Chapter 4

Automobile Ownership

4-1 Automobile Ads
4-2 Automobile Transactions
4-3 Automobile Insurance
4-4 Probability: The Basis of Insurance
4-5 Linear Automobile Depreciation
4-6 Historical and Exponential Depreciation
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CHAPTER OVERVIEW
This chapter offers nine lessons pertaining to the automobile. Students explore formulas of varying degrees of mathematical sophistication as they work on pricing structures, insurance issues, automobile depreciation, and data that can assist them in making wise and safe driving decisions.

What do you think Marshall McLuhan meant by his quote?

The car has become an article of dress without which we feel uncertain, unclad, and incomplete.
—Marshall McLuhan, Canadian educator and philosopher

What do you think?

Answers might include that the car is much more than a means of transportation. It has become a mode of self-expression as well as a mode of transportation. People pride themselves in automobile ownership; many even see it as a status symbol.

The automobile is part of the American way of life. Many people commute to jobs that require them to own a car. Some students drive several miles to school. Stores and businesses often line the perimeter of a town that could be miles from residential neighborhoods. When there is no mass transit system readily available to you, an automobile can provide necessary transportation.

Owning an automobile is a tremendous responsibility. The costs of gas, repairs, and insurance are high. Driving an automobile can also be dangerous. As a driver, you have a responsibility to yourself, your passengers, pedestrians, cyclists, and other motorists. Also, before embarking upon that first automobile purchase, you need to be aware of the finances of operating a car. Being equipped with all this knowledge will make your years on the road safer, less expensive, and more enjoyable.
How much does it cost to fill a car's gas tank today? Did your parents ever tell you stories about gas prices when they were young? Gas prices increased as the result of an oil embargo, and gas was relatively hard to find for many weeks in 1973. Gas stations closed during the hours they had no gas to sell. There were long lines in the stations that remained open. Can you imagine people having to wait in gas lines in 1973, irate that gas was scarce and gas prices had risen dramatically to over 50 cents per gallon?

The table shows the average price per gallon of gasoline from 1950 until 2015. Gas prices vary from region to region. They even differ from gas station to gas station, depending on the services the station provides and the neighborhood in which it is. Therefore, use the table as a general guide to gas prices.

Imagine what it would cost to fill a tank in any of the years listed in the table. Imagine what new cars cost! The first Corvette, in 1953, had a base price of $3,498.00. There were only 300 of these cars manufactured. It cost about $5.00 to fill its 18-gallon gas tank! The 1953 Corvette buyer had an easy time picking a color. The car came in one color only—white.

<table>
<thead>
<tr>
<th>Year</th>
<th>Price per Gallon ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1950</td>
<td>0.27</td>
</tr>
<tr>
<td>1955</td>
<td>0.30</td>
</tr>
<tr>
<td>1960</td>
<td>0.31</td>
</tr>
<tr>
<td>1965</td>
<td>0.31</td>
</tr>
<tr>
<td>1970</td>
<td>0.35</td>
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<td>1975</td>
<td>0.53</td>
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<tr>
<td>1980</td>
<td>1.13</td>
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<tr>
<td>1985</td>
<td>1.19</td>
</tr>
<tr>
<td>1990</td>
<td>1.13</td>
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<tr>
<td>1995</td>
<td>1.14</td>
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<tr>
<td>2000</td>
<td>1.66</td>
</tr>
<tr>
<td>2005</td>
<td>2.33</td>
</tr>
<tr>
<td>2010</td>
<td>2.84</td>
</tr>
<tr>
<td>2015</td>
<td>2.52</td>
</tr>
</tbody>
</table>
Objectives

- Compute the cost of classified ads for used cars.
- Compute the cost of sales tax on automobiles.

Key Terms

<table>
<thead>
<tr>
<th>Sales tax</th>
<th>Domain</th>
<th>Piecewise function</th>
<th>Split function</th>
<th>Cusp</th>
</tr>
</thead>
</table>

Warm-Up

\[ A(x) = \frac{x^2}{4} \sqrt{3} \]

Evaluate the function for each value of \( x \).

a. \( x = 6 \)

b. \( x = \sqrt{3} \)

c. \( x = n + 1 \)

How Do Buyers and Sellers Advertise for Automobiles?

Most teenagers cannot wait to get their own set of “wheels.” New cars are expensive, so for most people, their first car is a used car. You can buy used cars from a dealer or by looking at the classified ads in the newspaper or on the Internet.

Automotive ads online and in print media use abbreviations to save space and lower the cost of the ad. Take a look at a classified ad section in a newspaper or online and see how many of the abbreviations you understand.

Words such as \textit{mint} and \textit{immaculate} are often used to describe cars in excellent condition. A car with many options is often listed as \textit{loaded}. The thousands of miles the car has been driven is abbreviated as K. An ad that says “34K” tells you that the car has been driven a total of 34,000 miles. All of the options have an abbreviation. In the table at the right, review some of these abbreviations used in classified ads for used cars.

The asking price is usually given in the advertisement. \textit{Negotiable} means that the seller is willing to bargain with you. \textit{Firm} means that the owner is unwilling to negotiate. \textit{Sacrifice} means that the seller needs to sell the car quickly and believes that the price is lower than the car’s worth.

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By knowing what these expressions mean, you will be able to skim the ads and focus on the ones that describe the used car that would be best for you.

Skills and Strategies

Here you learn some of the steps that may be involved when buying or selling a used car. You can contact your state's Department of Motor Vehicles to find specific information about cars in your state. In many states the buyer of a used car must pay sales tax on the car. The sales tax is a percentage of the sale price paid to the government for sales of products and services.

EXAMPLE 1

Kerry purchased a used car for $7,400 and had to pay 8½% sales tax. How much tax did she pay?

**SOLUTION** To find the sales tax, multiply the price of the item by the sales tax rate, expressed as a decimal. Recall that 8½% can be written as 8.5%, and when changed to a decimal is equivalent to 0.085.

\[
\text{Sales tax} = \text{Price of item} \times \text{Sales tax rate} \\
= 7,400 \times 0.085 = 629.00
\]

Kerry must pay $629.00 in sales tax. This money goes to the state when you register the car, not the seller of the car. Be sure you consider the sales tax expense on a car you are planning to purchase. It can be thousands of dollars on a new car.

The sales tax rate in Mary Ann's state is 4%. If she purchases a car for \( x \) dollars, express the total cost of the car with sales tax algebraically.

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EXAMPLE 2

The cost of an automobile ad is determined by its length. John plans to sell his car and places a five-line ad in an auto enthusiasts’ publication. The publication charges $31 for the first two lines and $6 per extra line to run the ad for 1 week. What will John's ad cost to run for 2 weeks?

**SOLUTION** Subtract to find the number of lines over two lines.

\[
5 - 2 = 3
\]

Multiply 3 by $6 to find the cost of the extra three lines.

\[
3(6) = 18
\]

Add to find the cost of running the ad for 1 week.

\[
31 + 18 = 49
\]

Multiply by 2 to get the cost for the 2-week ad.

\[
49(2) = 98
\]

The ad will cost John $98.

Ramon plans to sell his car and places an ad with \( x \) lines. The newspaper charges \( y \) dollars for the first \( g \) lines and \( p \) dollars per extra line to run the ad for a week. If \( x > g \), express the cost of running the ad for a week.
EXAMPLE 3

Students are building their first piecewise function in this example. Point out that they can use the word “if” to replace the word “when.” Example 1 gives them practice interpreting pricing schedules for classified ads. Once students understand these price schedules, they are introduced to a topic usually taught in precalculus: piecewise (split) functions. They need to understand the role of the domain in these problems.

CHECK YOUR UNDERSTANDING

Answer

\[ c(x) = \begin{cases} 
38 & \text{when } x \leq 4 \\
38 + 6.25(x - 4) & \text{when } x > 4 
\end{cases} \]

Remind them to use two let statements:
- Let \( x \) represent the number of lines.
- Let \( c(x) \) represent the cost of the ad.

Jason works for the Glen Oaks News and is writing a program to compute ad costs. He needs to enter an algebraic representation of the costs of an ad. His company charges $42.50 for up to five lines for an automotive ad. Each additional line costs $7. Express the cost of an ad with \( x \) lines as a function of \( x \) algebraically.

SOLUTION

The algebraic representation of the automotive ad cost function requires two rules. One rule is for ads with five or fewer lines and the other rule is for ads with more than five lines. Recall from Chapter 1 that the domain is the set of values that can be input into a function.

You can view these two conditions as two different domains. You will find the equation for the cost when \( x \leq 5 \), and then find the equation for the cost when \( x > 5 \). These are the two different domains.

Let \( c(x) \) represent the cost of the classified ad. In this situation, \( x \) must be an integer. If the ad has five or fewer lines, the cost is $42.50.

\[ c(x) = 42.50 \text{ when } x \leq 5 \]

If the ad has more than five lines, the cost is $42.50 plus the cost of the lines over five lines. Note that the domain is given by the inequality that follows when in the statement of the function. If \( x \) is the number of lines, then the number of lines over five can be expressed as \( x - 5 \). These extra lines cost $7 each.

\[ c(x) = 42.50 + 7(x - 5) \text{ when } x > 5 \]

These two equations can be written in mathematical shorthand using a piecewise function. Piecewise functions are sometimes called split functions. A piecewise function gives a set of rules for each domain of the function. Notice that \( c(x) \) is computed differently depending on the value of \( x \). Here \( c(x) \) is expressed as a piecewise function.

\[ c(x) = \begin{cases} 
42.50 & \text{when } x \leq 5 \\
42.50 + 7(x - 5) & \text{when } x > 5 
\end{cases} \]

The domain is defined by the inequalities that follow when in the above statement.

CHECK YOUR UNDERSTANDING

Answer

The Smithtown News charges $38 for a classified ad that is four or fewer lines long. Each line above four lines costs an additional $6.25. Express the cost of an ad as a piecewise function.
EXAMPLE 4

Roxanne set up the following piecewise function, which represents the cost of an automotive ad from her hometown newspaper.

\[ c(x) = \begin{cases} 
41.55 & \text{when } x \leq 6 \\
41.55 + 5.50(x - 6) & \text{when } x > 6 
\end{cases} \]

If \( x \) is the number of lines in the ad, use words to express the price \( c(x) \) of an automotive ad from this paper.

SOLUTION Look at the two domains. Look at the function rule in the first line. The inequality \( x \leq 6 \) tells you that the cost is $41.55 if the number of lines is less than or equal to six.

Next, look at the second line. The expression \( x - 6 \) gives the number of lines over six. That expression is multiplied by 5.50, so the cost of each extra line must be $5.50. The inequality \( x > 6 \) tells you that the cost is $41.55 for the first six lines, and $5.50 for each line over six lines.

The following piecewise function gives the price \( p(w) \) of a classified ad in a classic car magazine. If \( w \) is the number of lines in the ad, use words to express the price \( p(w) \) of a classified ad from this paper.

\[ p(w) = \begin{cases} 
60 & \text{when } w \leq 5 \\
60 + 8(w - 5) & \text{when } w > 5 
\end{cases} \]

EXAMPLE 5

Graph the piecewise function Roxanne created in Example 4.

SOLUTION Use your graphing calculator to display functions with more than one domain.

\[ c(x) = \begin{cases} 
42.50 & \text{when } x \leq 5 \\
42.50 + 7(x - 5) & \text{when } x > 5 
\end{cases} \]

Notice that the graph is composed of two straight lines that meet at the point (6, 41.55). The point where the two lines meet is called a cusp because it resembles the sharp cusp on a tooth.

Find the coordinates of the cusp of the graph of the following piecewise function.

\[ c(x) = \begin{cases} 
42.50 & \text{when } x \leq 5 \\
42.50 + 7(x - 5) & \text{when } x > 5 
\end{cases} \]
## Applications

In auto sales, appearance is everything, or almost everything. It is certainly the most important single factor in a consumer’s decision to buy this or that make.

—Harley Earl, designer/inventor of the Corvette

1. Interpret the quote in the context of what you learned about buying and selling cars in this lesson. See margin.

2. The *North Shore News* charges $19.50 for a two-line automotive ad. Each additional line costs $7. How much does a six-line ad cost? $47.50

3. The *Antique Auto News* charges $10 for a 10-word classified ad. Each additional word costs $0.40. For an extra $40, a seller can include a photo in the ad. How much would a 20-word ad with a photo cost? $54

4. A local newspaper charges $g$ dollars for a four-line classified ad. Each additional line costs $d$ dollars. Write an expression for the cost of a seven-line ad. $g + 3d$

5. The *Auto Times* charges $g$ dollars for a classified ad with $m$ or less lines. Each additional line is $d$ dollars. If $x > m$, express the cost of an $x$-line ad algebraically. $g + d(x - m)$

6. Samantha purchased a used car for $4,200. Her state charges 4% tax for the car, $47 for registration, $50 for a new title certificate, and $35 for a state safety and emissions inspection. How much does Samantha need to pay for these extra charges, not including the price of the car? $300

7. Ralph placed a classified ad to sell his used SUV for $18,500. After 2 weeks, he didn’t sell the SUV, and the newspaper suggested lowering the price 5%. What would the new price be if Ralph reduced it according to the suggestion? $17,575

8. The *Bayside Bugle* charges by the word to run automotive ads. The newspaper charges $18 for the first 20 words and $0.35 for each additional word. How much would a 27-word ad cost? $20.45

9. A local publication charges by the character for its classified ads. Letters, numbers, spaces, and punctuation each count as one character. They charge $46 for the first 200 characters and $0.15 for each additional character.
   a. If $x$ represents the number of characters in the ad, express the cost $c(x)$ of an ad as a piecewise function. See Additional Answers.
   b. Graph the function from part a. See Additional Answers.
   c. Find the coordinates of the cusp in the graph in part b. (200, 46)

10. The *Kings Park Register* gives senior citizens a 10% discount on automotive ads. Mr. Quadrino, a senior citizen, is selling his car and wants to take out a four-line ad. The paper charges $6.50 per line. What is the price of the ad for Mr. Quadrino? $23.40

11. The *Good Ole Times* magazine charges for ads by the “column inch.” A column inch is as wide as one column, and it is 1 inch high. The cost is $67 per column inch. How much would the magazine charge to print a 2½-inch ad? $167.50
12. Leslie placed this ad in *Collector Car Monthly*.

1957 Chevrolet Nomad station wagon. Tropical Turquoise, 6 cyl. auto, PS, PW, AM/FM, repainted, rebuilt transmission, restored two-tone interior. Mint! Moving, sacrifice, $52,900. 555-4231

a. If the publication charges $48 for the first three lines and $5 for each extra line, how much will this ad cost Leslie? **$53**
b. Ruth buys the car for 8% less than the advertised price. How much does she pay? **$48,688**
c. Ruth must pay her state 6% sales tax on the sale. How much must she pay in sales tax? **$2,920.08**

13. *Online Car Auctioneer* charges a commission for classified ads. If the car sells, the seller is charged 4% of the *advertised* price, not of the price for which the car actually sells. If Barbara advertises her Cadillac for $12,000 and sells it for $11,200, how much must she pay for the ad? **$480**

14. The cost of an ad in a local paper is given by the piecewise function.

\[ c(x) = \begin{cases} 
38 & \text{when } x \leq 4 \\
38 + 6.25(x - 4) & \text{when } x > 4 
\end{cases} \]

a. Find the cost of a three-line ad. **$38**
b. Find the difference in cost between a one-line ad and a four-line ad. **$0**
c. Find the cost of a seven-line ad. **$56.75**
d. Graph this function on your graphing calculator. See margin.
e. Find the coordinates of the cusp from the graph in part d. **(4, 38)**

15. Express the following classified ad rate as a piecewise function. Use a let statement to identify what \( x \) and \( y \) represent. See margin.

$29 for the first five lines and $6.75 for each additional line.

16. The piecewise function describes a newspaper’s classified ad rates.

\[ y = \begin{cases} 
21.50 & \text{when } x \leq 3 \\
21.50 + 5(x - 3) & \text{when } x > 3 
\end{cases} \]

a. If \( x \) represents the number of lines, and \( y \) represents the cost, translate the function into words. See margin.
b. If the function is graphed, what are the coordinates of the cusp? **(3, 21.50)**

17. A local *coupon mailer* charges $11 for each of the first three lines of an ad and $5 for each additional line.

a. What is the price of a two-line ad? **$22**
b. What is the price of a five-line ad? **$43**
c. If \( x \) is the number of lines in the ad, express the cost \( c(x) \) of the ad as a piecewise function. See margin. **17c. \( c(x) = \begin{cases} 
11x & \text{when } x \leq 3 \\
33 + 5(x - 3) & \text{when } x > 3 
\end{cases} \)**

18. Ace Auto Repair needs a new mechanic, so they place a help-wanted ad. The *Position Posted* job website charges $15 to post, plus $2.50 for each of the first five lines and $8 for each additional line. If \( x \) is the number of lines in the ad, write a piecewise function for the cost of the ad, \( c(x) \). See margin. **18. \( c(x) = \begin{cases} 
15 + 2.5x & \text{when } x \leq 5 \\
27.50 + 8(x - 5) & \text{when } x > 5 
\end{cases} \)**
4-2 Automobile Transactions

Statistical thinking will one day be as necessary for efficient citizenship as the ability to read and write.
—H.G. Wells, English science fiction author

Objectives
- Compute quartiles and interquartile range.
- Create a frequency distribution from a set of data.
- Use box-and-whisker plots and stem-and-leaf plots to display information.

Key Terms
- Data
- Measures of central tendency
- Quartiles
- Lower quartile
- Upper quartile
- Subscript
- Interquartile range (IQR)
- Stem-and-leaf plot
- Box-and-whisker plot
- Box plot
- Modified box plot

Warm-Up
Laura has grades of 88, 92, 84, and 86 on the first four math tests. Find the fifth test grade for each situation.

a. She wants her average to be 90. 100
b. She wants her average to be at least 85. At least 75

How Can Statistics Help You Negotiate the Sale or Purchase of a Car?

You are planning to buy a used car. How can you tell what a reasonable price is for the car you want to buy? You can find a lot of information about used car prices on the Internet. You can also visit a used car dealer. The price of any car depends heavily on its condition and how desirable it is in the marketplace.

You will probably spend a few weeks shopping for your car. You can determine a reasonable price for a particular car by examining the prices of those and similar cars listed in classified ads.

The Kelley Blue Book (www.kbb.com) and Edmunds (edmunds.com) are two of many excellent sources on the Internet you can use to find the value of a used car. Ask questions as you do your research. You can contact sellers to find out about their cars. Be smart in your search and, if possible, bring a knowledgeable person with you when you go to test-drive a used car.

As you search, compile a list of advertised prices for the cars you want. Then, you can use statistics to help analyze the numbers, or data, that you compile. Measures of central tendency represent "typical" values for the data.

You will find less variability in the prices of new cars because all new cars are in the same condition. The price you will pay is based on the sticker price of the car. Different dealers can give different prices, and it is best to compare deals when buying a new car.
Skills and Strategies

Used car prices vary greatly, and a skilled negotiator will have an advantage when buying or selling a used car.

Quartiles

In Chapter 1 you learned about measures of central tendency and measures of dispersion, or spread. You also learned that outliers are items of data that vary widely from the bulk of the data. This definition is too vague for a mathematician. Here we extend our knowledge of measures of spread and introduce a formal definition of an outlier.

If you want to find out more about how the numbers are dispersed, you can use quartiles. Quartiles are four values represented by \( Q_1 \), \( Q_2 \), \( Q_3 \), and \( Q_4 \) that divide the distribution into four subsets that each contain 25% of the data.

**EXAMPLE 1**

Find the quartiles for the tire pressures of car tires at an auto clinic.

\[
15, 17, 21, 25, 31, 32, 32, 32, 34
\]

Tire pressure is measured in psi—pounds per square inch.

**SOLUTION** The numbers are in ascending order.

- \( Q_1 \) is the first quartile or lower quartile, and 25% of the numbers in the data set are at or below \( Q_1 \).
- \( Q_2 \) is the second quartile. Half the numbers are below \( Q_2 \) and half are above, so \( Q_2 \) is equal to the median.
- \( Q_3 \) is the third quartile, or upper quartile, and 75% of the numbers are at or below \( Q_3 \).
- \( Q_4 \) is the maximum value in the data set because 100% of the numbers are at or below that number.

The subscripts identify each quartile.

To find the quartiles, first find \( Q_2 \). Because \( Q_2 \) equals the median of the tire pressures, \( Q_2 = 31 \).

For \( Q_1 \), find the median of the numbers below the median, which are 15, 17, 21, and 25. The median of these numbers is \( Q_1 = 19 \).

Add, and then divide by 2.

\[
\frac{17 + 21}{2} = 19
\]

For \( Q_3 \), find the median of the numbers in the data set that are above the median, which are 32, 32, 32, and 34. The two middle numbers are 32, so \( Q_3 = 32 \).

The maximum value in the data set is 34. So, \( Q_4 = 34 \). The quartile values are \( Q_1 = 19 \), \( Q_2 = 31 \), \( Q_3 = 32 \), and \( Q_4 = 34 \).

You can use your graphing calculator to find quartiles.

**CHECK YOUR UNDERSTANDING**

What percent of the numbers in a data set are above \( Q_3 \)?

## Solution

**Answer: 25%**

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EXAMPLE 2
The interquartile range will be necessary to find the outliers, so students need to understand it before attempting Example 3. Students can think of the IQR as covering the “middle 50%.”

CHECK YOUR UNDERSTANDING
Answer 16

EXAMPLE 3
Students should memorize the formulas for the outlier boundaries. Explain that the mean is very sensitive to outliers, but the median is not. The range is very sensitive to outliers, but the interquartile range is not.

CHECK YOUR UNDERSTANDING
Answer Yes

EXAMPLE 2
What is the difference between $Q_1$ and $Q_3$ in the data set in Example 1?

**SOLUTION** The difference $Q_3 - Q_1$ is called the interquartile range (IQR). The interquartile range gives the range of the middle 50% of the numbers. A small interquartile range means that the middle 50% of the numbers are clustered together. A large interquartile range means that the middle 50% of the numbers are more spread out. To find the interquartile range, subtract. The interquartile range is $Q_3 - Q_1 = 32 - 19 = 13$.

Find the interquartile range for the data set shown below.

12, 12, 13, 15, 17, 20, 20, 23, 24, 24, 27, 31, 33, 34, 38.

EXAMPLE 3
Find the outlier(s) for these tire prices:
$45, 88, 109, 129, 146, 189, 202, 218, 545$

**SOLUTION** The interquartile range is used to identify outliers. Outliers may occur on the lower and/or upper end of the data set. The numbers are in ascending order. The median, $Q_2$, is $146$.

\[
Q_1 = \frac{88 + 109}{2} = 98.5 \quad Q_3 = \frac{202 + 218}{2} = 210
\]

IQR = $210 - 98.5 = 111.5$

Use $Q_1 - 1.5(IQR)$ to compute the boundary for lower outliers.

