We have just demonstrated in Chapter 6 how we can use our model of choice sets and tastes to illustrate optimal decision making by individuals such as consumers or workers. We now turn to the question of how such optimal decisions change when economic circumstances change. Since economic circumstances in this model are fully captured by the choice set, we could put this differently by saying that we will now ask how optimal choices change when income, endowments, or prices change.

As we proceed, it is important for us to keep in mind the difference between tastes and behavior. Behavior, or what we have been calling choice, emerges when tastes confront circumstances as individuals try to do the “best” they can given those circumstances. If I buy less wine because the price of wine has increased, my behavior has changed but my tastes have not. Wine still tastes the same as it did before, it just costs more. In terms of the tools we have developed, my indifference map remains exactly as it was. I simply move to a different indifference curve as my circumstances (i.e., the price of wine) change.

In the process of thinking about how behavior changes with economic circumstances, we will identify two conceptually distinct causes, known as income and substitution effects. At first it will seem like the distinction between these effects is abstract and quite unrelated to real-world issues we care about. As you will see later, however, this could not be further from the truth. Deep questions related to the efficiency of tax policy, the effectiveness of Social Security and health policy, and the desirability of different types of antipoverty programs are fundamentally rooted in questions related to income and substitution effects. While we are still in the stage of building tools for economic analysis, I hope you will be patient and bear with me as we develop an understanding of these tools.

Still, it may be useful to at least give an initial example to motivate the effects we will develop in this chapter, an example that will already be familiar to you if you have done end-of-chapter exercise 6.14. As you know, there is increasing concern about carbon-based emissions from automobiles, and an increased desire by policy makers to find ways of reducing such emissions. Many economists have long recommended the simple policy of taxing gasoline heavily in order to encourage consumers to find ways of conserving gasoline (by driving less and buying more fuel-efficient cars). The obvious concern with such a policy is that it imposes substantial hardship on households that rely heavily on their cars, particularly poorer households that would be hit pretty hard by such a tax. Some

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1Chapters 2 and 4 through 6 are required reading for this chapter. Chapter 3 is not necessary.

2This distinction was fully introduced into neoclassical economics by Sir John Hicks in his influential book, Value and Capital, originally published in 1939. We had previously mentioned him in part B of Chapter 5 as the economist who first derived a way to measure substitutability through “elasticities of substitution.” Hicks was awarded the Nobel Prize in Economics in 1972 (together with Ken Arrow).
economists have therefore proposed simply sending all tax revenues from such a gasoline tax back to taxpayers in the form of a tax refund. This has led many editorial writers to conclude that economists must be nuts; after all, if we send the money back to the consumers, wouldn’t they then just buy the same amount of gasoline as before since (at least on average) they would still be able to afford it? Economists may be nuts, but our analysis will tell us that they are also almost certainly right, and editorial writers are almost certainly wrong, when it comes to the prediction of how this policy proposal would change behavior. And the explanation lies fully in an understanding of substitution effects that economists understand and most noneconomists don’t think about. We’ll return to this in the conclusion to the chapter.

### 7A Graphical Exposition of Income and Substitution Effects

There are two primary ways in which choice sets (and thus our economic circumstances) can change: First, a change in our income or wealth might shift our budget constraints without changing their slopes, and thus without changing the opportunity costs of the various goods we consume. Second, individual prices in the economy—whether in the form of prices of goods, wages, or interest rates—may change and thus alter the slopes of our budget constraints and the opportunity costs we face. These two types of changes in choice sets result in different types of effects on behavior, and we will discuss them separately in what follows. First, we will look only at what happens to economic choices when income or wealth changes without a change in opportunity costs (Section 7A.1). Next, we will investigate how decisions are impacted when only opportunity costs change without a change in real wealth (Section 7A.2). Finally, we will turn to an analysis of what happens when changes in income and opportunity costs occur at the same time, which, as it turns out, is typically the case when relative prices in the economy change.

#### 7A.1 The Impact of Changing Income on Behavior

What happens to our consumption when our income increases because of a pay raise at work or when our wealth endowment increases because of an unexpected inheritance or when our leisure endowment rises due to the invention of some time-saving technology? Would we consume more shirts, pants, Coke, housing, and jewelry? Would we consume more of some goods and fewer of others, work more or less, save more or less? Would our consumption of all goods go up by the same proportion as our income or wealth?

The answer depends entirely on the nature of our tastes, and the indifference map that represents our tastes. For most of us, it is likely that our consumption of some goods will go up by a lot while our consumption of other goods will increase by less, stay the same, or even decline. The impact of changes in our income or wealth on our consumption decisions (in the absence of changes in opportunity costs) is known as the income or wealth effect.

The economics “lingo” is not entirely settled on whether to call this kind of an effect a “wealth” or an “income” effect, and we will use the two terms in the following way: Whenever we are analyzing a model where the size of the choice set is determined by exogenously given income, as in Chapter 2 and for the remainder of this chapter, we will refer to the impact of a change in income as an income effect. In models where the size of the choice set is determined by the value of an endowment, as in Chapter 3 and in the next chapter, we will refer to the impact of changes in that endowment as a wealth effect. What should be understood throughout, however, is that by both income and wealth effect we mean an impact on consumer decisions that arises from a parallel shift in the budget constraint, a shift that does not include a change in opportunity costs as captured by a change in the slope of the budget line.
7A.1.1 Normal and Inferior Goods

During my first few years in graduate school, my wife and I made relatively little money. Often, our budget would permit few extravagances, with dinners heavily tilted toward relatively cheap foods such as potatoes and pasta. When my wife’s business began to take off, our income increased considerably, and she observed one night over a nice steak dinner that we seemed to be eating a lot less pasta these days. Our consumption of pasta, it turned out, declined as our income went up, whereas our consumption of steak and other goods increased. How could this happen within the context of the general model that we have developed in the last few chapters?

Consider a simple model in which we put monthly consumption of boxes of pasta on the horizontal axis and the monthly consumption of pounds of steak on the vertical. My wife and I began with a relatively low income and experienced an increase in income as my wife’s business succeeded. This is illustrated by the outward shift in our budget constraint (from blue to magenta) in each of the panels of Graph 7.1. As we then add the indifference curves that contain our optimal choices under the two budget constraints, we get less pasta consumption at the higher income only if the tangency on the budget line occurs to the left of our tangency on the lower budget line. This is illustrated in panel (a) of Graph 7.1. Panel (b), on the other hand, illustrates the relationship between the two indifference curves if pasta consumption had remained unchanged with the increase in our income, while panel (c) illustrates the case had our pasta consumption increased with our income. This change in consumer behavior as exogenous income changes is called the income effect.

Since my wife observed that our consumption of pasta declined with an increase in our income, our preferences must look more like those in panel (a), where increased income has a negative impact on pasta consumption. We will then say that the income effect is negative whenever an increase in exogenous income (without a change in opportunity cost) results in less consumption, and goods whose consumption is characterized by negative income effects are called inferior goods. In contrast, we will say that the income effect is positive whenever an increase in exogenous income (without a change in opportunity cost) results in more consumption, and goods whose consumption is characterized by positive income effects are called normal goods. Panel
(c) of Graph 7.1 illustrates an example of what our preferences could look like if pasta were in fact a normal good for us.

Finally, panel (b) of Graph 7.1 illustrates an indifference map that gives rise to no income effect on our pasta consumption. Notice the following defining characteristic of this indifference map: The marginal rate of substitution is constant along the vertical line that connects points A and B. In Chapter 5, we called tastes that are represented by indifference curves whose marginal rates of substitution are constant in this way quasilinear (in pasta). The sequence of panels in Graph 7.1 then illustrates how quasilinear tastes are the only kinds of tastes that do not give rise to income effects for some good, and as such they represent the borderline case between normal and inferior goods.

It is worthwhile noting that whenever we observe a negative income effect on our consumption of one good, there must be a positive income effect on our consumption of a different good. After all, the increased income must be going somewhere, whether it is increased consumption of some good today or increased savings for consumption in the future. In Graph 7.1a, for instance, we observe a negative income effect on our consumption of pasta on the horizontal axis. At the same time, on the vertical axis we observe a positive income effect on our consumption of steak.

Is it also the case that whenever there is a positive income effect on our consumption of one good, there must be a negative income effect on our consumption of a different good?

Can a good be an inferior good at all income levels? (Hint: Consider the bundle (0,0).)

**7A.1.2 Luxuries and Necessities**  As we have just seen, quasilinear tastes represent one special case that divides two types of goods: normal goods whose consumption increases with income and inferior goods whose consumption decreases with income. The defining difference between these two types of goods is how consumption changes in an absolute sense as our income changes. A different way of dividing goods into two sets is to ask how our relative consumption of different goods changes as income changes. Put differently, instead of asking whether total consumption of a particular good increases or decreases with an increase in income, we could ask whether the fraction of our income spent on a particular good increases or decreases as our income goes up; i.e., whether our consumption increases relative to our income.

Consider, for instance, our consumption of housing. In each panel of Graph 7.2, we model choices between square footage of housing and “dollars of other goods.” As in the previous graph, we consider how choices will change as income doubles, with bundle A representing the optimal choice at the lower income and bundle B representing the optimal choice at the higher income. Suppose that in each panel, the individual spends 25% of her income on housing at bundle A. If housing remains a constant fraction of consumption as income increases, then the optimal consumption bundle B when income doubles would simply involve twice as much housing and twice as much “other good” consumption. This bundle would then lie on a ray emanating from the origin and passing through point A, as pictured in Graph 7.2b. If, on the other hand, the fraction of income allocated to housing declines as income rises, B would lie to the left of this ray (as in Graph 7.2a), and if the fraction of income allocated to housing increases as income rises, B would lie to the right of the ray (as in Graph 7.2c). It turns out that on average, people spend approximately 25% of their income on housing regardless of how much they make, which implies that tastes for housing typically look most like those in Graph 7.2b.
Economists have come to refer to goods whose consumption as a fraction of income declines with income as necessities while referring to goods whose consumption as a fraction of income increases with income as luxuries. The borderline tastes that divide these two classes of goods are tastes of the kind represented in Graph 7.2b, tastes that we defined as homothetic in Chapter 5. (Recall that we said tastes were homothetic if the marginal rates of substitution are constant along any ray emanating from the origin.) Thus, just as quasilinear tastes represent the borderline tastes between normal and inferior goods, homothetic tastes represent the borderline tastes between necessary and luxury goods.

**Exercise 7A.3** Are all inferior goods necessities? Are all necessities inferior goods? (*Hint:* The answer to the first is yes; the answer to the second is no.) Explain.

**Exercise 7A.4** At a particular consumption bundle, can both goods (in a two-good model) be luxuries? Can they both be necessities?

### 7A.2 The Impact of Changing Opportunity Costs on Behavior

Suppose my brother and I go off on a week-long vacation to the Cayman Islands during different weeks. He and I are identical in every way, same income, same tastes.³ Since there is no public transportation on the Cayman Islands, you only have two choices of what to do once you step off the airplane: you can either rent a car for the week, or you can take a taxi to your hotel and then rely on taxis for any additional transportation needs. After we returned home from our respective vacations, we compared notes and discovered that, although we had stayed at exactly the same hotel, I had rented a car whereas my brother had used only taxis.

³This assumption is for illustration only. Both my brother and I are horrified at the idea of anyone thinking we are identical, and he asked for this clarification in this text.
Which one of us do you think went on more trips away from our hotel? The difference between the number of car rides he and I took is what we will call a substitution effect.

