What you should learn
• Find domains of algebraic expressions.
• Simplify rational expressions.
• Add, subtract, multiply, and divide rational expressions.
• Simplify complex fractions and rewrite difference quotients.

Why you should learn it
Rational expressions can be used to solve real-life problems. For instance, in Exercise 102 on page A48, a rational expression is used to model the projected numbers of U.S. households banking and paying bills online from 2002 through 2007.

RATIONAL EXPRESSIONS

Domain of an Algebraic Expression
The set of real numbers for which an algebraic expression is defined is the domain of the expression. Two algebraic expressions are equivalent if they have the same domain and yield the same values for all numbers in their domain. For instance, 
\[(x + 1) + (x + 2) = x + 1 + x + 2 = x + x + 1 + 2 = 2x + 3.\]

Example 1 Finding the Domain of an Algebraic Expression

a. The domain of the polynomial
   \[2x^3 + 3x + 4\]
   is the set of all real numbers. In fact, the domain of any polynomial is the set of all real numbers, unless the domain is specifically restricted.

b. The domain of the radical expression
   \[\sqrt{x - 2}\]
   is the set of real numbers greater than or equal to 2, because the square root of a negative number is not a real number.

c. The domain of the expression
   \[\frac{x + 2}{x - 3}\]
   is the set of all real numbers except \(x = 3\), which would result in division by zero, which is undefined.

Now try Exercise 7.

The quotient of two algebraic expressions is a fractional expression. Moreover, the quotient of two polynomials such as
\[
\frac{1}{x}, \quad \frac{2x - 1}{x + 1}, \quad \text{or} \quad \frac{x^2 - 1}{x^2 + 1}
\]
is a rational expression.

Simplifying Rational Expressions

Recall that a fraction is in simplest form if its numerator and denominator have no factors in common aside from \(\pm 1\). To write a fraction in simplest form, divide out common factors.
\[
\frac{a}{b} \cdot \frac{c}{d} = \frac{a}{b}, \quad c \neq 0
\]
The key to success in simplifying rational expressions lies in your ability to factor polynomials. When simplifying rational expressions, be sure to factor each polynomial completely before concluding that the numerator and denominator have no factors in common.

**Example 2**  Simplifying a Rational Expression

Write \( \frac{x^2 + 4x - 12}{3x - 6} \) in simplest form.

**Solution**

\[
\frac{x^2 + 4x - 12}{3x - 6} = \frac{(x + 6)(x - 2)}{3(x - 2)}
\]

Factor completely.

\[
= \frac{x + 6}{3}, \quad x \neq 2
\]

Divide out common factors.

Note that the original expression is undefined when \( x = 2 \) (because division by zero is undefined). To make sure that the simplified expression is equivalent to the original expression, you must restrict the domain of the simplified expression by excluding the value \( x = 2 \).

**CHECK POINT**  Now try Exercise 33.

Sometimes it may be necessary to change the sign of a factor by factoring out \(-1\) to simplify a rational expression, as shown in Example 3.

**Example 3**  Simplifying Rational Expressions

Write \( \frac{12 + x - x^2}{2x^2 - 9x + 4} \) in simplest form.

**Solution**

\[
\frac{12 + x - x^2}{2x^2 - 9x + 4} = \frac{(4 - x)(3 + x)}{(2x - 1)(x - 4)}
\]

Factor completely.

\[
= \frac{- (x - 4)(3 + x)}{(2x - 1)(x - 4)}
\]

\[
= \frac{-3 + x}{2x - 1}, \quad x \neq 4
\]

Divide out common factors.

**CHECK POINT**  Now try Exercise 39.

In this text, when a rational expression is written, the domain is usually not listed with the expression. It is implied that the real numbers that make the denominator zero are excluded from the expression. Also, when performing operations with rational expressions, this text follows the convention of listing by the simplified expression all values of \( x \) that must be specifically excluded from the domain in order to make the domains of the simplified and original expressions agree. In Example 3, for instance, the restriction \( x \neq 4 \) is listed with the simplified expression to make the two domains agree. Note that the value \( x = \frac{1}{2} \) is excluded from both domains, so it is not necessary to list this value.
Operations with Rational Expressions

To multiply or divide rational expressions, use the properties of fractions discussed in Appendix A.1. Recall that to divide fractions, you invert the divisor and multiply.