$98.5 - 1.5(111.5) = -68.75$

Any number below $-68.75$ is an outlier. There are no lower outliers.

Use $Q_3 + 1.5(IQR)$ to compute the boundary for upper outliers.

$210 + 1.5(111.5) = 377.25$

Any number above $377.25$ is an upper outlier, so $545$ is an upper outlier.

The store that charged $545 for a tire in Example 3 had a sale and lowered its price to $399. Is the new price an upper outlier?
EXAMPLE 4

Rod was doing Internet research on the number of gasoline price changes per year in gas stations in his county. He found the following graph, called a stem-and-leaf plot. What are the mean and the median of this distribution?

```
1 1 2 3 7 9
2 0 3 6 6
3 8 8 9 9 9 9
4 0
5 2 2 4 5 5 6 7
6 3 4 4
7 2
```

SOLUTION A stem-and-leaf plot displays data differently than a frequency table. To read a stem-and-leaf plot, look at the legend at the bottom of the plot. In this plot, a number to the left of the vertical line represent the tens place digit, and is the stem. A number to the right of the vertical line represents the digit in the ones place. They are in ascending order, and are the leaves. The first row represents the following numbers.

11, 11, 12, 13, 17, 19

The second row represents the following numbers.

20, 23, 26, 26

The last row represents the number 72.

With one quick look at a stem-and-leaf plot, you can tell if there are many low numbers, many high numbers, or many numbers clustered in the center. Upon further investigation, you can find the total frequency and every item of data in the data set. This allows you to find the mean, median, mode, range, and quartile values.

By counting the leaves in the plot above, the entries on the right side of the vertical line, you find the frequency is 30. Add the numbers in the plot and divide by the total frequency to find the mean. The sum is 1,188.

Divide by 30 to find the mean.

\[
1,188 \div 30 = 39.6
\]

The stem-and-leaf plot presents numbers in ascending order. To find the median, locate the middle number. The frequency, 30, is even, so find the mean of the numbers in the 15th and 16th positions. The two middle numbers are both 39, so the median is 39.

Stem-and-leaf plots may have slightly different looks depending on the information displayed. A stem-and-leaf plot must include a legend or key that describes how to read it.

Find the range and the upper and lower quartiles in Example 3.

CHECK YOUR UNDERSTANDING

Answer

Range = 61;
\[Q_1 = 23; Q_3 = 55\]
EXAMPLE 5
Box-and-whisker plots should be drawn to scale whenever possible. Explain that the mean cannot be computed using the information from a box-and-whisker plot. Also the median is not necessarily midway between the first and third quartiles. This is a common misconception. You should provide several examples of boxplots where $Q_3 - Q_1$ does not equal $Q_2 - Q_1$, to show students all boxplots are not symmetrical.

CHECK YOUR UNDERSTANDING
Answer 75%

Rod, who was researching gas prices in Example 4, found another graph called a box-and-whisker plot, or box plot. It is shown below.

The box-and-whisker plot shows all four quartiles and the least number. It should be drawn to scale, so it changes shape depending on the distribution. Recall that the interquartile range can be computed by subtracting $Q_1$ from $Q_3$.

$$Q_3 - Q_1 = 55 - 23 = 32$$

The interquartile range is 32. That means 50% of all the gas prices are within this range. Notice that you can also find the range using a boxplot, but you cannot find the mean from a boxplot. You can use the statistics menu on your graphing calculator to draw a box-and-whisker plot.

Based on the box-and-whisker plot from Example 4, what percent of the gas stations had 55 or fewer price changes?

Modified Box-and-Whisker Plots
If you want to show outliers on a box plot, you can create a modified box plot. A modified box plot shows all the outliers as single points past the whiskers. If there were upper outliers, the modified box plot would have dots to the right of the whisker. Modified box plots give more information than standard box-and-whisker plots. Your calculator can draw modified box plots. Example 6 shows how you can interpret modified box-and-whisker plots.
EXAMPLE 6

The following box-and-whisker plot gives the purchase prices of the cars of 114 seniors at West High School. Are any of the car prices outliers?

SOLUTION

Quartiles are shown on the box plot, so you can find the interquartile range. The interquartile range is

$$\text{IQR} = Q_3 - Q_1 = 9,100 - 5,200 = 3,900$$

The boundary for lower outliers is

$$Q_1 - 1.5(\text{IQR}) = 5,200 - 1.5(3,900) = -650$$

There are no lower outliers.

The boundary for upper outliers is

$$Q_3 + 1.5(\text{IQR}) = 9,100 + 1.5(3,900) = 14,950$$

There is at least one upper outlier, the high price of $43,000. From this box plot, you cannot tell if there are any others because the box plot does not give all the original data. Box plots are drawn to scale, so the long whisker on the right means that there could be more than one outlier. In the following modified box plot, $43,000 is the only outlier. The greatest price less than $43,000 is $12,500, which is not an outlier.

If there were three different upper outliers, the modified box plot would have three dots to the right of the whisker.

Modified box plots give more information than standard box-and-whisker plots. Your calculator can draw modified box plots.

CHECK YOUR UNDERSTANDING

Answer: Yes

Examine the modified box plot. Is 400 an outlier?

4-2 Automobile Transactions 221
Applications

Statistical thinking will one day be as necessary for efficient citizenship as the ability to read and write.
—H.G. Wells, English science fiction author

1. Interpret the quote in the context of what you learned about statistics in this lesson.

2. The quartiles of a data set are $Q_1 = 50$, $Q_2 = 72$, $Q_3 = 110$, and $Q_4 = 140$. Find the interquartile range. 60

3. The following list of prices is for a used original radio for a 1955 Thunderbird. The prices vary depending on the condition of the radio.

$210, $210, $320, $200, $300, $10, $340, $300, $245, $325, $700, $250, $240, $200

a. Find the mean of the radio prices. Round to the nearest cent. $275
b. Find the median of the radio prices. $247.50
c. Find the mode of the radio prices. $200, $210, $300
d. Find the four quartiles. $Q_1 = 210; Q_2 = 247.50; Q_3 = 320; Q_4 = 700$
e. Find the interquartile range for this data set. 110
f. Find the boundary for lower outliers. Are there any lower outliers? $45; $10
g. Find the boundary for upper outliers. Are there any upper outliers? $485; $700

4. Bill is looking for original taillights for his 1932 Ford. The prices vary depending on the condition. He finds these prices: $450, $100, $180, $600, $300, $350, $300, and $400.

a. Find the four quartiles. $Q_1 = 240; Q_2 = 325; Q_3 = 425; Q_4 = 600$
b. Find the interquartile range. 185
c. Find the boundary for lower outliers. Are there any lower outliers? $-37.50; there are none
d. Find the boundary for upper outliers. Are there any upper outliers? $702.50; there are none

5. Create a list of numbers with two upper outliers and one lower outlier. Answers will vary.

6. Explain why you cannot find the range of a data set if you are given just the four quartiles. You need the minimum number to find the range, and that is not one of the quartiles.

7. Mitzi looked up prices of 13 used Chevrolet HHR “retro” trucks in the classified ads and found these prices: $8,500, $8,500, $8,500, $9,900, $10,800, $10,800, $11,000, $12,500, $12,500, $13,000, $13,000, $14,500, and $23,000.

a. Find the range. $14,500
b. Find the four quartiles. $Q_1 = 9,200; Q_2 = 11,000; Q_3 = 13,000; Q_4 = 23,000$
c. Find the interquartile range. $3,800
d. Find the boundary for upper outliers. $18,700
e. Find the boundary for lower outliers. $3,500
f. How many outliers are there? 1

g. Draw a modified box-and-whisker plot. Label it. See additional answers.
8. The Cold Spring High School student government polled randomly selected seniors and asked them how much money they spent on gas in the last week, rounded to the nearest dollar. The following stem-and-leaf plot shows the data they collected.

a. How many students were polled? 26
b. Find the mean to the nearest cent. $62.12
c. Find the median. $65
d. Find the mode. $53
e. Find the range. $67
f. Find the four quartiles. $Q_1 = 53; Q_2 = 65; Q_3 = 75; Q_4 = 84$
g. What percent of the students spent $53 or more on gas? 75%
h. Find the interquartile range. $22$
i. What percent of the students spent from $53 to $75 on gas? 50%
j. Find the boundary for lower outliers. $20
k. Find the boundary for upper outliers. $108
l. How many outliers are there? 1
m. Draw a modified box plot. See additional answers.
**4-3 Automobile Insurance**

Never lend your car to anyone to whom you have given birth.
—Erma Bombeck, humor writer

**Objectives**
- Identify different types of auto insurance coverage.
- Compute insurance costs.
- Compute payments on insurance claims.

**Key Terms**
- liable
- negligent
- automobile insurance
- premium
- claim
- liability insurance
- bodily injury liability (BI)
- property damage liability (PD)
- uninsured/underinsured motorist protection (UMP)
- personal injury protection (PIP)
- no-fault insurance
- comprehensive insurance
- collision insurance
- car-rental insurance
- emergency road service insurance
- surcharge
- deductible

**Warm-Up**
Write a linear equation for each of the following.

a. slope: $-\frac{2}{3}$  
y-intercept: $-8$

b. slope: 0  
y-intercept: $-8$

c. no slope  
x-intercept: $-8$

**Why is Having Auto Insurance So Important?**

Everyone, even a responsible driver, runs the risk of injuring themselves, hurting other people, and damaging property. By law, drivers are liable (responsible) to pay for the damages they cause with their automobiles. You could also be sued for being negligent (at fault) if you cause an accident.

Drivers purchase automobile insurance because most drivers cannot afford the costs that could result from an auto accident. In many states, it is also the law; you are unable to register your car without proof of auto insurance. An automobile insurance policy is a contract between a driver and an insurance company. The driver agrees to pay a fee (called the premium) and the company agrees to cover certain accident-related costs when the driver makes a claim (a request for money). Liability insurance is the most important coverage. Liability insurance protects you if your driving causes injury to others or to other people’s property. Today’s costs of medical care and auto body repair are high, and as a driver you are responsible for damage you cause, so having liability insurance is important. States set the minimum amount of coverage required for liability insurance. You can always opt to purchase more than the minimum state...
requirements. Insurance regulations vary by state. Liability insurance is required unless you can prove financial responsibility otherwise. Several types of coverages are available:

- **Bodily injury liability (BI)** covers bodily injury. If you are at fault in an automobile accident, you are responsible for paying the medical expenses of anyone injured in the accident. You can purchase as much BI liability as you want.

- **Property damage liability (PD)** pays for damage you cause to other people's property. You are financially responsible if you damage a telephone pole, fire hydrant, another car, or any other person's property. You can purchase as much PD liability insurance as you want.

- **Uninsured/underinsured motorist protection (UMP)** pays for injuries to you or your passengers caused by a driver who has no insurance or does not have enough liability insurance to cover your medical losses.

- **Personal injury protection (PIP)**, mandatory in some states, pays for any physical injuries you or your passengers sustain while in, on, around, or under the vehicle, even if you are not involved in a traffic accident. It compensates you regardless of who is at fault, so it is sometimes called no-fault insurance. Your PIP insurance will cover you and people injured in, on, around, or under your car for medical treatment and possibly loss of wages due to injury.

- **Comprehensive insurance** covers the repair or replacement of parts of your car damaged by vandalism, fire, flood, wind, earthquakes, falling objects, riots, hail, damage from trees, and other disasters. It also covers your car if it is stolen. If your car is older, comprehensive coverage may not be cost-effective.

- **Collision insurance** pays for the repair or replacement of your car if it's damaged in a collision with another vehicle or object or if it overturns, no matter who is at fault. If you took out a loan to purchase your car, the lender will probably require you to have collision coverage. If your car is older, collision coverage may not be a worthwhile expense.

- **Car-rental insurance** pays you for part of the cost of a rented car if your car is disabled because of a collision or comprehensive-covered repair.

- **Emergency road service insurance** pays for towing or road service when your car is disabled. Only the road service fee is covered. Gas, oil, parts, and labor are not covered.

Auto insurance companies are in business to make a profit. The company loses money if a high percentage of insured drivers make claims. It is your responsibility, and a very important one, to understand how your auto insurance policy protects you and other people on America's roads.

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**CLASS DISCUSSION**

Do you know how much auto body work costs? How much a fire hydrant or lamp post costs? What do you know about the cost of doctors and hospitals? Have students list what property damage they can cause with their cars.

If a driver who has PIP insurance causes an injury, and there are no lawsuits, then the PIP insurance, and not the BI insurance, takes care of covering the injuries. Students sometimes find this confusing, since it is easy to assume that “bodily injury” covers all injuries.

Discuss what specific items are covered by comprehensive insurance, and ask students if they personally know of any car damage caused by the listed mishaps. You might discuss the automobiles damaged and destroyed in New York City by September 11, 2001, attacks, and their coverage by comprehensive insurance.

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Once you learn more about auto insurance, you'll understand how you can save money and comparison-shop for different insurance policies.

**EXAMPLE 1**
Discuss the advantages of paying quarterly. Point out that the surcharge is paid with each quarterly payment.

**CHECK YOUR UNDERSTANDING**

**Answer** \( \frac{x}{2} + y \)

**EXAMPLE 2**
Stress that property damage liability insurance does not cover damage you do to your own car in an accident.

**CHECK YOUR UNDERSTANDING**

**Answer** \( x + y \)

This amount will be paid in full if the coverage limit on the property damage is greater than \( x + y \). The \( w \)-dollar damage is not covered under PD.

**Check Your Understanding**

Leon’s annual premium is \( x \) dollars. If he pays his premium semi-annually, there is a \( y \)-dollar surcharge on each semi-annual payment. Express the amount of his semi-annual payment algebraically.

**Check Your Understanding**

Keith ran his car into a telephone pole that had a bicycle leaning against it, which was also damaged. The pole will cost \( x \) dollars to fix, the bicycle will cost \( y \) dollars to replace, and there was \( w \) dollars damage to the car. Express algebraically the amount that can be claimed under Keith’s property damage liability insurance.

**Deductibles**
Collision and comprehensive insurance are sold on a deductible basis. When you purchase an automobile insurance policy, you must choose a deductible amount that will be part of the policy. The deductible is the amount the policyholder must pay before the insurance policy pays any money. The higher the deductible, the lower the premium the policyholder has to pay. Once an owner has paid the deductible amount, the insurance company pays the rest of the cost to get the repairs done. Collision and comprehensive insurance are sold on a deductible basis. Collision insurance only covers damage to the policyholder’s car, not property damage, or another driver’s vehicle. If a driver has a $500 deductible and the repairs to his car cost $2,200, the driver pays the first $500 and the insurance company pays the balance, $2,200 – 500, or $1,700.
Automobile Insurance

Bodily Injury and Property Damage

Bodily injury insurance coverage is expressed by two numbers divided by slashes. The first number is the maximum amount per accident the insurance company will pay (in thousands) to any one person who is hurt and sues you due to your driving negligence. The second number represents the maximum amount per accident your insurance company will pay (in thousands) in total to all people who make claims or sue as a result of the accident. Sometimes, bodily injury and property damage are combined into a three-number system with two slashes. The numbers 100/300/25 represent $100,000/$300,000 BI insurance and $25,000 PD insurance.

EXAMPLE 3
Peter has collision insurance with a $1,000 deductible. Peter backs his car into a lamppost and causes $4,300 worth of damage to the car. How much will his insurance company have to pay?

SOLUTION
Subtract the deductible, which is $1,000, because Peter must pay that amount.

\[ 4,300 - 1,000 = 3,300 \]

The company must pay $3,300.

EXAMPLE 4
Bob was in an auto accident caused by his negligence. He has 100/300 bodily injury insurance. The three people injured in the accident sued. One person was awarded $140,000, and each of the other two was awarded $75,000. How much does the insurance company pay?

SOLUTION
Bob has 100/300 BI, so the company only pays $100,000 to the person who was awarded $140,000. The other two injured persons were awarded a total of $150,000. Each was under $100,000. The most Bob’s company would pay out for any BI claim is $300,000.

\[ 100,000 + 75,000 + 75,000 = 250,000 \]

Since $250,000 < $300,000, the insurance company pays $250,000. The remaining $40,000 owed to one of the injured is Bob’s responsibility.

Check Your Understanding
Manuel has an x-dollar deductible on his comprehensive insurance. His car is stolen and never recovered. The value of his car is assessed by the insurance company at y dollars where y > x. How much will the insurance company pay him for his stolen car?

Joan has 50/100 BI liability insurance. She hits a school bus, and 28 children require medical attention. Each child is awarded $10,000 as a result of a lawsuit. How much will the insurance company pay in total?

Check Your Understanding
You can pose similar scenarios to students to make sure they understand the bodily injury numbering scheme. It will take several examples for all students to be clear on the different possibilities.

Answer
$100,000
EXAMPLE 5
This example will help show the difference between PIP and BI insurance. There were no lawsuits, so BI did not take effect since the person had no-fault (PIP) insurance.

CHECK YOUR UNDERSTANDING
Answer $280,000
Remind students that the $100,000 PIP coverage limit is per person, per accident.

EXAMPLE 5
Rodrigo has a policy with 50/150 BI, $50,000 PD, and $50,000 PIP. He causes an accident in which he hurts seven people in a minivan and four people in his own car, including himself. The total medical bills for all involved equal $53,233. How much does the insurance company pay?

SOLUTION
Rodrigo is covered by his PIP, which has a limit of $50,000 per person, per accident. PIP takes care of medical payments without regard to who is at fault. The company pays the entire $53,233, as long as no individual person requests more than $50,000. Notice that bodily injury coverage limits were not relevant in this scenario because there were no lawsuits, just medical claims covered under PIP.

Check Your Understanding
Pat has 50/100 BI liability insurance and $100,000 PIP insurance. She hurts 28 children in a school bus and is not sued. However, if each child needs $10,000 for medical care, how much will the insurance company pay in total for these medical claims?

Answer $280,000
Remind students that the $100,000 PIP coverage limit is per person, per accident.
Applications

Never lend your car to anyone to whom you have given birth.
—Erma Bombeck, humor writer

1. Interpret the quote in the context of what you learned about auto insurance in this section. See margin.

2. Rachel has $25,000 worth of property damage insurance. She causes $32,000 worth of damage to a sports car in an accident.
   a. How much of the damages will the insurance company have to pay? $25,000
   b. How much will Rachel have to pay? $7,000

3. Ronald bought a new car and received these price quotes from his insurance company.
   a. What is the annual premium? $1,467
   b. What is the semi-annual premium? $733.50
   c. How much less would Ronald’s semi-annual payments be if he dropped the optional collision insurance? $205

4. Gloria pays her insurance in three installments each year. The first payment is 40% of the annual premium, and each of the next two payments is 30% of the annual premium. If the annual premium is $924, find the amounts of the three payments. $369.60; $277.20; $277.20

5. Ruth has decided to drop her collision insurance because her car is getting old. Her total annual premium is $916, of which $170.60 covers collision insurance.
   a. What will her annual premium be after she drops collision insurance? $745.40
   b. What will her quarterly payments be after she drops collision coverage? $186.35

6. Gary has $10,000 worth of property damage insurance. He collides with two parked cars and causes $12,000 worth of damage. How much money must Gary pay after the insurance company pays its share? $2,000

7. Nico has a personal injury protection policy that covers each person in, on, around, or under his car for medical expenses as a result of an accident. Each person can collect up to $50,000. Nico is involved in an accident and three people are hurt. One person has $23,000 of medical expenses, one person has $500 worth of medical expenses, and Nico himself has medical expenses totaling $70,000. How much money must the insurance company pay out for these three people? $73,500

8. Leslie has comprehensive insurance with a $500 deductible on her van. On Halloween her van is vandalized, and the damages total $1,766. Leslie submits a claim to her insurance company.
   a. How much must Leslie pay for the repair? $500
   b. How much must the insurance company pay? $1,266

ANSWERS

1. Driving a car is a tremendous responsibility, and often the source of parent–teenager conflicts. Teenage drivers have the highest frequency of accidents, and it’s natural that parents worry about their childrens’ safety as well as protecting their cars.

TEACH Exercise 7
Under PIP, the fact that one person has a small claim has no effect on another person’s claim that exceeds the coverage limit. The person who exceeds the limit cannot receive funds from the “unused” money from the smaller claims.
9. Felix has $10,000 worth of property damage insurance and a $1,000 deductible collision insurance policy. He had a tire blowout while driving and crashed into a $1,400 fire hydrant. The crash caused $1,600 in damages to his car.
   a. Which insurance covers the damage to the fire hydrant? **Property damage**
   b. How much will the insurance company pay for the fire hydrant? **$1,400**
   c. Which insurance covers the damage to the car? **Collision**
   d. How much will the insurance company pay for the damage to the car? **$600**

10. Jared's car slides into a stop sign during an ice storm. There is x dollars damage to his car, where x > 1,000, and the stop sign will cost y dollars to replace. Jared has $25,000 worth of PD insurance, a $1,000 deductible on his collision and comprehensive insurance, and $50,000 no-fault insurance.
   a. Which insurance covers the damage to the sign? **Property damage**
   b. How much will his company pay for the stop sign? **$y, since y < $25,000**
   c. Which insurance covers the damage to his car? **Collision**
   d. How much will his company pay for the damage to the car? **$x - 1,000**

11. Eric must pay his p dollar annual insurance premium by himself. He works at a job after school.
   a. Express how much he must save each month to pay this premium algebraically. **\( \frac{P}{12} \)**
   b. If he gets into a few accidents and his company raises his insurance 15%, express how much he must save each month to meet this new premium algebraically. **\( 1.15 \left( \frac{P}{12} \right) \)**

12. Kaylee has 100/300/50 liability insurance and $50,000 PIP insurance. She drives through a stop sign, hits a telephone pole, and ricochets into a minivan with eight people inside. Some are seriously hurt and sue her. Others have minor injuries. Three passengers in Kaylee's car are also hurt.
   a. The pole will cost $7,000 to replace. Kaylee also did $6,700 worth of damage to the minivan. What insurance will cover this, and how much will the company pay? **Property damage; $13,700**
   b. The minivan's driver was a concert violinist. The injury to his hand means he can never work again. He sues for $4,000,000 and is awarded the money in court. What type of insurance covers this, and how much will the insurance company pay? **Bodily injury; $100,000**
   c. The minivan's driver (from part b) had medical bills totaling $60,000 from his hospital trip and physical therapy after the accident. What type of insurance covers this, and how much will the insurance company pay? **PIP; $50,000**
   d. The three passengers in Kaylee's car are hurt and each requires $12,000 worth of medical care. What insurance covers this, and how much will the company pay? **PIP; $36,000**

13. Julianne currently pays x dollars for her annual premium. She will be away at college for the upcoming year and will only use the car when she is home on vacations. Since she is using the car much less, she is less of a risk and her insurance company offers her a 35% discount for her annual premium. Express algebraically the amount she must save each month to pay the new, lower premium. **\( \frac{x - 0.35x}{12} \) or \( \frac{0.65x}{12} \)**

14. The Sundaram family just bought a third car. The annual premium would have been x dollars to insure the car, but they are entitled to a 10% discount since they have other cars insured with the company.
   a. Express their annual premium after the discount algebraically. **\( x - 0.1x \) or \( 0.9x \)**
   b. If they pay their premium quarterly and have to pay a y-dollar surcharge for this arrangement, express their quarterly payment algebraically. **\( \frac{0.9x}{4} + y \)**
15. Marc currently pays $x\text{ dollars per year for auto insurance. Next year, his rates are going to increase 15\%. If he completes a defensive driving course, the insurance company will lower his rate by } d\text{ dollars.}

a. Express his annual premium for next year algebraically if he completes the course. 
\[x + 0.15x - d, \text{ or } 1.15x - d\]

b. Express his semi-annual premium for next year algebraically if he does not complete the course. 
\[\frac{1.15x}{2}\]

16. The stem and leaf plot shown is called a back-to-back stem-and-leaf plot, and combines two stem and leaf plots. It gives the semi-annual premiums for the girls and boys in Van Buren High School who currently drive. The numbers between the two vertical lines represent the hundreds (left) and tens (right) digits. The numbers on the extreme left show the units digits for the girls. Notice they are written in ascending order as you move out from the middle. The numbers on the extreme right show the units digits for the boys.

a. How many girls at Van Buren HS drive? 17
b. How many boys at Van Buren HS drive? 13

c. Find the range of the annual premiums for all of the students. $36

17. The following stem-and-leaf plot gives the number of juniors who took a driver education course at Guy Patterson High School over the last two decades. Construct a box-and-whisker plot based on the data. See additional answers.