### 7A.2.1 Renting a Car versus Taking Taxis on Vacation

The answer jumps out straight away if we model the relevant aspects of the choice problem that my brother and I were facing when we arrived at the airport in the Cayman Islands. Basically, we were choosing the best way to travel by car during our vacation. We can model this choice by putting “miles travelled” on the horizontal axis and “dollars of other consumption” on the vertical. Depending on whether I rent a car or rely on taxis, I will face different budget constraints. If I rent a car, I end up paying a weekly rental fee that is the same regardless of how many miles I actually drive. I then have to pay only for the gas I use as I drive to different parts of the island. If I rely on taxis, on the other hand, I pay only for the miles I travel, but of course I pay a per mile cost that is higher than just the cost of gas. Translated into budget constraints with “miles driven” on the horizontal axis and “dollars of other consumption” on the vertical, this implies that my budget will have a higher intercept on the vertical axis if I choose to use taxis because I do not have to pay the fixed rental fee. At the same time, the slope of the budget constraint would be steeper if I chose to use taxis because each mile I travel has a higher opportunity cost.

The choice my brother and I faced when we arrived in the Cayman Islands is thus a choice between two different budget constraints, one with a higher intercept and steeper slope than the other, as depicted in Graph 7.3a. (If this looks familiar, it is because you may have done this in end-of-chapter exercise 2.6.) Since my brother and I are identical in every way and faced exactly the same choice, you can reasonably conclude that we were indifferent between these two modes of transportation (and thus between the two budget constraints). After all, if one choice was clearly better than the other, we should have ended up making the same choice.

Thus, although we made different choices, we must have ended up on the same indifference curve. (This statement—that we ended up on the same indifference curve—makes sense only because we know that my brother and I have the same tastes and thus the same map of indifference curves, and we have the same exogenous income.) Graph 7.3b therefore fits a single indifference curve tangent to the two budget constraints, illustrating that our optimal
choices on the two different budget constraints result in the same level of satisfaction. My brother’s optimal choice A then indicates fewer miles travelled than my optimal choice B.

The intuition behind the model’s prediction is straightforward. Once I sped off to my hotel in my rented car, I had to pay the rental fee no matter what else I did for the week. So, the opportunity cost or price of driving a mile (once I decided to rent a car) was only the cost of gasoline. My brother, on the other hand, faced a much higher opportunity cost since he had to pay taxi prices for every mile he travelled. Even though our choices made us equally well off, it is clear that my lower opportunity cost of driving led me to travel more miles and consume less of other goods than my brother.

Economists will often say that the flat weekly rental fee becomes a sunk cost as soon as I have chosen to rent a car. Once I have rented the car, there is no way for me to get back the fixed rental fee that I have agreed to pay, and it stays the same no matter what I do once I leave the rental car lot. So, the rental fee is never an opportunity cost of anything I do once I have rented the car. Such sunk costs, once they have been incurred, therefore do not affect economic decisions because our economic decisions are shaped by the trade-offs inherent in opportunity costs. We will return to the concept of sunk costs more extensively when we discuss producer behavior, and we will note in Chapter 29 that some psychologists quarrel with the economist’s conclusion that such costs should have no impact on behavior.

**7A.2.2 Substitution Effects**

The difference in my brother’s and my behavior in our Cayman Island example is what is known as a substitution effect. Substitution effects arise whenever opportunity costs or prices change. In our example, for instance, we analyzed the difference in consumer behavior when the price of driving changes, but the general intuition behind the substitution effect will be important for many more general applications throughout this book.

We will define a substitution effect more precisely as follows: The substitution effect of a price change is the change in behavior that results purely from the change in opportunity costs and not from a change in real income. By real income, we mean real welfare, so “no change in real income” should be taken to mean “no change in satisfaction” or “no change in indifference curves.” The Cayman Island example was constructed so that we could isolate a substitution effect clearly by focusing our attention on a single indifference curve or a single level of “real income.”

The fact that bundle B must lie to the right of bundle A is a simple matter of geometry: A steeper budget line fit tangent to an indifference curve must lie to the left of a shallower budget line that is tangent to the same indifference curve. The direction of a substitution effect is therefore always toward more consumption of the good that has become relatively cheaper and away from the good that has become relatively more expensive. Note that this differs from what we concluded about income effects whose direction depends on whether a good is normal or inferior.

**7A.2.3 How Large Are Substitution Effects?**

While the direction of substitution effects is unambiguous, the size of the effect is dependent entirely on the kinds of underlying tastes a consumer has. The picture in Graph 7.3b suggests a pretty clear and sizable difference between
the number of miles I drove and the number of miles my brother drove given that we faced different opportunity costs for driving while having the same level of satisfaction or welfare. But I could have equally well drawn the indifference curve with more curvature, and thus with less substitutability between miles driven and other consumption. *The less substitutability is built into a consumer’s tastes, the smaller will be substitution effects arising from changes in opportunity costs.*

For instance, consider the indifference curve in Graph 7.4b, an indifference curve with more curvature than that in Graph 7.4a and thus less built-in substitutability along the portion on which my brother and I are making our choices. Notice that, although the substitution effect points in the same direction as before, the effect is considerably smaller. Graph 7.4c illustrates the role played by the level of substitutability between goods even more clearly by focusing on the extreme case of perfect complements. Such tastes give rise to indifference curves that permit no substitutability between goods, leading to bundles A and B overlapping and a consequent disappearance of the substitution effect.

**True or False:** If you observed my brother and me consuming the same number of miles driven during our vacations, then our tastes must be those of perfect complements between miles driven and other consumption.

**Exercise 7A.6**

**7A.2.4 “Hicks” versus “Slutsky” Substitution** We have now defined the substitution effect as the change in consumption that is due to a change in opportunity cost without a change in “real income”; i.e., without a change in the indifference curve. This is sometimes called *Hicksian* substitution. A slightly different concept of a substitution effect arises when we ask how a change in opportunity costs alters a consumer’s behavior assuming that her ability to purchase the original bundle remains intact. This is called *Slutsky* substitution. It operates very similarly to Hicksian substitution, and we will therefore leave it to end-of-chapter exercise 7.11 to explore this further. We are also using the idea in exercise 7.11 (and its previous companion exercise 6.16) and 7.6 (as well as its previous companion exercise 6.9).
7A.3 Price Changes: Income and Substitution Effects Combined

As you were reading through the Cayman Island example, you may have wondered why I chose such an admittedly contrived story. The reason is that I wanted to follow our discussion of pure income effects (which occur in the absence of changes in opportunity costs) in Section 7A.1 with a discussion of pure substitution effects (which occur in the absence of any changes in real income or wealth) in Section 7A.2. Most real-world changes in opportunity costs, however, implicitly also give rise to changes in real income, causing the simultaneous operation of both income and substitution effects.

Let’s forget the Cayman Islands, then, and consider what happens when the price of a good that most of us consume goes up, as, for instance, the price of gasoline. When this happens, I can no longer afford to reach the same indifference curve as before if my exogenous income remains the same. Thus, not only do I face a different opportunity cost for gasoline but I also have to face the prospect of ending up with less satisfaction—or what we have called less “real” income—because I am doomed to operate on a lower indifference curve than before the price increase. Similarly, if the price of gasoline declines, I not only face a different opportunity cost for gasoline but will also end up on a higher indifference curve, and thus experience an increase in real income. A price change therefore typically results in both an income effect and a substitution effect. These can be conceptually disentangled even though they occur simultaneously, and it will become quite important for many policy applications to know the relative sizes of these conceptually different effects. You will see how this is important more clearly in later chapters. For now, we will simply focus on conceptually disentangling the two effects of price changes.

7A.3.1 An Increase in the Price of Gasoline

To model the impact of an increase in the price of gasoline on my behavior, we can once again put “miles driven” on the horizontal axis and “dollars of other consumption” on the vertical. An increase in the price of gasoline then causes an inward rotation of the budget line around the vertical intercept, as illustrated in Graph 7.5a. My optimal bundle prior to the price increase is illustrated by the tangency of the indifference curve at point A.
We can now begin our disentangling of income and substitution effects by asking how my consumption bundle would have changed had I only experienced the change in opportunity costs without a change in my real income. Put differently, we can ask how my consumption decision would change if I faced a new budget that incorporated the steeper slope implied by the price change but was large enough to permit me to be as satisfied as I was before the price change, large enough to keep me on my original indifference curve. This budget is illustrated as the green budget tangent to the indifference curve containing bundle A in Graph 7.5b and is called the compensated budget. A compensated budget for a price change is the budget that incorporates the new price but includes sufficient monetary compensation to make the consumer as well off as she was before the price change. If income is exogenous (as it is in our example), the compensated budget requires positive compensation when prices increase and negative compensation when prices decrease.

Graph 7.5b then looks very much like Graph 7.4b that illustrated a pure substitution effect for our Cayman Islands example. This is because we have imagined that I was provided sufficient compensation at the higher gasoline price to keep my real income constant in order to focus only on the change in my consumption that is due to the change in my opportunity costs along a single indifference curve. As in the Cayman example, we can then quickly see that consumption of gasoline is less at point B than at point A. When real income is unchanged, the substitution effect tells us that I will consume less gasoline because gasoline has become more expensive relative to other goods.

Rarely, however, will someone come to me and offer me compensation for a price change in real life. Rather, I will have to settle for a decrease in my real income when prices go up. In Graph 7.5c, we thus start with the compensated budget and ask how my actual consumption decision will differ from the hypothetical outcome B. Before answering this question, notice that the compensated budget and the final budget in Graph 7.5c have the same slope and thus differ only by the hypothetical compensation we have assumed when plotting the compensated budget. Thus when going from the compensated (green) to the final (magenta) budget, we are simply analyzing the impact of a change in my exogenous money income, or what we called a pure income effect in Section 7A.1.

Whether my optimal consumption of gasoline on my final budget line is larger or smaller than at point B then depends entirely on whether gasoline is a normal or an inferior good for me. We defined a normal good as one whose consumption moves in the same direction as changes in exogenous income, while we defined an inferior good as one whose consumption moved in the opposite direction of changes in exogenous income. Thus, the optimal bundle on the final budget might lie to the left of point B if gasoline is a normal good, and it might lie to the right of B if gasoline is an inferior good. In the latter case, it could lie in between A and B if the income effect is smaller than the substitution effect, or it might lie to the right of point A if the income effect is larger than the substitution effect. In Graph 7.5c, we illustrate the case where gasoline is a normal good, and the optimal final bundle C lies to the left of B. In this case, both income and substitution effects suggest that I will purchase less gasoline as the price of gasoline increases.

### 7A.3.2 Regular Inferior and Giffen Goods

Notice that we can conclude unambiguously that my consumption of gasoline will decline if its price increases whenever gasoline is a normal good (as is the case if bundle C in Graph 7.5c is my optimal final choice). This is because both the substitution and the income effect suggest declining consumption. If, on the other hand, gasoline is an inferior good for me, then my gasoline consumption could increase or decrease depending on whether my final consumption bundle lies between A and B as in Graph 7.6a or whether it lies to the right of A as in Graph 7.6b. We can therefore divide inferior goods into two subcategories: those whose consumption decreases with an increase in price and those whose consumption increases with an increase in price (when exogenous income remains constant). We will call the former regular inferior goods and the latter Giffen goods.

When initially introduced to the possibility that a consumer might purchase more of a good when its price goes up, students often misinterpret what economists mean by this. A common example that students will think of is that of certain goods that carry a high level of prestige...

precisely because everyone knows they are expensive. For instance, it may be true that some consumers who care about the prestige value of a BMW will be more likely to purchase BMWs as the price (and thus the prestige value) increases. This is not, however, the kind of behavior we have in mind when we think of Giffen goods. The person who attaches a prestige value to the price of a BMW is really buying two different goods when he or she buys this car: the car itself and the prestige value of the car. As the price of the BMW goes up, the car remains the same but the quantity of prestige value rises. So, a consumer who is more likely to buy BMWs as the price increases is not buying more of a single good but is rather buying a different mix of goods when the price of the BMW goes up. When the same consumer’s income falls (and the price of BMWs remains the same), the consumer would almost certainly be less likely to buy BMWs, which indicates that the car itself (with the prestige value held constant) is a normal good.

