This chapter uses marginal utility theory to discuss consumer choice. Sometimes budget constraint and indifference curve analysis is used instead, especially in upper-division economics courses. We examine this important topic in this appendix.

**THE BUDGET CONSTRAINT**

Societies have production possibilities frontiers, and individuals have budget constraints. The budget constraint is built on two prices and the individual’s income. To illustrate, consider O’Brien, who has a monthly income of $1,200. In a world of two goods, X and Y, O’Brien can spend his total income on X, he can spend his total income on Y, or he can spend part of his income on X and part on Y. Suppose the price of X is $100 and the price of Y is $80. Given this, if O’Brien spends his total income on X, he can purchase a maximum of 12 units; if he spends his total income on Y, he can purchase a maximum of 15 units. Locating these two points on a two-dimensional diagram and then drawing a line between them, as shown in Exhibit 1, gives us O’Brien’s budget constraint. Any point on the budget constraint, as well as any point below it, represents a possible combination (bundle) of the two goods available to O’Brien.

**Slope of the Budget Constraint**

The slope of the budget constraint has special significance. The absolute value of the slope represents the relative prices of the two goods, X and Y. In Exhibit 1, the slope, or \( P_X/P_Y \), is equal to 1.25, indicating that the relative price of 1 unit of X is 1.25 units of Y.

**Budget Constraint**

All the combinations or bundles of two goods a person can purchase given a certain money income and prices for the two goods.

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**Exhibit 1**

**The Budget Constraint**

An individual’s budget constraint gives us a picture of the different combinations (bundles) of two goods available to the individual. (We assume a two-good world; for a many-good world, we could put one good on one axis and “all other goods” on the other axis.) The budget constraint is derived by finding the maximum amount of each good an individual can consume (given his or her income and the prices of the two goods) and connecting these two points.
What Will Change the Budget Constraint?

The budget constraint is built on two prices and the individual’s income. This means that if any of the three variables changes (either of the prices or the individual’s income), the budget constraint changes. Not all changes are alike, however. Consider a fall in the price of good \( X \) from $100 to $60. With this change, the maximum number of units of good \( X \) purchasable with an income of $1,200 rises from 12 to 20. The budget constraint revolves away from the origin, as shown in Exhibit 2a. Notice that the number of O’Brien’s possible combinations of the two goods increases; there are more bundles of the two goods available after the price decrease than before.

Consider what happens to the budget constraint if the price of good \( X \) rises. If it goes from $100 to $150, the maximum number of units of good \( X \) falls from 12 to 8. The budget constraint revolves toward the origin. As a consequence, the number of bundles available to O’Brien decreases. We conclude that a change in the price of either good changes the slope of the budget constraint, with the result that relative prices and the number of bundles available to the individual also change.

We turn now to a change in income. If O’Brien’s income rises to $1,600, the maximum number of units of \( X \) rises to 16 and the maximum number of units of \( Y \) rises to 20. The budget constraint shifts rightward (away from the origin) and is parallel to the old budget constraint. As a consequence, the number of bundles available to O’Brien increases (Exhibit 2b). If O’Brien’s income falls from $1,200 to $800, the extreme end points on the budget constraint become 8 and 10 for \( X \) and \( Y \), respectively. The budget constraint shifts leftward (toward the origin) and is parallel to the old budget constraint. As a consequence, the number of bundles available to O’Brien falls (Exhibit 2b).

INDIFFERENCE CURVES

An individual can, of course, choose any bundle of the two goods on or below the budget constraint. We assume that she spends her total income and therefore chooses a point on the budget constraint. This raises two important questions: (1) Which bundle of the many bundles of the two goods does the individual choose? (2) How does the individual’s cho-
sen combination of goods change given a change in prices or income? Both questions can be answered by combining the budget constraint with the graphical expression of the individual's preferences—that is, indifference curves.