**Example 4**  
**Multiplying Rational Expressions**

\[
\frac{2x^2 + x - 6}{x^2 + 4x - 5} \cdot \frac{x^3 - 3x^2 + 2x}{4x^2 - 6x} = \frac{(2x-3)(x+2)}{(x+5)(x-1)} \cdot \frac{x(x-2)(x-1)}{2x(2x-3)}
\]

\[
= \frac{(x+2)(x-2)}{2(x+5)}, \quad x \neq 0, x \neq 1, x \neq \frac{3}{2}
\]

**CHECK Point**  
Now try Exercise 53.

In Example 4, the restrictions \(x \neq 0, x \neq 1, \text{ and } x \neq \frac{3}{2}\) are listed with the simplified expression in order to make the two domains agree. Note that the value \(x = -5\) is excluded from both domains, so it is not necessary to list this value.

**Example 5**  
**Dividing Rational Expressions**

\[
\frac{x^3 - 8}{x^2 - 4} \div \frac{x^2 + 2x + 4}{x^3 + 8} = \frac{x^3 - 8}{x^2 - 4} \cdot \frac{x^3 + 8}{x^2 + 2x + 4}
\]

\[
= \frac{(x-2)(x^2+2x+4)}{(x+2)(x-2)}, \quad \frac{(x+2)(x^2 - 2x + 4)}{(x^2 + 2x + 4)}
\]

\[
= x^2 - 2x + 4, \quad x \neq \pm 2
\]

**CHECK Point**  
Now try Exercise 55.

To add or subtract rational expressions, you can use the LCD (least common denominator) method or the basic definition

\[
\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}, \quad b \neq 0, d \neq 0.
\]

Basic definition

This definition provides an efficient way of adding or subtracting two fractions that have no common factors in their denominators.

**Example 6**  
**Subtracting Rational Expressions**

\[
\frac{x}{x - 3} - \frac{2}{3x + 4} = \frac{x(3x + 4) - 2(x - 3)}{(x - 3)(3x + 4)}
\]

\[
= \frac{3x^2 + 4x - 2x + 6}{(x - 3)(3x + 4)}
\]

\[
= \frac{3x^2 + 2x + 6}{(x - 3)(3x + 4)}
\]

**CHECK Point**  
Now try Exercise 65.

**WARNING/CAUTION**

When subtracting rational expressions, remember to distribute the negative sign to all the terms in the quantity that is being subtracted.
For three or more fractions, or for fractions with a repeated factor in the denominators, the LCD method works well. Recall that the least common denominator of several fractions consists of the product of all prime factors in the denominators, with each factor given the highest power of its occurrence in any denominator. Here is a numerical example.

\[
\frac{1}{6} + \frac{3}{4} - \frac{2}{3} = \frac{1 \cdot 2 + 3 \cdot 3 - 2 \cdot 4}{3 \cdot 4}
\]

The LCD is 12.

\[
= \frac{2 + 9 - 8}{12}
\]

\[
= \frac{3}{12}
\]

\[
= \frac{1}{4}
\]

Sometimes the numerator of the answer has a factor in common with the denominator. In such cases the answer should be simplified. For instance, in the example above, \(\frac{3}{12}\) was simplified to \(\frac{1}{4}\).

**Example 7** Combining Rational Expressions: The LCD Method

Perform the operations and simplify.

\[
\frac{3}{x - 1} - \frac{2}{x} + \frac{x + 3}{x^2 - 1}
\]

**Solution**

Using the factored denominators \((x - 1), x,\) and \((x + 1)(x - 1)\), you can see that the LCD is \(x(x + 1)(x - 1)\).

