1 Renting a Car versus Taking a Taxi

Suppose you know that my brother and I came to the Cayman Islands with $2,000 to spend on our vacations and that taxi rides cost $1 per mile. Letting \(x_1\) denote miles driven in Cayman and \(x_2\) “dollars of other consumption in Cayman,” we know that my brother’s budget line is \(2,000 = x_1 + x_2\) given that the price of “dollars of other consumption” is by definition also 1. Suppose we also know that my brother’s (and my own) tastes can be summarized by the Cobb–Douglas utility function \(u(x_1, x_2) = x_1^{0.1} x_2^{0.9}\). Doing our usual constrained optimization problem, we can then determine that my brother’s optimal consumption bundle is \(x_1 = 200\) and \(x_2 = 1,800\).

Set up my brother’s constrained optimization problem and solve it to check that his optimal consumption bundle is indeed equal to this.

Now suppose that I had lost my receipt for the rental car and no longer remember how much of a fixed fee I was charged to drive it for the week. All I do remember is that gasoline cost $0.20 per mile. From the information we have, we can calculate what the fixed rental car fee must have been in order for me to be just as well off renting a car as my brother was using taxis.

Specifically, we can calculate the value associated with my brother’s optimal indifference curve by simply plugging \(x_1 = 200\) and \(x_2 = 1,800\) into the utility function \(u(x_1, x_2) = x_1^{0.1} x_2^{0.9}\) to get a value of approximately 1,445. While this number has no inherent meaning since we cannot quantify utility objectively, we do know from our analysis in Section 7A.2.1 (and Graph 7.3) that...
I ended up on the same indifference curve, and thus with the same utility level as measured by the utility function that my brother and I share. This gives us enough information to find bundle \( B \)—my optimal bundle of miles driven and other consumption in Graph 7.3b—using a method that builds on the intuition that comes out of the graph. All we have to do is find the smallest possible choice set with a budget line that has the slope reflecting my lower opportunity cost for miles driven and is tangent to the indifference curve that my brother has achieved; i.e., the indifference curve associated with the utility value 1,445.

This can be formulated mathematically as the following problem: We would like to find the minimum expenditure necessary for achieving a utility value of 1,445 (as measured by the utility function \( u(x_1, x_2) = x_1^{0.1} x_2^{0.9} \)) given that my price for miles driven is 0.2 (while my price for “other consumption” remains at 1). Letting \( E \) stand for expenditure, we can state this formally as a constrained minimization problem:

\[
\min_{x_1, x_2} E = 0.2x_1 + x_2 \text{ subject to } x_1^{0.1} x_2^{0.9} = 1,445. \tag{7.9}
\]

Constrained minimization problems have the same basic structure as constrained maximization problems. The first part of (7.9) lets us know that we are trying to minimize a function by choosing the values for \( x_1 \) and \( x_2 \). The function we are trying to minimize, or what we call our objective function, then follows and is simply the equation for the budget constraint that we will end up with, which reflects the new opportunity cost of driving miles given that I have paid a fixed fee for my rental car and now face a lower opportunity cost for driving each mile. Finally, the last part of (7.9) tells us the constraint of our minimization problem: we are trying to reach the indifference curve associated with the value 1,445.

Finding the solution to a minimization problem is quite similar to finding the solution to a maximization problem. The reason for this similarity is most easily seen within the economic examples with which we are working. In our utility maximization problem, for instance, we are taking as fixed the budget line and trying to find the indifference curve that is tangent to that line. This is illustrated graphically in Graph 7.8a where a consumer faces a fixed budget line and tries to get to the highest possible indifference curve that still contains a bundle within the choice set.

**Graph 7.8:** Maximizing Utility with Budgets Fixed (a) versus Minimizing Expenditure with Utility Fixed (b)
defined by the fixed budget line. In the expenditure minimization problem defined in expression (7.9), on the other hand, we are taking the indifference curve as fixed and trying to find the smallest possible choice set given the opportunity costs of the goods. This is illustrated in Graph 7.8b where we are trying to reach a fixed indifference curve with the smallest possible choice set. In both cases, we are therefore trying to find a solution, a combination of $x_1$ and $x_2$, where an indifference curve is tangent to a budget line (assuming the problem does not have non-convexities or corner solutions).

For this reason, the same Lagrange Method that we have employed in solving maximization problems can be employed to solve our newly defined minimization problem. Again, we create the Lagrange function by combining the objective function with a second term that is equal to $\lambda$ times the constraint set to zero, only now the objective function is the budget constraint and the constraint is the indifference curve. Thus,

$$L(x_1, x_2, \lambda) = 0.2x_1 + x_2 + \lambda(1,445 - x_1^{0.1}x_2^{0.9}). \tag{7.10}$$

We then again take the first derivatives of $L$ with respect to the choice variables ($x_1$ and $x_2$) and $\lambda$ to get the first order conditions

$$\frac{\partial L}{\partial x_1} = 0.2 - 0.1\lambda x_1^{-0.9}x_2^{0.9} = 0,$n

$$\frac{\partial L}{\partial x_2} = 1 - 0.9\lambda x_1^{0.1}x_2^{-0.1} = 0,$$ \tag{7.11}

$$1,445 - x_1^{0.1}x_2^{0.9} = 0.$$

Solving the first two equations for $x_2$ we get

$$x_2 = \frac{0.9(0.2x_1)}{0.1} = 1.8x_1 \tag{7.12}$$

and plugging this into the third equation and solving for $x_1$, we get $x_1 = 851.34$. Finally, plugging this back into expression (7.12), we get $x_2 = 1,532.41$. This is point $B$ in Graph 7.3, which implies that I chose to drive approximately 851 miles in my rental car during my Cayman Island vacation while consuming approximately $1,532 in other goods.

We can now see how much the bundle $B$ costs by multiplying my optimal levels of $x_1$ and $x_2$ by the prices of those goods, 0.2 for $x_1$ and 1 for $x_2$, and adding these expenditures together:

$$E = 0.2(851.34) + 1(1,532.41) = 1,702.68. \tag{7.13}$$

Thus, bundle $B$ costs a total of $1,702.68. Since you know that I arrived in Cayman with $2,000, you know that the difference between my total money budget for my vacation and the total I spent on driving and other goods must be what I paid for the fixed rental car fee: $297.32. This is equal to the vertical distance labeled “rental car fee” in Graph 7.3a.

**7B.2.2 Substitution Effects** Notice that, in the process of making these calculations, we have identified the size of the substitution effect we treated graphically in Graph 7.3. Put differently, assuming tastes that can be represented by the utility function $u(x_1, x_2) = x_1^{0.1}x_2^{0.9}$, an individual who chooses to drive 200 miles while consuming $1,800 in other goods when the opportunity cost per mile is $1 will reduce his other consumption and substitute toward 851 miles driven when we keep his real wealth—or his real well-being—fixed and change the opportunity cost for driving a mile to $0.2.