18. Express the boundary for the upper outliers algebraically, using the modified box-and-whisker plot given below. 
\[d + 1.5(d - c)\]

19. A local insurance agent visits the high school and tells the students that his insurance company will give them a 10\% discount on liability insurance, PIP, comprehensive insurance, and collision insurance for 3 years if they take a defensive driving course. Maria spends $1,276 on her auto insurance annually.

a. If she takes the defensive driving course, how much will she save in a 3-year period? $382.80

b. Maria’s defensive driving course costs $55. Does she save enough in 1 year to pay for the course? Yes
Chapter 4
Automobile Ownership

Objectives
• Explain how to interpret two-way tables.
• Compute conditional probabilities based on two-way tables.
• Determine if two events are independent.
• Interpret Venn diagrams.
• Create Venn diagrams.

Key Terms
actuary
probability
event
two-way table
conditional probability
independent events
associated events
Venn diagrams

Warm-Up
Given the following set of car wash prices (in dollars), find the difference between the range and the interquartile range.
12, 13.50, 14, 15, 16, 18.50, 20, 20, 20, 21, 22.50, 22.50, 24, 24, 26, 26, 27.50

It is a capital mistake to theorize before one has data.
—Sir Arthur Conan Doyle, Scottish author (Sherlock Holmes novels)

How Does Probability Affect Your Auto Insurance?

Insurance companies classify drivers according to their age, sex, marital status, driving record, and locality. This may, at first, sound discriminatory, but it is not. An insurance company is in business to make a profit. As a result, they need to know what types of drivers are more likely to be involved in accidents because these drivers pose a greater risk of costing the company money. The number of accidents people in these categories get involved in allow mathematicians to look at frequencies and relative frequencies and to use the data to formulate probabilities.

A statistician, called an actuary, can predict how often customers, based on these criteria, will submit claims. Insurance companies use these predictions to create the premiums they charge to different classes of drivers. So, if in your age group, women pay less than men for the same car insurance coverage, it is because statistics show that women, as a group, are less financially risky to insure. It is not because of any personal characteristics any specific man or woman might have.

As a teenager you will pay higher rates than your parents because teens have proven to be a higher risk to insurance companies. Many insurance companies offer discounts for taking a driver’s education course, or for maintaining high grades in school. Why? Teens who’ve taken driver’s education or who have higher grades have proven to be less of a risk to insurance companies. It’s all about probability. Probability is the relative possibility that an event will occur. In this section you are introduced to some basic concepts of probability.
Skills and Strategies

You encounter probability when you play games. You may have rolled number cubes or picked cards from decks and tried to estimate your chances of a favorable outcome. You also encounter probability when you play computer games, but it’s often harder to see.

EXAMPLE 1
A pail contains seven red, six green, three orange, and four black marbles. One marble is to be drawn at random. What is the probability that the marble is red?

SOLUTION The outcome of an experiment is called an event. In this case, the outcome is “red.” Mathematicians use shorthand to express “the probability that the marble is red.” A capital $P$ is used along with parentheses, and the event is abbreviated with a capital letter. If we let $R$ represent the event “a red is drawn” then the probability that a red marble is drawn can be written as $P(R)$.

$$P(R) = \frac{\text{number of marbles that are red}}{\text{total number of marbles}} = \frac{7}{20} = 35\%$$

There is a 35% chance someone will pick a red marble in this experiment.

The driver’s education class at Monroe High School has 23 girls and 20 boys in it. If one person is selected at random to represent the school in a driver safety exposition, what is the probability that the person selected will be a girl?

Logan picks a black marble from the pail in Example 1, and doesn’t replace it in the pail. Brett will then randomly select a marble. What is the probability that Brett picks a green marble?

EXAMPLE 2
A factory is planning to rearrange the parking spaces in its parking lot to better serve the employees. The employees work in three different shifts and drive five different types of vehicles, as shown in the two-way table below.

<table>
<thead>
<tr>
<th>Type of Vehicle</th>
<th>Shift</th>
<th>2-Door Sedan (2D)</th>
<th>4-Door Sedan (4D)</th>
<th>SUV (S)</th>
<th>Pick-up Truck (PU)</th>
<th>Motorcycle (C)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Morning (M)</td>
<td>20</td>
<td>31</td>
<td>42</td>
<td>13</td>
<td>4</td>
<td>110</td>
<td></td>
</tr>
<tr>
<td>Day (D)</td>
<td>15</td>
<td>25</td>
<td>30</td>
<td>5</td>
<td>2</td>
<td>77</td>
<td></td>
</tr>
<tr>
<td>Night (N)</td>
<td>7</td>
<td>17</td>
<td>19</td>
<td>11</td>
<td>5</td>
<td>59</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>42</td>
<td>73</td>
<td>91</td>
<td>29</td>
<td>11</td>
<td>246</td>
<td></td>
</tr>
</tbody>
</table>

An employee is selected at random. What is the probability that an employee drives an SUV?
**SOLUTION** Use the Total columns to find the total number of employees, which is 246. Look for the SUV column to find that the total number of SUV drivers is 91. The probability can be found by creating a fraction.

\[
P(\text{SUV}) = \frac{\text{number of employees who are SUV drivers}}{\text{total number of employees}} = \frac{91}{246} = 37\%
\]

The probability the randomly selected employee drives an SUV is \(\frac{91}{246}\), or approximately 37%.

Using the table from Example 2, find the probability that a randomly selected employee is on the night shift.

**EXAMPLE 3**

This is the introduction to conditional probability. Have students make up conditional probability questions involving the table and have other students answer as part of a class discussion.

Show students that \(P(A|B)\) and \(P(B|A)\) are not the same using entries from the table.

<table>
<thead>
<tr>
<th>Type of Vehicle</th>
<th>2-Door Sedan (2D)</th>
<th>4-Door Sedan (4D)</th>
<th>SUV (S)</th>
<th>Pick-up Truck (PU)</th>
<th>Motorcycle (C)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Shift</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Morning (M)</td>
<td>20</td>
<td>31</td>
<td>42</td>
<td>13</td>
<td>4</td>
<td>110</td>
</tr>
<tr>
<td>Day (D)</td>
<td>15</td>
<td>25</td>
<td>30</td>
<td>5</td>
<td>2</td>
<td>77</td>
</tr>
<tr>
<td>Night (N)</td>
<td>7</td>
<td>17</td>
<td>19</td>
<td>11</td>
<td>5</td>
<td>59</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>42</td>
<td>73</td>
<td>91</td>
<td>29</td>
<td>11</td>
<td>246</td>
</tr>
</tbody>
</table>

The letter symbols can be used, along with a vertical line that stands for “given” to create a symbolic shorthand for “the probability the person drives a motorcycle given that they are on the night shift.” This can be written as \(P(C|N)\).

\[
P(C|N) = \frac{\text{number of night shift employees who drive motorcycles}}{\text{total number of night shift employees}} = \frac{5}{59} = 8\%
\]

Using the two-way table from Example 2, find the probability that a randomly selected employee works the night shift given that they drive an SUV. Express your answer in fraction form.
EXAMPLE 4

Justin is doing a statistical analysis to check if sports cars in his town are involved in more accidents than other types of cars. He polls 315 randomly selected parents of students in his high school and gets the results shown in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Sports Car (S)</th>
<th>Sedan (SD)</th>
<th>Sport Utility Vehicle (SUV)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Had an accident (A)</td>
<td>50</td>
<td>69</td>
<td>31</td>
<td>150</td>
</tr>
<tr>
<td>Never had an accident (N)</td>
<td>55</td>
<td>45</td>
<td>65</td>
<td>165</td>
</tr>
<tr>
<td>Total</td>
<td>105</td>
<td>114</td>
<td>96</td>
<td>315</td>
</tr>
</tbody>
</table>

Does the table support Justin's theory that sports cars get into more accidents?

SOLUTION Justin needs to examine two probabilities, \(P(A)\) and \(P(A|S)\). For all types of vehicles in his survey, the probability of getting into an accident is \(P(A)\).

\[
P(A) = \frac{\text{number of cars in accidents}}{\text{total number of cars}} = \frac{150}{315} \approx 48\%
\]

If we restrict to the set of sports cars, we can find the conditional probability \(P(A|S)\).

\[
P(A|S) = \frac{\text{number of sports cars that were in accidents}}{\text{total number of sports cars}} = \frac{50}{105} \approx 48\%
\]

Since \(P(A) = P(A|S)\), we call \(A\) and \(S\) independent events. Two events are independent if the probability of one occurring is unaffected by the occurrence of the other event. If \(P(A) = P(A|B)\) we say \(A\) and \(B\) are independent, and if \(P(A) \neq P(A|B)\) then we say \(A\) and \(B\) are associated events.

Since the probability of having had an accident is the same for the set of sports cars as it is for everyone in the entire survey, Justin did not get support for his theory. The percentage of sports car drivers who get into accidents is no different than the general percentage of people who got into accidents in Justin's survey.

Using the table from Example 4, explain whether or not \(N\) and \(SD\) are independent events.

CHECK YOUR UNDERSTANDING

Answer \(P(N) = \frac{165}{315}\), which is about 52%. \(P(N|SD) = \frac{45}{114}\), which is about 39%. Since \(P(N)\) and \(P(N|SD)\) are not equivalent, \(N\) and \(SD\) are not independent events.
EXAMPLE 5

Jhanvi is judging a classic car show. She is also preparing a report on the cars in the show. There were 135 cars at the show. Seventy of the cars had a manual transmission, 43 had air conditioning, 13 had both a manual transmission and air conditioning, and 35 had neither. A car will be selected at random to appear on the cover of the report. What is the probability that the car will have air conditioning, but not a manual transmission?

SOLUTION Jhanvi knows that "a picture is worth a thousand words," so she is using a specific type of graph called a Venn diagram to display some of her findings. The Venn diagram has different regions that represent each of the different combinations of categories. Jhanvi lets $M$ represent cars with a manual transmission and $A$ represent cars with air conditioning. She created the Venn diagrams on the right. Notice that the numbers in each region add up to the total number of cars, 135.

How can these Venn diagrams be interpreted? The section shaded in light blue below shows the 43 cars that have air conditioning. Notice that 13 of these cars are also in the manual transmission circle, so these 13 cars have both manual transmissions and air conditioning.

The 30 cars in the section shaded in dark blue below have air conditioning, but do not have manual transmissions. The 35 cars shown in the yellow shaded area do not have a manual transmission or air conditioning.

You can view Venn diagrams as schematic puzzles that give you information. The probability that a randomly selected car has air conditioning, but not a manual transmission, can be found using the numbers in the Venn diagram.

$$P(A) = \frac{\text{number of cars with air conditioning and no manual transmission}}{\text{total number of cars}} = \frac{30}{135}$$

$\approx 22\%$

There is a 22% chance that the car on the cover of the report will have air conditioning but not a manual transmission.

CHECK YOUR UNDERSTANDING

Answer $\frac{151}{176}$

EXTEND YOUR UNDERSTANDING

Answer 64

Check Your Understanding

The following Venn diagram described cars on the lot of Macelli’s Used Car Showroom. The letter $B$ is used if the car exterior is black, and $T$ is used if the interior color is tan.

What is the probability that a randomly selected car from Macelli’s lot is not black?

Extend Your Understanding

Using the Venn diagram from the previous Check Your Understanding problem, how many cars have a black exterior or have a tan interior?
EXAMPLE 6

Using the Venn diagram from the previous Check Your Understanding problem, what is the conditional probability that a randomly selected car is black given that it has a tan interior?

**SOLUTION** Look at the two distinct regions in the circle representing the tan interior color. Out of the 47 cars with tan interiors, 8 are black, and we use these numbers to form the fraction that represents the probability.

$$P(B|T) = \frac{\text{number of black cars with tan interiors}}{\text{total number of cars with tan interiors}} = \frac{8}{47} \approx 17\%$$

The conditional probability that the car is black given it has a tan interior is approximately 17%.

EXAMPLE 7

The amount of money the Derham Insurance Company has paid out in comprehensive claims is normally distributed with mean $887 and standard deviation $301. Derham is doing a survey of their customers. If a person who made a comprehensive claim with Derham is randomly selected, what is the probability that their claim was less than $1,200?

**SOLUTION** In Section 1-2 you learned about relative frequency. Relative frequencies can also be interpreted as probabilities.

- If 22% of the students in a school are freshmen, then the probability of randomly selecting a freshman from this school is also 22%.
- If 79% of the students at Bayside High School have collision insurance on their cars, then the probability that a randomly selected student has collision insurance is 79%.

In Section 1-4 you learned how to use the normal curve and the normal curve table to find relative frequencies for normally distributed variables. Recall that the total area under the normal curve is 1, and that the areas under the normal curve represent relative frequencies. Take a look back at that section to refresh your memory on how to use the table to find areas under the normal curve.

Sketch the normal curve and label the mean 887 and the region less than 1,200 that we are interested in.

Convert the raw score of 1,200 to a z-score using the z-score formula.

$$z = \frac{x - \mu}{\sigma}$$

Substitute.

$$z = \frac{1200 - 887}{301} \approx 1.04$$

Look at the table on page 718 for the area below 1.04 under the normal curve. The area is 0.8508. Therefore, the probability of randomly selecting a person who made a comprehensive claim less than $1,200 is approximately 85%.
Check Your Understanding

CHECK YOUR UNDERSTANDING

Answer 11%

EXAMPLE 8

Students may have encountered ratio problems in a previous algebra course.

Find the probability that a randomly selected person made a comprehensive claim between $1,200 and $1,400. Round to the nearest percent.

EXAMPLE 8

There are 80 people who attend classes at Vetrone’s Driving School. The ratio of boys to girls is 2:3. A student will be randomly selected to compete in a driver safety contest. What is the probability that a girl is picked?

SOLUTION Since the ratio is 2:3, the number of boys can be represented as 2x and the number of girls can be represented as 3x.

Use an equation to represent the fact that the total is 80.

\[ 2x + 3x = 80 \]

Combine like terms.

\[ 5x = 80 \]

Divide both sides by 5.

\[ x = 16 \]

The number of girls is 3x, so substitute.

\[ 3x = 48 \]

There are 48 girls. The probability of a girl being selected is \( P(G) \).

\[ P(G) = \frac{\text{number of girls}}{\text{total number of students}} = \frac{48}{80} \approx 60\% \]

The probability that a girl will be selected is 60%.

The ratio of seniors who are licensed drivers to seniors who are not licensed at Glen Cove High School is 7:3. If there are 190 seniors, find the probability that a randomly selected senior will be a licensed driver.

CHECK YOUR UNDERSTANDING

Answer \( \frac{7}{10} \)
1. Read the quote at the beginning of this section. Interpret the quote in terms of what you have learned about probability. See margin.

2. The ratio of hybrid cars to gasoline-powered cars in the parking lot of a local office building is 3:11. If there are 280 cars in the lot and one is to be selected at random to get the reserved parking space nearest the building, find the probability the car selected is a hybrid. Express your answer as a fraction. \(\frac{3}{14}\)

3. There are five solids called Platonic solids. Each Platonic solid has congruent faces and congruent angles, and they are shown below. Use the figures to compute the probabilities. Express your answers as fractions.

   a. An octahedron has eight faces. If the octahedron faces are labeled with the numbers 1–8, and it is rolled, find the probability that it will land on a prime number. \(\frac{1}{2}\)
   
   b. The icosahedron has 20 congruent triangular faces. If they are labeled 1–20, and the icosahedron is rolled, what is the probability that it lands on a multiple of 3? \(\frac{3}{10}\)
   
   c. The dodecahedron has 12 congruent pentagonal faces. If they are labeled 1–12, and the dodecahedron is rolled, find the probability that it lands on a multiple of 3 and 2. \(\frac{1}{6}\)
   
   d. A tetrahedron has four congruent triangular faces labeled 10, 20, 30, and 40. Let \(T\) represent “the tetrahedron was rolled.” Let \(S\) represent the event “lands on a multiple of 7.” Find \(P(S \mid T)\). 0

4. The distribution of ages of licensed drivers shopping at the Kalish County Mall is normally distributed with mean 41 and standard deviation 7. A person will be selected at random. Round your answers to the nearest hundredth.
   
   a. What is the probability that the person is age 48 or older? 0.16
   
   b. What is the probability that the person is age 48 or younger? 0.84
   
   c. What is the probability that the person is between ages 41 and 48? 0.34
   
   d. What is the probability that the person selected will be less than 27 years old? 0.02
5. The following table shows music preferences found by a survey of the faculty at a local university. Express your answers in fraction form.

<table>
<thead>
<tr>
<th>Country Music (C)</th>
<th>Rock Music (R)</th>
<th>Oldies (O)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northern U.S. (N)</td>
<td>11</td>
<td>88</td>
<td>49</td>
</tr>
<tr>
<td>Southern U.S. (S)</td>
<td>70</td>
<td>50</td>
<td>44</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>81</strong></td>
<td><strong>138</strong></td>
<td><strong>93</strong></td>
</tr>
</tbody>
</table>

a. Find the probability that a randomly selected person from this group likes country music. \[\frac{81}{312}\]
b. What is the probability that a randomly selected person from this group likes rock music and is from the North? \[\frac{88}{312}\]
c. Find the probability that a randomly selected person from this group likes oldies given that they are from the South. \[\frac{44}{164}\]
d. Find \(P(R)\) in decimal form. Round to two decimal places. 0.44

e. Find \(P(S)\) in decimal form. Round to two decimal places. 0.53

f. Find \(P(R|S)\) and explain if events \(R\) and \(S\) are independent or associated events. See margin.

g. Zoe and Lisa are having a disagreement about conditional probability. Zoe thinks that \(P(A|B) = P(B|A)\) for any two events \(A\) and \(B\). Lisa says that \(P(A|B)\) and \(P(B|A)\) do not have to be equal. Pick two events from the table in problem 5 and determine who is correct. Answers vary.

6. The following Venn diagram describes cars sold last summer at Penny’s Autoland. The letter \(F\) represents four-wheel drive, \(S\) represents satellite radio, and \(R\) represents remote start.

a. How many people bought cars with four-wheel drive? 39

b. How many people bought cars with satellite radio and remote start? 17286

c. How many people purchased cars with all three options? 7

d. How many people purchased cars with exactly one of these options? 55

e. How many people purchased cars at Penny’s Autoland last summer? 106

f. A buyer from this group is selected at random to receive lifetime free oil changes. What is the probability that they purchased remote start?

g. One person from the group that bought four-wheel drive is going to be selected at random. Find \(P(R|F)\).

h. Determine if \(F\) and \(R\) are independent events and explain your answer. See margin.

i. Examine the purple and yellow shaded regions on the right. Describe the difference between the types of options the two regions represent. The yellow region represents people who bought four-wheel drive and remote start. The purple region excludes those who purchased satellite radio in addition to the other two options.
If the automobile had followed the same development cycle as the computer, a Rolls Royce would today cost $100 [and] get a million miles per gallon.

—Michael Moncur, Internet consultant

**Objective**

- Write, interpret, and graph a straight line depreciation equation.

**Key Terms**

depreciate, appreciate

---

**What is the Value of Your Car?**

Most cars **depreciate**; that is, their value becomes less than their purchase price over time. Some collectible cars increase in value over time, or **appreciate**. The simplest form of depreciation is **straight line depreciation**. When a car loses the same amount of value each year, the scatter plot that models this depreciation appears linear. In reality, cars do not depreciate by the same amount each year. But, when calculating depreciation for the IRS tax forms, linear depreciation formulas are used. Therefore, we begin this section by examining linear automobile depreciation. By determining the equation of this linear model, you can find the value of the car at any time in its lifespan. There are many factors contributing to the depreciation of an automobile. The condition of the car, mileage, and make of the car are only a few of those factors. The straight line depreciation equation is a mathematical model that can be used as a starting point in examining auto depreciation.

In previous math courses, you used the intercepts of linear equations when graphing linear functions. Traditionally, the horizontal axis is called the **x-axis** and the vertical axis is called the **y-axis**. Both the **x-intercept** and the **y-intercept** are numbers, not coordinates. The location on the graph of the horizontal **x-intercept** always has the form \((a, 0)\) and the location on the graph of the vertical **y-intercept** always has the form \((0, b)\). In addition to intercepts, straight lines also have slope. The **slope** of the line is the numerical value for the inclination or declination of that line. It is expressed as a ratio of the change in the value of **y** over the change in the value of **x** from one point on the line to another. Using those variable names, the slope of a line would be represented by the following ratio.

\[
\text{Slope} = \frac{\text{Change in } y\text{-value}}{\text{Change in } x\text{-value}}
\]
If the coordinates of the two points are \((x_1, y_1)\) and \((x_2, y_2)\), then the slope can be modeled mathematically by the following ratio.

\[
\text{Slope ratio} = \frac{y_2 - y_1}{x_2 - x_1}
\]

The independent variable in a car’s depreciation equation is \(\text{time in years}\) and the dependent variable is \(\text{car value}\). By identifying the intercepts and slope of a straight line depreciation model, you will be able to determine the equation that represents the depreciation.