\[
\frac{3}{x - 1} - \frac{2}{x} + \frac{x + 3}{(x + 1)(x - 1)}
\]

\[
= \frac{3(x)(x + 1)}{x(x + 1)(x - 1)} - \frac{2(x + 1)(x - 1)}{x(x + 1)(x - 1)} + \frac{(x + 3)(x)}{x(x + 1)(x - 1)}
\]

\[
= \frac{3(x)(x + 1) - 2(x + 1)(x - 1) + (x + 3)(x)}{x(x + 1)(x - 1)}
\]

\[
= \frac{3x^2 + 3x - 2x^2 + 2 + x^2 + 3x}{x(x + 1)(x - 1)}
\]

\[
= \frac{3x^2 - 2x^2 + x^2 + 3x + 3x + 2}{x(x + 1)(x - 1)}
\]

\[
= \frac{3x^2 + 6x + 2}{x(x + 1)(x - 1)}
\]

\[
= \frac{2(x^2 + 3x + 1)}{x(x + 1)(x - 1)}
\]

**CHECKPOINT** Now try Exercise 67.
Complex Fractions and the Difference Quotient

Fractional expressions with separate fractions in the numerator, denominator, or both are called complex fractions. Here are two examples.

\[
\frac{\frac{1}{x}}{x^2 + 1} \quad \text{and} \quad \frac{\frac{1}{x}}{x^2 + 1}
\]

To simplify a complex fraction, combine the fractions in the numerator into a single fraction and then combine the fractions in the denominator into a single fraction. Then invert the denominator and multiply.

**Example 8** Simplifying a Complex Fraction

\[
\frac{\frac{2}{x} - 3}{1 - \frac{1}{x} - 1} = \frac{\frac{2 - 3x}{x}}{\frac{1(x - 1) - 1}{x - 1}} \quad \text{Combine fractions.}
\]

\[
= \frac{\frac{2 - 3x}{x}}{\frac{x - 2}{x - 1}} \quad \text{Simplify.}
\]

\[
= \frac{2 - 3x}{x} \cdot \frac{x - 1}{x - 2} \quad \text{Invert and multiply.}
\]

\[
= \frac{(2 - 3x)(x - 1)}{x(x - 2)}, \quad x \neq 1
\]

**CHECKPOINT** Now try Exercise 73.

Another way to simplify a complex fraction is to multiply its numerator and denominator by the LCD of all fractions in its numerator and denominator. This method is applied to the fraction in Example 8 as follows.

\[
\frac{\frac{2}{x} - 3}{1 - \frac{1}{x} - 1} = \frac{\frac{2}{x} - 3}{\frac{1}{x} - 1} \cdot \frac{x(x - 1)}{x(x - 1)} \quad \text{LCD is } x(x - 1).
\]

\[
= \frac{\frac{2 - 3x}{x}}{\frac{x - 2}{x - 1}} \cdot \frac{x(x - 1)}{x(x - 1)} \quad \text{Invert and multiply.}
\]

\[
= \frac{(2 - 3x)(x - 1)}{x(x - 2)}, \quad x \neq 1
\]
The next three examples illustrate some methods for simplifying rational expressions involving negative exponents and radicals. These types of expressions occur frequently in calculus.

To simplify an expression with negative exponents, one method is to begin by factoring out the common factor with the smaller exponent. Remember that when factoring, you subtract exponents. For instance, in $3x^{-5/2} + 2x^{-3/2}$ the smaller exponent is $-\frac{3}{2}$ and the common factor is $x^{-\frac{5}{2}}$.

$$3x^{-5/2} + 2x^{-3/2} = x^{-5/2}[3(1) + 2x^{-3/2-(-5/2)}]$$
$$= x^{-5/2}(3 + 2x)$$
$$= \frac{3 + 2x}{x^{5/2}}$$

**Example 9** **Simplifying an Expression**

Simplify the following expression containing negative exponents.

$$x(1 - 2x)^{-3/2} + (1 - 2x)^{-1/2}$$

**Solution**

Begin by factoring out the common factor with the smaller exponent.

$$x(1 - 2x)^{-3/2} + (1 - 2x)^{-1/2} = (1 - 2x)^{-3/2}[x + (1 - 2x)^{-1/2-(-3/2)}]$$
$$= (1 - 2x)^{-3/2}[x + (1 - 2x)^{1}]$$
$$= \frac{1 - x}{(1 - 2x)^{3/2}}$$

**CHECKPoint** Now try Exercise 81.

A second method for simplifying an expression with negative exponents is shown in the next example.