**7B.2.3 The Size of Substitution Effects** By using a Cobb–Douglas utility function to represent tastes in the previous example, we have chosen a utility function that we know (from our discussion of Constant Elasticity of Substitution (CES) utility functions in
Chapter 5) has an elasticity of substitution equal to 1. The answers we calculated relate directly to this property of Cobb–Douglas utility functions. In fact, we can verify that the function \( u(x_1, x_2) = x_1^{0.1} x_2^{0.9} \) has an elasticity of substitution of 1 using our answers as we determined the bundles associated with points \( A \) and \( B \) in Graph 7.3. Recall the formula for an elasticity of substitution:

\[
\text{Elasticity of Substitution} = \left| \frac{\% \Delta (x_2/x_1)}{\% \Delta MRS} \right|. \tag{7.14}
\]

Bundle \( A \), my brother’s optimal bundle, is \((200, 1800)\), while bundle \( B \), my optimal bundle, is \((851.34, 1532.41)\). My brother’s ratio of \( x_2/x_1 \) is therefore equal to 1,800/200, or 9, while my ratio of \( x_2/x_1 \) is 1,532.41/851.34 or 1.8. In going from \( A \) to \( B \) on the same indifference curve, the change in the ratio \( x_2/x_1 \), \( \Delta (x_2/x_1) \), is therefore equal to \(-7.2\). The \( \% \Delta (x_2/x_1) \) is just the change in the ratio \( (x_2/x_1) \) divided by the original level of \( (x_2/x_1) \) at bundle \( A \); i.e.,

\[
\% \Delta \left( \frac{x_2}{x_1} \right) = \frac{\Delta (x_2/x_1)}{x_2^4/x_1^4} = \frac{-7.2}{9} = -0.8. \tag{7.15}
\]

Similarly, the \( MRS \) at bundle \( A \) is equal to the slope of my brother’s budget line, which is equal to 1 given that he faces a cost per mile of $1. My \( MRS \) at bundle \( B \), on the other hand, is equal to the slope of my budget line, which is equal to \(-0.2\) given that I face a cost per mile of only $0.20. The \( \% \Delta MRS \) as we go from \( A \) to \( B \) is therefore the change in the \( MRS \) divided by the original \( MRS \) at bundle \( A \); i.e.,

\[
\% \Delta MRS = \frac{\Delta MRS}{MRS_A} = 0.8. \tag{7.16}
\]

Plugging (7.15) and (7.16) into the equation for an elasticity of substitution in expression (7.14), we get an elasticity of substitution equal to 1. Thus, when the marginal rate of substitution of the indifference curve in Graph 7.3 changed by 80% (from \(-1\) to \(-0.2\)), the ratio of other consumption to miles driven also changed by 80% (from 9 to 1.8). It is the elasticity of substitution that is embedded in the utility function that determined the size of the substitution effect we calculated!

This relates directly to the intuition we built in Graph 7.4, where we showed how substitution effects get larger as the degree of substitutability, or the elasticity of substitution in our more mathematical language, changes. Were we to substitute utility functions with elasticities of substitution different from those in Cobb–Douglas utility functions, we would therefore calculate substitution effects that were larger or smaller depending on whether the elasticity of substitution imbedded into those utility functions was greater or smaller.

Consider, for instance, the CES utility function with \( \rho = -0.5 \), which implies an elasticity of substitution of 2 (rather than 1 as in the Cobb–Douglas case where \( \rho = 0 \)). More precisely, suppose that the utility function my brother and I share is

\[
u(x_1, x_2) = (0.25x_1^{0.5} + 0.75x_2^{0.5})^2, \tag{7.17}\]

and suppose again that our money budget for our Cayman vacation is $2,000 and the per mile cost is $1 for taxis and $0.20 for rental cars.\(^8\) My brother’s optimization problem is then

\[
\max_{x_1, x_2} (0.25x_1^{0.5} + 0.75x_2^{0.5})^2 \quad \text{subject to} \quad x_1 + x_2 = 2,000, \tag{7.18}\]

which you can verify results in an optimal consumption bundle of \( x_1 = 200 \) and \( x_2 = 1,800 \) just as it did in our previous example. Thus, point \( A \) remains unchanged. The indifference

\(^8\)The exponents in equation (7.17) are positive because \( \rho \) is negative and each exponent in the CES utility function has a negative sign in front of it.
curve on which point \( A \) lies, however, differs substantially from that in the previous example because of the different elasticity of substitution embedded in equation (7.17). When you plug the optimal bundle for my brother back into the utility function (7.17) you can calculate that he operates on an indifference curve giving him utility of 1,250 as measured by this utility function. We could then repeat our analysis of calculating bundle \( B \) by solving the problem analogous to the one we stated in expression (7.9) but adapted to the model we are now working with:

\[
\min E = 0.2x_1 + x_2 \quad \text{subject to} \quad (0.25x_1^{0.5} + 0.75x_2^{0.5})^2 = 1,250. \tag{7.19}
\]

You can again verify on your own that this results in an optimal bundle \( B \) of \( x_1 = 2,551.02 \) and \( x_2 = 918.37 \), which implies a substitution effect much larger than the one we found with the Cobb–Douglas utility function. This is because we have built a greater elasticity of substitution into the utility function of equation (7.17) than we had in our previous Cobb–Douglas utility function. The difference between the two scenarios is illustrated graphically in Graph 7.9.

---

How much did I pay in a fixed rental car fee in order for me to be indifferent in this example to taking taxis? Why is this amount larger than in the Cobb–Douglas case we calculated earlier?

---

Table 7.1 on the next page summarizes the outcome of similar calculations for CES utility functions with different elasticities of substitution. In each case, the remaining parameters of the CES utility function are set to ensure that my brother’s optimal choice remains the same: 200 miles driven and $1,800 in other consumption.9

---

9 More precisely, the utility function \( u(x_1, x_2) = \alpha(x_1^\rho + (1 - \alpha)x_2^\rho)^{-1/\rho} \) was used for these calculations, with \( \rho \) set as indicated in the first column of the table and \( \alpha \) adjusted to ensure that point \( A \) remains at (200,1800).
Table 7.1: $u(x_1, x_2) = (ax_1^p + (1 - a)x_2^q)^{-1/p}$

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>Elasticity of Subst.</th>
<th>Substitution Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.5$</td>
<td>2</td>
<td>2,351.02 More Miles Driven at B than at A</td>
</tr>
<tr>
<td>$0.0$</td>
<td>1</td>
<td>651.34 More Miles Driven at B than at A</td>
</tr>
<tr>
<td>$0.5$</td>
<td>0.67</td>
<td>337.28 More Miles Driven at B than at A</td>
</tr>
<tr>
<td>$1.0$</td>
<td>0.50</td>
<td>222.53 More Miles Driven at B than at A</td>
</tr>
<tr>
<td>$5.0$</td>
<td>0.167</td>
<td>57.55 More Miles Driven at B than at A</td>
</tr>
<tr>
<td>$10.0$</td>
<td>0.091</td>
<td>29.67 More Miles Driven at B than at A</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.000</td>
<td>0.00 More Miles Driven at B than at A</td>
</tr>
</tbody>
</table>