Real Giffen goods are quite different, and we rarely observe them in the real world. Economists have struggled for literally centuries to find examples; this is how rare they are. At the end of the 19th century, Alfred Marshall (1842–1924), one of the great economists of that century, included a hypothetical example in his economics textbook and attributed it to Robert Giffen, a contemporary of his. 

Over the years, a variety of attempts to find credible historical examples that are not hypothetical have been discredited, although a recent paper demonstrates that rice in poor areas of China may indeed be a Giffen good there.

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5 While an increase in the price still causes an increase in the consumption of the physical good we observe, such goods are examples of what is known as Veblen Goods after Thorstein Veblen (1857–1929) who hypothesized that preferences for certain goods intensify as price increases, which can cause what appear to be increases in consumption as price goes up. You can think through this more carefully in end-of-chapter exercise 7.9, where you are asked to explain an increase in the consumption of Gucci accessories when the price increases. In Chapter 21, we revisit Veblen goods in end-of-chapter exercise 21.5 in the context of network externalities.

6 To quote from his text: “As Mr. Giffen has pointed out, a rise in the price of bread makes so large a drain on the resources of the poorer labouring families ... that they are forced to curtail their consumption of meat and the more expensive farinaceous foods: and bread being still the cheapest food which they can get and will take, they consume more, and not less of it.” A. Marshall, Principles of Economics (MacMillan: London, 1895). While Robert Giffen (1837–1910) was a highly regarded economist and statistician, it appears no one has located a reference to the kinds of goods that are named after him in any of his own writings, only in Marshall’s.

Chapter 7. Income and Substitution Effects in Consumer Goods Markets

A friend of mine in graduate school once told me a story that is the closest example I have ever personally heard of a real Giffen good. He came from a relatively poor family in the Midwest where winters get bitterly cold and where they heated their home with a form of gasoline. Every winter, they would spend a month over Christmas with relatives in Florida. One year during the 1973 energy crisis, the price of gasoline went up so much that they decided they could not afford to go on their annual vacation in Florida. So, they stayed in the Midwest and had to heat their home for one additional month. While they tried to conserve on gasoline all winter, they ended up using more than usual because of that extra month. Thus, their consumption of gasoline went up precisely because the price of gasoline went up and the income effect outweighed the substitution effect. This example, as well as the recent research on rice in China, both illustrate that, in order to find the “Giffen behavior” of increasing consumption with an increase in price, it must be that the good in question represents a large portion of a person’s income to begin with, with a change in price therefore causing a large income effect. It furthermore must be the case that there are no very good substitutes for the good in order for the substitution effect to remain small. Given the variety of substitutable goods in the modern world and the historically high standard of living, it therefore seems very unlikely that we will find much “Giffen behavior” in the part of the world that has risen above subsistence income levels.

Can you re-tell the Heating Gasoline-in-Midwest story in terms of income and substitution effects in a graph with “yearly gallons of gasoline consumption” on the horizontal axis and “yearly time on vacation in Florida” on the vertical?

Exercise 7A.7*

7A.3.3 Income and Substitution Effects for Pants and Shirts Now let’s return to our example from Chapter 2: My wife sends me to Wal-Mart with a fixed budget to buy pants and shirts. Since I know how much Wal-Mart charges for pants and shirts, I enter the store already having solved for my optimal bundle. Now suppose that one of the greeters at Wal-Mart hands me a 50% off coupon for pants, effectively decreasing the price of pants I face. We already know that this will lead to an outward rotation of my budget as shown in Graph 7.7a. Armed with the new information presented in this chapter, however, we can now predict how my consumption of pants and shirts will change depending on whether pants and shirts are normal, regular inferior, or Giffen goods.

Graph 7.7: Inferring the Type of Good from Observed Choices

![Graph 7.7: Inferring the Type of Good from Observed Choices](image-url)
First, we isolate once again the substitution effect by drawing my (green) compensated budget under the new price in Graph 7.7b. Notice that the “compensation” in this case is negative: In order to keep my “real income” (i.e., my indifference curve) constant and concentrate only on the impact of the change in opportunity costs, you would have to take away some of the money my wife had given me. As always, the substitution effect, the shift from A to B, indicates that I will switch away from the good that has become relatively more expensive (shirts) and toward the good that has become relatively cheaper (pants).

In Graph 7.7c, we then focus on what happens when we switch from the hypothetical optimum on the compensated (green) budget to our new optimum on the final (magenta) budget. Since this involves no change in opportunity costs, we are left with a pure income effect as we jump from the optimal point B on the compensated budget line to the final optimum on the final budget constraint. Suppose we know that both shirts and pants are normal goods for me. This would tell me that, when I experience an increase in income from the compensated to the final budget, I will choose to consume more pants and shirts than I did at point B. If shirts are inferior and pants are normal, I will consume more pants and fewer shirts than at B; and if pants are inferior and shirts are normal, I will consume fewer pants and more shirts. Given that I am restricted in this example to consuming only shirts and pants, it cannot be the case that both goods are inferior because this would imply that I consume fewer pants and fewer shirts on my final budget than I did at point B, which would put me at a bundle to the southwest of B. Since “more is better,” I would not be at an optimum given that I can move to a higher indifference curve from there.

Now suppose that you know not only that pants are an inferior good but also that pants are a Giffen good. The definition of a Giffen good implies that I will consume less of the good as its price decreases when exogenous income remains unchanged. Thus, I would end up consuming not just fewer pants than at point B but also fewer than at point A. Notice that this is the only scenario under which we would not have to first find the substitution effect; if we know something is a Giffen good and we know its price has decreased, we immediately know that consumption will decrease as well. In each of the other scenarios, however, we needed to find the compensated optimum B before being able to apply the definition of normal or inferior goods.

Finally, suppose you know that shirts rather than pants are a Giffen good. Remember that in order to observe a Giffen good, we must observe a price change for that good (with exogenous income constant) since Giffen goods are goods whose consumption moves in the same direction as price (when income is exogenous and unchanged). In this example, we did not observe a price change for shirts, which means that we cannot usefully apply the definition of a Giffen good to predict how consumption will change. Rather, we can simply note that, since all Giffen goods are also inferior goods, I will consume fewer shirts as my income increases from the compensated budget to the final budget. Thus, knowing that shirts are Giffen tells us nothing more in this example than knowing that shirts are inferior goods.

Exercise 7A.8
Replicate Graph 7.7 for an increase in the price of pants (rather than a decrease).

7B
The Mathematics of Income and Substitution Effects

In this section, we will now begin to explore income and substitution effects mathematically. I say that we will “begin” doing this because our exploration of these effects will become deeper as we move through the next few chapters. For now, we will try to illustrate how to relate the intuitions developed in part A of this chapter most directly to some specific mathematics, and in the process we will build
the tools for a more general treatment later on. As you read through this section, you will undoubtedly get lost a bit unless you sit with pencil and paper and follow the calculations we undertake closely on your own. As you do this, you will begin to get a feel for how we can use the various mathematical concepts introduced thus far to identify precisely the points $A$, $B$, and $C$ that appear in our graphs of this chapter. It might help you even more to then reread the chapter and construct simple spreadsheets in a program like Microsoft Excel, which is precisely how I kept track of the different numerical answers that are presented in the text as I wrote this section. Setting up such spreadsheets will give you a good feel for how the mathematics of consumer choice works for specific examples.

7B.1 The Impact of Changing Income on Behavior

In Chapter 6, we solved the consumer’s constrained optimization problem for specific economic circumstances; i.e., for specific prices and incomes. In Section 7A.1, we became interested in how consumer behavior changes when exogenous income changes, and we discovered that the answer depends on the nature of the underlying map of indifference curves. We will now translate some of this analysis from Section 7A.1 into the mathematical optimization language we developed in Chapter 6.

7B.1.1 Inferior and Normal Goods

Consider, for instance, the example of pasta and steak we introduced in Section 7A.1.1, and suppose my wife and I had discovered that our consumption of pasta remained unchanged as our income increased (as depicted in Graph 7.1b). Suppose that the price of a box of pasta is $2 and the price of a pound of steak is $10, and suppose we let boxes of pasta be denoted by $x_1$ and pounds of steak by $x_2$. We know from our discussion in Section 7A.1.1 that pasta consumption can remain constant as income increases only if the underlying tastes are quasilinear in pasta; i.e., when utility functions can be written as $u(x_1, x_2) = v(x_1) + x_2$. For an income level $I$ and for tastes that can be described by a utility function $u(x_1, x_2)$, the constrained optimization problem can then be written as

$$
\max_{x_1, x_2} u(x_1, x_2) = v(x_1) + x_2 \text{ subject to } 2x_1 + 10x_2 = I,
$$

with a corresponding Lagrange function

$$
\mathcal{L}(x_1, x_2, \lambda) = v(x_1) + x_2 + \lambda (I - 2x_1 - 10x_2).
$$

Taking the first two first order conditions, we get

$$
\begin{align*}
\frac{\partial \mathcal{L}}{\partial x_1} &= \frac{dv(x_1)}{dx_1} - 2\lambda = 0, \\
\frac{\partial \mathcal{L}}{\partial x_2} &= 1 - 10\lambda = 0.
\end{align*}
$$

The second of the expressions in (7.3) can then be rewritten as $\lambda = 1/10$, which, when substituted into the first expression in (7.3), gives

$$
\frac{dv(x_1)}{dx_1} = \frac{1}{5}. \quad (7.4)
$$

Notice that the left-hand side of (7.4) is just a function of $x_1$, whereas the right-hand side is just a real number, which implies that, when we have a specific functional form for the function $v$, we can solve for $x_1$ as just a real number. For instance, if $u(x_1, x_2) = \ln x_1 + x_2$ (implying $v(x_1) = \ln x_1$), expression (7.4) becomes

$$
\frac{1}{x_1} = \frac{1}{5} \text{ or } x_1 = 5. \quad (7.5)
$$
When the underlying tastes are quasilinear, the optimal quantity of pasta \((x_1)\) is therefore 5 (when prices of pasta and steak are 2 and 10) and is thus always the same regardless of what value the exogenous income \(I\) takes in the optimization problem (7.1). Put differently, the variable \(I\) simply drops out of the analysis as we solve for \(x_1\). Thus, borderline normal/inferior goods have no income effects.

This is not true, of course, for tastes that cannot be represented by quasilinear utility functions. Consider, for instance, the same problem but with underlying tastes that can be represented by the Cobb–Douglas utility function \(u(x_1, x_2) = x_1^a x_2^{1-a}\). The Lagrange function is then

\[
\mathcal{L}(x_1, x_2, \lambda) = x_1^a x_2^{1-a} + \lambda(I - 2x_1 - 10x_2),
\]

and the first order conditions for this problem are

\[
\frac{\partial \mathcal{L}}{\partial x_1} = a x_1^{a-1} x_2^{1-a} - 2\lambda = 0,
\]

\[
\frac{\partial \mathcal{L}}{\partial x_2} = (1 - a)x_1^a x_2^{-a} - 10\lambda = 0,
\]

\[
\frac{\partial \mathcal{L}}{\partial \lambda} = I - 2x_1 - 10x_2 = 0.
\]

Adding \(2\lambda\) to both sides of the first equation and \(10\lambda\) to both sides of the second equation, and then dividing these equations by each other, we get \(a x_1/(1 - a)x_1 = 1/5\) or \(x_2 = (1 - a)x_1/5a\). Substituting this into the third equation of expression (7.7) and solving for \(x_1\), we get

\[
x_1 = \frac{\alpha I}{2}.
\]

Thus, for the underlying Cobb–Douglas tastes specified here, the optimal consumption of pasta \((x_1)\) depends on income, with higher income leading to greater consumption of pasta. Cobb–Douglas tastes (as well as all other homothetic tastes) therefore represent tastes for normal goods as depicted in Graph 7.1c.