**Skills and Strategies**

Here you learn how to find and use a straight line depreciation equation.

**EXAMPLE 1**

People say that a new car depreciates tremendously the moment it is driven off the car lot. The original price of the car is established at year 0 (before it is driven off the lot). If points on the depreciation line are represented by (year, value), then the point (0, 27,000) must lie on the line. If the car has no value after 12 years, then the point (12, 0) must also lie on the line. Ask students where these points would be located on a coordinate grid. Do they determine a unique line?

**CHECK YOUR UNDERSTANDING**

Answer (0, \(D\)) and (\(T\), 0)

A car sells for \(D\) dollars and depreciates to a value of 0 after \(T\) years. If this car straight line depreciates, what are the coordinates of the intercepts of the straight line depreciation equation?
EXAMPLE 2

Determine the slope of the straight line depreciation equation for the situation in Example 1.

SOLUTION Two points determine a line, so you only need two points to determine the slope of a line. Let the coordinates of the y-intercept be the first point. That is, \((x_1, y_1) = (0, 27,000)\). Let the coordinates of the x-intercept be the second point. That is, \((x_2, y_2) = (12, 0)\).

Use the slope ratio.

\[
\frac{y_2 - y_1}{x_2 - x_1}
\]

Substitute and simplify.

\[
\frac{0 - 27,000}{12 - 0} = \frac{-27,000}{12} = -2,250
\]

The slope of the depreciation line is \(-2,250\).

CHECK YOUR UNDERSTANDING

Answer \(\frac{D}{T}\)

Check Your Understanding

EXAMPLE 3

Write the straight line depreciation equation that models the situation in which a car is purchased for \(D\) dollars and totally depreciates after \(T\) years.

SOLUTION The general form for the equation of a straight line is

\[
y = mx + b
\]

where \(m\) represents the slope of the line and \(b\) represents the y-intercept.

The slope is \(-2,250\) and the y-intercept is 27,000. Therefore, the straight line depreciation equation is

\[
y = -2,250x + 27,000
\]

To graph the equation on a graphing calculator, first determine an appropriate graphing window. Use your maximum and minimum x- and y-values as a starting point. Choose x- and y-values that are larger than the maximum values you have determined so that you get a complete picture of the graph. One such pair could be a maximum of $30,000 on the y-axis and 15 on the x-axis as shown in the graph. Because time and car value are both positive numbers, the minimum x- and y-values will be zero.

CHECK YOUR UNDERSTANDING

Answer The equation is \(y = -2,000x + 22,000\). The graph is

Check Your Understanding

Write and graph the straight line depreciation equation for a car that was purchased for $22,000 and totally depreciates after 11 years.
EXAMPLE 4
Alert students that the time is in months, but the variable, $x$, represents time in years. Elicit from them how to change 66 months into years.

EXAMPLE 5
Students must first calculate a specified car value, substitute that value for $y$, and then manipulate the equation in order to solve for $x$ (the time in years).

CHECK YOUR UNDERSTANDING
Answer $y = -2,055x + 18,495$;
$y = -2,055(5.5) + 18,495$

A car sells for $18,495 and straight line depreciates to zero after 9 years. Write the straight line depreciation equation for this car and an expression for the value of the car after $W$ months.

EXAMPLE 5
The straight line depreciation equation for a car is $y = -4,000x + 32,000$. In approximately how many years will the car’s value decrease by 25%?

SOLUTION The original value of the car is the $y$-intercept, 32,000. You must determine the actual value of the car after it has dropped by 25%. This can be done in two ways.

You can find 25% of the original value of the car and then subtract that amount from the original value.

$0.25 \times 32,000 = 8,000$
$32,000 - 8,000 = 24,000$

The value is $24,000.

You are trying to determine a length of time. Solve the depreciation equation for $x$.

Use the depreciation equation.

$y = -4,000x + 32,000$

Substitute 24,000 for $y$.

$24,000 = -4,000x + 32,000$

Subtract 32,000 from each side.

$-8,000 = -4,000x$

Divide each side by $-4,000$.

$2 = x$

The car will depreciate by 25% after 2 years.
Automobile Expense Function

In Examples 1–5, you worked with depreciation functions to determine the value of a car at a given point in time. You now examine expense functions that will indicate the cumulative amount of money spent at any given point in time. You can create an expense function for an automobile. While there are many expenses that contribute to the running and upkeep of a car, for the purposes here, the expense function is composed of the fixed expense down payment that you make when you purchase a car and the variable expense monthly payment that you make to the lending institution. Looking at the linear expense and depreciation functions simultaneously will give you insight into the value of your automotive investment.

EXAMPLE 6

Celine bought a new car for $33,600. She made a $4,000 down payment and pays $560 each month for 5 years to pay off her loan. She knows from her research that the make and model of the car she purchased straight line depreciates to zero over 10 years.

a. Create an expense and depreciation function.

b. Graph these functions on the same axes.

c. Interpret the region before, at, and after the intersection point.

SOLUTION

a. Let x represent time in months and y represent dollars. Celine’s expense function is the sum of her monthly payments over this time period, 560x, and her initial down payment, $4,000.

Expense function

\[ y = 560x + 4,000 \]

Since the time, x, is in months rather than years, you will need to express Celine’s depreciation function in terms of months as well. Celine’s car totally depreciates after 10 years, or 120 months. To determine her monthly depreciation amount, divide the original car value by 120.

\[ \frac{33,600}{120} = 280 \]

Celine’s car depreciates $280 per month. To calculate the slope of the depreciation equation, use coordinates of the intercepts (0, 33600) and (120, 0).

Slope

\[ \frac{0 - 33,600}{120 - 0} = \frac{-33,600}{120} = -280 \]

The slope of the depreciation equation is –280. Notice that the slope is the negative of the monthly depreciation amount. The straight line depreciation function for Celine’s car is as follows.

Depreciation function

\[ y = -280x + 33,600 \]

b. Determine an appropriate graphing window by using the intercepts for both functions to set up the horizontal and vertical axes. Graph both functions as shown.
c. Using a graphing calculator, the coordinates of the intersection point, rounded to the nearest hundredth, are (35.24, 23733.33). This means that after a little more than 35 months, both Celine’s expenses and the car’s value are the same. In the region before the intersection point, the expenses are lower than the value of the car. The region after the intersection point indicates a period of time that the value of the car is less than what she has invested in it.

How might the expense function be altered so that it reflects a more accurate amount spent over time? What effect might that have on the graphs?

EXAMPLE 7

Suppose that your car straight line depreciates monthly over time. You know that after 12 months your car was worth $24,600. According to an online car value calculator, after 23 months, your car is now worth $21,850. How much does the car depreciate each month?

**SOLUTION**

This problem can be modeled using an arithmetic sequence. Recall from Section 2-3 that an arithmetic sequence is a progression in which any two consecutive numbers have a common difference. If you were to list the car values as a sequence, the common difference would represent the monthly depreciation amount. The term $a_{12}$ represents the car value at 12 months and $a_{23}$ represents the car value at 23 months.

Let $a_{12} = 24,600$ and $a_{23} = 21,850$. The formula for any number $a_n$ in an arithmetic sequence is given by $a_n = a_1 + (n - 1)d$, where $d$ is the common difference.

\[
\begin{align*}
    a_{12} &= 24,600 \\
    a_{12} &= a_1 + (12 - 1)d \\
    24,600 &= a_1 + (12 - 1)d \\

    a_{23} &= 21,850 \\
    a_{23} &= a_1 + (23 - 1)d \\
    21,850 &= a_1 + (23 - 1)d
\end{align*}
\]

Set up a system of linear equations.

**Simplify.**

\[
\begin{align*}
    24,600 &= a_1 + (12 - 1)d \\
    24,600 &= a_1 + 11d
\end{align*}
\]

**Simplify.**

\[
\begin{align*}
    21,850 &= a_1 + (23 - 1)d \\
    21,850 &= a_1 + 22d
\end{align*}
\]

**Subtract.**

\[
\begin{align*}
    24,600 &= a_1 + 11d \\
    -(21,850 &= a_1 + 22d) \\
    2,750 &= -11d \\
    -250 &= d
\end{align*}
\]

The common difference is –250. This can be interpreted as a drop in car value of $250 each month.

A car straight line depreciates monthly over time. After 8 months the car is worth $29,520. After 18 months the car is worth $26,420. How much does this car depreciate each month?
Applications

If the automobile had followed the same development cycle as the computer, a Rolls Royce would today cost $100 [and] get a million miles per gallon.
—Michael Moncur, Internet consultant

1. How might those words apply to what you have learned in this lesson? See margin

2. Delia purchased a new car for $25,350. This make and model straight line depreciates to zero after 13 years.
   a. Identify the coordinates of the x- and y-intercepts for the depreciation equation. (0, 25,350) and (13, 0)
   b. Determine the slope of the depreciation equation. −1,950
   c. Write the straight line depreciation equation that models this situation. y = −1,950x + 25,350
   d. Draw the graph of the straight line depreciation equation. See additional answers.

3. Vince purchased a used car for $11,200. This make and model used car straight line depreciates to zero after 7 years.
   a. Identify the coordinates of the x- and y-intercepts for the depreciation equation. (0, 11,200) and (7, 0)
   b. Determine the slope of the depreciation equation. −1,600
   c. Write the straight line depreciation equation that models this situation. y = −1,600x + 11,200
   d. Draw the graph of the straight line depreciation equation. See additional answers

4. Examine the straight line depreciation graph for a car.
   a. At what price was the car purchased? $28,000
   b. After how many years does the car totally depreciate? 10 years
   c. Write the equation of the straight line depreciation graph shown. y = −2,800x + 28,000

5. The straight line depreciation equation for a luxury car is y = −3,400x + 85,000.
   a. What is the original price of the car? $85,000
   b. How much value does the car lose per year? $3,400
   c. How many years will it take for the car to totally depreciate? 25

6. The straight line depreciation equation for a motorcycle is y = −2,150x + 17,200.
   a. What is the original price of the motorcycle? $17,200
   b. How much value does the motorcycle lose per year? $2,150
   c. How many years will it take for the motorcycle to totally depreciate? 8

7. The straight line depreciation equation for a car is y = −2,750x + 22,000.
   a. What is the car worth after 5 years? $8,250
   b. What is the car worth after 8 years? $0
   c. Suppose that A represents a length of time in years when the car still has value. Write an algebraic expression to represent the value of the car after A years. −2,750 A + 22,000

ANSWERS

1. As computer technology has become more sophisticated, the price of computers has drastically dropped. However, that is not the case for the automobile. The implication is that automotive engineers should have done better.

2. Exercise 2
   Impress upon students that the slope represents depreciation and therefore must be negative.

3. Exercise 3
   The intercepts that have been identified in 3a must satisfy the equation that the students create in 3c. Ask students to verify the accuracy of their work by testing the points in the equation.

See additional answers.
8. The straight line depreciation equation for a car is \( y = -2,680x + 26,800 \).
   a. How much is the car worth after 48 months? \$16,080
   b. How much is the car worth after 75 months? \$10,050
   c. Suppose that \( M \) represents the length of time in months when the car still has value. Write an algebraic expression to represent the value of this car after \( M \) months. \(-2,680 \left( \frac{M}{12} \right) + 26,800\)

9. The graph of a straight line depreciation equation is shown.
   a. Use the graph to approximate the value of the car after 4 years. \$12,800
   b. Use the graph to approximate the value of the car after 5 years. \$9,600
   c. Use the graph to approximate when the car will be worth half its original value. 4 years

10. A car is originally worth \$34,450.
    It takes 13 years for this car to totally depreciate.
    a. Write the straight line depreciation equation for this situation. \( y = -2,650x + 34,450 \)
    b. How long will it take for the car to be worth half its value? 6.5 years
    c. How long will it take for the car to be worth \$10,000? Round your answer to the nearest tenth of a year. 9.2 years

11. The original price of a car is entered into spreadsheet cell A1 and the length of time in years it takes to totally depreciate is entered into cell B1.
    a. Write the spreadsheet formula that calculates the amount that the car depreciates each year. \( =A1/B1 \)
    b. The spreadsheet user is instructed to enter a length of time in years that is within the car’s lifetime in cell C1. Write the spreadsheet formula that will calculate the car’s value after that period of time. \( =(-A1/B1)*C1+A1 \)

12. The original price of a car is entered into spreadsheet cell A1 and the annual depreciation amount in cell B1.
    a. Write the spreadsheet formula to determine the number of years it will take for the car to totally depreciate. \( =A1/B1 \)
    b. The spreadsheet user is instructed to enter a car value in cell D1. Write the spreadsheet formula to compute how long it will take for the car to depreciate to that value. \( =(D1-A1)/(B1) \)
    c. The spreadsheet user is instructed to enter a percent into cell E1. Write the spreadsheet formula to compute the length of time it will take for the car to decrease by that percent. \( =((100-E1)/100)*A1-A1)/(B1) \)

13. Winnie purchased a new car for \$54,000. She has determined that it straight line depreciates to zero over 10 years. When she purchased the car, she made an \$8,000 down payment and financed the rest with a 4-year loan at 4.875%.
    You can use the monthly payment formula from the last chapter to determine the monthly payment to the nearest cent. Depreciation: \( y = -450x + 54,000 \)
    a. Create an expense and depreciation function. Expense: \( y = 1,056.74x + 8,000 \)
    b. Graph these functions on the same axes. See additional answers.
    c. Interpret the region before, at, and after the intersection point in light of the context of this situation. See Margin

14. Milo’s car straight line depreciates monthly over time. He knew that after 7 months his car was worth \$26,930. According to an online car value calculator, after 30 months, he determined that his car was worth \$19,800.
    How much does his car depreciate each month? \$310
Driving a brand new car feels like riding around in an open billfold with the dollars flapping by your ears as they fly out the window.

—Grey Livingston

How Does Your Car Lose Its Value?

In the previous lesson, you examined the automobile depreciation where the car lost the same amount of dollar value each year. That may not always be the case. You can often get a good idea of how a car loses its value by looking at prices from the past. This information is known as historical data, and the devaluation of a car when using this type of data is called historical depreciation.

There are many websites that list the prices of used cars. One well-known site is Kelley Blue Book (kbb.com). Before the Internet, the Kelley Blue Book was an actual blue book of historical car prices updated yearly that could be used to determine the current value of a used car. Today, the website gives the same information in a much easier-to-access format.

Examine the data for a used Chevrolet Corvette two-door coupe in good condition. The table shows the age of the car in years and the value of the car at that time. The prices quoted are for cars with similar usage for their age and offered for sale in the same geographic location.

<table>
<thead>
<tr>
<th>Age</th>
<th>Value ($)</th>
<th>Age</th>
<th>Value ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24,230</td>
<td>6</td>
<td>15,245</td>
</tr>
<tr>
<td>2</td>
<td>22,355</td>
<td>7</td>
<td>14,075</td>
</tr>
<tr>
<td>3</td>
<td>20,645</td>
<td>8</td>
<td>13,100</td>
</tr>
<tr>
<td>4</td>
<td>18,070</td>
<td>9</td>
<td>12,325</td>
</tr>
<tr>
<td>5</td>
<td>16,265</td>
<td>10</td>
<td>11,525</td>
</tr>
</tbody>
</table>

The scatter plot of this data is shown. Notice that it is not linear, but rather appears to be curved. The car values seem to have a greater drop at the beginning of the car’s lifetime and less as each year passes. Notice that the depreciation is not constant from year to year. This scatter plot models an exponential decay function. Rather than the value decreasing by the same dollar amount each year, it decreases by the
The extent to which the exponential depreciation model fits the historical data varies from situation to situation. Here you learn how to determine and use an exponential depreciation model.

**EXAMPLE 1**

Each number in List 2 is a percentage of the number that precedes it. The exponential regression formula calculates that percentage and assigns it to $b$. The graph would be closer to the data points.

**SOLUTION**

The exponential depreciation function (or equation) can be determined using exponential regression calculated by hand, by computer software, or by a graphing calculator. When you use the statistics feature on a graphing calculator, the data is entered into two lists as shown. (Note that only 7 of the 10 data are shown on the calculator screen.) The independent variable is the age of the car, and the dependent variable is the car value.

The exponential regression equation is displayed in the graphing calculator screen at the bottom. Notice that the general form of the exponential regression equation used by the calculator is slightly different than the one introduced on the previous page. For ease of use, the numbers are rounded here to the nearest hundredth. Using the format $y = ab^x$, where $a = 25,921.87$ and $b = 0.92$, the exponential depreciation function is $y = 25,921.87(0.92)^x$. The graph of this function, superimposed over the scatter plot, appears to be a good fit.

### Check Your Understanding

How might a better-fitting exponential depreciation equation look when superimposed over the same scatter plot? The graph would be closer to the data points.
EXAMPLE 2

What is the depreciation percentage for the 10 years of car prices as modeled by the exponential depreciation equation found in Example 1?

SOLUTION The exponential decay function was introduced as $y = A(1 - r)^t$. The graphing calculator uses the format $y = ab^x$. The functions are identical given that $b = 1 - r$.

Use the equation and solve for $r$.

$\frac{b}{1} = 1 - r$

Subtract 1 from each side.

$\frac{b - 1}{1} = 1 - r - 1$

Simplify.

$\frac{b - 1}{-1} = -r$

Divide each side by -1.

$\frac{1}{b - 1} = \frac{-1}{-1}$

Simplify.

$1 - b = r$

Since $b$ is approximately 0.92, then $1 - 0.92 = 0.08$. The Corvette depreciated by about 8% per year.

EXAMPLE 3

Eamon purchased a 4-year-old car for $16,400. When the car was new, it sold for $23,000. Find the depreciation rate to the nearest tenth of a percent.

SOLUTION Let $r$ equal the depreciation rate expressed as a decimal. The exponential depreciation formula for this situation is $16,400 = 23,000(1 - r)^4$. Notice that the variable $r$ is in the base of an exponential expression. To solve for $r$, you must first isolate that expression.

Use the exponential depreciation formula.

$16,400 = 23,000(1 - r)^4$

Divide each side by 23,000.

$\frac{16,400}{23,000} = \frac{23,000(1 - r)^4}{23,000}$

Simplify.

$\frac{16,400}{23,000} = (1 - r)^4$

To solve for $r$, you need to undo the exponent of 4 to which the expression $1 - r$ has been raised by raising each side of the equation to the reciprocal of 4, or $\frac{1}{4}$.

To simplify a power raised to an exponent, multiply the exponents. The exponent on the right side of the equation is 1.

$\left(\frac{16,400}{23,000}\right)^{\frac{1}{4}} = (1 - r)^{\frac{4}{4}}$

$\left(\frac{16,400}{23,000}\right)^{\frac{1}{4}} = 1 - r$

Simplify.

$\left(\frac{16,400}{23,000}\right)^{\frac{1}{4}} = 1 - r$

Subtract 1 from each side.

$\left(\frac{16,400}{23,000}\right)^{\frac{1}{4}} - 1 = - r$
Chapter 4
Automobile Ownership

Example 4
This example lends itself to a graphical solution. The algebraic solution is beyond the scope of this course. The intersection of the exponential depreciation equation and the expense equation yields the point at which the values are the same.

Point out that the original statement of the expense equation uses time in months. Therefore, the monthly payment of $400 must be converted to a yearly payment of $4,800 in order that both equations can be graphed on the same axes.

A car originally sells for \( D \) dollars. After \( A \) years, the value of the car has dropped exponentially to \( P \) dollars. Write an algebraic expression for the exponential depreciation rate expressed as a decimal.

\[
1 - \left( \frac{P}{D} \right)^\frac{1}{A}
\]

Example 4
A car originally sold for $26,600. It depreciates exponentially at a rate of 5.5% per year. When purchasing the car, Richard put $6,000 down and pays $400 per month to pay off the balance. After how many years will his car value equal the amount he paid to date for the car?

SOLUTION This problem is similar to Example 6 in Section 4-5. To find the solution, you need to set up both an expense equation and a depreciation equation. The exponential depreciation equation is

\[
y = 26,600(1 - 0.055)^x
\]

where \( x \) represents time in years.

The expense equation is

\[
y = 400x + 6,000
\]

where \( x \) represents the number of months that have passed.

To graph these two equations on the same axes, the independent variable in each equation must represent the same unit of time.

If you let \( x \) represent time in years, then to make the expense equation work, you need to determine the yearly payment rather than the monthly payment. Over the course of the year, Richard will have paid 400(12), or $4,800, in car payments.

The new yearly expense equation is

\[
y = 4,800x + 6,000
\]

where \( x \) is time in years.

Use the equations to determine an appropriate viewing window to use on your graphing calculator. Find the coordinates of the point of intersection.

After approximately 3.3 years (about 40 months), Richard will have paid about $22,022.74 toward his loan payments and the car will have a value of that same amount.
EXAMPLE 5

A car exponentially depreciates at a rate of 6% per year. Akiya purchased a 5-year-old car for $18,000. What was the original price of the car when it was new?

SOLUTION

Use the exponential depreciation equation.

\[ y = A(1 - r)^x \]

Substitute 18,000 for \( y \), 0.06 for \( r \), and 5 for \( x \).

\[ 18,000 = A(1 - 0.06)^5 \]

Simplify.

\[ 18,000 = A(0.94)^5 \]

Divide each side by \((0.94)^5\).

\[ \frac{18,000}{(0.94)^5} = \frac{A(0.94)^5}{0.94^5} \]

Simplify and calculate to the nearest cent.

\[ 24,526.37 = A \]

The original price of this car was approximately $24,526.37.

EXAMPLE 6

Leah and Josh bought a used car valued at $20,000. When this car was new, it sold for $24,000. If the car depreciates exponentially at a rate of 8% per year, approximately how old is the car?