7B.3 Price Changes: Income and Substitution Effects Combined

Finally, we concluded in Section 7A.3 that most price changes involve both income and substitution effects because they involve both a change in our real wealth (or our optimal indifference curve) and a change in opportunity costs. We can then employ all the mathematical tools we have built thus far to identify income and substitution effects when prices change. In the following, we will consider once again the case of me shopping at Wal-Mart for pants ($x_1$) and shirts ($x_2$), as we did in Section 7A.3.3, to demonstrate how we can identify these effects separately. Throughout, we will assume that I have $200 to spend and that the price of shirts is $10, and we will focus on what happens when the price of pants, $p_1$, changes. We will assume (unrealistically) in this section that it is possible to consume fractions of shirts and pants. If this bothers you, you may feel more comfortable thinking of more continuous goods, such as nuts and candy from the bulk food isle where one can scoop as little or as much into a bag, instead of pants and shirts.

Suppose first that my tastes can once again be represented by a Cobb–Douglas utility function

$$u(x_1, x_2) = x_1^{0.5}x_2^{0.5}. \quad (7.20)$$

My constrained maximization problem at Wal-Mart is then

$$\max_{x_1, x_2} x_1^{0.5}x_2^{0.5} \text{ subject to } p_1x_1 + 10x_2 = 200. \quad (7.21)$$

Solving this in the usual way gives us the optimal bundle

$$x_1 = \frac{100}{p_1} \quad \text{and} \quad x_2 = 10. \quad (7.22)$$

Exercise 7B.3

Check to see that this solution is correct.

Initially, I face a price of $20 per pair of pants, which implies that my optimal bundle is 5 pants and 10 shirts. Then I discover that my wife gave me a 50% off coupon for pants, effectively reducing the price of pants from $20 to $10. As a result of this decrease in the price of pants, my optimal consumption bundle changes from (5,10) to (10,10). This is illustrated in Graph 7.10a, with bundle $A$ representing my original optimal bundle and bundle $C$ representing my new optimal bundle.
In order to decompose this change in my behavior into income and substitution effects, we have to calculate how my consumption would have changed had I faced the same change in opportunity costs without experiencing an increase in real wealth; i.e., without having shifted to a higher indifference curve. Thus, we need to employ the method we developed in the previous section to identify how much money I would have to give up when I received the coupon to be able to be just as well off as I was originally without the coupon. Notice that this is exactly analogous to our example involving my brother and me in the Cayman Islands where we wanted to identify how much the fixed rental car fee must have been in order for me to be just as well off as my brother was using taxis. In both cases, we have a fixed indifference curve, and we are trying to find the smallest possible choice set that will give me a fixed utility level when my opportunity costs change.

In Graph 7.10b, we illustrate the problem of finding the substitution effect graphically. We begin by drawing the indifference curve \( U^A \) that contains bundle \( A \) and the (magenta) budget line that I have with the coupon. Then we shift this budget line inward, keeping the slope and thus the new opportunity cost fixed, until only a single point on the indifference curve remains within the choice set. This process identifies bundle \( B \) on the compensated (green) budget, the bundle I would choose if I faced the opportunity costs under the coupon but had lost just enough of my money to be just as well off as I was originally when I consumed bundle \( A \).

Mathematically, we state the process graphed in Graph 7.10b as a constrained minimization problem in which we are trying to minimize my total expenditures (or my money budget) subject to the constraint that I would like to consume on the indifference curve that contains bundle \( A \).

We can write this as follows:

\[
\min_{x_1, x_2} E = 10x_1 + 10x_2 \quad \text{subject to} \quad x_1^{0.5} x_2^{0.5} = U^A, \tag{7.23}
\]

where \( U^A \) represents the level of utility I attained at bundle \( A \). This level of utility can be calculated using the utility function \( x_1^{0.5} x_2^{0.5} \) by simply plugging the bundle \( A (x_1 = 5, x_2 = 10) \) into the function, which gives us \( U^A \approx 7.071 \). Solving this minimization problem using the Lagrange Method illustrated in our Cayman example in the previous section, we get

\[
x_1 = x_2 \approx 7.071. \tag{7.24}
\]
The total expenditure required to consume this bundle at prices \( p_1 = p_2 = 10 \) is \( 141.42 \), which implies that you could take \( 58.58 \) out of my initial \( 200 \) and give me a 50% off coupon and I would be just as well off as I was without the coupon and with my initial \( 200 \). Put differently, my “real income” is \( 58.58 \) higher when I get the coupon because that is how much you could take from me once I get the coupon without changing my well-being. The compensated budget (which keeps utility constant) is therefore \( 141.42 \).

Combining Graphs 7.10a and 7.10b into a single graph, we then get Graph 7.10c showing bundles \( A, B, \) and \( C \) with the values we have calculated for each of these bundles. The substitution effect is the movement from \( A \) to \( B \), while the income effect, reflecting the change in my behavior that is solely due to the fact that I am \( 58.58 \) “richer” when I receive the coupon, is the movement from \( B \) to \( C \).

Just as was true for substitution effects we identified in the Cayman Islands example, the size of the substitution effect here once again arises from the degree of substitutability of the goods as captured by the shape of indifference curves and the form of the utility function. Similarly, the size of the income effect depends on the underlying nature of tastes and the degree to which pants and shirts represent normal or inferior goods.

Suppose, for instance, that my tastes could be represented by the quasilinear utility function

\[
\begin{align*}
    u(x_1, x_2) &= 6x_1^{0.5} + x_2, \\
    (7.25)
\end{align*}
\]

Setting up the maximization problem analogous to (7.21) gives

\[
\begin{align*}
    \max_{x_1, x_2} 6x_1^{0.5} + x_2 \quad \text{subject to} \quad p_1x_1 + 10x_2 &= 200, \\
    (7.26)
\end{align*}
\]

which you can verify solves to

\[
\begin{align*}
    x_1 &= \frac{900}{p_1^2} \quad \text{and} \quad x_2 = \frac{20p_1 - 90}{p_1}. \\
    (7.27)
\end{align*}
\]

Thus, when the price of pants is \( 20 \), we get an optimal bundle \((2.25, 15.5)\), and when the price falls to \( 10 \) due to the coupon, we get an optimal bundle \((9, 11)\). Total utility without the coupon is found by plugging \( x_1 = 2.25 \) and \( x_2 = 15.5 \) into equation (7.25), which gives utility equal to \( 24.5 \). This then permits us to find the substitution effect by solving the constrained minimization problem

\[
\begin{align*}
    \min_{x_1, x_2} E &= 10x_1 + 10x_2 \quad \text{subject to} \quad 6x_1^{0.5} + x_2 = 24.5, \\
    (7.28)
\end{align*}
\]

which gives \( x_1 = 9 \) and \( x_2 = 6.5 \). Thus (ignoring the fact that it is difficult to consume fractions of pants) the substitution effect changes my consumption of pants from my original \( 2.25 \) to \( 9 \), and the income effect causes no additional change in my consumption for pants. This lack of an income effect of course arises because tastes that are quasilinear in a particular good (in this case, pants) do not exhibit income effects for that good; such goods are borderline normal/inferior goods.\(^{10}\)