Finally, none of the utility functions we have discussed thus far represent tastes for inferior goods. This is because such tastes are difficult to capture in simple mathematical functions, in part because there are no tastes such that a particular good is always an inferior good. To see this, imagine beginning with zero income, thus consuming the origin in our graphs. Now suppose I give you $10. Since we cannot consume negative amounts of goods, it is not possible for you to consume less pasta than you did before I gave you $10, and it is therefore not possible to have tastes that represent inferior goods around the origin of our graphs. All goods are therefore normal or borderline normal/inferior goods at least around the bundle \((0,0)\). Goods can be inferior only for some portion of an indifference map, and this logical conclusion makes it difficult to represent such tastes in simple utility functions.

7B.1.2 Luxury Goods and Necessities

We also defined in Section 7A.1.2 the terms luxury goods and necessities, with borderline goods between the two represented by homothetic tastes. We know from our discussion of homothetic tastes in Chapter 5 that such tastes have the feature that the marginal rates of substitution stay constant along linear rays emanating from the origin, and it is this feature of such tastes that ensures that, when exogenous income is increased by \(x\%\) (without a change in opportunity costs), our consumption of each good also increases by \(x\%\), leaving the ratio of our consumption of one good relative to the other unchanged.

For instance, in equation (7.8), we discovered that my optimal consumption of pasta is equal to \(aI/2\) when my tastes are captured by the Cobb–Douglas function \(u(x_1, x_2) = x_1^a x_2^{1-a}\), when the price of pasta is $2 and the price of steak is $10 and when my income is given by \(I\). When plugging this value into the budget constraint for \(x_1\) and solving for \(x_2\), we can also determine that
my optimal consumption of steak is \((1 - \alpha)I/10\). Thus, the ratio \((x_1/x_2)\) of my pasta consumption to my steak consumption under these economic circumstances is \(5\alpha/(1 - \alpha)\). Put differently, my consumption of pasta relative to steak is independent of income. Since we know that Cobb–Douglas utility functions represent homothetic tastes, this simply confirms what our intuition already tells us: both pasta and steak are borderline luxury/necessity goods when the underlying tastes can be represented by Cobb–Douglas utility functions.

Again, this is not true for all types of tastes. If my tastes could be represented by the quasilinear utility function \(u(x_1, x_2) = \ln x_1 + x_2\), we concluded in expression (7.5) that my optimal consumption of pasta would be equal to 5 boxes regardless of my income level (assuming, of course, that I had at least enough income to cover that much pasta consumption). Plugging this into the budget constraint for and solving for , we also get that my optimal steak consumption is \(I/10\); i.e., my optimal steak consumption is a function of my income whereas my optimal pasta consumption is not. Put differently, my consumption of pasta relative to my consumption of steak declines with income, making pasta a necessity (and steak a luxury good).

### 7B.2 The Impact of Changing Opportunity Costs on Behavior

We introduced the concept of a substitution effect in Section 7A.2 by focusing on a particular example in which my brother chose to use taxis for transportation on his Cayman Islands vacation whereas I rented a car. To really focus on the underlying ideas, we assumed that my brother and I were identical in every way, allowing us to infer from the fact that we made two different choices that he and I were indifferent between renting a car and using taxis when we arrived at the airport in Cayman. The choice we made was one of choosing one of two budget constraints between “miles driven” and “other consumption” on our vacation. Renting a car requires a large fixed payment (thus reducing the level of other consumption that is possible if little or no driving occurs) but has the advantage of making additional miles cheap. Using taxis, on the other hand, involves no fixed payment but makes additional miles more expensive. Graph 7.3a illustrated the resulting choice sets, and Graph 7.3b illustrated a substitution effect from the different opportunity costs arising from those choice sets.

#### 7B.2.1 Renting a Car versus Taking a Taxi

Suppose you know that my brother and I came to the Cayman Islands with \(2,000\) to spend on our vacations and that taxi rides cost \$1 per mile. Letting \(x_1\) denote miles driven in Cayman and \(x_2\) “dollars of other consumption in Cayman,” we know that my brother’s budget line is \(2,000 = x_1 + x_2\) given that the price of “dollars of other consumption” is by definition also \(1\). Suppose we also know that my brother’s (and my own) tastes can be summarized by the Cobb–Douglas utility function \(u(x_1, x_2) = x_1^{0.1}x_2^{0.9}\). Doing our usual constrained optimization problem, we can then determine that my brother’s optimal consumption bundle is \(x_1 = 200\) and \(x_2 = 1,800\).

Set up my brother’s constrained optimization problem and solve it to check that his optimal consumption bundle is indeed equal to this.

Now suppose that I had lost my receipt for the rental car and no longer remember how much of a fixed fee I was charged to drive it for the week. All I do remember is that gasoline cost \$0.20 per mile. From the information we have, we can calculate what the fixed rental car fee must have been in order for me to be just as well off renting a car as my brother was using taxis.

Specifically, we can calculate the value associated with my brother’s optimal indifference curve by simply plugging \(x_1 = 200\) and \(x_2 = 1,800\) into the utility function \(u(x_1, x_2) = x_1^{0.1}x_2^{0.9}\) to get a value of approximately 1,445. While this number has no inherent meaning since we cannot quantify utility objectively, we do know from our analysis in Section 7A.2.1 (and Graph 7.3) that
I ended up on the same indifference curve, and thus with the same utility level as measured by the utility function that my brother and I share. This gives us enough information to find bundle \( B \)—my optimal bundle of miles driven and other consumption in Graph 7.3b—using a method that builds on the intuition that comes out of the graph. All we have to do is find the smallest possible choice set with a budget line that has the slope reflecting my lower opportunity cost for miles driven and is tangent to the indifference curve that my brother has achieved; i.e., the indifference curve associated with the utility value 1,445.

This can be formulated mathematically as the following problem: We would like to find the minimum expenditure necessary for achieving a utility value of 1,445 (as measured by the utility function \( u(x_1, x_2) = x_1^{0.1} x_2^{0.9} \)) given that my price for miles driven is 0.2 (while my price for “other consumption” remains at 1). Letting \( E \) stand for expenditure, we can state this formally as a constrained minimization problem:

\[
\min_{x_1, x_2} E = 0.2x_1 + x_2 \text{ subject to } x_1^{0.1} x_2^{0.9} = 1,445. \tag{7.9}
\]

Constrained minimization problems have the same basic structure as constrained maximization problems. The first part of (7.9) lets us know that we are trying to minimize a function by choosing the values for \( x_1 \) and \( x_2 \). The function we are trying to minimize, or what we call our objective function, then follows and is simply the equation for the budget constraint that we will end up with, which reflects the new opportunity cost of driving miles given that I have paid a fixed fee for my rental car and now face a lower opportunity cost for driving each mile. Finally, the last part of (7.9) tells us the constraint of our minimization problem: we are trying to reach the indifference curve associated with the value 1,445.

Finding the solution to a minimization problem is quite similar to finding the solution to a maximization problem. The reason for this similarity is most easily seen within the economic examples with which we are working. In our utility maximization problem, for instance, we are taking as fixed the budget line and trying to find the indifference curve that is tangent to that line. This is illustrated graphically in Graph 7.8a where a consumer faces a fixed budget line and tries to get to the highest possible indifference curve that still contains a bundle within the choice set.

**Graph 7.8:** Maximizing Utility with Budgets Fixed (a) versus Minimizing Expenditure with Utility Fixed (b)
defined by the fixed budget line. In the expenditure minimization problem defined in expression (7.9), on the other hand, we are taking the indifference curve as fixed and trying to find the smallest possible choice set given the opportunity costs of the goods. This is illustrated in Graph 7.8b where we are trying to reach a fixed indifference curve with the smallest possible choice set. In both cases, we are therefore trying to find a solution, a combination of \( x_1 \) and \( x_2 \), where an indifference curve is tangent to a budget line (assuming the problem does not have non-convexities or corner solutions).

For this reason, the same Lagrange Method that we have employed in solving maximization problems can be employed to solve our newly defined minimization problem. Again, we create the Lagrange function by combining the objective function with a second term that is equal to \( \lambda \) times the constraint set to zero, only now the objective function is the budget constraint and the constraint is the indifference curve. Thus,

\[
\mathcal{L}(x_1, x_2, \lambda) = 0.2x_1 + x_2 + \lambda(1,445 - x_1^{0.1}x_2^{0.9}).
\] (7.10)

We then again take the first derivatives of \( \mathcal{L} \) with respect to the choice variables \( (x_1 \text{ and } x_2) \) and \( \lambda \) to get the first order conditions

\[
\frac{\partial \mathcal{L}}{\partial x_1} = 0.2 - 0.1\lambda x_1^{-0.9}x_2^{0.9} = 0,
\]

\[
\frac{\partial \mathcal{L}}{\partial x_2} = 1 - 0.9\lambda x_1^{0.1}x_2^{-0.1} = 0, \quad (7.11)
\]

\[
1,445 - x_1^{0.1}x_2^{0.9} = 0.
\]

Solving the first two equations for \( x_2 \) we get

\[
x_2 = \frac{0.9(0.2x_1)}{0.1} = 1.8x_1
\] (7.12)

and plugging this into the third equation and solving for \( x_1 \), we get \( x_1 = 851.34 \). Finally, plugging this back into expression (7.12), we get \( x_2 = 1,532.41 \). This is point \( B \) in Graph 7.3, which implies that I chose to drive approximately 851 miles in my rental car during my Cayman Island vacation while consuming approximately $1,532 in other goods.

We can now see how much the bundle \( B \) costs by multiplying my optimal levels of \( x_1 \) and \( x_2 \) by the prices of those goods, 0.2 for \( x_1 \) and 1 for \( x_2 \), and adding these expenditures together:

\[
E = 0.2(851.34) + 1(1,532.41) = 1,702.68. \quad (7.13)
\]

Thus, bundle \( B \) costs a total of $1,702.68. Since you know that I arrived in Cayman with $2,000, you know that the difference between my total money budget for my vacation and the total I spent on driving and other goods must be what I paid for the fixed rental car fee: $297.32. This is equal to the vertical distance labeled “rental car fee” in Graph 7.3a.

### 7B.2.2 Substitution Effects

Notice that, in the process of making these calculations, we have identified the size of the substitution effect we treated graphically in Graph 7.3. Put differently, assuming tastes that can be represented by the utility function \( u(x_1, x_2) = x_1^{0.1}x_2^{0.9} \), an individual who chooses to drive 200 miles while consuming $1,800 in other goods when the opportunity cost per mile is $1 will reduce his other consumption and substitute toward 851 miles driven when we keep his real wealth—or his real well-being—fixed and change the opportunity cost for driving a mile to $0.2.