SOLUTION

You need to solve for the variable \( x \) in the exponential depreciation equation

\[ y = A(1 - r)^x \]

In Chapter 2, you learned that solving for an exponent requires the use of logarithms. The length of time, \( x \), can be found as follows.

\[ y = A(1 - r)^x \]

Divide both sides by \( A \).

\[ \frac{y}{A} = \frac{A(1 - r)^x}{A} \]

Simplify.

\[ \frac{y}{A} = (1 - r)^x \]

Use the One-to-One Property of logarithms to take the log of both sides of the equation.

\[ \log\left(\frac{y}{A}\right) = \log((1 - r)^x) \]

Use the Power Property of logarithms.

\[ \log\left(\frac{y}{A}\right) = x\log(1 - r) \]

Divide both sides by \log\((1 - r)\).

\[ \frac{\log\left(\frac{y}{A}\right)}{\log(1 - r)} = \frac{x\log(1 - r)}{\log(1 - r)} \]

Simplify.

\[ \frac{\log\left(\frac{y}{A}\right)}{\log(1 - r)} = x \]

CHECK YOUR UNDERSTANDING

Answer

Approximately $31,176.59
Because $y$ equals the value of the car after $x$ years, $y = 20,000$. The new car price, $A$, is $24,000$. The variable $r$ represents the depreciation rate expressed as a decimal. Therefore, $r = 0.08$.

Substitute and calculate: \[
\frac{\log \left( \frac{20,000}{24,000} \right)}{\log (1 - 0.08)} = x
\]

\[
2.19 = x
\]

At the time of the purchase, the car was about 2.19 years old.

**Example 7**

This is the first time that a geometric sequence has been introduced. It is important to develop the common ratio by first looking to see if the sequence has a common difference. Once the common difference test fails, students then look to see if the common ratio applies as the relationship between terms. Before working through the problem, you might want to have students create and share arithmetic and geometric sequences, identifying the common differences and ratios where applicable.

How old would the car in Example 4 be had it been purchased at half its original value?

**Solution**

According to an online company that supplies used car history reports, a certain make and model car has the following values based on its age. The original price of the car was $24,000.

$24,000 \quad 21,600 \quad 19,440 \quad 17,496 \quad 15,746.40 \quad 14,171.76$

If the pattern continues, what would the car be worth when it is 6 years old?

**Solution**

This problem can be modeled by examining the sequence of values. In Section 2-3, you learned that consecutive terms of an arithmetic sequence have a common difference. Since cars that straight line depreciate do so at the same amount each year, the sequence of those depreciated car prices would form an arithmetic sequence. Look for a common difference between consecutive terms in the car values above.

\[
24,000 - 21,600 = 2,400 \\
21,600 - 19,440 = 2,160
\]

Stop right here. The difference between the first two terms and the next two terms is not the same. So the sequence of car values is not arithmetic and therefore this is not a straight line depreciation situation. To check whether the car depreciates exponentially, you can try to model the values by a geometric sequence, which is also called a geometric progression. In a geometric sequence, there is a relationship between any two consecutive numbers but it is not one determined by a common difference. Rather, each term can be multiplied by the same number to yield the term that follows it. This is called the common ratio, since the ratio of a term to the one that comes before it is the same for the entire geometric sequence. As with arithmetic sequences, subscripts are used to indicate where in the sequence a specific term is. The first term is denoted $a_1$, and the $n^{th}$ term is denoted $a_n$. The common ratio is denoted by $r$. A geometric sequence can be finite, which means it ends, or infinite, which means it never ends. Here are two examples of geometric sequences:

3, 6, 12, 24, 48, 96... The common ratio is 2. $\frac{6}{3} = \frac{12}{6} = \frac{24}{12} = \frac{48}{24} \ldots = 2$

100, 50, 25, 12.5, 6.25 The common ratio is 0.5. $\frac{50}{100} = \frac{25}{50} = \frac{12.5}{25} = \frac{6.25}{12.5} = 0.5$
In algebraic terms,
\[ a_n = a_1 r^{n-1} \]

Thus, \( a_2 = a_1 r \), \( a_3 = a_1 r^2 \), \( a_4 = a_1 r^3 \), and so on.

Look at the pattern that emerges. \( a_n = a_1 r^{n-1} \)

That is the formula for generating the terms of any geometric sequence if you know the starting value and the common ratio.

Examine the sequence of car values.

\[
\begin{align*}
&\$24,000 \quad \$21,600 \quad \$19,440 \quad \$17,496 \quad \$15,746.40 \quad \$14,171.76 \\
& a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6
\end{align*}
\]

We need to determine if the ratio of the car values is common to all consecutive prices.

\[
\frac{21,600}{24,000} = \frac{19,440}{21,600} = \frac{17,496}{19,440} = \frac{15,746.40}{17,496} = \frac{14,171.76}{15,746.40}
\]

Yes it does!

The common ratio is 0.9. Therefore the general term for this geometric sequence is

\[ a_n = a_1 (0.9)^{n-1} \]

Notice that \( a_6 = \$14,171.76 \), which represents the value of the car at 5 years of age. Therefore the seventh entry in this geometric sequence will represent the value of the car at 6 years of age. The value of the 6-year-old car would be represented by \( a_7 \). According to the formula,

\[
\begin{align*}
& a_7 = a_1 (0.9)^{7-1} \\
& a_7 = 24,000(0.9)^6 \\
& a_7 = 24,000(0.531441) = 12,754.584
\end{align*}
\]

The value of the 6-year-old car would be $12,754.58.

Compare the general formula for the terms of a geometric sequence with the exponential depreciation equation.

\[
\text{geometric sequence formula} \quad a_n = a_1 r^{n-1} \quad \text{where} \quad r \quad \text{is the common ratio}
\]

\[
\text{exponential depreciation formula} \quad Y = A(1 - r)^x \quad \text{where} \quad r \quad \text{is the depreciation rate}
\]

By knowing the common ratio for the car values in Example 7, you can determine the depreciation rate by equating the two bases.

\[
\begin{align*}
0.9 &= 1 - r \\
0.9 - 1 &= 1 - r - 1 \\
-0.1 &= -r
\end{align*}
\]

Divide both sides by \(-1\).

\[ 0.1 = r \]

Since \( r = 0.1 \) in the context of exponential depreciation, the depreciation rate is 10%. The car values listed above showed a 10% depreciation from year to year.

The original price of a car is $20,000. The car’s yearly depreciation is shown by the first four terms of this geometric sequence. $20,000 \quad $17,200 \quad $14,792 \quad $12,721.12. What is the depreciation rate for this car? What will the car be worth when it is 5 years old?
1. Automobile depreciation is a given. As we drive, our cars lose monetary value. Livingston likens it to an open billfold in the wind. Money keeps flying out of it.

Driving a brand new car feels like riding around in an open billfold with the dollars flapping by your ears as they fly out the window.
—Grey Livingston,

ANSWERS

9a.

9b. $y = 17,895.97(0.89)^x$

10a.

10b. $y = 42,228.36(0.89)^x$

1. How might the quote apply to what you have learned? See margin.

2. Seamus bought a car that originally sold for $40,000. It exponentially depreciates at a rate of 7.75% per year. Write the exponential depreciation equation for this car. $y = 40,000(1 - 0.0775)^x$

3. Shannon’s new car sold for $28,000. Her online research indicates that the car will depreciate exponentially at a rate of 5.25% per year. Write the exponential depreciation formula for Shannon’s car. $y = 28,000(1 - 0.0525)^x$

4. Chris purchased a used car for $19,700. The car depreciates exponentially by 10% per year. How much will the car be worth after 6 years? Round your answer to the nearest penny. $10,469.39$

5. Laura’s new car cost her $21,000. She was told that this make and model depreciates exponentially at a rate of $0.085$ per year. How much will her car be worth after 100 months? $9,903.32$

6. Luisa purchased a used car for $D$ dollars. The car depreciates exponentially at a rate of $E$% per year. Write an expression for the value of the car in 5 years, in $A$ years, and in $M$ months. $D(1 - \frac{E}{100})^5; D(1 - \frac{E}{100})^A; D(1 - \frac{E}{100})^{\frac{M}{12}}$.

7. A graphing calculator has determined this exponential regression equation based on car value data: $y = a\cdot b^x$, $a = 20,952.11$, and $b = 0.785$. What is the rate of depreciation for this car? How much is this car worth after 6 years; 78 months; $w$ years? 21.5%; $4,902.82; $4,343.91; $20,952.11 \times 0.785^w$

8. A graphing calculator has determined this exponential regression equation based on car value data: $y = a\cdot b^x$, $a = 18,547.23$, and $b = 0.8625$. What is the rate of depreciation for this car? How much is this car worth after 6 years; 78 months; $w$ months? 13.75%; $7,635.43; $7,091.09; $18,547.23 \times 0.8625^{\frac{w}{12}}$

9. The historical prices of a car are recorded for 11 years as shown.
   a. Construct a scatter plot for the data. See margin.
   b. Determine the exponential depreciation equation that models this data. Round to the nearest hundredth. See margin.
   c. Determine the depreciation rate. Approx. 11%
   d. Predict the value of this car after 3½ years. $11,902.01$

10. The historical values of a car are recorded for 17 years as shown.
   a. Construct a scatter plot for the data. See margin.
   b. Determine the exponential depreciation formula that models this data. Round to the nearest hundredth. See margin.
   c. Determine the depreciation rate. Approx. 11%
   d. Predict the value of this car after 140 months. $10,843.12$

11. Raphael purchased a 3-year-old car for $16,000. He was told that this make and model depreciates exponentially at a rate of 5.45% per year. What was the original price value of the car when it was new? $18,929.34$
12. The car that Diana bought is 8 years old. She paid $6,700. This make and model depreciates exponentially at a rate of 14.15% per year. What was the original price of the car when it was new? $22,706.62

13. Chaz bought a 2-year-old car. He paid $D$ dollars. This make and model depreciates at a rate of $E$ percent per year. Write an expression for the original selling price of the car when it was new.

\[
\frac{D}{1 - \left(1 - \frac{E}{100}\right)^x}
\]

14. What is the exponential depreciation rate, expressed as a percent to the nearest tenth of a percent, for a car that originally sells for $30,000 when new but exponentially depreciates after 5 years to $18,700? 9%

15. What is the exponential depreciation rate, expressed as a percent to the nearest tenth of a percent, for a car that originally sells for $52,000 when new but exponentially depreciates to $45,000 after 32 months? 5.3%

16. A new car sells for $27,300. It exponentially depreciates at a rate of 6.1% to $22,100. How long did it take for the car to depreciate to this amount? Round your answer to the nearest tenth of a year. 3.4 years

17. Amber bought a used car valued at $16,000. When this car was new, it was sold for $28,000. If the car depreciates exponentially at a rate of 9% per year, approximately how old is the car? 5.9 years

18. A car originally sold for $25,900. It depreciates exponentially at a rate of 8.2% per year. Nina put $10,000 down and pays $550 per month. After how many years will her car value equal the amount she paid for the car to that point? What will that value be? $y = 25900(1 - .082)^x; y = 6600x + 10000. 1.83 years and $22,131.10

19. Chantel's car originally sold for $46,600. It depreciates exponentially at a rate of 10.3% per year. Chantel put $12,000 down and pays $800 per month to pay off the balance. After how many years will her car value equal the amount she paid to date for the car? What will that value be? $y = 46000(1 - .103)^x; y = 9600x + 12000; 2.5 years; $35,651.67

20. When sold as a new car in the 1950s, the price of a specific classic car was $13,074. It depreciated in value over the next few years. Then, in 1977, something interesting began to happen. As seen in this table of values.

<table>
<thead>
<tr>
<th>Year</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1977</td>
<td>$6,500</td>
</tr>
<tr>
<td>1987</td>
<td>$10,500</td>
</tr>
<tr>
<td>1997</td>
<td>$22,500</td>
</tr>
<tr>
<td>2007</td>
<td>$47,800</td>
</tr>
<tr>
<td>2017</td>
<td>$94,000</td>
</tr>
</tbody>
</table>

a. Construct a scatter plot for the data. Let 1977 be Year 1, 1987 be Year 11, 1997 be Year 21, and so on. What do you notice about the trend?

b. Find an exponential regression equation that models this situation. Round the numbers to the nearest hundredth. See margin.

c. What kind of a rate has been used? What is the value of that rate to the nearest tenth of a percent? Exponential appreciation rate; 7%

21. A car vehicle price history for a certain make and model contains the following list of yearly price values:

$21,000 $18,900 $17,010 $15,309 $13,778.1 $12,400.29

The original price of the car was $21,000. It exponentially depreciated to $18,900 after 1 year and continued depreciating by the same percentage each year thereafter. What will the value of the car be after 8 years? $9,039.81

22. Examine this geometric sequence of depreciating car values:

$20,000 $18,400 $16,928 $15,573.76

If the original price of the car was $20,000, what is the depreciation rate? What will the car be worth after 6 years? $12,127.10
4-7 Driving Data

Is it sufficient that you have learned to drive the car, or shall we look and see what is under the hood?
Most people go through life without ever knowing.
—June Singer, analyst and writer

Objectives
• Write, interpret, and use the distance formula.
• Use the formula for the relationship between distance, fuel economy, and gas usage.

Key Terms
odometer
trip odometer
speedometer
fuel economy measurement
miles per gallon (mpg)
kilometers per liter (km/L)
English Standard System
metric system
distance formula
currency exchange rate

Warm-Up
The volume of a cone is
\[ V = \frac{1}{3} \pi r^2 h, \]
where \( r \) is the radius and \( h \) is the height.
1. Solve the formula for the radius, \( r \).
2. Solve the formula for the height, \( h \).

EXAMINE THE QUESTION
This is a very important question that will help you understand what students see as significant driving information. Newer cars give a variety of dashboard data that are helpful in knowing about average speed, time traveled, fuel efficiency, temperature, and so on. This question goes beyond the dashboard numbers and asks students to come up with other data that might at some time save their lives.

CLASS DISCUSSION
Based on the equivalencies stated here, which is a greater distance—a mile or a kilometer? If a sign read “100 miles to the Canadian border,” would the number of kilometers be greater than 100 or less than 100?

What Data Are Important to a Driver?
The dashboard of an automobile is an information center. It supplies data on fuel, speed, time, and engine-operating conditions. It can also give information on the inside and outside temperature. Many cars now have a camera that shows what’s behind you when the car is in reverse. Some cars have a navigation system built into the car’s electronics, as well as optional Internet access. This can help the driver find destinations or map out alternate routes. Your cell phone can be wirelessly connected to your car so that you can use your phone hands-free. There have been many advances in the information that the driver has available to make trips safer, smarter, and more energy efficient.

The **odometer** indicates the distance a car has traveled since it left the factory. All automobiles have either an electronic or mechanical odometer. Some dashboard odometers can give readings in both miles and kilometers. An electronic odometer gives the readings digitally. In older cars, a mechanical odometer consists of a set of cylinders that turn to indicate the distance traveled. Many cars also have a **trip odometer**, which can be reset at the beginning of each trip. The trip odometer gives you the accumulated distance traveled on a particular trip. The **speedometer** tells you the rate at which the car is traveling. The rate, or speed, is reported in miles per hour (mi/h or mph) or kilometers per hour (km/h or kph).

Drivers are concerned not only with distance traveled and speed, but also with the amount of gasoline used. Gasoline is sold by the gallon or the liter. Over the past 20 years, the price of gasoline has changed dramatically.
Economizing on fuel is a financial necessity. Car buyers are usually interested in fuel economy measurements. These are calculated in miles per gallon (mi/g or mpg) or kilometers per liter (km/L). In order to understand these fuel economy measurements, it is necessary to have a good sense of distances in both the English Standard System of measurement used in the United States and the metric system of measurement used in most countries throughout the world.

A mile equals 5,280 feet. A meter is a little more than 39 inches. Driving distances are not reported in feet or meters, but in miles and kilometers. A kilometer is equal to 1,000 meters. Miles and kilometers can be compared as follows.

1 kilometer = 0.621371 mile
1 mile = 1.60934 kilometers

The distance from Seattle, Washington, to Vancouver, British Columbia, is about 176 kilometers, or 110 miles. Miles per gallon is a unit of measurement that gives the number of miles a car can be driven on one gallon of gas. A car that gets 28 mpg can travel about 28 miles on one gallon. A car that gets 11.9 km/L can travel about 11.9 kilometers on one liter. There are about 3.8 liters in a gallon and 0.26 gallons in a liter. When shopping for a new car, always ask for the fuel estimate.

Skills and Strategies

A smart automobile owner is aware that a working knowledge of driving data can help reduce the costs of automobile ownership. Here you learn how to use and interpret driving data.

EXAMPLE 1

A car travels at an average rate of speed of 50 miles per hour for 6 hours. How far does this car travel?

SOLUTION The distance that a car travels is a function of its speed and the time traveled. This relationship is shown in the distance formula.

\[ D = R \times T \]

where \( D \) represents the distance traveled, \( R \) represents the rate at which the car is traveling, and \( T \) is the time in hours.

Substitute 50 for \( D \) and 6 for \( T \).

\[ D = 50 \times 6 \]

Calculate.

\[ D = 300 \]

The car travels 300 miles.

A car is traveling at \( R \) miles per hour for \( M \) minutes. Write an algebraic expression for the distance traveled.

CHECK YOUR UNDERSTANDING

Answer \( R \frac{M}{60} \)
EXAMPLE 2

Byron lives in New York and will be attending college in Atlanta, Georgia. The driving distance between the two cities is 883 miles. Byron knows that the speed limits on the highways he will travel vary from 50 mi/h to 65 mi/h. He figures that he will average about 60 mi/h on his trip. At this average rate, for how long will he be driving? Express your answer rounded to the nearest tenth of an hour and to the nearest minute.

SOLUTION

Use the distance formula. \[ D = R \times T \]

Divide each side by \( R \).

\[ \frac{D}{R} = \frac{R \times T}{R} \]

Simplify. \[ \frac{D}{R} = T \]

Substitute 883 for \( D \) and 60 for \( R \).

\[ \frac{883}{60} = T \]

Calculate. \[ 14.7166666667 \]

The answer is a nonterminating, repeating decimal, as indicated by the bar over the digit 6. The time rounded to the nearest tenth of an hour is 14.7 hours.

If you are using a calculator and the display reads 14.71666667, the calculator has rounded the last digit, but it stores the repeating decimal in its memory. Because you know that the exact time is between 14 and 15 hours, use only the decimal portion of the answer. Once the answer is on the calculator screen, subtract the whole-number portion.

\[ 14.7166666667 - 14 = 0.7166666667 \]

The number of sixes displayed will depend on the accuracy of your calculator. There are 60 minutes in an hour, so multiply by 60.

\[ 0.7166666667 \times 60 = 43 \]

The decimal portion of the hour is 43 minutes. Jack will be driving for 14 hours and 43 minutes.

CHECK YOUR UNDERSTANDING

Answer 24 hours and 9 minutes

Danielle drove from Atlanta, Georgia, to Denver, Colorado, which is a distance of 1,401 miles. If she averaged 58 miles per hour on her trip, how long is her driving time to the nearest minute?
EXAMPLE 3
Kate left Albany, New York, and traveled to Montreal, Quebec. The distance from Albany to the Canadian border is approximately 176 miles. The distance from the Canadian border to Montreal, Quebec, is approximately 65 kilometers. If the entire trip took her about 3¾ hours, what was her average speed for the trip?

SOLUTION Kate’s average speed can be reported in miles per hour or kilometers per hour. To report her speed in miles per hour, convert the entire distance to miles. To change 65 kilometers to miles, multiply by the conversion factor 0.621371.

\[
65 \times 0.621371 = 40.389115
\]

The distance from the Canadian border to Montreal is approximately 40.4 miles. Kate’s total driving distance is the sum of the distances from Albany to the Canadian border and from the Canadian border to Montreal.

\[
176 + 40.4 = 216.4 \text{ miles}
\]

Now, solve for the rate. Let \( D = 216.4 \) and \( T = 3.75 \).

Use the distance formula.

\[
D = R \times T
\]

Divide each side by \( T \).

\[
\frac{D}{T} = \frac{R \times T}{T}
\]

Simplify.

\[
\frac{D}{T} = R
\]

Substitute 216.4 for \( D \) and 3.75 for \( T \).

\[
\frac{216.4}{3.75} = R
\]

Calculate.

\[
57.7 \approx R
\]

Kate traveled at approximately 58 miles per hour.

Follow the same reasoning to determine her speed in kilometers per hour. To change the portion of the trip reported in miles to kilometers, multiply 176 by the conversion factor 1.60934.

\[
176 \times 1.60934 = 283.2
\]

There are approximately 283.2 kilometers in 176 miles.
The distance from Albany to Montreal is 283.2 + 65, or 348.2 kilometers.

Let \( D = 348.2 \) and \( T = 3.75 \) in the distance formula.

\[
\frac{348.2}{3.75} = R
\]

\[
92.853\ldots \approx R
\]

Kate traveled approximately 93 kilometers per hour.

In Example 3 above, could Kate’s km/h have been calculated by multiplying her miles per hour by the conversion factor? Explain your answer.
EXAMPLE 4
This example provides an opportunity to show how to utilize the distance formula in another way. Students need to understand that the rate they are using in this problem is not a speed. It is a comparison of distance to gallons of fuel rather than distance to time.

CHECK YOUR UNDERSTANDING
Answer \( \frac{500}{g} \)

EXAMPLE 5
You can take this opportunity to discuss using a car for business. Students will likely think that the company will reimburse Barbara the money she spent on gasoline. You can explain that businesses reimburse on a set rate per mile driven. Lead the discussion so that students understand using your car for business costs more than just the money you spend for gasoline.

EXAMPLE 5
Juan has a hybrid car that averages 40 miles per gallon. His car has a 12-gallon tank. How far can he travel on one full tank of gas?

SOLUTION The distance traveled can also be expressed as a function of the fuel economy measurement and the number of gallons used.

\[
\text{Distance} = \text{miles per gallon} \times \text{gallons}
\]

\[
\text{Distance} = \text{kilometers per liter} \times \text{liters}
\]

Therefore, the distance that Juan can travel on one tank of gas is the product of his miles per gallon and the tank size in gallons.

\[
\text{Distance} = 40 \times 12 = 480 \text{ miles}
\]

When traveling at an average rate of 40 mpg, one full tank of gas in Juan's hybrid car can take him 480 miles.

Lily drove a total of 500 miles on \( g \) gallons of gas. Express her fuel economy measurement in miles per gallon as an algebraic expression.

EXAMPLE 5
When Barbara uses her car for business, she must keep accurate records so that she will be reimbursed for her car expenses. When she started her trip, the odometer read 23,787.8. When she ended the trip it read 24,108.6. Barbara's car gets 32 miles per gallon. Her tank was full at the beginning of the trip. When she filled the tank, it cost her $24.06. What price did she pay per gallon of gas on this fill-up?

SOLUTION Begin by computing the distance Barbara traveled. Find the difference between her ending and beginning odometer readings.

\[
24,108.6 - 23,787.8 = 320.8
\]

Barbara traveled 320.8 miles.

Since Barbara's car gets 32 mpg, you can determine the number of gallons of gas used on the trip with the formula

\[
D = M \times G
\]

where \( D \) is the distance traveled, \( M \) is the miles per gallon, and \( G \) is the number of gallons used.