\(^{10}\)A small caveat to this is that such tastes do exhibit income effects in the quasilinear good when there are corner solutions. This is explored in more detail in end-of-chapter exercise 7.5.
CONCLUSION

We have begun in this chapter to discuss the important concepts of income and substitution effects in the context of consumer goods markets. In our mathematical section, we furthermore began to calculate income and substitution effects for some very specific examples in order to illustrate how the graphs of Section 7A related to the mathematical ideas we have dealt with thus far. A more general theory of consumer behavior will emerge from the building blocks of the optimization model we have laid, but we will not have completed the building of this theory until Chapter 10. Before doing so, we will now first translate the concepts of income and substitution effects in consumer goods markets to similar ideas that emerge in labor and capital markets (Chapter 8). We will then illustrate in Chapters 9 and 10 how our notions of demand and consumer surplus relate directly to income and substitution effects as introduced here.

There is no particular reason why it should be fully apparent to you at this point why these concepts are important. The importance will become clearer as we apply them in exercises and as we turn to some real-world issues later on. We did, however, raise one example in the introduction, and we can now make a bit more sense of it. We imagined a policy in which the government would reduce consumption of gasoline by taxing it heavily, only to turn around and distribute the revenues from the tax in the form of rebate checks. For many, including some very smart columnists and politicians, such a combination of a gasoline tax and rebate makes no sense; on average, they argue, consumers would receive back as much as they paid in gasoline taxes, and as a result, they would not change their behavior.11 Now that we have isolated income and substitution effects, however, we can see why economists think such a tax/rebate program will indeed curb gasoline consumption: The tax raises the price of gasoline and thus gives rise to income and substitution effects that (assuming gasoline is a normal good) both result in less consumption of gasoline. The rebate, on the other hand, does not change prices back; it simply causes incomes to rise above where they would otherwise have been after the tax. Thus, the rebate only causes an income effect in the opposite direction. The negative income effect from the increase in the price should be roughly offset by the positive income effect from the tax rebate, which leaves us with a substitution effect that unambiguously implies a decrease in gasoline consumption.

END-OF-CHAPTER EXERCISES

7.1 Here, we consider some logical relationships between preferences and types of goods.

A. Suppose you consider all the goods that you might potentially want to consume.

a. Is it possible for all these goods to be luxury goods at every consumption bundle? Is it possible for all of them to be necessities?

b. Is it possible for all goods to be inferior goods at every consumption bundle? Is it possible for all of them to be normal goods?

c. True or False: When tastes are homothetic, all goods are normal goods.

d. True or False: When tastes are homothetic, some goods could be luxuries while others could be necessities.

e. True or False: When tastes are quasi-linear, one of the goods is a necessity.

11This argument was in fact advanced by opponents of such a policy advocated by the Carter administration in the late 1970s, a proposal that won only 35 votes (out of 435) in the U.S. House of Representatives. It is not the only argument against such policies. For instance, some have argued that a gasoline tax would be too narrow, and that the goals of such a tax would be better advanced by a broad-based carbon tax on all carbon-emitting activity.

*conceptually challenging

**computationally challenging

†solutions in Study Guide
f. True or False: In a two-good model, if the two goods are perfect complements, they must both be normal goods.

g. True or False: In a three-good model, if two of the goods are perfect complements, they must both be normal goods.

B. In each of the following cases, suppose that a person whose tastes can be characterized by the given utility function has income \( I \) and faces prices that are all equal to 1. Illustrate mathematically how his or her consumption of each good changes with income, and use your answer to determine whether the goods are normal or inferior, luxuries or necessities.

a. \( u(x_1, x_2) = x_1x_2 \)

b. \( u(x_1, x_2) = x_1 + \ln x_2 \)

c. \( u(x_1, x_2) = \ln x_1 + \ln x_2 \)

d. \( u(x_1, x_2, x_3) = 2\ln x_1 + \ln x_2 + 4\ln x_3 \)

e. \( u(x_1, x_2) = 2x_1^{0.5} + \ln x_2 \)

7.2 Suppose you have an income of $24 and the only two goods you consume are apples \( (x_1) \) and peaches \( (x_2) \). The price of apples is $4 and the price of peaches is $3.

A. Suppose that your optimal consumption is 4 peaches and 3 apples.

a. Illustrate this in a graph using indifference curves and budget lines.

b. Now suppose that the price of apples falls to $2 and I take enough money away from you to make you as happy as you were originally. Will you buy more or fewer peaches?

c. In reality, I do not actually take income away from you as described in (b), but your income stays at $24 after the price of apples fell. I observe that, after the price of apples fell, you did not change your consumption of peaches. Can you conclude whether peaches are an inferior or normal good for you?

B. Suppose that your tastes can be characterized by the function \( u(x_1, x_2) = x_1^{a}x_2^{1-a} \).

a. What value must \( a \) take in order for you to choose 3 apples and 4 peaches at the original prices?

b. What bundle would you consume under the scenario described in A(b)?

c. How much income can I take away from you and still keep you as happy as you were before the price change?

d. What will you actually consume after the price increase?

7.3 Consider once again my tastes for Coke and Pepsi and my tastes for right and left shoes (as described in end-of-chapter exercise 6.2).

A. On two separate graphs—one with Coke and Pepsi on the axes, the other with right shoes and left shoes—replicate your answers to end-of-chapter exercise 6.2A(a) and (b). Label the original optimal bundles \( A \) and the new optimal bundles \( C \).

a. In your Coke/Pepsi graph, decompose the change in consumer behavior into income and substitution effects by drawing the compensated budget and indicating the optimal bundle \( B \) on that budget.

b. Repeat (a) for your right shoes/left shoes graph.

B. Now consider the following utility functions: \( u(x_1, x_2) = \min\{x_1, x_2\} \) and \( u(x_1, x_2) = x_1 + x_2 \). Which of these could plausibly represent my tastes for Coke and Pepsi, and which could represent my tastes for right and left shoes?

a. Use the appropriate function to assign utility levels to bundles \( A \), \( B \), and \( C \) in your graph from 7.3A(a).

b. Repeat this for bundles \( A \), \( B \), and \( C \) for your graph in 7.3A(b).

7.4 Return to the case of our beer and pizza consumption from end-of-chapter exercise 6.3.

A. Again, suppose you consume only beer and pizza (sold at prices \( p_1 \) and \( p_2 \) respectively) with an exogenously set income \( I \). Assume again some initial optimal (interior) bundle \( A \).
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a. In 6.3A(b), can you tell whether beer is normal or inferior? What about pizza?
b. When the price of beer goes up, I notice that you consume less beer. Can you tell whether beer is a normal or an inferior good?
c. When the price of beer goes down, I notice you buy less pizza. Can you tell whether pizza is a normal good?
d. When the price of pizza goes down, I notice you buy more beer. Is beer an inferior good for you? Is pizza?
e. Which of your conclusions in part (d) would change if you knew pizza and beer are very substitutable?