### 7B.2.3 The Size of Substitution Effects

By using a Cobb–Douglas utility function to represent tastes in the previous example, we have chosen a utility function that we know (from our discussion of Constant Elasticity of Substitution (CES) utility functions in
Chapter 5) has an elasticity of substitution equal to 1. The answers we calculated relate directly to this property of Cobb–Douglas utility functions. In fact, we can verify that the function \( u(x_1, x_2) = x_1^{0.1}x_2^{0.9} \) has an elasticity of substitution of 1 using our answers as we determined the bundles associated with points A and B in Graph 7.3. Recall the formula for an elasticity of substitution:

\[
\text{Elasticity of Substitution} = \left| \frac{\% \Delta (x_2/x_1)}{\% \Delta MRS} \right|.
\]

(7.14)

Bundle A, my brother’s optimal bundle, is (200, 1800), while bundle B, my optimal bundle, is (851.34, 1532.41). My brother’s ratio of \( x_2/x_1 \) is therefore equal to 1,800/200, or 9, while my ratio of \( x_2/x_1 \) is 1,532.41/851.34 or 1.8. In going from A to B on the same indifference curve, the change in the ratio \( x_2/x_1 \), \( \Delta(x_2/x_1) \), is therefore equal to \(-7.2\). The \( \% \Delta(x_2/x_1) \) is just the change in the ratio \( (x_2/x_1) \) divided by the original level of \( (x_2/x_1) \) at bundle A; i.e.,

\[
\% \Delta \left( \frac{x_2}{x_1} \right) = \frac{\Delta(x_2/x_1)}{(x_2/x_1)^A} = \frac{-7.2}{9} = -0.8.
\]

(7.15)

Similarly, the \( MRS \) at bundle A is equal to the slope of my brother’s budget line, which is equal to \(-1\) given that he faces a cost per mile of $1. My \( MRS \) at bundle B, on the other hand, is equal to the slope of my budget line, which is equal to \(-0.2\) given that I face a cost per mile of only $0.20. The \( \% \Delta MRS \) as we go from A to B is therefore the change in the \( MRS \) divided by the original \( MRS \) at bundle A; i.e.,

\[
\% \Delta MRS = \frac{\Delta MRS}{MRS^A} = 0.8.
\]

(7.16)

Plugging (7.15) and (7.16) into the equation for an elasticity of substitution in expression (7.14), we get an elasticity of substitution equal to 1. Thus, when the marginal rate of substitution of the indifference curve in Graph 7.3 changed by 80% (from \(-1\) to \(-0.2\)), the ratio of other consumption to miles driven also changed by 80% (from 9 to 1.8). It is the elasticity of substitution that is embedded in the utility function that determined the size of the substitution effect we calculated!

This relates directly to the intuition we built in Graph 7.4, where we showed how substitution effects get larger as the degree of substitutability, or the elasticity of substitution in our more mathematical language, changes. Were we to substitute utility functions with elasticities of substitution different from those in Cobb–Douglas utility functions, we would therefore calculate substitution effects that were larger or smaller depending on whether the elasticity of substitution imbedded into those utility functions was greater or smaller.

Consider, for instance, the CES utility function with \( \rho = -0.5 \), which implies an elasticity of substitution of 2 (rather than 1 as in the Cobb–Douglas case where \( \rho = 0 \)). More precisely, suppose that the utility function my brother and I share is

\[
u(x_1, x_2) = (0.25x_1^{0.5} + 0.75x_2^{0.5})^2,
\]

(7.17)

and suppose again that our money budget for our Cayman vacation is $2,000 and the per mile cost is $1 for taxis and $0.20 for rental cars.\(^8\) My brother’s optimization problem is then

\[
\max \ (0.25x_1^{0.5} + 0.75x_2^{0.5})^2 \text{ subject to } x_1 + x_2 = 2,000,
\]

(7.18)

which you can verify results in an optimal consumption bundle of \( x_1 = 200 \) and \( x_2 = 1,800 \) just as it did in our previous example. Thus, point A remains unchanged. The indifference

\(^8\)The exponents in equation (7.17) are positive because \( \rho \) is negative and each exponent in the CES utility function has a negative sign in front of it.
curve on which point \( A \) lies, however, differs substantially from that in the previous example because of the different elasticity of substitution embedded in equation (7.17). When you plug the optimal bundle for my brother back into the utility function (7.17) you can calculate that he operates on an indifference curve giving him utility of 1,250 as measured by this utility function. We could then repeat our analysis of calculating bundle \( B \) by solving the problem analogous to the one we stated in expression (7.9) but adapted to the model we are now working with:

\[
\min E = 0.2x_1 + x_2 \text{ subject to } \left(0.25x_1^{0.5} + 0.75x_2^{0.5}\right)^2 = 1,250. \tag{7.19}
\]

You can again verify on your own that this results in an optimal bundle \( B \) of \( x_1 = 2,551.02 \) and \( x_2 = 918.37 \), which implies a substitution effect much larger than the one we found with the Cobb–Douglas utility function. This is because we have built a greater elasticity of substitution into the utility function of equation (7.17) than we had in our previous Cobb–Douglas utility function. The difference between the two scenarios is illustrated graphically in Graph 7.9.

---

**Exercise 7B.2**

How much did I pay in a fixed rental car fee in order for me to be indifferent in this example to taking taxis? Why is this amount larger than in the Cobb–Douglas case we calculated earlier?

Table 7.1 on the next page summarizes the outcome of similar calculations for CES utility functions with different elasticities of substitution. In each case, the remaining parameters of the CES utility function are set to ensure that my brother’s optimal choice remains the same: 200 miles driven and $1,800 in other consumption.

---

More precisely, the utility function \( u(x_1, x_2) = (ax_1^p + (1 - ax_2^q)^{-1/p} \) was used for these calculations, with \( p \) set as indicated in the first column of the table and \( a \) adjusted to ensure that point \( A \) remains at (200,1800).
Table 7.1: Substitution Effects as Elasticity of Substitution Changes

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>Elasticity of Subst.</th>
<th>Substitution Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5</td>
<td>2</td>
<td>2,351.02 More Miles Driven at $B$ than at $A$</td>
</tr>
<tr>
<td>0.0</td>
<td>1</td>
<td>651.34 More Miles Driven at $B$ than at $A$</td>
</tr>
<tr>
<td>0.5</td>
<td>0.67</td>
<td>337.28 More Miles Driven at $B$ than at $A$</td>
</tr>
<tr>
<td>1.0</td>
<td>0.50</td>
<td>222.53 More Miles Driven at $B$ than at $A$</td>
</tr>
<tr>
<td>5.0</td>
<td>0.167</td>
<td>57.55 More Miles Driven at $B$ than at $A$</td>
</tr>
<tr>
<td>10.0</td>
<td>0.091</td>
<td>29.67 More Miles Driven at $B$ than at $A$</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.000</td>
<td>0.00 More Miles Driven at $B$ than at $A$</td>
</tr>
</tbody>
</table>

7B.3 Price Changes: Income and Substitution Effects Combined

Finally, we concluded in Section 7A.3 that most price changes involve both income and substitution effects because they involve both a change in our real wealth (or our optimal indifference curve) and a change in opportunity costs. We can then employ all the mathematical tools we have built thus far to identify income and substitution effects when prices change. In the following, we will consider once again the case of me shopping at Wal-Mart for pants ($x_1$) and shirts ($x_2$), as we did in Section 7A.3.3, to demonstrate how we can identify these effects separately. Throughout, we will assume that I have $200 to spend and that the price of shirts is $10, and we will focus on what happens when the price of pants, $p_1$, changes. We will assume (unrealistically) in this section that it is possible to consume fractions of shirts and pants. If this bothers you, you may feel more comfortable thinking of more continuous goods, such as nuts and candy from the bulk food isle where one can scoop as little or as much into a bag, instead of pants and shirts.

Suppose first that my tastes can once again be represented by a Cobb–Douglas utility function

\[ u(x_1, x_2) = (\alpha x_1^p + (1 - \alpha)x_2^p)^{-1/\rho} \]

My constrained maximization problem at Wal-Mart is then

\[
\max_{x_1, x_2} x_1^{0.5}x_2^{0.5} \text{ subject to } p_1x_1 + 10x_2 = 200. \]

Solving this in the usual way gives us the optimal bundle

\[
x_1 = \frac{100}{p_1} \text{ and } x_2 = 10. \]

Exercise 7B.3

Check to see that this solution is correct.

Initially, I face a price of $20 per pair of pants, which implies that my optimal bundle is 5 pants and 10 shirts. Then I discover that my wife gave me a 50% off coupon for pants, effectively reducing the price of pants from $20 to $10. As a result of this decrease in the price of pants, my optimal consumption bundle changes from (5,10) to (10,10). This is illustrated in Graph 7.10a, with bundle $A$ representing my original optimal bundle and bundle $C$ representing my new optimal bundle.
In order to decompose this change in my behavior into income and substitution effects, we have to calculate how my consumption would have changed had I faced the same change in opportunity costs without experiencing an increase in real wealth; i.e., without having shifted to a higher indifference curve. Thus, we need to employ the method we developed in the previous section to identify how much money I would have to give up when I received the coupon to be able to be just as well off as I was originally without the coupon. Notice that this is exactly analogous to our example involving my brother and me in the Cayman Islands where we wanted to identify how much the fixed rental car fee must have been in order for me to be just as well off as my brother was using taxis. In both cases, we have a fixed indifference curve, and we are trying to find the smallest possible choice set that will give me a fixed utility level when my opportunity costs change.

In Graph 7.10b, we illustrate the problem of finding the substitution effect graphically. We begin by drawing the indifference curve $U^A$ that contains bundle $A$ and the (magenta) budget line that I have with the coupon. Then we shift this budget line inward, keeping the slope and thus the new opportunity cost fixed, until only a single point on the indifference curve remains within the choice set. This process identifies bundle $B$ on the compensated (green) budget, the bundle I would choose if I faced the opportunity costs under the coupon but had lost just enough of my money to be just as well off as I was originally when I consumed bundle $A$.

Mathematically, we state the process graphed in Graph 7.10b as a constrained minimization problem in which we are trying to minimize my total expenditures (or my money budget) subject to the constraint that I would like to consume on the indifference curve that contains bundle $A$.

We can write this as follows:

$$\min_{x_1, x_2} E = 10x_1 + 10x_2 \text{ subject to } x_1^{0.5}x_2^{0.5} = U^A, \quad (7.23)$$

where $U^A$ represents the level of utility I attained at bundle $A$. This level of utility can be calculated using the utility function $x_1^{0.5}x_2^{0.5}$ by simply plugging the bundle $A$ ($x_1 = 5, x_2 = 10$) into the function, which gives us $U^A \approx 7.071$. Solving this minimization problem using the Lagrange Method illustrated in our Cayman example in the previous section, we get

$$x_1 = x_2 \approx 7.071. \quad (7.24)$$
The total expenditure required to consume this bundle at prices $p_1 = p_2 = 10$ is $141.42$, which implies that you could take $58.58$ out of my initial $200$ and give me a $50\%$ off coupon and I would be just as well off as I was without the coupon and with my initial $200$. Put differently, my “real income” is $58.58$ higher when I get the coupon because that is how much you could take from me once I get the coupon without changing my well-being. The compensated budget (which keeps utility constant) is therefore $141.42$.

Combining Graphs 7.10a and 7.10b into a single graph, we then get Graph 7.10c showing bundles $A$, $B$, and $C$ with the values we have calculated for each of these bundles. The substitution effect is the movement from $A$ to $B$, while the income effect, reflecting the change in my behavior that is solely due to the fact that I am $58.58$ “richer” when I receive the coupon, is the movement from $B$ to $C$.

Just as was true for substitution effects we identified in the Cayman Islands example, the size of the substitution effect here once again arises from the degree of substitutability of the goods as captured by the shape of indifference curves and the form of the utility function. Similarly, the size of the income effect depends on the underlying nature of tastes and the degree to which pants and shirts represent normal or inferior goods.