**Constructing an Indifference Curve**

Is it possible to be indifferent between two bundles of goods? Yes, it is. Suppose bundle \( A \) consists of 2 pairs of shoes and 6 shirts and bundle \( B \) consists of 3 pairs of shoes and 4 shirts. A person who is indifferent between these two bundles is implicitly saying that it doesn't matter which bundle he has; one is as good as the other. He is likely to say this, though, only if he receives equal total utility from the two bundles. If this were not the case, he would prefer one bundle to the other.

If we tabulate all the different bundles from which the individual receives equal utility, we have an **indifference set**. We can then plot the data in the indifference set and draw an **indifference curve**. Consider the indifference set illustrated in Exhibit 3a. There are four bundles of goods, \( A–D \); each bundle gives the same total utility as every other bundle. These equal-utility bundles are plotted in Exhibit 3b. Connecting these bundles in a two-dimensional space gives us an indifference curve.

**Characteristics of Indifference Curves**

Indifference curves for goods have certain characteristics that are consistent with reasonable assumptions about consumer behavior.

1. **Indifference curves are downward-sloping (from left to right).** The assumption that consumers always prefer more of a good to less requires that indifference curves slope downward left to right. Consider the alternatives to downward-sloping: vertical, horizontal, and upward-sloping (left to right). A horizontal or vertical curve would combine bundles of goods some of which had more of one good and no less of another good than other bundles (Exhibit 4a–b). (If bundle \( B \) contains more of one good and no less of another good than bundle \( A \), would an individual be indifferent between the two bundles? No, he or she wouldn't. Individuals prefer more to...
less.) An upward-sloping curve would combine bundles of goods some of which had more of both goods than other bundles (Exhibit 4c). A simpler way of putting it is to say that indifference curves are downward-sloping because a person has to get more of one good in order to maintain his or her level of satisfaction (utility) when giving up some of another good.

2. **Indifference curves are convex to the origin.** This implies that the slope of the indifference curve becomes flatter as we move down and to the right along the indifference curve. For example, at 8 units of milk (point A in Exhibit 3b), the individual is willing to give up 3 units of milk to get an additional unit of orange juice (and thus move to point B). At point B, where she has 5 units of milk, she is willing to give up only 2 units of milk to get an additional unit of orange juice (and thus move to point C). Finally, at point C, with 3 units of milk, she is now willing to give up only 1 unit of milk to get an additional unit of orange juice. We conclude that the more of one good that an individual has, the more units he or she will give up to get an additional unit of another good; the less of one good that an individual has, the fewer units he or she will give up to get an additional unit of another good. Is this reasonable? The answer is yes. Our observation is a reflection of diminishing marginal utility at work. As the quantity of a good consumed increases, the marginal utility of that good decreases; therefore we reason that the more of one good an individual has, the more units he or she can (and will) sacrifice to get an additional unit of another good and still maintain total utility. Stated differently, if the law of diminishing marginal utility did not exist then it would not make sense to say that indifference curves of goods are convex to the origin.

An important peripheral point about marginal utilities is that the absolute value of the slope of the indifference curve—which is called the **marginal rate of substitution**—represents the ratio of the marginal utility of the good on the horizontal axis to the marginal utility of the good on the vertical axis:

\[
\frac{MU_{\text{good on horizontal axis}}}{MU_{\text{good on vertical axis}}}
\]

**Marginal Rate of Substitution**
The amount of one good an individual is willing to give up to obtain an additional unit of another good and maintain equal total utility.