B. Suppose, as you did in end-of-chapter exercise 6.3B, that your tastes over beer \(x_1\) and pizza \(x_2\) can be summarize by the utility function \(u(x_1, x_2) = x_1^\alpha + x_2\). If you have not already done so, calculate the optimal quantity of beer and pizza consumption as a function of \(p_1, p_2\), and \(I\).

a. Illustrate the optimal bundle \(A\) when \(p_1 = 2, p_2 = 10\) and weekly income \(I = 180\). What numerical label does this utility function assign to the indifference curve that contains bundle \(A\)?
b. Using your answer, show that both beer and pizza are normal goods when your tastes can be summarized by this utility function.
c. Suppose the price of beer goes up to $4. Illustrate your new optimal bundle and label it \(C\).
d. How much beer and pizza would you buy if you had received just enough of a raise to keep you just as happy after the increase in the price of beer as you were before (at your original income of $180)? Illustrate this as bundle \(B\).
e. How large was your salary increase in (d)?
f. Now suppose the price of pizza \(p_2\) falls to $5 (and suppose the price of beer and your income are $2 and $180 as they were originally at bundle \(A\)). Illustrate your original budget, your new budget, the original optimum \(A\), and the new optimum \(C\) in a graph.
g. Calculate the income effect and the substitution effect for both pizza and beer consumption from this change in the price of pizza. Illustrate this in your graph.
h. True or False: Since income and substitution effects point in opposite directions for beer, beer must be an inferior good.

7.5† Return to the analysis of my undying love for my wife expressed through weekly purchases of roses (as introduced in end-of-chapter exercise 6.4).

A. Recall that initially roses cost $5 each and, with an income of $125 per week, I bought 25 roses each week. Then, when my income increased to $500 per week, I continued to buy 25 roses per week (at the same price).

a. From what you observed thus far, are roses a normal or an inferior good for me? Are they a luxury or a necessity?
b. On a graph with weekly roses consumption on the horizontal and “other goods” on the vertical, illustrate my budget constraint when my weekly income is $125. Then illustrate the change in the budget constraint when income remains $125 per week and the price of roses falls to $2.50. Suppose that my optimal consumption of roses after this price change rises to 50 roses per week and illustrate this as bundle \(C\).
c. Illustrate the compensated budget line and use it to illustrate the income and substitution effects.
d. Now consider the case where my income is $500 and, when the price changes from $5 to $2.50, I end up consuming 100 roses per week (rather than 25). Assuming quasilinearity in roses, illustrate income and substitution effects.
e. True or False: Price changes of goods that are quasilinear give rise to no income effects for the quasilinear good unless corner solutions are involved.

B. Suppose again, as in 6.4B, that my tastes for roses \(x_1\) and other goods \(x_2\) can be represented by the utility function \(u(x_1, x_2) = \beta x_1^\alpha + x_2\).

a. If you have not already done so, assume that \(p_2\) is by definition equal to 1, let \(\alpha = 0.5\) and \(\beta = 50\), and calculate my optimal consumption of roses and other goods as a function of \(p_1\) and \(I\).
b. The original scenario you graphed in 7.5A(b) contains corner solutions when my income is $125 and the price is initially $5 and then $2.50. Does your previous answer allow for this?

c. Verify that the scenario in your answer to 7.5A(d) is also consistent with tastes described by this utility function; i.e., verify that $A$, $B$, and $C$ are as you described in your answer.

**7.6 Everyday Application: Housing Price Fluctuations: Part 2**

Suppose, as in end-of-chapter exercise 6.9, you have $400,000 to spend on “square feet of housing” and “all other goods.” Assume the same is true for me.

**A.** Suppose again that you initially face a $100 per square foot price for housing, and you choose to buy a 2,000-square-foot house.

a. Illustrate this on a graph with square footage of housing on the horizontal axis and other consumption on the vertical. Then suppose, as you did in exercise 6.9, that the price of housing falls to $50 per square foot after you bought your 2,000-square-foot house. Label the square footage of the house you would switch to $h_B$.

b. Is $h_B$ smaller or larger than 2,000 square feet? Does your answer depend on whether housing is normal, regular inferior, or Giffen?

c. Now suppose that the price of housing had fallen to $50 per square foot before you bought your initial 2,000-square-foot house. Denote the size of house you would have bought $h_C$ and illustrate it in your graph.

d. Is $h_C$ larger than $h_B$? Is it larger than 2,000 square feet? Does your answer depend on whether housing is a normal, regular inferior, or Giffen good?

e. Now consider me. I did not buy a house until the price of housing was $50 per square foot, at which time I bought a 4,000-square-foot house. Then the price of housing rises to $100 per square foot. Would I sell my house and buy a new one? If so, is the new house size $h_{B'}$ larger or smaller than 4,000 square feet? Does your answer depend on whether housing is normal, regular inferior, or Giffen for me?

f. Am I better or worse off?

g. Suppose I had not purchased at the low price but rather purchased a house of size $h_{C'}$ after the price had risen to $100 per square foot. Is $h_{C'}$ larger or smaller than $h_B$? Is it larger or smaller than 4,000 square feet? Does your answer depend on whether housing is normal, regular inferior, or Giffen for me?

**B.** Suppose both you and I have tastes that can be represented by the utility function $u(x_1, x_2) = x_1^{0.5}x_2^{0.5}$, where $x_1$ is square feet of housing and $x_2$ is “dollars of other goods.”

a. Calculate the optimal level of housing consumption $x_1$ as a function of per square foot housing prices $p_i$ and income $I$.

b. Verify that your initial choice of a 2,000-square-foot house and my initial choice of a 4,000-square-foot house was optimal under the circumstances we faced (assuming we both started with $400,000).

c. Calculate the values of $h_B$ and $h_C$ as they are described in A(a) and (c).

d. Calculate $h_{B'}$ and $h_{C'}$ as they are described in A(e) and (g).

e. Verify your answer to A(f).

**7.7 Everyday Application: Turkey and Thanksgiving**

Every Thanksgiving, my wife and I debate about how we should prepare the turkey we will serve (and will then have left over). On the one hand, my wife likes preparing turkeys the conventional way: roasted in the oven where it has to cook at 350 degrees for 4 hours or so. I, on the other hand, like to fry turkeys in a big pot of peanut oil heated over a powerful flame outdoors. The two methods have different costs and benefits. The conventional way of cooking turkeys has very little set-up cost (since the oven is already there and just has to be turned on) but a relatively large time cost from then on. (It takes hours to cook.) The frying method, on the other hand, takes some set-up (dragging out the turkey fryer, pouring gallons of peanut oil, etc., and then later the cleanup associated with it), but turkeys cook predictably quickly in just 3.5 minutes per pound.