Suppose, for instance, that my tastes could be represented by the quasilinear utility function

\[ u(x_1, x_2) = 6x_1^{0.5} + x_2. \] (7.25)

Setting up the maximization problem analogous to (7.21) gives

\[ \max_{x_1, x_2} 6x_1^{0.5} + x_2 \text{ subject to } p_1x_1 + 10x_2 = 200, \] (7.26)

which you can verify solves to

\[ x_1 = \frac{900}{p_1^2} \text{ and } x_2 = \frac{20p_1 - 90}{p_1}. \] (7.27)

Thus, when the price of pants is 20, we get an optimal bundle $(2.25, 15.5)$, and when the price falls to 10 due to the coupon, we get an optimal bundle $(9, 11)$. Total utility without the coupon is found by plugging $x_1 = 2.25$ and $x_2 = 15.5$ into equation (7.25), which gives utility equal to 24.5. This then permits us to find the substitution effect by solving the constrained minimization problem

\[ \min_{x_1, x_2} E = 10x_1 + 10x_2 \text{ subject to } 6x_1^{0.5} + x_2 = 24.5, \] (7.28)

which gives $x_1 = 9$ and $x_2 = 6.5$. Thus (ignoring the fact that it is difficult to consume fractions of pants) the substitution effect changes my consumption of pants from my original 2.25 to 9, and the income effect causes no additional change in my consumption for pants. This lack of an income effect of course arises because tastes that are quasilinear in a particular good (in this case, pants) do not exhibit income effects for that good; such goods are borderline normal/inferior goods.10

10A small caveat to this is that such tastes do exhibit income effects in the quasilinear good when there are corner solutions. This is explored in more detail in end-of-chapter exercise 7.5.
CONCLUSION

We have begun in this chapter to discuss the important concepts of income and substitution effects in the context of consumer goods markets. In our mathematical section, we furthermore began to calculate income and substitution effects for some very specific examples in order to illustrate how the graphs of Section 7A related to the mathematical ideas we have dealt with thus far. A more general theory of consumer behavior will emerge from the building blocks of the optimization model we have laid, but we will not have completed the building of this theory until Chapter 10. Before doing so, we will now first translate the concepts of income and substitution effects in consumer goods markets to similar ideas that emerge in labor and capital markets (Chapter 8). We will then illustrate in Chapters 9 and 10 how our notions of demand and consumer surplus relate directly to income and substitution effects as introduced here.

There is no particular reason why it should be fully apparent to you at this point why these concepts are important. The importance will become clearer as we apply them in exercises and as we turn to some real-world issues later on. We did, however, raise one example in the introduction, and we can now make a bit more sense of it. We imagined a policy in which the government would reduce consumption of gasoline by taxing it heavily, only to turn around and distribute the revenues from the tax in the form of rebate checks. For many, including some very smart columnists and politicians, such a combination of a gasoline tax and rebate makes no sense; on average, they argue, consumers would receive back as much as they paid in gasoline taxes, and as a result, they would not change their behavior.

Now that we have isolated income and substitution effects, however, we can see why economists think such a tax/rebate program will indeed curb gasoline consumption: The tax raises the price of gasoline and thus gives rise to income and substitution effects that (assuming gasoline is a normal good) both result in less consumption of gasoline. The rebate, on the other hand, does not change prices back; it simply causes incomes to rise above where they would otherwise have been after the tax. Thus, the rebate only causes an income effect in the opposite direction. The negative income effect from the increase in the price should be roughly offset by the positive income effect from the tax rebate, which leaves us with a substitution effect that unambiguously implies a decrease in gasoline consumption.

END-OF-CHAPTER EXERCISES

7.1 Here, we consider some logical relationships between preferences and types of goods.

A. Suppose you consider all the goods that you might potentially want to consume.
   a. Is it possible for all these goods to be luxury goods at every consumption bundle? Is it possible for all of them to be necessities?
   b. Is it possible for all goods to be inferior goods at every consumption bundle? Is it possible for all of them to be normal goods?
   c. True or False: When tastes are homothetic, all goods are normal goods.
   d. True or False: When tastes are homothetic, some goods could be luxuries while others could be necessities.
   e. True or False: When tastes are quasi-linear, one of the goods is a necessity.

Exercise 7B.6

Using the previous calculations, plot graphs similar to Graph 7.10 illustrating income and substitution effects when my tastes can be represented by the utility function $u(x_1, x_2) = 6x_1^{0.5} + x_2$.

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11This argument was in fact advanced by opponents of such a policy advocated by the Carter administration in the late 1970s, a proposal that won only 35 votes (out of 435) in the U.S. House of Representatives. It is not the only argument against such policies. For instance, some have argued that a gasoline tax would be too narrow, and that the goals of such a tax would be better advanced by a broad-based carbon tax on all carbon-emitting activity.

*conceptually challenging
**computationally challenging
†solutions in Study Guide
f. True or False: In a two-good model, if the two goods are perfect complements, they must both be normal goods.

g. True or False: In a three-good model, if two of the goods are perfect complements, they must both be normal goods.

B. In each of the following cases, suppose that a person whose tastes can be characterized by the given utility function has income 1 and faces prices that are all equal to 1. Illustrate mathematically how his or her consumption of each good changes with income, and use your answer to determine whether the goods are normal or inferior, luxuries or necessities.

a. \( u(x_1, x_2) = x_1 x_2 \)

b. \( u(x_1, x_2) = x_1 + \ln x_2 \)

c. \( u(x_1, x_2) = \ln x_1 + \ln x_2 \)

d. \( u(x_1, x_2, x_3) = 2 \ln x_1 + \ln x_2 + 4 \ln x_3 \)

e. True or False: In a three-good model, if two of the goods are perfect complements, they must both be normal goods.

7.2 Suppose you have an income of $24 and the only two goods you consume are apples \((x_1)\) and peaches \((x_2)\). The price of apples is $4 and the price of peaches is $3.

A. Suppose that your optimal consumption is 4 peaches and 3 apples.

a. Illustrate this in a graph using indifference curves and budget lines.

b. Now suppose that the price of apples falls to $2 and I take enough money away from you to make you as happy as you were originally. Will you buy more or fewer peaches?

c. In reality, I do not actually take income away from you as described in (b), but your income stays at $24 after the price of apples falls. I observe that, after the price of apples fell, you did not change your consumption of peaches. Can you conclude whether peaches are an inferior or normal good for you?

B. Suppose that your tastes can be characterized by the function \( u(x_1, x_2) = x_1^{a} x_2^{(1-a)} \).

a. What value must \( a \) take in order for you to choose 3 apples and 4 peaches at the original prices?

b. What bundle would you consume under the scenario described in A(b)?

c. How much income can I take away from you and still keep you as happy as you were before the price change?

d. What will you actually consume after the price increase?

7.3 Consider once again my tastes for Coke and Pepsi and my tastes for right and left shoes (as described in end-of-chapter exercise 6.2).

A. On two separate graphs—one with Coke and Pepsi on the axes, the other with right shoes and left shoes—replicate your answers to end-of-chapter exercise 6.2A(a) and (b). Label the original optimal bundles \( A \) and the new optimal bundles \( C \).

a. In your Coke/Pepsi graph, decompose the change in consumer behavior into income and substitution effects by drawing the compensated budget and indicating the optimal bundle \( B \) on that budget.

b. Repeat (a) for your right shoes/left shoes graph.

B. Now consider the following utility functions: \( u(x_1, x_2) = \min\{x_1, x_2\} \) and \( u(x_1, x_2) = x_1 + x_2 \). Which of these could plausibly represent my tastes for Coke and Pepsi, and which could represent my tastes for right and left shoes?

a. Use the appropriate function to assign utility levels to bundles \( A \), \( B \), and \( C \) in your graph from 7.3A(a).

b. Repeat this for bundles \( A \), \( B \), and \( C \) for your graph in 7.3A(b).

7.4 Return to the case of our beer and pizza consumption from end-of-chapter exercise 6.3.

A. Again, suppose you consume only beer and pizza (sold at prices \( p_1 \) and \( p_2 \) respectively) with an exogenously set income \( I \). Assume again some initial optimal (interior) bundle \( A \).
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a. In 6.3A(b), can you tell whether beer is normal or inferior? What about pizza?
b. When the price of beer goes up, I notice that you consume less beer. Can you tell whether beer is a normal or an inferior good?
c. When the price of beer goes down, I notice you buy less pizza. Can you tell whether pizza is a normal good?
d. When the price of pizza goes down, I notice you buy more beer. Is beer an inferior good for you? Is pizza?
e. Which of your conclusions in part (d) would change if you knew pizza and beer are very substitutable?

B. Suppose, as you did in end-of-chapter exercise 6.3B, that your tastes over beer \((x_1)\) and pizza \((x_2)\) can be summarize by the utility function \(u(x_1, x_2) = x_1^a x_2^b\). If you have not already done so, calculate the optimal quantity of beer and pizza consumption as a function of \(p_1, p_2,\) and \(I\).

a. Illustrate the optimal bundle \(A\) when \(p_1 = 2, p_2 = 10\) and weekly income \(I = 180\). What numerical label does this utility function assign to the indifference curve that contains bundle \(A\)?
b. Using your answer, show that both beer and pizza are normal goods when your tastes can be summarized by this utility function.
c. Suppose the price of beer goes up to $4. Illustrate your new optimal bundle and label it \(C\).
d. How much beer and pizza would you buy if you had received just enough of a raise to keep you just as happy after the increase in the price of beer as you were before (at your original income of $180)? Illustrate this as bundle \(B\).
e. How large was your salary increase in (d)?
f. Now suppose the price of pizza \((p_2)\) falls to $5 (and suppose the price of beer and your income are $2 and $180 as they were originally at bundle \(A\)). Illustrate your original budget, your new budget, the original optimum \(A\), and the new optimum \(C\) in a graph.
g. Calculate the income effect and the substitution effect for both pizza and beer consumption from this change in the price of pizza. Illustrate this in your graph.
h. True or False: Since income and substitution effects point in opposite directions for beer, beer must be an inferior good.

7.5† Return to the analysis of my undying love for my wife expressed through weekly purchases of roses (as introduced in end-of-chapter exercise 6.4).

A. Recall that initially roses cost $5 each and, with an income of $125 per week, I bought 25 roses each week. Then, when my income increased to $500 per week, I continued to buy 25 roses per week (at the same price).

a. From what you observed thus far, are roses a normal or an inferior good for me? Are they a luxury or a necessity?
b. On a graph with weekly roses consumption on the horizontal and “other goods” on the vertical, illustrate my budget constraint when my weekly income is $125. Then illustrate the change in the budget constraint when income remains $125 per week and the price of roses falls to $2.50. Suppose that my optimal consumption of roses after this price change rises to 50 roses per week and illustrate this as bundle \(C\).
c. Illustrate the compensated budget line and use it to illustrate the income and substitution effects.
d. Now consider the case where my income is $500 and, when the price changes from $5 to $2.50, I end up consuming 100 roses per week (rather than 25). Assuming quasilinearity in roses, illustrate income and substitution effects.
e. True or False: Price changes of goods that are quasilinear give rise to no income effects for the quasilinear good unless corner solutions are involved.

B. Suppose again, as in 6.4B, that my tastes for roses \((x_1)\) and other goods \((x_2)\) can be represented by the utility function \(u(x_1, x_2) = \beta x_1^a + x_2\).

a. If you have not already done so, assume that \(p_2\) is by definition equal to 1, let \(a = 0.5\) and \(\beta = 50\), and calculate my optimal consumption of roses and other goods as a function of \(p_1\) and \(I\).
b. The original scenario you graphed in 7.5A(b) contains corner solutions when my income is $125 and the price is initially $5 and then $2.50. Does your previous answer allow for this?
c. Verify that the scenario in your answer to 7.5A(d) is also consistent with tastes described by this utility function; i.e., verify that \( A, B, \) and \( C \) are as you described in your answer.

### 7.6 Everyday Application: Housing Price Fluctuations: Part 2

Suppose, as in end-of-chapter exercise 6.9, you have $400,000 to spend on “square feet of housing” and “all other goods.” Assume the same is true for me.