**Indifference Curves for Goods Do Not Look Like This**
(a) Bundle B has more milk and no less orange juice than bundle A, so an individual would prefer B to A and not be indifferent between them. (b) Bundle B has more orange juice and no less milk than bundle A, so an individual would prefer B to A and not be indifferent between them. (c) Bundle B has more milk and more orange juice than bundle A, so an individual would prefer B to A and not be indifferent between them.
Let’s look carefully at the words in italics. First, we said that the absolute value of the slope of the indifference curve is the marginal rate of substitution. The marginal rate of substitution (MRS) is the amount of one good an individual is willing to give up to obtain an additional unit of another good and maintain equal total utility. For example, in Exhibit 3b, we see that moving from point A to point B, the individual is willing to give up 3 units of milk to get an additional unit of orange juice, with total utility remaining constant (between points A and B). The marginal rate of substitution is therefore 3 units of milk for 1 unit of orange juice in the area between points A and B. And as we said, the absolute value of the slope of the indifference curve, the marginal rate of substitution, is equal to the ratio of the $MU$ of the good on the horizontal axis to the $MU$ of the good on the vertical axis. How can this be? Well, if it is true that an individual giving up 3 units of milk and receiving 1 unit of orange juice maintains her total utility, it follows that (in the area under consideration) the marginal utility of orange juice is approximately three times the marginal utility of milk. In general terms

$$\text{Absolute value of the slope of the indifference curve} = \frac{MU_{\text{good on horizontal axis}}}{MU_{\text{good on vertical axis}}}$$

3. **Indifference curves that are farther from the origin are preferable because they represent larger bundles of goods.** Exhibit 3b shows only one indifference curve. However, different bundles of the two goods exist and have indifference curves passing through them. These bundles have less of both goods or more of both goods than those in Exhibit 3b. Illustrating a number of indifference curves on the same diagram gives us an **indifference curve map**. Strictly speaking, an indifference curve map represents a number of indifference curves for a given individual with reference to two goods. A “mapping” is illustrated in Exhibit 5.

**Indifference Curve Map**

Represents a number of indifference curves for a given individual with reference to two goods.

**An Indifference Map**

A few of the many possible indifference curves are shown. Any point in the two-dimensional space is on an indifference curve. Indifference curves farther away from the origin represent greater total utility than those closer to the origin.
Notice that although only five indifference curves have been drawn, many more could have been added. For example, there are many indifference curves between $I_1$ and $I_2$.

Also notice that the farther away from the origin an indifference curve is, the higher total utility it represents. You can see this by comparing point $A$ on $I_1$ and point $B$ on $I_2$. At point $B$, there is the same amount of orange juice as at point $A$ but more milk. Point $B$ is therefore preferable to point $A$, and because $B$ is on $I_2$ and $A$ is on $I_1$, $I_2$ is preferable to $I_1$. The reason for this is simple: An individual receives more utility at any point on $I_2$ (because more goods are available) than at any point on $I_1$.

4. **Indifference curves do not cross (intersect).** Indifference curves do not cross because individuals’ preferences are **transitive**. Consider the following example. If Kristin prefers Coca-Cola to Pepsi-Cola and she also prefers Pepsi-Cola to root beer, then it follows that she prefers Coca-Cola to root beer. If she preferred root beer to Coca-Cola, she would be contradicting her earlier preferences. To say that an individual has transitive preferences means that he or she maintains a logical order of preferences during a given time period. Consider what indifference curves that crossed would represent. In Exhibit 6, indifference curves $I_1$ and $I_2$ intersect at point $A$. Notice that point $A$ lies on both $I_1$ and $I_2$. Comparing $A$ and $B$, we hold that the individual must be indifferent between them because they lie on the same indifference curve. The same holds for $A$ and $C$. But if the individual is indifferent between $A$ and $B$ and between $A$ and $C$, it follows that she must be indifferent between $B$ and $C$. But $C$ has more of both goods than $B$, and thus the individual will not be indifferent between $B$ and $C$; she will prefer $C$ to $B$. We cannot have transitive preferences and make sense of crossing indifference curves. We can, however, have transitive preferences and make sense of non-crossing indifference curves. We go with the latter.

**THE INDIFFERENCE MAP AND THE BUDGET CONSTRAINT COME TOGETHER**

At this point, we bring the indifference map and the budget constraint together to illustrate consumer equilibrium. We have the following facts: (1) The individual has a budget constraint. (2) The absolute value of the slope of the budget constraint is the relative prices of the two goods under consideration, say, $P_X/P_Y$. (3) The individual has an indifference map. (4) The absolute value of the slope of the indifference curve at any point is the marginal rate of substitution, which is equal to the marginal utility of one good divided by the marginal utility of another good; for example, $MU_X/MU_Y$.