**A.** As a household, we seem to be indifferent between doing it one way or another; sometimes we use the oven, sometimes we use the fryer. But we have noticed that we cook much more turkey, several turkeys, as a matter of fact, when we use the fryer than when we use the oven.
Chapter 7. Income and Substitution Effects in Consumer Goods Markets

7.8* Business Application: Sam’s Club and the Marginal Consumer: Superstores like Costco and Sam’s Club serve as wholesalers to businesses but also target consumers who are willing to pay a fixed fee in order to get access to the lower wholesale prices offered in these stores. For purposes of this exercise, suppose that you can denote goods sold at superstores as \( x_1 \) and “dollars of other consumption” as \( x_2 \).

A. Suppose all consumers have the same homothetic tastes over \( x_1 \) and \( x_2 \), but they differ in their income. Every consumer is offered the same option of either shopping at stores with somewhat higher prices for \( x_1 \) or paying the fixed fee \( c \) to shop at a superstore at somewhat lower prices for \( x_1 \).

   a. On a graph with \( x_1 \) on the horizontal axis and \( x_2 \) on the vertical, illustrate the regular budget (without a superstore membership) and the superstore budget for a consumer whose income is such that these two budgets cross on the 45-degree line. Indicate on your graph a vertical distance that is equal to the superstore membership fee \( c \).

   b. Now consider a consumer with twice that much income. Where will this consumer’s two budgets intersect relative to the 45-degree line?

   c. Suppose consumer 1 (from part (a)) is just indifferent between buying and not buying the superstore membership. How will her behavior differ depending on whether or not she buys the membership?

---

B.** Suppose that, if we did not cook turkeys, we could consume 100 units of “other consumption,” but the time it takes to cook turkeys takes away from that consumption. Setting up the turkey fryer costs \( c \) units of consumption and waiting 3.5 minutes (which is how long it takes to cook 1 pound of turkey) costs 1 unit of consumption. Roasting a turkey involves no set-up cost, but it takes 5 times as long to cook per pound. Suppose that tastes can be characterized by the CES utility function \( u(x_1, x_2) = (0.5x_1^p + 0.5x_2^p)^{-1/p} \) where \( x_1 \) is pounds of turkey and \( x_2 \) is “other consumption.”

   a. What are the two budget constraints I am facing?

   b. Can you calculate how much turkey someone with these tastes will roast (as a function of \( p \))? How much will the same person fry? (Hint: Rather than solving this using the Lagrange Method, use the fact that you know the \( MRS \) is equal to the slope of the budget line and recall from Chapter 5 that, for a CES utility function of this kind, \( MRS = -(x_2/x_1)^{1-1/p} \).

   c. Suppose my family has tastes with \( p = 0 \) and my friend’s with \( p = 1 \). If each of us individually roasts turkeys this Thanksgiving, how much will we each roast?

   d. How much utility will each of us get (as measured by the relevant utility function)? (Hint: In the case where \( p = 0 \), the exponent \( 1/p \) is undefined. Use the fact that you know that when \( p = 0 \) the CES utility function is Cobb–Douglas.)

   e. Which family is happier?

   f. If we are really indifferent between roasting and frying, what must \( c \) be for my family? What must it be for my friend’s family? (Hint: Rather than setting up the usual minimization problem, use your answer to (b) to determine \( c \) by setting utility equal to what it was for roasting.)

   g. Given your answers so far, how much would we each have fried had we chosen to fry instead of roast (and we were truly indifferent between the two because of the different values of \( c \) we face)?

   h. Compare the size of the substitution effect you have calculated for my family and that you calculated for my friend’s family and illustrate your answer in a graph with pounds of turkey on the horizontal and other consumption on the vertical. Relate the difference in the size of the substitution effect to the elasticity of substitution.
d. If consumer 1 was indifferent between buying and not buying the superstore membership, can you tell whether consumer 2 (from part (b)) is also indifferent? (Hint: Given that tastes are homothetic and identical across consumers, what would have to be true about the intersection of the two budgets for the higher income consumer in order for the consumer also to be indifferent between them?)

e. True or False: Assuming consumers have the same homothetic tastes, there exists a “marginal” consumer with income \( \bar{T} \) such that all consumers with income greater than \( \bar{T} \) will buy the superstore membership and no consumer with income below \( \bar{T} \) will buy that membership.

f. True or False: By raising \( c \) and/or \( p_1 \), the superstore will lose relatively lower income customers and keep high income customers.

g. Suppose you are a superstore manager and you think your store is overcrowded. You’d like to reduce the number of customers while at the same time increasing the amount each customer purchases. How would you do this?

B. Suppose you manage a superstore and you are currently charging an annual membership fee of $50. Since \( x_3 \) is denominated in dollar units, \( p_2 = 1 \). Suppose that \( p_1 = 1 \) for those shopping outside the superstore, but your store sells \( x_1 \) at 0.95. Your statisticians have estimated that your consumers have tastes that can be summarized by the utility function \( u(x_1, x_2) = x_1^{0.15} x_2^{0.85} \).

a. What is the annual discretionary income (that could be allocated to purchasing \( x_1 \) and \( x_2 \)) of your “marginal” consumer?

b. Can you show that consumers with more income than the marginal consumer will definitely purchase the membership while consumers with less income will not? (Hint: Calculate the income of the marginal consumer as a function of \( c \) and show what happens to income that makes a consumer marginal as \( c \) changes.)

c. If the membership fee is increased from $50 to $100, how much could the superstore lower \( p_1 \) without increasing membership beyond what it was when the fee was $50 and \( p_1 \) was 0.95?

79* Business Application: Are Gucci Products Giffen Goods? We defined a Giffen good as a good that consumers (with exogenous incomes) buy more of when the price increases. When students first hear about such goods, they often think of luxury goods such as expensive Gucci purses and accessories. If the marketing departments for firms like Gucci are very successful, they may find a way of associating prestige with “prestige” in the minds of consumers, and this may allow them to raise the price and sell more products. But would that make Gucci products Giffen goods? The answer, as you will see in this exercise, is no.