**Example 10** **Simplifying an Expression with Negative Exponents**

$$\frac{(4 - x^2)^{1/2} + x^2(4 - x^2)^{-1/2}}{4 - x^2}$$

$$= \frac{(4 - x^2)^{1/2} + x^2(4 - x^2)^{-1/2}}{4 - x^2} \cdot \frac{(4 - x^2)^{1/2}}{(4 - x^2)^{1/2}}$$

$$= \frac{(4 - x^2)^{1} + x^2(4 - x^2)^{0}}{(4 - x^2)^{3/2}}$$

$$= \frac{4 - x^2 + x^2}{(4 - x^2)^{3/2}}$$

$$= \frac{4}{(4 - x^2)^{3/2}}$$

**CHECKPoint** Now try Exercise 83.
Rewriting a Difference Quotient

The following expression from calculus is an example of a difference quotient.

\[
\frac{\sqrt{x+h} - \sqrt{x}}{h}
\]

Rewrite this expression by rationalizing its numerator.

**Solution**

\[
\frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}
\]

\[
= \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})}
\]

\[
= \frac{h}{h(\sqrt{x+h} + \sqrt{x})}
\]

\[
= \frac{1}{\sqrt{x+h} + \sqrt{x}}, \quad h \neq 0
\]

Notice that the original expression is undefined when \( h = 0 \). So, you must exclude \( h = 0 \) from the domain of the simplified expression so that the expressions are equivalent.

**Example 11**

You can review the techniques for rationalizing a numerator in Appendix A.2.

**Difference quotients**, such as that in Example 11, occur frequently in calculus. Often, they need to be rewritten in an equivalent form that can be evaluated when \( h = 0 \). Note that the equivalent form is not simpler than the original form, but it has the advantage that it is defined when \( h = 0 \).

**VOCABULARY:** Fill in the blanks.

1. The set of real numbers for which an algebraic expression is defined is the ________ of the expression.
2. The quotient of two algebraic expressions is a fractional expression and the quotient of two polynomials is a ________ ________.
3. Fractional expressions with separate fractions in the numerator, denominator, or both are called ________ fractions.
4. To simplify an expression with negative exponents, it is possible to begin by factoring out the common factor with the ________ exponent.
5. Two algebraic expressions that have the same domain and yield the same values for all numbers in their domains are called ________.
6. An important rational expression, such as \( \frac{(x+h)^2 - x^2}{h} \), that occurs in calculus is called a ________ ________.
SKILLS AND APPLICATIONS

In Exercises 7–22, find the domain of the expression.

7. \(3x^2 - 4x + 7\)  
8. \(2x^2 + 5x - 2\)  
9. \(4x^3 + 3, \ x \geq 0\)  
10. \(6x^2 - 9, \ x > 0\)  
11. \(\frac{1}{3 - x}\)  
12. \(\frac{x + 6}{3x + 2}\)  
13. \(\frac{x^2 - 1}{x^2 - 2x + 1}\)  
14. \(\frac{x^2 - 5x + 6}{x^2 - 4}\)  
15. \(\frac{x^2 - 2x - 3}{x^2 - 6x + 9}\)  
16. \(\frac{x^2 - x - 12}{x^3 - 8x + 16}\)  
17. \(\sqrt{x + 7}\)  
18. \(\sqrt{4 - x}\)  
19. \(\sqrt{2x - 5}\)  
20. \(\sqrt{4x + 5}\)  
21. \(\frac{1}{\sqrt{x - 3}}\)  
22. \(\frac{1}{\sqrt{x + 2}}\)

In Exercises 23 and 24, find the missing factor in the numerator such that the two fractions are equivalent.

23. \(\frac{5}{2x} = \frac{5(\underline{\text{___}})}{6x^2}\)  
24. \(\frac{3}{4} = \frac{3(\underline{\text{___}})}{4(x + 1)}\)

In Exercises 25–42, write the rational expression in simplest form.