With this information, what is the necessary condition for consumer equilibrium? Obviously, the individual will try to reach a point on the highest indifference curve she can reach. This point will be where the slope of the budget constraint is equal to the slope of an indifference curve (or where the budget constraint is tangent to an indifference curve). At this point, consumer equilibrium is established and the following condition holds:

$$\text{Slope of budget constraint} = \text{slope of indifference curve}$$

$$\frac{P_X}{P_Y} = \frac{MU_X}{MU_Y}$$

**Transitivity**
The principle whereby if $A$ is preferred to $B$, and $B$ is preferred to $C$, then $A$ is preferred to $C$.

**Exhibit 6**

**Crossing Indifference Curves Are Inconsistent With Transitive Preferences**

Point $A$ lies on both indifference curves $I_1$ and $I_2$. This means that the individual is indifferent between $A$ and $B$ and between $A$ and $C$, which results in her (supposedly) being indifferent between $B$ and $C$. But individuals prefer “more to less” (when it comes to goods) and, thus, would prefer $C$ to $B$. We cannot have transitive preferences and make sense of crossing indifference curves.
This condition is met in Exhibit 7 at point $E$. Note that this condition looks similar to the condition for consumer equilibrium earlier in this chapter. By rearranging the terms in the condition, we get\(^1\)

$$\frac{MU_x}{P_x} = \frac{MU_Y}{P_Y}$$

FROM INDIFFERENCE CURVES TO A DEMAND CURVE

We can now derive a demand curve within a budget constraint–indifference curve framework. Exhibit 8a shows two budget constraints, one reflecting a $10 price for good $X$ and the other reflecting a $5 price for good $X$. Notice that as the price of $X$ falls, the consumer moves from point $A$ to point $B$. At $B$, 35 units of $X$ are consumed; at $A$, 30 units of $X$ were consumed. We conclude that a lower price for $X$ results in greater consumption of $X$. By plotting the relevant price and quantity data, we derive a demand curve for good $X$ in Exhibit 8b.

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1. Start with $P_x/P_y = MU_x/MU_y$ and cross multiply. This gives $P_x MU_y = P_y MU_x$. Next divide both sides by $P_x$. This gives $MU_y = P_x MU_y / P_x$. Finally, divide both sides by $P_y$. This gives $MU_y/P_y = MU_x/P_x$. 

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From Indifference Curves to a Demand Curve

(a) At a price of $10 for good X, consumer equilibrium is at point A with the individual consuming 30 units of X. As the price falls to $5, the budget constraint moves outward (away from the origin), and the consumer moves to point B and consumes 35 units of X. Plotting the price-quantity data for X gives a demand curve for X in (b).

Appendix Summary

> A budget constraint represents all combinations of bundles of two goods a person can purchase given a certain money income and prices for the two goods.

> An indifference curve shows all the combinations or bundles of two goods that give an individual equal total utility.

> Indifference curves are downward-sloping, convex to the origin, and do not cross. The farther away from the origin an indifference curve is, the greater total utility it represents for the individual.

> Consumer equilibrium is at the point where the slope of the budget constraint equals the slope of the indifference curve.

> A demand curve can be derived within a budget constraint-indifference curve framework.

Questions and Problems

1. Diagram the following budget constraints:
   a. Income = $4,000; \( P_X = $50 \); \( P_Y = $100 \)
   b. Income = $3,000; \( P_X = $25 \); \( P_Y = $200 \)
   c. Income = $2,000; \( P_X = $40 \); \( P_Y = $150 \)

2. Explain why indifference curves (a) are downward-sloping, (b) are convex to the origin, and (c) do not cross.

3. Explain why consumer equilibrium is equivalent whether using marginal utility analysis or using indifference curve analysis.

4. Derive a demand curve using indifference curve analysis.