A. Suppose we model a consumer who cares about the “practical value and style of Gucci products,” dollars of other consumption, and the “prestige value” of being seen with Gucci products. Denote these as \( x_1, x_2, \) and \( x_3 \) respectively.

a. The consumer only has to buy \( x_1 \) and \( x_2 \)—the prestige value \( x_3 \) comes with the Gucci products. Let \( p_1 \) denote the price of Gucci products and \( p_2 = 1 \) be the price of dollars of other consumption. Illustrate the consumer’s budget constraint (assuming an exogenous income \( I \)).

b. The prestige value of Gucci purchases, \( x_3 \), is something an individual consumer has no control over. If \( x_3 \) is fixed at a particular level \( \bar{x}_3 \), the consumer therefore operates on a two-dimensional slice of her three-dimensional indifference map over \( x_1, x_2, \) and \( x_3 \). Draw such a slice for the indifference curve that contains the consumer’s optimal bundle \( A \) on the budget from part (a).

c. Now suppose that Gucci manages to raise the prestige value of its products and thus \( x_3 \) that comes with the purchase of Gucci products. For now, suppose they do this without changing \( p_1 \). This implies you will shift to a different two-dimensional slice of your three-dimensional indifference map. Illustrate the new two-dimensional indifference curve that contains \( A \). Is the new \( MRS \) at \( A \) greater or smaller in absolute value than it was before?

d.* Would the consumer consume more or fewer Gucci products after the increase in prestige value?

e. Now suppose that Gucci manages to convince consumers that Gucci products become more desirable the more expensive they are. Put differently, the prestige value \( x_3 \) is linked to \( p_1 \), the price of the Gucci products. On a new graph, illustrate the change in the consumer’s budget as a result of an increase in \( p_1 \).
f. Suppose that our consumer increases her purchases of Gucci products as a result of the increase in the price \( p_1 \). Illustrate two indifference curves: one that gives rise to the original optimum \( A \) and another that gives rise to the new optimum \( C \). Can these indifference curves cross?

g. Explain why, even though the behavior is consistent with what we would expect if Gucci products were a Giffen good, Gucci products are not a Giffen good in this case.

h. In a footnote in the chapter, we defined the following: A good is a Veblen good if preferences for the good change as price increases, with this change in preferences possibly leading to an increase in consumption as price increases. Are Gucci products a Veblen good in this exercise?

B. Consider the same definition of \( x_1, x_2, \) and \( x_3 \) as in part A. Suppose that the tastes for our consumer can be captured by the utility function \( u(x_1, x_2, x_3) = \alpha x_3 \ln x_1 + x_2 \).

a. Set up the consumer’s utility maximization problem, keeping in mind that \( x_3 \) is not a choice variable.

b. Solve for the optimal consumption of \( x_1 \) (which will be a function of the prestige value \( x_3 \)).

c. Is \( x_1 \) normal or inferior? Is it Giffen?

d. Now suppose that prestige value is a function of \( p_1 \). In particular, suppose that \( x_3 = p_1 \). Substitute this into your solution for \( x_1 \). Will consumption increase or decrease as \( p_1 \) increases?

e. How would you explain that \( x_1 \) is not a Giffen good despite the fact that its consumption increases as \( p_1 \) goes up?

7.10 **Policy Application: Tax Deductibility and Tax Credits**: In end-of-chapter exercise 2.17, you were asked to think about the impact of tax deductibility on a household’s budget constraint.

A. Suppose we begin in a system in which mortgage interest is not deductible and then tax deductibility of mortgage interest is introduced.

a. Using a graph (as you did in exercise 2.17) with “square feet of housing” on the horizontal axis and “dollars of other consumption” on the vertical, illustrate the direction of the substitution effect.

b. What kind of good would housing have to be in order for the household to consume less housing as a result of the introduction of the tax deductibility program?

c. On a graph that contains both the before and after deductibility budget constraints, how would you illustrate the amount of subsidy the government provides to this household?

d. Suppose the government provided the same amount of money to this household but did so instead by simply giving it to the household as cash back on its taxes (without linking it to housing consumption). Will the household buy more or less housing?

e. Will the household be better or worse off?

f. Do your answers to (d) and (e) depend on whether housing is normal, regular inferior, or Giffen?

g. Under tax deductibility, will the household spend more on other consumption before or after tax deductibility is introduced? Discuss your answer in terms of income and substitution effects and assume that “other goods” is a normal good.

h. If you observed that a household consumes more in “other goods” after the introduction of tax deductibility, could that household’s tastes be quasilinear in housing? Could they be homothetic?

B. Households typically spend about a quarter of their after-tax income \( I \) on housing. Let \( x_1 \) denote square feet of housing and let \( x_2 \) denote other consumption.

a. If we represent a household’s tastes with the Cobb–Douglas function \( u(x_1, x_2) = x_1^\alpha x_2^{1-\alpha} \), what should \( \alpha \) be?

b. Using your answer about the value of \( \alpha \), and letting the price per square foot of housing be denoted as \( p_1 \), derive the optimal level of housing consumption (in terms of \( I, p_1 \), and \( t \)) under a tax deductibility program that implicitly subsidizes a fraction \( t \) of a household’s housing purchase.

c. What happens to housing consumption and other good consumption under tax deductibility as a household’s tax bracket (i.e., their tax rate \( t \)) increases?
d. Determine the portion of changed housing consumption that is due to the income effect and the portion that is due to the substitution effect.

e. Calculate the amount of money the government is spending on subsidizing this household’s mortgage interest.

f. Now suppose that, instead of a deductibility program, the government simply gives the amount you calculated in (e) to the household as cash. Calculate the amount of housing now consumed and compare it with your answer under tax deductibility.

7.11 Policy Application: Substitution Effects and Social Security Cost of Living Adjustments: In end-of-chapter exercise 6.16, you investigated the government’s practice for adjusting Social Security income for seniors by ensuring that the average senior can always afford to buy some average bundle of goods that remains fixed. To simplify the analysis, let us again assume that the average senior consumes only two different goods.

A. Suppose that last year our average senior optimized at the average bundle \( A \) identified by the government, and begin by assuming that we denominate the units of \( x_1 \) and \( x_2 \) such that last year \( p_1 = p_2 = 1 \).

a. Suppose that \( p_1 \) increases. On a graph with \( x_1 \) on the horizontal and \( x_2 \) on the vertical axis, illustrate the compensated budget and the bundle \( B \) that, given your senior’s tastes, would keep the senior just as well off at the new price.

b. In your graph, compare the level of income the senior requires to get to bundle \( B \) with the income required to get him back to bundle \( A \).

c. What determines the size of the difference in the income necessary to keep the senior just as well off when the price of good 1 increases as opposed to the income necessary for the senior still to be able to afford bundle \( A \)?

d. Under what condition will the two forms of compensation be identical?

e. You should recognize the move from \( A \) to \( B \) as a pure substitution effect as we have defined it in this chapter. Often this substitution effect is referred to as the *Hicksian substitution effect*, defined as the change in behavior when opportunity costs change but the consumer receives sufficient compensation to remain just as happy. Let \( B' \) be the consumption bundle the average senior would choose when compensated so as to be able to afford the original bundle \( A \). The movement from \( A \) to \( B' \) is often called the *Slutsky substitution effect*, defined as the change in behavior when opportunity costs change but the consumer receives sufficient compensation to be able to stay at the original consumption bundle. **True or False:** The government could save money by using Hicksian rather than Slutsky substitution principles to determine appropriate cost of living adjustments for Social Security recipients.

f. **True or False:** Hicksian and Slutsky compensation get closer to one another the smaller the price changes.