**A.** Suppose again that you initially face a $100 per square foot price for housing, and you choose to buy a 2,000-square-foot house.

a. Illustrate this on a graph with square footage of housing on the horizontal axis and other consumption on the vertical. Then suppose, as you did in exercise 6.9, that the price of housing falls to $50 per square foot after you bought your 2,000-square-foot house. Label the square footage of the house you would switch to \( h_B \).

b. Is \( h_B \) smaller or larger than 2,000 square feet? Does your answer depend on whether housing is normal, regular inferior, or Giffen?

c. Now suppose that the price of housing had fallen to $50 per square foot before you bought your initial 2,000-square-foot house. Denote the size of house you would have bought \( h_C \) and illustrate it in your graph.

d. Is \( h_C \) larger than \( h_B \)? Is it larger than 2,000 square feet? Does your answer depend on whether housing is normal, regular inferior, or Giffen good?

e. Now consider me. I did not buy a house until the price of housing was $50 per square foot, at which time I bought a 4,000-square-foot house. Then the price of housing rises to $100 per square foot. Would I sell my house and buy a new one? If so, is the new house size \( h_B' \) larger or smaller than 4,000 square feet? Does your answer depend on whether housing is normal, regular inferior, or Giffen for me?

f. Am I better or worse off?

g. Suppose I had not purchased at the low price but rather purchased a house of size \( h_C' \) after the price had risen to $100 per square foot. Is \( h_C' \) larger or smaller than \( h_B' \)? Is it larger or smaller than 4,000 square feet? Does your answer depend on whether housing is normal, regular inferior, or Giffen for me?

**B.** Suppose both you and I have tastes that can be represented by the utility function \( u(x_1, x_2) = x_1^{0.5} x_2^{0.5} \), where \( x_1 \) is square feet of housing and \( x_2 \) is “dollars of other goods.”

a. Calculate the optimal level of housing consumption \( x_1 \) as a function of per square foot housing prices \( p_1 \) and income \( I \).

b. Verify that your initial choice of a 2,000-square-foot house and my initial choice of a 4,000-square-foot house was optimal under the circumstances we faced (assuming we both started with $400,000).

c. Calculate the values of \( h_B \) and \( h_C \) as they are described in A(a) and (c).

d. Calculate \( h_B' \) and \( h_C' \) as they are described in A(e) and (g).

e. Verify your answer to A(f).

### 7.7 Everyday Application: Turkey and Thanksgiving

Every Thanksgiving, my wife and I debate about how we should prepare the turkey we will serve (and will then have left over). On the one hand, my wife likes preparing turkeys the conventional way: roasted in the oven where it has to cook at 350 degrees for 4 hours or so. I, on the other hand, like to fry turkeys in a big pot of peanut oil heated over a powerful flame outdoors. The two methods have different costs and benefits. The conventional way of cooking turkeys has very little set-up cost (since the oven is already there and just has to be turned on) but a relatively large time cost from then on. (It takes hours to cook.) The frying method, on the other hand, takes some set-up (dragging out the turkey fryer, pouring gallons of peanut oil, etc., and then later the cleanup associated with it), but turkeys cook predictably quickly in just 3.5 minutes per pound.

**A.** As a household, we seem to be indifferent between doing it one way or another; sometimes we use the oven, sometimes we use the fryer. But we have noticed that we cook much more turkey, several turkeys, as a matter of fact, when we use the fryer than when we use the oven.
a. Construct a graph with “pounds of cooked turkeys” on the horizontal and “other consumption” on the vertical. (“Other consumption” here is not denominated in dollars as it normally is but rather in some consumption index that takes into account the time it takes to engage in such consumption.) Think of the set-up cost for frying turkeys and the waiting cost for cooking them as the main costs that are relevant. Can you illustrate our family’s choice of whether to fry or roast turkeys at Thanksgiving as a choice between two “budget lines”?

b. Can you explain the fact that we seem to eat more turkey around Thanksgiving whenever we pull out the turkey fryer as opposed to roasting the turkey in the oven?

c. We have some friends who also struggle each Thanksgiving with the decision of whether to fry or roast, and they, too, seem to be indifferent between the two options. But we have noticed that they only cook a little more turkey when they fry than when they roast. What is different about them?

B.** Suppose that, if we did not cook turkeys, we could consume 100 units of “other consumption,” but the time it takes to cook turkeys takes away from that consumption. Setting up the turkey fryer costs \( c \) units of consumption and waiting 3.5 minutes (which is how long it takes to cook 1 pound of turkey) costs 1 unit of consumption. Roasting a turkey involves no set-up cost, but it takes 5 times as long to cook per pound. Suppose that tastes can be characterized by the CES utility function

\[
U(x_1, x_2) = (0.5x_1^p + 0.5x_2^p)^{-1/p}
\]

where \( x_1 \) is pounds of turkey and \( x_2 \) is “other consumption.”

a. What are the two budget constraints I am facing?

b. Can you calculate how much turkey someone with these tastes will roast (as a function of \( p \)?) How much will the same person fry? (Hint: Rather than solving this using the Lagrange Method, use the fact that you know the MRS is equal to the slope of the budget line and recall from Chapter 5 that, for a CES utility function of this kind, \( MRS = -(x_2/x_1)^{1-1/p} \).)

c. Suppose my family has tastes with \( p = 0 \) and my friend’s with \( p = 1 \). If each of us individually roasts turkeys this Thanksgiving, how much will we each roast?

d. How much utility will each of us get (as measured by the relevant utility function) (Hint: In the case where \( p = 0 \), the exponent \( 1/p \) is undefined. Use the fact that you know that when \( p = 0 \) the CES utility function is Cobb–Douglas.)

e. Which family is happier?

f. If we are really indifferent between roasting and frying, what must \( c \) be for my family? What must it be for my friend’s family? (Hint: Rather than setting up the usual minimization problem, use your answer to (b) to determine \( c \) by setting utility equal to what it was for roasting.)

g. Given your answers so far, how much would we each have fried had we chosen to fry instead of roast (and we were truly indifferent between the two because of the different values of \( c \) we face)?

h. Compare the size of the substitution effect you have calculated for my family and that you calculated for my friend’s family and illustrate your answer in a graph with pounds of turkey on the horizontal and other consumption on the vertical. Relate the difference in the size of the substitution effect to the elasticity of substitution.

7.8* Business Application: Sam’s Club and the Marginal Consumer: Superstores like Costco and Sam’s Club serve as wholesalers to businesses but also target consumers who are willing to pay a fixed fee in order to get access to the lower wholesale prices offered in these stores. For purposes of this exercise, suppose that you can denote goods sold at superstores as \( x_1 \) and “dollars of other consumption” as \( x_2 \).

A. Suppose all consumers have the same homothetic tastes over \( x_1 \) and \( x_2 \), but they differ in their income. Every consumer is offered the same option of either shopping at stores with somewhat higher prices for \( x_1 \) or paying the fixed fee \( c \) to shop at a superstore at somewhat lower prices for \( x_1 \).

a. On a graph with \( x_1 \) on the horizontal axis and \( x_2 \) on the vertical, illustrate the regular budget (without a superstore membership) and the superstore budget for a consumer whose income is such that these two budgets cross on the 45-degree line. Indicate on your graph a vertical distance that is equal to the superstore membership fee \( c \).

b. Now consider a consumer with twice that much income. Where will this consumer’s two budgets intersect relative to the 45-degree line?

c. Suppose consumer 1 (from part (a)) is just indifferent between buying and not buying the superstore membership. How will her behavior differ depending on whether or not she buys the membership?
d. If consumer 1 was indifferent between buying and not buying the superstore membership, can you tell whether consumer 2 (from part (b)) is also indifferent? (Hint: Given that tastes are homothetic and identical across consumers, what would have to be true about the intersection of the two budgets for the higher income consumer in order for the consumer also to be indifferent between them?)

e. True or False: Assuming consumers have the same homothetic tastes, there exists a “marginal” consumer with income \( I \) such that all consumers with income greater than \( I \) will buy the superstore membership and no consumer with income below \( I \) will buy that membership.

f. True or False: By raising \( c \) and/or \( p_1 \), the superstore will lose relatively lower income customers and keep high income customers.

g. Suppose you are a superstore manager and you think your store is overcrowded. You’d like to reduce the number of customers while at the same time increasing the amount each customer purchases. How would you do this?

B. Suppose you manage a superstore and you are currently charging an annual membership fee of $50. Since \( x_2 \) is denominated in dollar units, \( p_2 = 1 \). Suppose that \( p_1 = 1 \) for those shopping outside the superstore, but your store sells \( x_1 \) at 0.95. Your statisticians have estimated that your consumers have tastes that can be summarized by the utility function \( u(x_1, x_2) = x_1^{0.15} x_2^{0.85} \).

a. What is the annual discretionary income (that could be allocated to purchasing \( x_1 \) and \( x_2 \)) of your “marginal” consumer?

b. Can you show that consumers with more income than the marginal consumer will definitely purchase the membership while consumers with less income will not? (Hint: Calculate the income of the marginal consumer as a function of \( c \) and show what happens to income that makes a consumer marginal as \( c \) changes.)

c. If the membership fee is increased from $50 to $100, how much could the superstore lower \( p_1 \) without increasing membership beyond what it was when the fee was $50 and \( p_1 \) was 0.95?

79* Business Application: Are Gucci Products Giffen Goods? We defined a Giffen good as a good that consumers (with exogenous incomes) buy more of when the price increases. When students first hear about such goods, they often think of luxury goods such as expensive Gucci purses and accessories. If the marketing departments for firms like Gucci are very successful, they may find a way of associating price with “prestige” in the minds of consumers, and this may allow them to raise the price and sell more products. But would that make Gucci products Giffen goods? The answer, as you will see in this exercise, is no.

A. Suppose we model a consumer who cares about the “practical value and style of Gucci products,” dollars of other consumption, and the “prestige value” of being seen with Gucci products. Denote these as \( x_1, x_2, \) and \( x_3 \) respectively.

a. The consumer only has to buy \( x_1 \) and \( x_2 \)—the prestige value \( x_1 \) comes with the Gucci products. Let \( p_1 \) denote the price of Gucci products and \( p_2 = 1 \) be the price of dollars of other consumption. Illustrate the consumer’s budget constraint (assuming an exogenous income \( I \)).

b. The prestige value of Gucci purchases, \( x_1 \), is something an individual consumer has no control over. If \( x_3 \) is fixed at a particular level \( x_3^* \), the consumer therefore operates on a two-dimensional slice of her three-dimensional indifference map over \( x_1, x_2, \) and \( x_3 \). Draw such a slice for the indifference curve that contains the consumer’s optimal bundle \( A \) on the budget from part (a).

c. Now suppose that Gucci manages to raise the prestige value of its products and thus \( x_3 \) that comes with the purchase of Gucci products. For now, suppose they do this without changing \( p_1 \). This implies you will shift to a different two-dimensional slice of your three-dimensional indifference map. Illustrate the new two-dimensional indifference curve that contains \( A \). Is the new MRS at \( A \) greater or smaller in absolute value than it was before?

d. Would the consumer consume more or fewer Gucci products after the increase in prestige value?

e. Now suppose that Gucci manages to convince consumers that Gucci products become more desirable the more expensive they are. Put differently, the prestige value \( x_3 \) is linked to \( p_1 \), the price of the Gucci products. On a new graph, illustrate the change in the consumer’s budget as a result of an increase in \( p_1 \).
f. Suppose that our consumer increases her purchases of Gucci products as a result of the increase in the price \( p_1 \). Illustrate two indifference curves: one that gives rise to the original optimum \( A \) and another that gives rise to the new optimum \( C \). Can these indifference curves cross?

g. Explain why, even though the behavior is consistent with what we would expect if Gucci products were a Giffen good, Gucci products are not a Giffen good in this case.

h. In a footnote in the chapter, we defined the following: A good is a Veblen good if preferences for the good change as price increases, with this change in preferences possibly leading to an increase in consumption as price increases. Are Gucci products a Veblen good in this exercise?

B. Consider the same definition of \( x_1, x_2, \) and \( x_3 \) as in part A. Suppose that the tastes for our consumer can be captured by the utility function \( u(x_1, x_2, x_3) = \alpha x_3^3 \ln x_1 + x_2 \).

a. Set up the consumer’s utility maximization problem, keeping in mind that \( x_3 \) is not a choice variable.

b. Solve for the optimal consumption of \( x_1 \) (which will be a function of the prestige value \( x_3 \)).

c. Is \( x_1 \) normal or inferior? Is it Giffen?

d. Now suppose that prestige value is a function of \( p_1 \). In particular, suppose that \( x_3 = p_1 \). Substitute this into your solution for \( x_1 \). Will consumption increase or decrease as \( p_1 \) increases?

e. How would you explain that \( x_1 \) is not a Giffen good despite the fact that its consumption increases as \( p_1 \) goes up?