Use the formula. \( D = M \times G \)

Substitute 320.8 for \( D \) and 32 for \( M \).

\[
320.8 = 32G
\]

Divide each side by 32.

\[
\frac{320.8}{32} = \frac{32G}{32}
\]

Simplify.

\[
\frac{320.8}{32} = G
\]

Calculate.

\[
10.025 = G
\]
EXAMPLE 6

David is driving in Mexico on his vacation. He notices that gas costs 14.25 Mexican pesos per liter. What is this equivalent to in U.S. dollars?

**SOLUTION** David must find the current currency exchange rate. The currency exchange rate is a number that expresses the price of one country's currency calculated in another country's currency. Up-to-date exchange rates are available on the Internet. Currency codes are often used when referring to exchange rates. The currency code for U.S. dollars is USD and the currency code for Mexican pesos is MXN.

David needs to know what 1 USD is worth in MXN. For the time of his travel, 1 USD = 18.66 MXN. Divide the value of the foreign currency paid for gas by the exchange rate.

\[
\frac{14.25}{18.66} = 0.76
\]

Each liter would cost him about 76 cents U.S. He knows there are approximately 3.8 liters in a gallon, so he can multiply 0.76 \times 3.8 to determine the equivalent gas price if it was purchased with U.S. dollars per gallon.

The price of 14.25 Mexican pesos per liter is approximately $2.89 per gallon.

On a trip through Canada, Angie noticed that the average price of gas per liter was 1.28 Canadian dollars. If 1 USD is equivalent to approximately 1.39 Canadian dollars (CAD), what is the equivalent gas price per gallon in USD?

**CHECK YOUR UNDERSTANDING**

Answer: Approx. $3.50 per gallon

Students may have difficulty understanding when to multiply and when to divide to convert properly. Walking through this example using dimensional analysis will help them visualize the conversion process, because they can see how the units cancel.

Barbara used 10.025 gallons of gas on this trip. If her total gas bill was $24.06, divide this total amount by the number of gallons used to get the price per gallon paid.

\[
\frac{24.06}{10.025} = \text{price per gallon}
\]

Barbara paid $2.40 per gallon for this fill-up.
EXAMPLE 7
Example 6 converts Mexican pesos per liter to U.S. dollars per gallon. Example 7 converts U.S. dollars per gallon to Mexican pesos per liter. Explaining to students that these are inverse operations may help them to more easily understand the conversions.

EXAMPLE 7
David knows that the price of gas in his hometown is about $2.30 per gallon. How can he compare this price to the price paid in Example 6 for a liter?

SOLUTION
David needs to express the U.S. gas price as a price in USD per liter. There are approximately 3.8 liters in a gallon. Divide the price per gallon by 3.8 to determine the price per liter in USD.

\[2.30 \div 3.8 = 0.61\]

His hometown gas price is equivalent to about 0.61 USD per liter. So gas is more expensive in Mexico, $0.61 < $0.76.

To compare the prices in pesos, multiply the amount in USD by the exchange rate. Exchange rate was 18.66.

\[0.61 \times 18.66 = 11.38\]

The gas in his hometown would sell for about 11.38 MXN. Just as the comparison in USD showed, the comparison in MXN shows that at this particular time, gas is more expensive in Mexico because 11.38 < 14.25.

CHECK YOUR UNDERSTANDING

Answer Approx. $0.92 per liter

In the Example 6 Check Your Understanding, Angie knew that the price of gas in her hometown was $2.50 per gallon. What is the equivalent price in Canadian dollars per liter?
Is it sufficient that you have learned to drive the car, or shall we look and see what is under the hood? Most people go through life without ever knowing.
—June Singer, analyst and writer

1. How might the quote apply to what you have learned? See margin.

2. Carlo travels for 3 hours on the highway. His average speed is 55 mi/h. How far does he travel? 165 miles

3. Yolanda is planning a 778-mile trip to visit her daughter in Maryland. She plans to average 50 miles per hour. At that speed, approximately how long will the trip take? Express your answer to the nearest tenth of an hour. Then express your answer to the nearest minute. 15.6 hr; 15 h, 36 min

4. Steve's SUV has a 17-gallon gas tank. The SUV gets an estimated 24 miles per gallon. Approximately how far can the SUV run on half a tank of gas? 204 miles

5. Becky is planning a 2,100-mile trip to St. Louis to visit a college. Her car averages 30 miles per gallon. About how many gallons will her car use on the trip? 70

6. Robbie's car gets $M$ miles per gallon. Write an algebraic expression that represents the number of gallons he would use when traveling 270 miles. $\frac{270}{M}$

7. Michael used his car for business last weekend. When he reports the exact number of miles he traveled, the company will pay him 52 cents for each mile. At the beginning of the weekend, the odometer in Michael's car read 74,902.6 miles. At the end of the weekend, it read 75,421.1 miles.
   a. How many miles did Michael drive during the weekend? 518.5 miles
   b. How much money should his company pay him for the driving? $269.62$

8. Lenny's car gets approximately 20 miles per gallon. He is planning a 750-mile trip.
   a. About how many gallons of gas should Lenny plan to buy? 37.5 gallons
   b. At an average price of $2.20 per gallon, how much should Lenny expect to spend for gas? $82.50$

9. Francoise's car gets about 11 kilometers per liter. She is planning a 1,200-kilometer trip.
   a. About how many liters of gas should Francoise plan to buy? Round your answer to the nearest liter. 109 liters
   b. At an average price of $0.89 per liter, how much should Francoise expect to spend for gas? $97.01$

10. Nola's car gets approximately 42 miles per gallon. She is planning to drive $x$ miles to visit her friends.
    a. What expression represents the number of gallons of gas she should expect to buy? $\frac{x}{42}$
    b. At an average price of $2.38 per gallon, write an expression for the amount that Nola will spend for gas. $\frac{2.38x}{42}$

11. Jason uses his car for business. He must keep accurate records so his company will reimburse him for his car expenses. When he started his trip, the odometer read 42,876.1. When he ended the trip, it read 43,156.1. Jason's car gets 35 miles per gallon. His tank was full at the beginning of the trip. When he filled the tank, it cost $17.20. What price did he pay per gallon of gas on this fill-up? $2.15$

ANSWERS

1. Answers will vary. Although this analyst is probably using a car as a metaphor, she does infer that it isn't enough to just know how to drive a car but to know all of the important information about the car as well.

TEACH

Exercise 7
Remind students that the distance traveled must be a positive number.

Exercises 8 and 9
These problems do not require students to convert between the two systems.
12. Complete the chart for entries a–l.

<table>
<thead>
<tr>
<th>Number of gallons purchased</th>
<th>Price per gallon</th>
<th>Total gas cost</th>
<th>Number of people in carpool</th>
<th>Gas cost per person</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>$1.99</td>
<td>a. $19.90</td>
<td>4</td>
<td>g. $4.98</td>
</tr>
<tr>
<td>12</td>
<td>$2.08</td>
<td>b. $24.96</td>
<td>5</td>
<td>h. $4.99</td>
</tr>
<tr>
<td>17</td>
<td>$2.15</td>
<td>c. $36.55</td>
<td>3</td>
<td>i. $12.18</td>
</tr>
<tr>
<td>26</td>
<td>$2.30</td>
<td>d. $59.80</td>
<td>6</td>
<td>j. $9.97</td>
</tr>
<tr>
<td>15</td>
<td>D</td>
<td>e. $15D</td>
<td>4</td>
<td>k. $4 $15D</td>
</tr>
<tr>
<td>17</td>
<td>G</td>
<td>f. $GP</td>
<td>C</td>
<td>l. $GP</td>
</tr>
</tbody>
</table>

13. Alexandra uses her car for business. She knows that her tank was full when she started her business trip, but she forgot to write down the odometer reading at the beginning of the trip. When the trip was over, the odometer read 13,020.5. Alexandra’s car gets 25 miles per gallon. When she filled up the tank with gas that cost $2.45 per gallon, her total bill for the trip was $35.28. Determine Alexandra’s beginning odometer reading. 12,713.25

14. Bill left Burlington, Vermont, and traveled to Ottawa, Ontario, the capital of Canada. The distance from Burlington to the Canadian border is approximately 42 miles. The distance from the Canadian border to Ottawa is approximately 280 kilometers. If it took him 4.3 hours to complete the trip, what was his average speed in miles per hour? 50 mph

15. A car averages 56 mi/h on a trip.
   a. Write an equation that shows the relationship between distance, rate, and time for this situation. \( D = 56T \)
   b. Let time be the independent variable and distance be the dependent variable. Draw and label the graph of this equation. See additional answers.
   c. Use the graph to determine approximately how far this car would travel after 14 hours. Approx. 800 miles.
   d. Use the graph to determine the approximate length of time a 500-mile trip would take. Approx. 9 hours

16. A spreadsheet has been started for the user to enter information in the cells noted below.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Starting odometer reading in A1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Ending odometer reading in A2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Fuel efficiency measure in mpg in A3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Duration of trip in A4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Gas price per gallon in A5</td>
<td></td>
</tr>
</tbody>
</table>

   a. Write a formula to calculate the speed of the car for the trip in cell C1.
   b. Write a formula to calculate the number of gallons of gas used in cell C2.
   c. Write a formula to calculate the total cost of gas for the trip in cell C3.

Use the following information to complete Exercises 17–22. Round all answers to two decimal places.

- 1 USD = 1.37 Canadian dollars (CAD)
- 1 USD = 113.59 Japanese yen (JPY)
- 1 USD = 0.90 euros (EUR)
- 1 USD = 15.38 South African rands (ZAR)
- 1 USD = 1.40 Australian dollars (AUD)
- 1 USD = 0.99 Swiss franc (CHF)
17. Complete the chart. See margin.

<table>
<thead>
<tr>
<th>USD</th>
<th>CAD</th>
<th>EUR</th>
<th>AUD</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.80</td>
<td>a.</td>
<td>b.</td>
<td>c.</td>
</tr>
<tr>
<td>15.75</td>
<td>d.</td>
<td>e.</td>
<td>f.</td>
</tr>
<tr>
<td>20.00</td>
<td>g.</td>
<td>h.</td>
<td>i.</td>
</tr>
<tr>
<td>178.50</td>
<td>j.</td>
<td>k.</td>
<td>l.</td>
</tr>
<tr>
<td>250.00</td>
<td>m.</td>
<td>n.</td>
<td>p.</td>
</tr>
<tr>
<td>5500.00</td>
<td>q.</td>
<td>r.</td>
<td>s.</td>
</tr>
</tbody>
</table>

18. Complete the chart.

<table>
<thead>
<tr>
<th>Foreign Currency</th>
<th>USD Equivalent</th>
<th>Foreign Currency</th>
<th>USD Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>85 CAD</td>
<td>a.</td>
<td>130 CHF</td>
<td>d.</td>
</tr>
<tr>
<td>1000 EUR</td>
<td>b.</td>
<td>222 ZAR</td>
<td>e.</td>
</tr>
<tr>
<td>500 AUD</td>
<td>c.</td>
<td>36 JPY</td>
<td>f.</td>
</tr>
</tbody>
</table>

19. Reid will be driving through Spain this summer. He did some research and knows that the average price of gas in Spain is approximately 1.21 euros per liter.
   a. What is this amount equivalent to in U.S. dollars? **Approx. $1.34 USD/L**
   b. What is this rate equivalent to in U.S. dollars per gallon? **Approx. $5.09 USD/gal**

20. Shyla will be driving through South Africa. She has found that the average price of gas in Johannesburg is about 0.82 South African rands per liter.
   a. What is this amount equivalent to in U.S. dollars? **Approx. $0.05 USD**
   b. What is this rate equivalent to in U.S. dollars per gallon? **Approx. $0.19 USD per gallon**

21. Brenda will be driving through Europe. She plans to pay an average price of $\text{h}$ euros per liter for gasoline.
   a. What is this amount equivalent to in U.S. dollars? **$\frac{3.8h}{0.69}$ USD/L**
   b. What is this rate equivalent to in U.S. dollars per gallon? **$\frac{3.8h}{0.69}$ USD/gal**

22. While Willie traveled in India, he paid an average of 65.73 Indian rupees (INR) for a liter of gas.
   a. What expression represents the price of this gas in U.S. dollars if the exchange rate was $\frac{3.8}{x}$ USD/INR?
   b. What is this rate equivalent to in U.S. dollars per gallon? **$\frac{3.8(65.73)}{x}$ USD/gal**
   c. If Willie spent about $90, how many gallons of gas did he buy? **$90 \div \frac{3.8(65.73)}{x}$**
   d. If Willie spent about $90, how many liters of gas did he buy? **$90 \div \frac{65.73}{x}$**
4-8 Driving Safety Data

Nowhere in this country should we have laws that permit drinking and driving or drinking in vehicles that are on American highways. This is not rocket science. We know how to prevent this. . . .

—Byron Dorgan, U.S. senator

Objectives

- Calculate reaction time and distance in the English Standard System.
- Calculate and use the braking distance in both the English Standard and metric systems.
- Calculate and use the total stopping distance in both the English Standard and metric systems.

Key Terms

- reaction time
- reaction distance
- braking distance
- total stopping distance

EXAMINE THE QUESTION

In the last lesson, students used data to become more informed drivers. In this lesson, a working knowledge of mathematics is important for students to have their beliefs tested about the interaction between distance, rate, and time for safety purposes.

Warm-Up

Write each function in two equivalent ways. You can partially or fully expand the function.

a. \( f(x) = (x + 1)(x + 1) \)
   \( f(x) = (x + 1)^2 \)

b. \( g(x) = (x + 1)^2(x - 1); g(x) = (x^2 - 2x + 1)(x - 1); \)
   \( g(x) = x^3 - 3x^2 + 3x - 1 \)

CLASS DISCUSSION

After reviewing the definitions of reaction time, reaction distance, and braking distance, elicit from students their best guess as to how long it takes to react, how far the car travels in that time, and how far the car travels when the brakes are applied before it stops.

How Can You Use Mathematics to Become a Safer Driver?

Although a dashboard can give you much information about the car’s ability to go, it gives less information about the car’s ability to stop. It takes time to stop a moving car safely. Even during the time your foot switches from the gas pedal to the brake pedal, the car continues to travel.

The average, alert driver takes from approximately three-quarters of a second to 1½ seconds to switch from the gas pedal to the brake pedal. This time is the reaction time. During the reaction time, the car travels a greater distance than most people realize. That distance is the reaction distance. The distance a car travels while braking to a complete stop once the brakes are applied is the braking distance. Braking distance is also affected by road conditions and the condition of the car. Most people think they can stop on a dime. In reality, that is far from the truth. Take a look at these facts:

- There are 5,280 feet in a mile.
- A car traveling 55 mi/h covers 55 miles in 1 hour.
- A car traveling 55 mi/h covers 55 \( \times 5,280 \) or 290,400 feet in 1 hour.
- A car traveling 55 mi/h covers 290,400 \( \div 60 \) or 4,840 feet in 1 minute.
- A car traveling 55 mi/h covers 4,840 \( \div 60 \) or 80.67 feet in 1 second.

Suppose that your reaction time is about 1 second. That is, it takes you 1 second from the time you realize that you have to brake to the time you actually apply your foot to the brake pedal. When traveling at 55 mi/h, in that 1 second of time, you travel about 81 feet.
By thinking about these facts, you can understand how speeding, tailgating, texting while driving, and driving while intoxicated can cost you in damages or even lives!

**Skills and Strategies**

Here you learn how to make driving decisions based on reaction and braking distances.

**EXAMPLE 1**

What is the reaction distance for a car traveling approximately 48 miles per hour?

**SOLUTION**
The reaction distance is the approximate distance covered in the time it takes an average driver to switch from the gas pedal to the brake pedal.

Research has determined that the average driver takes from 0.75 to 1.5 seconds to react.

A car traveling at 55 mi/h travels about 81 feet per second.

Let \( x \) = the distance traveled during the driver’s reaction time of 0.78 seconds.

Write a proportion.

\[
\frac{81}{1} = \frac{x}{0.75}
\]

Multiply each side by 0.75.

\[
\frac{81}{1} \times 0.75 = \frac{x}{0.75} \times 0.75
\]

Simplify.

\( 81 \times 0.75 = x \)

Calculate.

\( 60.75 = x \)

Let \( x \) = the distance traveled during a driver’s reaction time of 1.5 seconds.

Write a proportion.

\[
\frac{81}{1} = \frac{x}{1.5}
\]

Multiply each side by 1.5.

\[
\frac{81}{1} \times 1.5 = \frac{x}{1.5} \times 1.5
\]

Simplify.

\( 81 \times 1.5 = x \)

Calculate.

\( 121.5 = x \)

If the average person’s reaction time ranges from 0.75 to 1.5 seconds, the average person’s reaction distance when traveling at 55 mi/h ranges from 60.75 to 121.5 feet. That’s quite a span in the short time it takes for a person to apply the brakes.

The reaction distances and times are used to give you a sense of how far the car will go during the reaction time. A conservative rule of thumb for the reaction distance is that a car travels about 1 foot for each mile per hour of speed.

Therefore, a car traveling at 48 mi/h has a reaction distance of approximately 48 feet.

A car is traveling at 65 mi/h. Approximately how far will it travel during the average reaction time?

**CHECK YOUR UNDERSTANDING**

**Answer** 65 feet
EXAMPLE 2
Impress upon students the need for the correct placement of parentheses. Here, there is a big difference between 0.1s^2 and (0.1s)^2.

CHECK YOUR UNDERSTANDING
Answer 211.25 ft. Road conditions such as rain, snow, and ice affect the efficiency of the car’s brakes. Tire pressure, driver impairment, and other factors greatly affect the braking distance of a car.

EXAMPLE 3
Total stopping distance takes into account the reaction distance and the braking distance. It is the sum of both amounts and indicates how far a car will go (on average) from the moment that a driver realizes the need to stop to the moment that the car actually comes to a complete stop.

CHECK YOUR UNDERSTANDING
Answer Using the formula, the total stopping distance is 276.25 feet.

EXAMPLE 2
What is the approximate braking distance for a car traveling at 48 mi/h?

SOLUTION The general formula for the braking distance is

\[
\frac{s^2}{20}
\]

where \(s\) represents the speed of the car. The formula can also be expressed as

\[
(0.1 \times s)^2 \times 5.
\]

Notice the four expressions below are equivalent.

\[
(0.1 \times s)^2 \times 5 = \left(\frac{1}{10} \times s\right)^2 \times 5 = \left(\frac{s}{10}\right)^2 \times 5 = \frac{5s^2}{100} = \frac{s^2}{20}
\]

Each of the expressions yields the braking distance when \(s = 48\).

\[
\frac{s^2}{20} = \frac{48^2}{20} = 115.2 \quad \text{or} \quad (0.1 \times s)^2 \times 5 = (0.1 \times 48)^2 \times 5 = 115.2
\]

Once the brakes are applied, on average, a car traveling at 48 mi/h will come to a complete stop after the car has traveled approximately 115.2 feet. This is the braking distance. Keep in mind that this is only the distance the car traveled from the moment the brakes were applied.

Check Your Understanding

What is the approximate braking distance of a car traveling at 65 mph? What factors also need to be taken into account that might add to or subtract from the braking distance?

EXAMPLE 3
Rachel is driving 48 mi/h on a one-lane highway. She sees an accident directly ahead of her about 200 feet away. Will she be able to stop in time?

SOLUTION The total stopping distance from the moment a driver realizes the need to stop to the time that the car is no longer moving is the sum of the reaction distance and the braking distance.

\[
\text{Total stopping distance} = \text{Reaction distance} + \text{Braking distance}
\]

Since the reaction distance of a car traveling at \(s\) miles per hour is approximated by using a distance of \(s\) feet, the formula can be represented by either of the following.

\[
s + (0.1 \times s)^2 \times 5 \quad \text{or} \quad s + \frac{s^2}{20}
\]

Rachel's total stopping distance is \(48 + 115.2 = 163.2\) feet. The accident is 200 feet away, so she should be able to stop in time.

Check Your Understanding

What is the total stopping distance for a car traveling at 65 mi/h?
EXAMPLE 4

Desireé is traveling through Canada. The speedometer in her rented car indicates kilometers per hour, and all of the road signs give distances in kilometers. She knows that one kilometer is equal to 1,000 meters and one meter is a little more than 3 feet. Determine Desireé’s total stopping distance in meters if she is traveling 88 kilometers per hour.

**SOLUTION** Since 1 kilometer = 0.6213712 miles, 88 kilometers per hour can be expressed in miles per hour by multiplying 88 by the conversion factor.

\[
88 \times 0.621371 = 54.680648
\]

88 km/h = 54.68 mi/h

Evaluate the total stopping distance formula \( s + (0.1 \times s)^2 \times 5 \) when \( s = 54.68 \).

\[
s + (0.1 \times s)^2 \times 5 = 54.68 + (0.1 \times 54.68)^2 \times 5 = 204.17512 \text{ feet}
\]

There are approximately 0.3048 meters in 1 foot.

Multiply the stopping distance in feet by this conversion factor.

\[
204.17512 \times 0.3048 = 62.23 \text{ meters}
\]

The approximate stopping distance of Desireé’s car is 62.23 meters.

Notice that this gives an answer that has been determined through various stages of rounding since you used rounded versions of answers and conversion factors along the way.

There is a formula that can be used to determine the total stopping distance directly. Let \( s \) represent the speed in kilometers per hour.

\[
\text{Total stopping distance in meters} = \frac{s^2}{170} + \frac{s}{5}
\]

Substitute \( s = 88 \).

\[
\frac{88^2}{170} + \frac{88}{5} \approx 63.15 \text{ meters}
\]

Notice that the two answers, 62.23 meters and 63.15 meters, are very close to each other.

A car is traveling at 78 km/h. What is the total stopping distance in meters? Round your answer to the nearest hundredth of a meter.

CHECK YOUR UNDERSTANDING

**Answer** Approx. 51.39 meters

**Extend Your Understanding**

Toni’s car is traveling 75 km/h. Randy’s car is behind Toni’s car and is traveling 72 km/h. Toni notices a family of ducks crossing the road 50 meters ahead of her. Will she be able to stop before she reaches the ducks? What is the least distance that Randy’s car can be from Toni’s car to avoid hitting her car, if he reacts as soon as he sees her brakes?

**Answer** Yes; 3.2 meters
Applications

Nowhere in this country should we have laws that permit drinking and driving or drinking in vehicles that are on American highways. This is not rocket science. We know how to prevent this. . . .