25. \(\frac{15x^2}{10x}\)  
26. \(\frac{18y^2}{60y^3}\)  
27. \(\frac{3xy}{xy + x}\)  
28. \(\frac{2x^2y}{xy - y}\)  
29. \(\frac{4y - 8y^2}{10y - 5}\)  
30. \(\frac{9x^2 + 9x}{2x + 2}\)  
31. \(\frac{x - 5}{10 - 2x}\)  
32. \(\frac{12 - 4x}{x - 3}\)  
33. \(\frac{y^2 - 16}{y + 4}\)  
34. \(\frac{x^2 - 25}{5 - x}\)  
35. \(\frac{x^3 + 5x^2 + 6x}{x^2 - 4}\)  
36. \(\frac{x^2 + 8x - 20}{x^2 + 11x + 10}\)  
37. \(\frac{x^2 - 7y + 12}{y^2 + 3y - 18}\)  
38. \(\frac{x^2 - 7x + 6}{x^2 + 11x + 10}\)  
39. \(\frac{x^2 - 2x + x^3 - 3x^2}{x^2 - 4}\)  
40. \(\frac{x^2 - 9}{x^3 + x^2 - 9x - 9}\)  
41. \(\frac{z^3 - 8}{z^2 + 2z + 4}\)  
42. \(\frac{y^3 - 2y^2 + 3y}{y^3 + 1}\)

Error Analysis  Describe the error.

43. \(\frac{5x}{2x + 4} = \frac{5x^3}{2x^3 + 4}\)  
44. \(\frac{x^2 + 25x}{x^2 - 2x - 15} = \frac{x(x^2 + 25)}{(x - 5)(x + 3)}\)

In Exercises 45 and 46, complete the table. What can you conclude?

45. \[
\begin{array}{c|cccccc}
\text{x} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
\text{x} - 2x - 3 & \boxed{2x + 1} & \boxed{x} & \boxed{3} & \boxed{4} & \boxed{5} & \boxed{6} \\
\text{x} + 1 & \boxed{0} & \boxed{1} & \boxed{2} & \boxed{3} & \boxed{4} & \boxed{5} & \boxed{6} \\
\end{array}
\]

46. \[
\begin{array}{c|cccccc}
\text{x} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
\text{x} - 3 & \boxed{2x + 3} & \boxed{x} & \boxed{3} & \boxed{4} & \boxed{5} & \boxed{6} \\
\text{x}^2 - x - 6 & \boxed{1} & \boxed{x + 2} & \boxed{3} & \boxed{4} & \boxed{5} & \boxed{6} \\
\end{array}
\]

GEOMETRY  In Exercises 47 and 48, find the ratio of the area of the shaded portion of the figure to the total area of the figure.

47. [Diagram of a circle with a sector shaded]

48. [Diagram of a rectangle with a triangle shaded]

In Exercises 49–56, perform the multiplication or division and simplify.

49. \(\frac{5}{x - 1} \cdot \frac{x - 1}{25(x - 2)}\)  
50. \(\frac{x + 13}{x^3(3 - x)} \div \frac{x(x - 3)}{5}\)  
51. \(\frac{r}{r - 1} \div \frac{r^2}{r^2 - 1}\)  
52. \(\frac{4y - 16}{5y + 15} \div \frac{4 - y}{2y + 6}\)  
53. \(\frac{t^2 - t - 6}{t^2 + 6t + 9} \div \frac{t + 3}{t^2 - 4}\)  
54. \(\frac{x^2 + xy - 2y^2}{x^3 + x^2y} \cdot \frac{x}{x^2 + 3xy + 2y^2}\)  
55. \(\frac{x^2 - 36}{x} \div \frac{x^3 - 6x^2}{x^2 + x}\)  
56. \(\frac{x^2 - 14x + 49}{x^2 - 49} \div \frac{3x - 21}{x + 7}\)
In Exercises 57–68, perform the addition or subtraction and simplify.

57. \(6 - \frac{5}{x + 3}\)  
58. \(\frac{3}{x - 1} - 5\)

59. \(\frac{5}{x - 1} + \frac{x}{x - 1}\)  
60. \(\frac{2x - 1}{x + 3} + \frac{1 - x}{x + 3}\)

61. \(\frac{3}{x - 2} + \frac{5}{2 - x}\)  
62. \(\frac{2x - 5}{x - 5} - \frac{5}{x - 5}\)

63. \(\frac{4}{2x + 1} - \frac{x}{x + 2}\)  
64. \(\frac{2}{x - 3} + \frac{5x}{3x + 4}\)

65. \(\frac{1}{x^2 - x - 2} - \frac{x}{x^2 - 5x + 6}\)  
66. \(\frac{2}{x^2 - x - 2} + \frac{10}{x^2 + 2x - 8}\)