7.10 Policy Application: Tax Deductibility and Tax Credits: In end-of-chapter exercise 2.17, you were asked to think about the impact of tax deductibility on a household’s budget constraint.

A. Suppose we begin in a system in which mortgage interest is not deductible and then tax deductibility of mortgage interest is introduced.

a. Using a graph (as you did in exercise 2.17) with “square feet of housing” on the horizontal axis and “dollars of other consumption” on the vertical, illustrate the direction of the substitution effect.

b. What kind of good would housing have to be in order for the household to consume less housing as a result of the introduction of the tax deductibility program?

c. On a graph that contains both the before and after deductibility budget constraints, how would you illustrate the amount of subsidy the government provides to this household?

d. Suppose the government provided the same amount of money to this household but did so instead by simply giving it to the household as cash back on its taxes (without linking it to housing consumption). Will the household buy more or less housing?

e. Will the household be better or worse off?

f. Do your answers to (d) and (e) depend on whether housing is normal, regular inferior, or Giffen?

g. Under tax deductibility, will the household spend more on other consumption before or after tax deductibility is introduced? Discuss your answer in terms of income and substitution effects and assume that “other goods” is a normal good.

h. If you observed that a household consumes more in “other goods” after the introduction of tax deductibility, could that household’s tastes be quasilinear in housing? Could they be homothetic?

B. **Households typically spend about a quarter of their after-tax income \( I \) on housing. Let \( x_1 \) denote square feet of housing and let \( x_2 \) denote other consumption.

a. If we represent a household’s tastes with the Cobb–Douglas function \( u(x_1, x_2) = x_1^\alpha x_2^{1-\alpha} \), what should \( \alpha \) be?

b. Using your answer about the value of \( \alpha \), and letting the price per square foot of housing be denoted as \( p_1 \), derive the optimal level of housing consumption (in terms of \( I, \ p_1, \) and \( t \)) under a tax deductibility program that implicitly subsidizes a fraction \( t \) of a household’s housing purchase.

c. What happens to housing consumption and other good consumption under tax deductibility as a household’s tax bracket (i.e., their tax rate \( t \)) increases?
d. Determine the portion of changed housing consumption that is due to the income effect and the portion that is due to the substitution effect.

e. Calculate the amount of money the government is spending on subsidizing this household’s mortgage interest.

f. Now suppose that, instead of a deductibility program, the government simply gives the amount you calculated in (e) to the household as cash. Calculate the amount of housing now consumed and compare it with your answer under tax deductibility.

7.11 Policy Application: Substitution Effects and Social Security Cost of Living Adjustments: In end-of-chapter exercise 6.16, you investigated the government’s practice for adjusting Social Security income for seniors by ensuring that the average senior can always afford to buy some average bundle of goods that remains fixed. To simplify the analysis, let us again assume that the average senior consumes only two different goods.

A. Suppose that last year our average senior optimized at the average bundle \( A \) identified by the government, and begin by assuming that we denote the units of \( x_1 \) and \( x_2 \) such that last year \( p_1 = p_2 = 1 \).

a. Suppose that \( p_1 \) increases. On a graph with \( x_1 \) on the horizontal and \( x_2 \) on the vertical axis, illustrate the compensated budget and the bundle \( B \) that, given your senior’s tastes, would keep the senior just as well off at the new price.

b. In your graph, compare the level of income the senior requires to get to bundle \( B \) with the income required to get him back to bundle \( A \).

c. What determines the size of the difference in the income necessary to keep the senior just as well off when the price of good 1 increases as opposed to the income necessary for the senior still to be able to afford bundle \( A \)?

d. Under what condition will the two forms of compensation be identical?

e. You should recognize the move from \( A \) to \( B \) as a pure substitution effect as we have defined it in this chapter. Often this substitution effect is referred to as the **Hicksian substitution effect**, defined as the change in behavior when opportunity costs change but the consumer receives sufficient compensation to remain just as happy. Let \( B' \) be the consumption bundle the average senior would choose when compensated so as to be able to afford the original bundle \( A \). The movement from \( A \) to \( B' \) is often called the **Slutsky substitution effect**, defined as the change in behavior when opportunity costs change but the consumer receives sufficient compensation to be able to afford to stay at the original consumption bundle. True or False: The government could save money by using Hicksian rather than Slutsky substitution principles to determine appropriate cost of living adjustments for Social Security recipients.

f. True or False: Hicksian and Slutsky compensation get closer to one another the smaller the price changes.

B. Now suppose that the tastes of the average senior can be captured by the Cobb–Douglas utility function \( u(x_1, x_2) = x_1 x_2 \), where \( x_2 \) is a composite good (with price by definition equal to \( p_2 = 1 \)). Suppose the average senior currently receives Social Security income \( I \) (and no other income) and with it purchases bundle \( (x_1^A, x_2^A) \).

a. Determine \( (x_1^I, x_2^I) \) in terms of \( I \) and \( p_1 \).

b. Suppose that \( p_1 \) is currently \$1 and \( I \) is currently \$2,000. Then \( p_1 \) increases to \$2. How much will the government increase the Social Security check given how it is actually calculating cost of living adjustments? How will this change the senior’s behavior?

c. How much would the government increase the Social Security check if it used Hicksian rather than Slutsky compensation? How would the senior’s behavior change?

d.* Can you demonstrate mathematically that Hicksian and Slutsky compensation converge to one another as the price change gets small and diverge from each other as the price change gets large?

e. We know that Cobb–Douglas utility functions are part of the CES family of utility functions, with the elasticity of substitution equal to 1. Without doing any math, can you estimate the range of how much Slutsky compensation can exceed Hicksian compensation with tastes that lie within the CES family? (Hint: Consider the extreme cases of elasticities of substitution.)
7.12 Policy Application: Fuel Efficiency, Gasoline Consumption, and Gas Prices: Policy makers frequently search for ways to reduce consumption of gasoline. One straightforward option is to tax gasoline, thereby encouraging consumers to drive less and switch to more fuel-efficient cars.

A.* Suppose that you have tastes for driving and for other consumption, and assume throughout that your tastes are homothetic.

a. On a graph with monthly miles driven on the horizontal and “monthly other consumption” on the vertical axis, illustrate two budget lines: one in which you own a gas-guzzling car, which has a low monthly payment (that has to be made regardless of how much the car is driven) but high gasoline use per mile; the other in which you own a fuel-efficient car, which has a high monthly payment that has to be made regardless of how much the car is driven but uses less gasoline per mile. Draw this in such a way that it is possible for you to be indifferent between owning the gas-guzzling and the fuel-efficient car.

b. Suppose you are indeed indifferent. With which car will you drive more?

c. Can you tell with which car you will use more gasoline? What does your answer depend on?

d. Now suppose that the government imposes a tax on gasoline, and this doubles the opportunity cost of driving both types of cars. If you were indifferent before the tax was imposed, can you now say whether you will definitively buy one car or the other (assuming you waited to buy a car until after the tax is imposed)? What does your answer depend on? (Hint: It may be helpful to consider the extreme cases of perfect substitutes and perfect complements before deriving your general conclusion to this question.)

e. The empirical evidence suggests that consumers shift toward more fuel-efficient cars when the price of gasoline increases. True or False: This would tend to suggest that driving and other good consumption are relatively complementary.

f. Suppose an increase in gasoline taxes raises the opportunity cost of driving a mile with a fuel-efficient car to the opportunity cost of driving a gas guzzler before the tax increase. Will someone who was previously indifferent between a fuel-efficient and a gas-guzzling car now drive more or less in a fuel-efficient car than he did in a gas guzzler prior to the tax increase? (Continue with the assumption that tastes are homothetic.)

B. Suppose your tastes were captured by the utility function \( u(x_1, x_2) = 0.5x_1^{0.5} - 0.5x_2^{0.5} \), where \( x_1 \) stands for miles driven and \( x_2 \) stands for other consumption. Suppose you have $600 per month of discretionary income to devote to your transportation and other consumption needs and that the monthly payment on a gas guzzler is $200. Furthermore, suppose the initial price of gasoline is $0.10 per mile in the fuel-efficient car and $0.20 per mile in the gas guzzler.

a. Calculate the number of monthly miles driven if you own a gas guzzler.

b. Suppose you are indifferent between the gas guzzler and the fuel-efficient car. How much must the monthly payment for the fuel-efficient car be?

c. Now suppose that the government imposes a tax on gasoline that doubles the price per mile driven of each of the two cars. Calculate the optimal consumption bundle under each of the new budget constraints.

d. Do you now switch to the fuel-efficient car?

e. Consider the utility function you have worked with so far as a special case of the CES family \( u(x_1, x_2) = (0.5x_1^p + 0.5x_2^p)^{-1/p} \). Given what you concluded in A(d) of this question, how would your answer to B(d) change as \( p \) changes?

7.13 Policy Application: Public Housing and Housing Subsidies: In exercise 2.14, you considered two different public housing programs in parts A(a) and (b), one where a family is simply offered a particular apartment for a below-market rent and another where the government provides a housing price subsidy that the family can use anywhere in the private rental market.

A. Suppose we consider a family that earns $1,500 per month and either pays $0.50 per square foot in monthly rent for an apartment in the private market or accepts a 1,500-square-foot government public housing unit at the government’s price of $500 per month.

a. On a graph with square feet of housing and “dollars of other consumption,” illustrate two cases where the family accepts the public housing unit, one where this leads them to consume less housing than they otherwise would and another where it leads them to consume more housing than they otherwise would.
b. If we use the members of the household’s own judgment about the household’s well-being, is it always the case that the option of public housing makes the participating households better off?

c. If the policy goal behind public housing is to increase the housing consumption of the poor, is it more or less likely to succeed the less substitutable housing and other goods are?

d. What is the government’s opportunity cost of owning a public housing unit of 1,500 square feet? How much does it therefore cost the government to provide the public housing unit to this family?

e. Now consider instead a housing price subsidy under which the government tells qualified families that it will pay some fraction of their rental bills in the private housing market. If this rental subsidy is set so as to make the household just as well off as it was under public housing, will it lead to more or less consumption of housing than if the household chooses public housing?

f. Will giving such a rental subsidy cost more or less than providing the public housing unit? What does your answer depend on?

g. Suppose instead that the government simply gave cash to the household. If it gave sufficient cash to make the household as well off as it is under the public housing program, would it cost the government more or less than $250? Can you tell whether under such a subsidy the household consumes more or less housing than under public housing?

B. Suppose that household tastes over square feet of housing \((x_1)\) and dollars of other consumption \((x_2)\) can be represented by 
\[
 u(x_1, x_2) = \alpha \ln x_1 + (1 - \alpha) \ln x_2.
\]

a. Suppose that empirical studies show that we spend about a quarter of our income on housing. What does that imply about \(\alpha\)?

b. Consider a family with income of $1,500 per month facing a per square foot price of 
\(p_1 = 0.50\). For what value of \(\alpha\) would the family not change its housing consumption when offered the 1,500-square-foot public housing apartment for $500?

c. Suppose that this family has \(\alpha\) as derived in B(a). How much of a rental price subsidy would the government have to give to this family in order to make it as well off as the family is with the public housing unit?

d. How much housing will the family rent under this subsidy? How much will it cost the government to provide this subsidy?

e. Suppose the government instead gave the family cash (without changing the price of housing). How much cash would it have to give the family in order to make it as happy?

f. If you are a policy maker whose aim is to make this household happier at the least cost to the taxpayer, how would you rank the three policies? What if your goal was to increase the household’s housing consumption?