—Byron Dorgan, U.S. senator

1. Explain how the quote can be interpreted from what you have learned. See margin.

2. There are 5,280 feet in a mile. Round answers to the nearest unit.
   a. How many miles does a car traveling at 65 mi/h go in 1 hour? 65
   b. How many feet does a car traveling at 65 mi/h go in 1 hour? 343,200
   c. How many feet does a car traveling at 65 mi/h go in 1 minute? 5,720
   d. How many feet does a car traveling at 65 mi/h go in 1 second? 95

3. There are 5,280 feet in a mile. Round answers to the nearest unit.
   a. How many miles does a car traveling at 42 mi/h go in 1 hour? 42
   b. How many feet does a car traveling at 42 mi/h go in 1 hour? 221,760
   c. How many feet does a car traveling at 42 mi/h go in 1 minute? 3,696
   d. How many feet does a car traveling at 42 mi/h go in 1 second? 62
   e. How many miles does a car traveling at x mi/h go in 1 hour? x
   f. How many feet does a car traveling at x mi/h go in 1 hour? 5,280x
   g. How many feet does a car traveling at x mi/h go in 1 minute? See margin.
   h. How many feet does a car traveling at x mi/h go in 1 second? See margin.

4. Determine the distance covered by a car traveling 80 km/h for each unit and time given. Round answers to the nearest unit.
   a. kilometers in 1 hour 80
   b. meters in 1 hour 80,000
   c. meters in 1 minute 1,333
   d. meters in 1 second 22

5. Determine the distance covered by a car traveling 55 km/h for each unit and time given. Round answers to the nearest unit.
   a. kilometers in 1 hour 55
   b. meters in 1 hour 55,000
   c. meters in 1 minute 917
   d. meters in 1 second 15

6. Determine the distance covered by a car traveling x km/h for each unit and time given.
   a. kilometers in 1 hour x
   b. meters in 1 hour 1,000x
   c. meters in 1 minute See margin.
   d. meters in 1 second See margin.

7. Mindy is driving 32 mi/h as she nears an elementary school. A first-grade student runs into the street after a soccer ball, and Mindy reacts in about three-quarters of a second. What is her approximate reaction distance? 32 feet

8. Determine the distance covered by a car traveling 68 mi/h for each unit and time given. Round answers to the nearest unit.
   a. miles in 1 hour 68
   b. feet in 1 hour 359,040
   c. feet in 1 minute 5,984
   d. feet in 1 second 100
9. Edward is driving 52 mi/h on a one-lane road. He must make a quick stop because there is a stalled car ahead.
   a. What is his approximate reaction distance? 52 feet
   b. What is his approximate braking distance? 135.2 feet
   c. About how many feet does the car travel from the time he switches pedals until the car has completely stopped (total stopping distance)? 187.2 feet

10. Complete the chart for entries a–j. See margin.

<table>
<thead>
<tr>
<th>Speed</th>
<th>Reaction Distance</th>
<th>Braking Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 mi/h</td>
<td>a.</td>
<td>f.</td>
</tr>
<tr>
<td>30 mi/h</td>
<td>b.</td>
<td>g.</td>
</tr>
<tr>
<td>20 mi/h</td>
<td>c.</td>
<td>h.</td>
</tr>
<tr>
<td>15 mi/h</td>
<td>d.</td>
<td>i.</td>
</tr>
<tr>
<td>5 mi/h</td>
<td>e.</td>
<td>j.</td>
</tr>
</tbody>
</table>

11. David is driving on the highway at the legal speed limit of 70 mi/h. He notices that there is an accident up ahead approximately 200 feet away. His reaction time is approximately \( \frac{3}{4} \) of a second. Is he far enough away to bring the car safely to a complete stop? Explain your answer. See margin.

12. Martine is driving on an interstate at 70 km/h. She sees a traffic jam about 50 meters ahead and needs to bring her car to a complete stop before she reaches that point. Her reaction time is approximately \( \frac{3}{4} \) of a second. Is she far enough away from the traffic jam to safely bring the car to a complete stop? Explain. See margin.

13. Model the total stopping distance by the equation \( y = \frac{x^2}{20} + x \), where \( x \) represents the speed in miles per hour and \( y \) represents the total stopping distance in feet.
   a. Graph this equation for the values of \( x \), where \( x \leq 70 \) mi/h. See additional answers.
   b. Use the graph to approximate the stopping distance for a car traveling at 53 mi/h. About 190 feet
   c. Use the graph to approximate the speed for a car that stops completely after 70 feet. About 28 mi/hr

14. Model the total stopping distance by the equation \( y = \frac{x^2}{170} + \frac{x}{5} \), where \( x \) represents the speed in km/h and \( y \) represents the total stopping distance in meters.
   a. Graph this equation for the values of \( x \), where \( x \leq 100 \) km/h. See additional answers.
   b. Use the graph to approximate the stopping distance for a car traveling at 60 km/h. About 33 m
   c. Use the graph to approximate the speed for a car that stops completely after 60 meters. About 85 km/h

15. A spreadsheet user inputs a speed in miles per hour into cell A1.
   a. Write a formula that would enter the approximate equivalent of that speed in km/h in cell A2. \(-A1*1.60934\)
   b. Write a spreadsheet formula that would enter the approximate total stopping distance in feet in cell A3. \(=A1^2/20 + A1\)
   c. Write a spreadsheet formula that would enter the approximate total stopping distance in kilometers in cell A4. \(=(A2^2)/170 + A2/5\)
**4-9 Accident Investigation Data**

**It takes 8,460 bolts to assemble an automobile, and one nut to scatter it all over the road.**

—Author Unknown

**Objectives**
- Determine the minimum skid speed using the skid mark formula.
- Determine the minimum skid speed using the yaw mark formula.

**Key Terms**
- accident reconstructionist
- skid mark
- shadow skid mark
- antilock braking system (ABS)
- yaw mark
- skid speed formula
- drag factor
- braking efficiency
- skid distance

**Warm-Up**

Let \( f(x) = (x - 2)/7 \). Let \( g(x) = 5/x \). Solve the equation \( f(x) = g(x) \).

**What Data Might a Car Leave Behind at the Scene of an Accident?**

Auto accidents happen. Many times it is clear who is at fault, but that may not always be the case. When fault is uncertain, it is up to the authorities to get detailed and accurate information from witnesses and each of the parties involved. It may be necessary to examine the data that was left behind at the scene. That data is interpreted by accident reconstructionists, who have knowledge of both crime-scene investigations and mathematics that can help them understand the circumstances surrounding the accident.

Reconstructionists pay very close attention to the marks left on the road by the tires of a car. A **skid mark** is a mark that a tire leaves on the road when it is in a locked mode, that is, when the tire is not turning but the car is continuing to move. When the driver first applies the brakes, the skid mark is light and is a **shadow skid mark**. This mark darkens until the car comes to a complete stop either on its own or in a collision.

Cars with an **antilock brake system (ABS)** do not allow the wheels to continuously lock. In cars equipped with this feature, the driver feels a pulsing vibration on the brake pedal and that pedal moves up and down when the brake is fully engaged. The skid marks left by a car with ABS look like uniform dashed lines on the pavement. A driver without ABS may try to simulate that effect by pumping the brakes. The skid marks left by these cars are also dashed, but they are not uniform in length.
When a car enters a skid and the brakes lock (or lock intermittently), the driver cannot control the steering. Therefore, the skid is usually a straight line. The vehicle continues to move straight ahead as the brakes lock, making the tire marks straight. When the vehicle slips sideways while at the same time continuing in a forward motion, the tire marks appear curved. These are called yaw marks.

Taking skid and yaw measurements, as well as other information from the scene, can allow reconstructionists to compute the speed of the car when it entered the skid. The formulas used are often presented in court and are recognized for their strength in modeling real-world automobile accidents.

Skills and Strategies

Here you learn how to use the skid and yaw formulas to examine the circumstances surrounding an automobile accident.

The skid speed formula is

\[ S = \sqrt{30 \cdot D \cdot f \cdot n} = \sqrt{30Dfn} \]

where \( S \) is the speed of the car when entering the skid, \( D \) is the skid distance, \( f \) is the drag factor, and \( n \) is the braking efficiency.

Before using the equation, it is important that you understand its component parts. The number 30 is a constant; it is part of the equation and does not change from situation to situation. Simply put, the drag factor is the pull of the road on the tires. It is a number that represents the amount of friction that the road surface contributes. Many accident reconstructionists perform drag factor tests with a piece of equipment known as a drag sled. The table lists acceptable ranges of drag factors for the road surfaces.

<table>
<thead>
<tr>
<th>Road Surface</th>
<th>Drag Factor Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement</td>
<td>0.55–1.20</td>
</tr>
<tr>
<td>Asphalt</td>
<td>0.50–0.90</td>
</tr>
<tr>
<td>Gravel</td>
<td>0.40–0.80</td>
</tr>
<tr>
<td>Snow</td>
<td>0.10–0.55</td>
</tr>
<tr>
<td>Ice</td>
<td>0.10–0.25</td>
</tr>
</tbody>
</table>

The skid distance is a function of the number and lengths of the skid marks left at the scene. If there are four marks of equal length, then that amount is used. But, if the lengths are different or there are fewer than four skid marks, then the average of the lengths is used in the formula. If there is only one skid mark, that length is used.

Finally, you need to know the braking efficiency of the car. This number is determined by an examination of the rear and front wheel brakes. It can vary from 0% efficiency (no brakes at all) to 100% efficiency (brakes are in excellent condition). The braking efficiency value is expressed as a decimal when used in the formula.
EXAMPLE 1
Make sure students realize that there are many factors other than drag, brake efficiency, and road surface that can affect skid speed. But this formula is recognized by many law enforcement personnel as a good means of making an approximate determination.

EXAMPLE 2
Melissa was traveling at 50 mi/h on a concrete road with a drag factor of 1.2. Her brakes were working at 90% efficiency. To the nearest tenth of a foot, what would you expect the average length of the skid marks to be if she applied her brakes in order to come to a full stop?

SOLUTION
You are asked to find the skid distance given the speed, the drag factor, and the braking efficiency.

Use the skid speed formula.

\[ S = \sqrt{30Dfn} \]

Substitute 50 for \( D \), 0.78 for \( f \) and 1.0 for \( n \).

\[ S = \sqrt{30 \times 50 \times 0.78 \times 1.0} \]

Simplify.

\[ S = \sqrt{1,872} \]

Take the square root.

\[ S \approx 43.3 \]

The car was traveling at approximately 43.3 miles per hour. The driver was exceeding the speed limit and could be fined.

CHECK YOUR UNDERSTANDING
Answer \( \sqrt{\frac{30xyz}{100}} \)

Check Your Understanding

EXAMPLE 1
A car is traveling on an asphalt road with a drag factor of 0.78. The speed limit on this portion of the road is 35 mi/h. The driver just had his car in the shop and his mechanic informed him that the brakes were operating at 100% efficiency. The driver must make an emergency stop when he sees an obstruction in the road ahead of him. His car leaves four distinct skid marks each 80 feet in length. What is the minimum speed the car was traveling when it entered the skid? Round your answer to the nearest tenth. Was the driver exceeding the speed limit when entering the skid?

SOLUTION
Determine the car speed.

Use the skid speed formula.

\[ S = \sqrt{30Dfn} \]

Substitute 80 for \( D \), 0.78 for \( f \) and 1.0 for \( n \).

\[ S = \sqrt{30 \times 80 \times 0.78 \times 1.0} \]

Simplify.

\[ S = \sqrt{1,872} \]

Take the square root.

\[ S \approx 43.3 \]

The car was traveling at approximately 43.3 miles per hour. The driver was exceeding the speed limit and could be fined.

A portion of road has a drag factor of \( x \). A car with a \( y \) percent braking efficiency is approaching a traffic jam ahead, causing the driver to apply the brakes for an immediate stop. The car leaves four distinct skid marks of \( z \) feet each. Write an expression for determining the minimum speed of the car when entering into the skid.

EXAMPLE 2
In this example, students are asked to find the value of a variable under the square root sign. It is therefore necessary for them to have a working understanding of how to undo the effect of a square root by squaring both sides of the equation. Before attempting the problem solution, you may want to have students do some numerical examples of undoing a square root by squaring.

Melissa was traveling at 50 mi/h on a concrete road with a drag factor of 1.2. Her brakes were working at 90% efficiency. To the nearest tenth of a foot, what would you expect the average length of the skid marks to be if she applied her brakes in order to come to a full stop?

SOLUTION
You are asked to find the skid distance given the speed, the drag factor, and the braking efficiency.

Use the skid speed formula.

\[ S = \sqrt{30Dfn} \]

Substitute 50 for \( S \), 1.2 for \( f \) and 0.9 for \( n \).

\[ S = \sqrt{30 \times 50 \times 1.2 \times 0.9} \]

Simplify the expression under the radical.

\[ 50 = \sqrt{324D} \]

It is necessary to solve for a variable that is under a radical sign. To undo the square root, square both sides.

\[ (50)^2 = (\sqrt{324D})^2 \]

Simplify.

\[ 2,500 = 324D \]

Divide each side by 32.4.

\[ \frac{2,500}{32.4} = \frac{324D}{32.4} \]

Simplify.

\[ \frac{77.2}{1} = D \]

Under the given conditions, you would expect the average of the skid marks to be approximately 77.2 feet.
Neil is traveling on a road at $M$ miles per hour when he slams his foot on the brake pedal in order to avoid hitting a car up ahead. He is traveling on a gravel road with a drag factor of $A$ and his brakes are operating at 100% efficiency. His car leaves three skid marks of length $x$, $y$, and $z$, respectively. Write an algebraic expression that represents the drag factor, $A$.

**Yaw Marks**

Examine how the minimum speed can be determined from the data available by measuring the yaw marks. If $S$ is the minimum speed, $f$ is the drag factor, and $r$ is the radius of the arc of the yaw mark, the most basic formula is

$$S = \sqrt{15fr}$$

To identify a radius, you must be able to find the center of the circle of which the arc is part. Here is how reconstructionists do that. First, they select two points on the outer rim of the arc and connect them with a chord. A **chord** is the line segment that connects two points on an arc or circle as shown.

The center of the chord is located and a perpendicular line segment is drawn from that center to the arc, creating a right angle. This short line segment is the **middle ordinate**.

Reconstructionists use the following formula to determine the radius.

$$r = \frac{C^2}{8M} + \frac{M}{2}$$

where $r$ is the radius of the yaw arc, $C$ is the length of the chord, and $M$ is the length of the middle ordinate.

**EXAMPLE 3**

An accident reconstructionist took measurements from yaw marks left at a scene. Using a 43-foot-long chord, she determined that the middle ordinate measured approximately 4 feet. The drag factor for the road surface was determined to be 0.8. Determine the radius of the curved yaw mark to the nearest tenth of a foot. Determine the minimum speed that the car was going when the skid occurred to the nearest tenth.

**SOLUTION** Solve for $r$ by substituting 43 for $C$ and 4 for $M$ in the equation.

$$r = \frac{C^2}{8M} + \frac{M}{2}$$

$$r = \frac{43^2}{8 \cdot 4} + \frac{4}{2}$$

$$r = 59.8$$

The radius of the curve is approximately 59.8 feet.

Solve for $S$ by substituting $r = 59.8$ and $f = 0.8$ in the equation.

$$S = \sqrt{15fr}$$

$$S = \sqrt{15 \cdot 0.8 \cdot 59.8}$$

$$S = 26.8$$

The car entered the skid with an approximate minimum speed of 26.8 miles per hour.
Sir Isaac Newton (1643–1727) was one of the most influential physicists and mathematicians in history. According to Newton’s first law of motion, a body at rest tends to stay at rest and a body in motion tends to stay in motion unless acted on by an outside force. Here you examine an application of Newton’s first law, known as projectile motion. An important application of quadratic equations is projectile motion, a type of motion in which an object is thrown, shot, or ejected close to Earth’s surface and traces a path along a parabola.

**EXAMPLE 4**

An auto accessory maker wants to test a new type of roof strap. Cargo is secured to the roof of a car. The car is remotely operated and travels at 50 mi/h before hitting a concrete block head-on. It was hoped that the straps could withstand the force of the impact. The cargo was secured at a height of 5.5 feet above the ground. Unfortunately, as soon as the front of the car hit the concrete block, the straps snapped, and the cargo went “flying” off the top of the roof in the path illustrated below.

How long did it take the cargo to hit the ground? What horizontal distance will it travel?

**SOLUTION** The path the cargo takes when ejected off the top of the roof upon impact is known as the *trajectory*. Examine these two parabolic trajectories.

The trajectory on the left illustrates a situation where an object is thrown, shot, or ejected upward. The trajectory on the right occurs when the object is thrown, shot, or ejected horizontally from a certain height. In both cases, the force of gravity ultimately brings the object to the ground. The situation in this example is modeled by the half-parabola pictured on the right and by the two following quadratic equations that have been created using Newton’s laws.

Equation 1: Here the time that the cargo travels, \( t \), is the independent variable and the height, \( y \), at any given moment is the dependent variable since the

Determine the minimum speed of a car at the point the brakes are immediately applied to avoid a collision based on a yaw mark chord measuring 62.4 feet and a middle ordinate measuring 5 feet. The drag factor of the road surface is 1.2. Round your answer to the nearest tenth.

**CHECK YOUR UNDERSTANDING**

**Answer** Using both the radius and the skid speed formulas, the minimum skid speed is approximately 42.4 mph.

*The study of projectile motion extends far beyond the automobile examples given here. You might want to have students research other real-world applications where the projectile motion formulas are used.*

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height of the object is dependent on the amount of time that has passed since impact. The equation is for an object falling from a height of 5.5 feet.

\[ y = -16.1t^2 + 5.5 \]

Equation 2: Here the horizontal distance \( x \), traveled by the cargo is the independent variable and the height, \( y \), is the dependent variable.

\[ y = -0.003x^2 + 5.5 \]

To determine how long after impact it takes for the cargo to hit the ground, let \( y = 0 \) in the first equation. This makes sense because at that moment the cargo will have a height of 0.

\[ 0 = -16.1t^2 + 5.5 \]

Solve for \( t \). Subtract 5.5 from both sides of the equation.

\[ -5.5 = -16.1t^2 \]

Divide both sides by \(-16.1\).

\[ -\frac{5.5}{-16.1} = t^2 \]

Simplify. Round to the nearest thousandth.

\[ 0.3416 \approx t^2 \]

Solve for \( t \) by taking the square root of each side.

\[ \sqrt{0.3416} = \sqrt{t^2} \]

\[ \pm 0.58 \approx t \]

Since \( t \) represents time, disregard the negative solution. The cargo will hit the ground approximately 0.58 seconds after the head-on impact.

To find the horizontal distance that the cargo travels, determine the \( x \) value when \( y \) is equal to zero in equation 2. Since the cargo will be on the ground, it will have no height, so \( y = 0 \).

\[ y = -0.003x^2 + 5.5 \]

\[ 0 = -0.003x^2 + 5.5 \]

Solve for \( x \). Subtract 5.5 from both sides of the equation.

\[ -5.5 = -0.003x^2 \]

Divide both sides by \(-0.003\) and simplify to the nearest thousandth.

\[ 1,833.333 = x^2 \]

Take the square root of both sides of the equation. Round to the nearest thousandth.

\[ \sqrt{1,833.333} = \sqrt{x^2} \]

\[ \pm 42.8174 = x \]

Use only the positive answer since distance is positive in this situation. Round to the nearest whole number.

\[ 43 \approx x \]

The cargo traveled approximately 43 horizontal feet upon impact.

Had the car above been traveling at 35 mi/h before impact, the two projectile motion equations would be \( y = -16.1t^2 + 5.5 \) (this equation remains the same) and \( y = -0.006x^2 + 5.5 \). How long would it have taken the cargo to hit the ground? What horizontal distance would it have traveled?

Projectile motion equations can assist in accident reconstruction work. Newton’s laws definitely underscore the need for securely strapping down cargo on a roof, for all riders in a car wearing seat belts, and to avoid putting objects on the back window ledge.
TEACH

Exercises 2–4
These problems should be completed in order together. Students work through the numerical and then algebraic representation of tire skid mark distances.

ANSWERS

1. Clearly the quote is said in jest, but there is a great deal of truth in it. It highlights the fact that it is very easy to have an accident, and drivers should always be alert and aware.

5. The police were correct since according to the formula, Rona's minimum skid speed was approximately 39.7 miles per hour.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Enter the road surface drag factor in B1.</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Enter the braking efficiency as a decimal in B2.</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Enter the number of skid marks on the road in B3.</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Enter lengths of skid marks. If fewer than 4 skid marks, enter measures and zero in the remaining cell(s).</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Skid mark #1 – cell B5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Skid mark #2 – cell B6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Skid mark #3 – cell B7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Skid mark #4 – cell B8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Calculated skid distance</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Minimum skid speed</td>
<td></td>
</tr>
</tbody>
</table>

6. In the spreadsheet, the prompts for entering data are in column A. The user enters the data in column B.

a. Write the spreadsheet formula that will calculate the skid distance in cell B9: \( =\text{sum}(B5:B8)/B3 \)

b. Write the spreadsheet formula that will calculate the minimum skid speed in cell B10. The format for finding a square root in a spreadsheet is \( =\text{sqrt}(\text{number or expression}) \).

c. Verify the accuracy of your formula for the following input values: drag factor, 0.6; braking efficiency, 0.8; and two skid marks, 45.3 ft and 48.2 ft. 25.9 mi/h

7. Ravi was driving on an asphalt road with a drag factor of 0.75. His brakes were working at 85% efficiency. He hit the brakes in order to avoid a dog that ran out in front of his car. Two of his tires made skid marks of 36 ft and 45 ft, respectively. What was the minimum speed Ravi was going at the time he went into the skid? 27.8 mi/hr
8. A car leaves four skid marks each 50 feet in length. The drag factor for the road is 0.9. Let \( x \) represent the braking efficiency.
   a. What is the range of values that can be substituted for \( x \)? \( 0 \leq x \leq 1 \) (0% to 100%)
   b. Let the speed be represented by the variable \( y \) and \( x \) represent the braking efficiency. Write the skid speed equation in terms of \( x \) and \( y \). \( y = \sqrt{30 \cdot 50 \cdot 0.9x} = \sqrt{1350x} \)
   c. Graph the skid speed equation using the braking efficiency as the independent variable and the skid speed as the dependent variable. See additional answers.
   d. Use your graph to estimate the skid speed for braking efficiencies of 20%, 40%, 60%, 80%, and 100%. 20%: 16 mph; 40%: 23 mph; 60%: 28 mph; 80%: 33 mph; 100%: 37 mph

9. A car is traveling at 57 mi/h before it enters into a skid. The drag factor of the road surface is 1.1, and the braking efficiency is 100%. How long might the average skid mark be to the nearest tenth of a foot? 98.5 feet

10. Steve is driving at 35 mi/h when he makes an emergency stop. His brakes lock and his tires leave four skid marks of equal length. The drag factor for the road surface was 0.97 and his brakes were operating at 90% efficiency. How long might the skid marks be to the nearest foot? 47 feet

11. Marielle was in an accident. She was traveling down a road at 36 mi/h when she slammed on her brakes. Her tires left two skid marks that averaged 50 ft in length with a difference of 4 ft between them. Her brakes were operating at 80% efficiency at the time of the accident.
   a. What was the possible drag factor of this road surface? 1.08
   b. What were the lengths of each skid mark? 52 feet and 48 feet

12. An accident reconstructionist takes measurements of the yaw marks at the scene of an accident. What is the radius of the curve if the middle ordinate measures 4.8 feet when using a chord with a length of 42 ft? Round your answer to the nearest tenth of a foot. 48.3 feet

13. The measure of the middle ordinate of a yaw mark is 6 ft. The radius of the arc is 70 ft. What was the length of the chord used in this situation? Round the answer to the nearest tenth of a foot. 56.7 feet

14. The following measurements from yaw marks left at the scene of an accident were taken by law enforcement officers. Using a 31-ft-long chord, the middle ordinate measured approximately 3 ft. The drag factor for the road surface is 1.02.
   a. Determine the radius of the yaw mark to the nearest tenth of a foot. 41.5 feet
   b. Determine the minimum speed that the car was going when the skid occurred to the nearest tenth. 25.2 mi/h

15. Juanita is an accident reconstruction expert. She measured a 70-ft chord from the outer rim of the yaw mark on the road surface. The middle ordinate measured 9 ft in length. The drag factor of the road surface was determined to be 1.13.
   a. Determine the radius of the yaw mark to the nearest tenth of a foot. 72.6 feet
   b. Determine the minimum speed that the car was going when the skid occurred to the nearest tenth. 35.1 mi/h

16. The formula used to determine the radius of the yaw mark arc is derived from a geometric relationship about two intersecting chords in a circle. In the figure, chords \( \overline{AB} \) and \( \overline{CD} \) intersect at point \( E \) in the circle. The product of the two segment lengths making up chord \( \overline{AB} \), \( AE \times EB \), is equal to the product of the two segment lengths making up chord \( \overline{CD} \), \( CE \times ED \).