B. Now suppose that the tastes of the average senior can be captured by the Cobb–Douglas utility function \( u(x_1, x_2) = x_1 x_2 \), where \( x_2 \) is a composite good (with price by definition equal to \( p_2 = 1 \)). Suppose the average senior currently receives Social Security income \( I \) (and no other income) and with it purchases bundle \( (x_1^A, x_2^A) \).

a. Determine \( (x_1^A, x_2^A) \) in terms of \( I \) and \( p_1 \).

b. Suppose that \( p_1 \) is currently $1 and \( I \) is currently $2,000. Then \( p_1 \) increases to $2. How much will the government increase the Social Security check given how it is actually calculating cost of living adjustments? How will this change the senior’s behavior?

c. How much would the government increase the Social Security check if it used Hicksian rather than Slutsky compensation? How would the senior’s behavior change?

d.* Can you demonstrate mathematically that Hicksian and Slutsky compensation converge to one another as the price change gets small and diverge from each other as the price change gets large?

e. We know that Cobb–Douglas utility functions are part of the CES family of utility functions, with the elasticity of substitution equal to 1. Without doing any math, can you estimate the range of how much Slutsky compensation can exceed Hicksian compensation with tastes that lie within the CES family? (**Hint:** Consider the extreme cases of elasticities of substitution.)
7.12** Policy Application: Fuel Efficiency, Gasoline Consumption, and Gas Prices: Policy makers frequently search for ways to reduce consumption of gasoline. One straightforward option is to tax gasoline, thereby encouraging consumers to drive less and switch to more fuel-efficient cars.

A.* Suppose that you have tastes for driving and for other consumption, and assume throughout that your tastes are homothetic.

a. On a graph with monthly miles driven on the horizontal and “monthly other consumption” on the vertical axis, illustrate two budget lines: one in which you own a gas-guzzling car, which has a low monthly payment (that has to be made regardless of how much the car is driven) but high gasoline use per mile; the other in which you own a fuel-efficient car, which has a high monthly payment that has to be made regardless of how much the car is driven but uses less gasoline per mile. Draw this in such a way that it is possible for you to be indifferent between owning the gas-guzzling and the fuel-efficient car.

b. Suppose you are indeed indifferent. With which car will you drive more?

c. Can you tell with which car you will use more gasoline? What does your answer depend on?

d. Now suppose that the government imposes a tax on gasoline, and this doubles the opportunity cost of driving both types of cars. If you were indifferent before the tax was imposed, can you now say whether you will definitively buy one car or the other (assuming you waited to buy a car until after the tax is imposed)? What does your answer depend on? (Hint: It may be helpful to consider the extreme cases of perfect substitutes and perfect complements before deriving your general conclusion to this question.)

e. The empirical evidence suggests that consumers shift toward more fuel-efficient cars when the price of gasoline increases. True or False: This would tend to suggest that driving and other good consumption are relatively complementary.

f. Suppose an increase in gasoline taxes raises the opportunity cost of driving a mile with a fuel-efficient car to the opportunity cost of driving a gas guzzler before the tax increase. Will someone who was previously indifferent between a fuel-efficient and a gas-guzzling car now drive more or less in a fuel-efficient car than he did in a gas guzzler prior to the tax increase? (Continue with the assumption that tastes are homothetic.)

B. Suppose your tastes were captured by the utility function \( u(x_1, x_2) = \frac{0.5}{x_1^{0.5}} \), where \( x_1 \) stands for miles driven and \( x_2 \) stands for other consumption. Suppose you have $600 per month of discretionary income to devote to your transportation and other consumption needs and that the monthly payment on a gas guzzler is $200. Furthermore, suppose the initial price of gasoline is $0.10 per mile in the fuel-efficient car and $0.20 per mile in the gas guzzler.

a. Calculate the number of monthly miles driven if you own a gas guzzler.

b. Suppose you are indifferent between the gas guzzler and the fuel-efficient car. How much must the monthly payment for the fuel-efficient car be?

c. Now suppose that the government imposes a tax on gasoline that doubles the price per mile driven of each of the two cars. Calculate the optimal consumption bundle under each of the new budget constraints.

d. Do you now switch to the fuel-efficient car?

e. Consider the utility function you have worked with so far as a special case of the CES family \( u(x_1, x_2) = (0.5x_1^p + 0.5x_2^p)^{-1/p} \). Given what you concluded in A(d) of this question, how would your answer to B(d) change as \( p \) changes?

7.13 Policy Application: Public Housing and Housing Subsidies: In exercise 2.14, you considered two different public housing programs in parts A(a) and (b), one where a family is simply offered a particular apartment for a below-market rent and another where the government provides a housing price subsidy that the family can use anywhere in the private rental market.

A. Suppose we consider a family that earns $1,500 per month and either pays $0.50 per square foot in monthly rent for an apartment in the private market or accepts a 1,500-square-foot government public housing unit at the government’s price of $500 per month.

a. On a graph with square feet of housing and “dollars of other consumption,” illustrate two cases where the family accepts the public housing unit, one where this leads them to consume less housing than they otherwise would and another where it leads them to consume more housing than they otherwise would.
b. If we use the members of the household’s own judgment about the household’s well-being, is it always the case that the option of public housing makes the participating households better off?

c. If the policy goal behind public housing is to increase the housing consumption of the poor, is it more or less likely to succeed the less substitutable housing and other goods are?

d. What is the government’s opportunity cost of owning a public housing unit of 1,500 square feet? How much does it therefore cost the government to provide the public housing unit to this family?

e. Now consider instead a housing price subsidy under which the government tells qualified families that it will pay some fraction of their rental bills in the private housing market. If this rental subsidy is set so as to make the household just as well off as it was under public housing, will it lead to more or less consumption of housing than if the household chooses public housing?

f. Will giving such a rental subsidy cost more or less than providing the public housing unit? What does your answer depend on?

g. Suppose instead that the government simply gave cash to the household. If it gave sufficient cash to make the household as well off as it is under the public housing program, would it cost the government more or less than $250? Can you tell whether under such a subsidy the household consumes more or less housing than under public housing?

B. Suppose that household tastes over square feet of housing \((x_1)\) and dollars of other consumption \((x_2)\) can be represented by 
\[ u(x_1, x_2) = \alpha \ln x_1 + (1 - \alpha) \ln x_2. \]

a. Suppose that empirical studies show that we spend about a quarter of our income on housing. What does that imply about \(\alpha\)?

b. Consider a family with income of $1,500 per month facing a per square foot price of \(p_1 = 0.50\). For what value of \(\alpha\) would the family not change its housing consumption when offered the 1,500-square-foot public housing apartment for $500?

c. Suppose that this family has \(\alpha\) as derived in B(a). How much of a rental price subsidy would the government have to give to this family in order to make it as well off as the family is with the public housing unit?

d. How much housing will the family rent under this subsidy? How much will it cost the government to provide this subsidy?

e. Suppose the government instead gave the family cash (without changing the price of housing). How much cash would it have to give the family in order to make it as happy?

f. If you are a policy maker whose aim is to make this household happier at the least cost to the taxpayer, how would you rank the three policies? What if your goal was to increase the household’s housing consumption?