In the next figure, the yaw mark is continued as a dotted line to form a complete circle. A chord is drawn connecting two points on the yaw mark. The middle ordinate is also drawn. The length of the middle ordinate is \( M \) and the length of the chord is \( CD \). The middle ordinate cuts the chord into
two equal pieces with each half of the chord \( \frac{CD}{2} \) units in length. The radius of the circle has length \( r \) as shown in the diagram. Applying the property to the two intersecting chords in this diagram, you get \( AE \times EB = CE \times ED \).

**a.** From the diagram, \( CE = \frac{CD}{2}, ED = \frac{CD}{2}, \) and \( EB = M \). You need to determine the length of the segment \( AE \). Notice that \( AB = 2r \). (It is a diameter, which equals the length of two radii.) Also notice that \( AE = AB - EB \). Write an algebraic expression that represents the length of \( AE \).

**b.** Write the algebraic expression for the product of the segments of a chord that applies to this situation. Do not simplify.

**c.** Simplify the side of the equation that represents the product of the segments of chord \( CD \). Write the new equation.

**d.** Solve the equation for \( r \) by isolating the variable \( r \) on one side of the equation. Show your work. Compare your answer with the radius formula.

17. In the spreadsheet, the prompts for entering data are in column A. The user enters the data in column B.

**a.** Write the spreadsheet formula that will calculate the radius in cell B4. \( \sqrt{\frac{15 \times B1 \times B4}{2}} \)

**b.** Write the spreadsheet formula that will calculate the minimum skid speed in cell B5. The formula for finding that speed is found by taking the square root of the product of 15, the drag factor, and the radius. \( \sqrt{15 \times B1 \times B4} \)

**c.** Verify the accuracy of your formula for the following input values: drag factor, 0.97; chord length, 47 ft; and middle ordinate, 5 feet. Approx. 29 mi/h

18. Ghada works for an insurance company as an accident reconstruction expert. She measured a 52-ft chord from the outer rim of the yaw mark on the road surface. The middle ordinate measured \( x \) ft in length. The drag factor of the road surface was determined to be 1.05.

**a.** What is the expression for the radius of the yaw mark?

**b.** Determine the expression for the minimum speed that the car was going when the skid occurred.

19. Cargo is tied with rope on the roof of a 4.5-foot-tall car. The car travels down a road at 40 mi/h and hits a concrete barrier and the rope snaps, allowing the cargo to propel forward. Find the time it takes for the cargo to hit the ground and the horizontal distance the cargo travels. Let \( y \) represent height in feet, let \( x \) represent horizontal distance in feet, and let \( t \) represent time in seconds.

**Equation 1:** \( y = -16.1t^2 + 4.5 \) Approx. 0.53 seconds

**Equation 2:** \( y = -0.0047x^2 + 4.5 \) Approx. 30.94 feet

20. A minivan is traveling at 80.5 kilometers per hour. Cargo is strapped to the roof at a height of 1.75 meters. The car hits a concrete barrier, and the cargo is ejected from the roof. Use the following two equations to determine how long it takes for the cargo to hit the ground and how far it travels in the horizontal direction.

Let \( y \) represent height in meters, \( x \) represent horizontal distance in meters, and \( t \) represent time in seconds.

**Equation 1:** \( y = -4.9t^2 + 1.75 \) Approx. 0.6 seconds

**Equation 2:** \( y = -0.0081x^2 + 1.75 \) Approx. 14.7 meters
You Write the Story!!

The graph below illustrates the impact that both moderate and fast sales growth of electric vehicles around the world would be expected to have on automotive gasoline consumption.

Write a short news-type article centered on this graph. You can find an electronic copy at www.cengage.com/financial_alg2e. Copy it and paste it into your article.

Examine the equation below used for exponential depreciation calculations. Look through this chapter and your notes to help you write a problem that could be modeled by the equation.

$$19,777.84 = 24,800(1 - 0.055)^4$$

1. Go to a new car dealership. Pick out a car and make a list of the options you would order. Find the price of the car, the price of each option, and the total cost. Compute the sales tax and make a complete list of any extra charges for delivery. Report your findings to the class.

2. Pick any used car you would like to own. Make a list of the options you would like the car to have. Search for used car prices on the Internet and find out what the car is worth. Print pages that summarize the car and its value. Visit a local insurance agent, and find out the cost of insurance for the car. Display your findings on a poster.
3. Visit your local motor vehicle department website. Make a list of the forms needed to register a car and get license plates. If possible, get sample copies of each form. Show and explain each form to the class.

4. Pick a road trip you would like to make. Estimate the gas cost for a car of your choice. Get hotel prices for any overnight stays and estimate food expenses. Get the full cost of staying at your destination. Present all your data in a spreadsheet or PowerPoint presentation.

5. Talk to your teacher about having an insurance agent speak to your class. Have the class submit questions about automobile insurance. Provide a copy of questions to the agent before the talk.

6. Write an ad to sell a used car. Contact several newspapers to find the price of both a print and online ad for 1 week. Report your findings to the class.

7. Choose a new or used car that you would like to own. For that car, choose one of the following repair jobs: complete brake job or complete exhaust system replacement. Go to a garage or repair shop and get a price estimate for the job. Be sure to include all parts and labor. Then go to an auto supply store and find out what each of the parts would cost. Compare the garage or repair shop’s estimate of parts and labor to the cost of repairing the car yourself.

8. Interview a local auto insurance agent. Find out when premiums must be paid, the types of discounts offered, insurance that is mandatory in your state, optional insurance that is available, and any other questions you can think of. Summarize your interview in a report.

9. Go online and find the cost of renting a car of your choice for 2 weeks. Research mileage charges and drop-off fees. Pay particular attention to the loss damage waiver they offer. If you rent a car, you will be asked if you want to pay a loss damage waiver. This will reduce your liability for physical damage to the car. This type of insurance is expensive. Certain credit cards provide this coverage if you charge the rental on that card. Go online and get contact information for two credit card companies. Contact them and ask which of their cards includes coverage for the loss damage waiver for a rented car. Prepare your findings in a PowerPoint presentation.

10. Instead of buying a new automobile, some people like to lease their new cars. A lease is like a long-term rental. The lease agreement has many stipulations in it about allowable mileage, origination fees, early termination, gap insurance, and excess wear and tear. Go online and do research about the details of leasing a car. Find approximate costs for different models. Learn the advantages and disadvantages of leasing. Interview a relative, friend, or neighbor who leased their car and ask them about the leasing experience. Include your findings on a poster or in a slide show.

11. Flamboyant cars have graced movie and television screens for decades. Do some research to compile a list of famous cars. Give the make, model, and year of each car. Include information on where these cars are now and the highest price paid for each car as it changed owners. Add photos and other interesting facts about each car. Present your information on a poster or electronic slide show.
12. A nomograph, or nomogram, is a chart that graphs the relationships among three quantities. Nomographs have been used in many fields such as medicine, physics, information technology, geology, and more. One such nomograph charts the fuel economy relationship—distance is equal to the miles per gallon fuel consumption of a car times the number of gallons used. Research the creation and usages of nomographs and find one that relates to fuel economy. Write a short description of this nomograph, explaining how it works and how it can be helpful to drivers. Include an example of the nomograph.

13. For years, if you wanted to travel by car, you could either own, rent, lease, or hire a cab or a limousine. Recently, another option has become available in many cities. Car-sharing services have grown in popularity because they make traveling by car easier than ever before. Research three different car-sharing services. What plans do they offer? Do you need to become a member? If so, is there a membership fee? Who pays for insurance? Who pays for gas? Is there a mileage limit per trip? Once you have all of this information, compare prices for a 70-mile trip. Find some online reviews and include in your report what customers are saying about each service.

14. In this chapter you learned about automobile ownership and maintenance. Do you have a specific interest related to any of the topics discussed? For this project you will design your own project assignment based on your interests. Read all of the Reality Check projects to get a basic idea of what comprises a project assignment. When you have decided on a project topic and plan, download the project proposal form at www.cengage.com/financial_alg2e and carefully complete the project information. Create a project assignment for a project you would like to complete. You will need to get it approved by your teacher before undertaking the project. Upon teacher approval, complete the project you have created.

Take another look at the gas price table. As you marvel at how inexpensive gas prices may have seemed, remember one word— inflation. If you research the price of gas on the Internet and look at prices adjusted for inflation, you will be surprised. In 1960, the median annual income of U.S. families was $5,600. So the $5.00 it took to fill your gas tank was a significant outlay of cash. Nevertheless, it is always fascinating to look at price changes through history.

1. Using the data from the table of gas prices, draw a scatter plot on a sheet of graph paper. See margin.
2. Go online and find out the average cost of a gallon of gas today. What is the ratio of today's gas price to the 1950 gas price from the table? Answers vary.
3. Add today's cost to your scatter plot. Answers vary.
4. Draw a smooth curve that, by eye, looks like the best fit to the points on your scatter plot. Answers vary.
5. Go online and look up the median U.S. income for 1950 and for last year. What is the ratio of today's median U.S. income compared to the median income for 1950? Answers vary.
6. Find out the base price of this year's Corvette. What is the ratio of the base price of this year's Corvette compared to the price of the 1953 Corvette, $3,498.00? Answers vary.
Applications

1. A local publication charges by the character for automotive ads. Letters, numbers, spaces, and punctuation count as characters. They charge $34 for the first 100 characters, and $0.09 for each additional character. If $x$ represents the number of characters, express the cost $c(x)$ of an ad as a piecewise function. Graph the function. See margin.

2. *Classic Car Monthly* charges $49 for a three-line classified ad. Each additional line costs $9.50. For an extra $30, a seller can include a photo. How much would a five-line ad with a photo cost? $98

3. A local newspaper charges $d$ dollars for a three-line classified ad. Each additional line costs $a$ dollars. Express the cost of a six-line ad algebraically. $d + 3a$

4. The straight line depreciation equation for a car is $y = -2400x + 36,000$.
   a. What is the original price of the car? $36,000$
   b. How much value does the car lose per year? $2,400$
   c. How many years will it take for the car to totally depreciate? 15 years

5. A car is originally worth $43,500. It takes 12 years for this car to totally depreciate.
   a. Write the straight line depreciation equation that models this situation.
   b. How long will it take for the car to be worth one-quarter of its original price? 9 years
   c. How long will it take for the car to be worth $20,000? Round your answer to the nearest tenth of a year. 6.5 years

6. Prices for used stainless-steel side trim for a 1957 Chevrolet convertible are $350, $350, $390, $400, $500, $500, $500, $600, $650, $725, $800, $850, $900, and $1,700. The prices vary depending on condition.
   a. Find the mean of the trim prices to the nearest dollar. $658$
   b. Find the median of the trim prices. $550$
   c. Find the mode of the trim prices. $500$
   d. Find the four quartiles for these data. $Q_1 = 400; Q_2 = 550; Q_3 = 800; Q_4 = 1,700$
   e. Find the interquartile range for these data. $400$
   f. Find the boundary for the lower outliers. Are there any lower outliers?
   g. Find the boundary for the upper outliers. Are there any upper outliers?
   h. Draw a modified box-and-whisker plot. See additional answers.

7. Kathy purchased a new car for $37,800. From her research she has determined that it straight line depreciates over 14 years. She made a $7,000 down payment and pays $710 per month for her car loan.
   a. Create an expense and depreciation function where $x$ represents the number of months. Depreciation: $y = -225x + 37,800$; Expense: $y = 710x + 7,000$
   b. Graph these functions on the same axes. See additional answers.
   c. Interpret the region before, at, and after the intersection point in the context of this situation. See margin.

8. Grahamsville High School recently polled its teachers to see how many miles they drive to work each day. At the left is a stem-and-leaf plot of the results.
   a. How many teachers were polled? 25
   b. Find the mean to the nearest mile. 40
   c. Find the median. 38
   d. Find the mode(s). 19, 20, 36, 37, 55, 59, 62
   e. Find the range. 51
   f. Find the four quartiles. $Q_1 = 21.5; Q_2 = 38; Q_3 = 57; Q_4 = 62$
   g. What percent of the teachers travel more than 38 miles to work? 48%
   h. Find the interquartile range. 35.5
   i. What percent of the teachers travel from 38 to 57 miles to work? 28%

ANSWERS

7c. A graphing tool shows that the coordinates of the intersection point, rounded to the nearest hundredth, are $(32.92, 30388.24)$. This means that after a little less than 33 months, both the expenses-to-date and the car’s value are the same. In the region before the intersection point, the expenses are lower than the value of the car. But, the region after the intersection point indicates a period of time that the value of the car is less than what was invested in it.

5a. $y = -3.625x + 43,500$

6f. $-200$; there are none.
6g. $1,400$; yes; $1,700$
9. Stewart has $25,000 worth of property damage insurance and a $1,000 deductible collision insurance policy. He crashed into a fence when his brakes failed and did $7,000 worth of damage to the fence. The crash caused $3,600 in damages to his car.
   a. Which insurance covers the damage to the fence? Property damage
   b. How much will the insurance company pay for the fence? $7,000
   c. Stewart's car still was drivable after the accident. On the way home from the accident, he hit an empty school bus and did $20,000 worth of damage to the bus and $2,100 worth of damage to his car. How much will the insurance company pay for the damage to the bus? $20,000
   d. Which insurance covers the damage to Stewart's car? Collision
   e. How much will the insurance company pay for the damage to the car? $19,000

10. The historical prices of a car with the same make, model, and features are recorded for a period of 10 years in the table at the right.
   a. Construct a scatter plot for the data. See margin.
   b. Determine the exponential depreciation formula that models this data. Round all numbers to the nearest hundredth. \( y = 31,985.36(0.91)^x \)
   c. Determine the depreciation rate to the nearest percent. Approx. 9%
   d. Use the model equation to predict the value of this car after 66 months. Round to the nearest thousand dollars. Approx. $19,000

11. Gina has 250/500/50 liability insurance and $50,000 PIP insurance. One afternoon, she changed lanes too quickly, hit the metal guard rail, and then hit a tour bus. Four people are seriously hurt and sue her. Twenty others have minor injuries. Gina's boyfriend, who was in her car, was also hurt.
   a. The guard rail will cost $2,000 to replace. Gina also did $9,700 worth of damage to the bus. What insurance will cover this, and how much will the company pay? Property damage; $11,700
   b. The bus driver severed his hand and cannot drive a bus again. He sues for $2,500,000 and is awarded $1,750,000 in court. What type of insurance covers this? How much will the insurance company pay? $250,000 under BI
   c. The bus driver (from part b) had medical bills totaling $90,000 from an operation after the accident. What type of insurance covers this, and how much will the insurance company pay? $50,000 under PIP
   d. Gina's boyfriend requires $19,000 worth of medical attention. What insurance covers this, and how much will the company pay? $19,000 under PIP

12. Jerome just purchased a 4-year-old car for $12,000. He was told that this make and model depreciates exponentially at a rate of 5.8% per year. What was the original price to the nearest hundred dollars? $15,200

13. The following two-way table displays information about favorite sports cars that resulted from a survey given to all students at Shore High School.

<table>
<thead>
<tr>
<th></th>
<th>Corvette (C)</th>
<th>Porsche (P)</th>
<th>Ferrari (F)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boys (B)</td>
<td>90</td>
<td>60</td>
<td>120</td>
<td>270</td>
</tr>
<tr>
<td>Girls (G)</td>
<td>110</td>
<td>141</td>
<td>79</td>
<td>330</td>
</tr>
<tr>
<td>Total</td>
<td>200</td>
<td>201</td>
<td>199</td>
<td>600</td>
</tr>
</tbody>
</table>

   a. Find \( P(B) \), \( P(C) \), \( P(B|C) \) and explain whether or not events \( B \) and \( C \) are independent. See margin.
   b. What is the probability that a randomly selected student from this school is a boy? \( \frac{270}{600} = \frac{9}{20} \)
   c. What is the probability that a randomly selected student from this school prefers Corvettes, given that the student is a girl? \( \frac{1}{3} \)
14. Using yearly car values, a graphing calculator has calculated the following exponential regression equation: \( y = ab^x \), \( a = 28,158.50 \), \( b = 0.815 \).
   a. What is the rate of depreciation for this car? 18.5%
   b. How much is this car worth to the nearest dollar after 6 years? $8,252
   c. How much is this car worth to the nearest hundred dollars after 39 months? $14,500
   d. How much is this car worth after \( y \) years? \( \frac{28,158.50}{0.815^y} \)

15. Jonathan's car gets approximately 25 miles per gallon. He is planning a 980-mile trip. About how many gallons of gas will his car use for the trip? At an average price of $2.50 per gallon, how much should Jonathan expect to spend for gas? Round to the nearest ten dollars. 39.2 gallons; $100

16. Ann's car gets about 12 kilometers per liter of gas. She is planning a 2,100-kilometer trip. To the nearest liter, how many liters of gas should Ann plan to buy? At an average price of $0.71 per liter, how much should Ann expect to spend for gas? 175 liters; $124.25

17. Max is driving 42 miles per hour. A dog runs into the street and Max reacts in about three-quarters of a second. What is his approximate reaction distance? 42 feet

18. Tricia is driving 64 miles per hour on an interstate highway. She must make a quick stop because there is an emergency vehicle ahead.
   a. What is her approximate reaction distance? Round to the nearest foot. 64 feet
   b. What is her approximate braking distance? Round to the nearest foot. 204.8 feet
   c. About how many feet does the car travel from the time she starts to switch pedals until the car has completely stopped, or her total stopping distance? 268.8 feet

19. Marlena is driving on an interstate at 65 km/h. She sees a traffic jam about 30 meters ahead and needs to bring her car to a complete stop before she reaches that point. Her reaction time is approximately ¾ of a second. Is she far enough away from the traffic jam to safely bring the car to a complete stop? Explain. See margin.

20. Richie was driving on an asphalt road that had a 40 mi/h speed limit. A bicyclist veered into his lane, causing him to slam on his brakes. His tires left three skid marks of 69 ft, 70 ft, and 74 ft. The road had a drag factor of 0.95. His brakes were operating at 98% efficiency. The police gave Richie a ticket for speeding. Richie insisted that he was driving under the speed limit. Who is correct? Explain. See margin.

21. A car was traveling at 52 mi/h before it enters into a skid. It was determined that the drag factor of the road surface is 1.05, and the braking efficiency is 80%. How long might the average skid mark be to the nearest tenth of a foot for this situation? 107.3 feet

22. A reconstructionist took measurements from yaw marks left at the scene of an accident. Using a 46-ft chord, the middle ordinate measured approximately 6 ft. The drag factor for the road surface was 0.95. Determine the radius of the yaw mark to the nearest tenth of a foot. Determine the minimum speed when the skid occurred to the nearest tenth of a mile. 47.1 feet; 25.9 mi/h

ANSWERS
19. She does not have enough room to stop. Marlena’s total stopping distance is 37.85 meters, which is more than the distance to the traffic jam.
20. The police were correct, since according to the formula, Richie’s minimum skid speed was approximately 44.53 miles per hour.
23. A car vehicle price history for a certain make and model contains the following list of yearly price values: $30,000 $28,500 $27,075 $25,721.25. The original price of the car was $30,000. It exponentially depreciated to $28,500 after 1 year and continued depreciating by the same percentage each year thereafter. What will the value of the car be after 7 years? $20,950.12

24. Five years ago, a certain make and model of a car now considered to be a classic had a selling price of $26,000. Examine this geometric sequence representing the yearly appreciation in the price of the car since then: $26,000 $31,200 $37,440 $44,928 $53,913.60 $64,696.32. If this continues the same way, what would you expect to pay for this classic car 5 years from now? $160,985.15

25. Cargo is tied with rope on the roof of a 4.5-foot-tall car. The car is traveling down a road at 42 mph and hits a concrete barrier. The rope snaps, allowing the cargo to propel forward. Use the equations $y = -16.1t^2 + 4.75$ and $y = -0.0042x^2 + 4.75$ where $y$ represents height in feet, $x$ represents horizontal distance in feet, and $t$ represents time in seconds, to find the time it takes for the cargo to hit the ground and the horizontal distance it travels. 0.54; 33.6 feet

26. A car is traveling at 68 km/h. Cargo is strapped to the roof at a height of 1.6 m. The car hits a concrete barrier, and the cargo is horizontally ejected off the roof. Use these two equations to determine how long it takes for the cargo to hit the ground and how far it travels in the horizontal direction. $y = -4.9t^2 + 1.6$ and $y = -0.0136x^2 + 1.6$ where $y$ represents the height in meters, $x$ represents the horizontal distance in meters, and $t$ represents the time in seconds. 0.57; 10.85 meters