67. \(-\frac{1}{x} + \frac{2}{x^2 + 1} + \frac{1}{x^3 + x}\)  
68. \(\frac{2}{x + 1} + \frac{2}{x - 1} + \frac{1}{x^2 - 1}\)

**ERROR ANALYSIS** In Exercises 69 and 70, describe the error.

69. \(\frac{x + 4}{x + 2} \cdot \frac{3x - 8}{x + 2} = \frac{x + 4 - 3x - 8}{x + 2} = -\frac{2x - 4}{x + 2} = -\frac{2(x + 2)}{x + 2} = -2\)

70. \(\frac{6 - x}{x(x + 2)} + \frac{x + 2}{x^2} + \frac{8}{x^2(x + 2)} = \frac{x(6 - x) + (x + 2)^2 + 8}{x^2(x + 2)} = \frac{6x - x^2 + x^2 + 4 + 8}{x^2(x + 2)} = \frac{6(x + 2)}{x^2(x + 2)} = \frac{6}{x^2}\)

In Exercises 71–76, simplify the complex fraction.

71. \(\frac{x - 1}{2} \div (x - 2)\)  
72. \(\frac{x - 4}{\frac{x}{4} - \frac{x}{x}}\)

73. \(\frac{x^2}{(x + 1)^2}\)  
74. \(\frac{x}{(x - 1)^2}\)

75. \(\frac{\sqrt{x} - \frac{1}{2\sqrt{x}}}{\sqrt{x}}\)  
76. \(\frac{\sqrt{x^2 + 1} - \sqrt{t^2 + 1}}{t^2}\)

In Exercises 77–82, factor the expression by removing the common factor with the smaller exponent.

77. \(x^5 - 2x^{-2}\)  
78. \(x^5 - 5x^{-3}\)

79. \(\frac{x^2(x + 1)^{-5} - (x^2 + 1)^{-4}}{3}\)  
80. \(2x(x - 5)^{-3} - 4x^2(x - 5)^{-4}\)

81. \(2x^2(x - 1)^{-1/2} - 5(x - 1)^{-1/2}\)  
82. \(4x^3(2x - 1)^{-3/2} - 2x(2x - 1)^{-1/2}\)

In Exercises 83 and 84, simplify the expression.

83. \(\frac{3x^{1/3} - x^{-2/3}}{3x^{-2/3}}\)  
84. \(-\frac{x^3(1 - x^2)^{-1/2} - 2x(1 - x^2)^{1/2}}{x^4}\)

In Exercises 85–88, simplify the difference quotient.

85. \(\frac{\frac{1}{x + h} - \frac{1}{x}}{h}\)  
86. \(\frac{\frac{1}{x + h} - \frac{1}{x}}{h}\)

87. \(\frac{\frac{1}{x + h} - \frac{1}{x}}{h}\)  
88. \(\frac{\frac{1}{x + h} - \frac{1}{x}}{h}\)

In Exercises 89–94, simplify the difference quotient by rationalizing the numerator.

89. \(\sqrt{x + 2} - \sqrt{x}\)  
90. \(\sqrt{x - 3} - \sqrt{z}\)

91. \(\sqrt{x + 3} - \sqrt{x}\)  
92. \(\sqrt{x + 5} - \sqrt{3}\)

93. \(\sqrt{x + h + 1} - \sqrt{x + 1}\)  
94. \(\sqrt{x + h - 2} - \sqrt{x - 2}\)

**PROBABILITY** In Exercises 95 and 96, consider an experiment in which a marble is tossed into a box whose base is shown in the figure. The probability that the marble will come to rest in the shaded portion of the box is equal to the ratio of the shaded area to the total area of the figure. Find the probability.

95. \(x + \frac{1}{2}\)  
96. \(x + \frac{4}{x + 2}\)

97. **RATE** A digital copier copies in color at a rate of 50 pages per minute.

(a) Find the time required to copy one page.
(b) Find the time required to copy $x$ pages.
(c) Find the time required to copy 120 pages.

98. RATE After working together for $r$ hours on a common task, two workers have done fractional parts of the job equal to $\frac{r}{3}$ and $\frac{r}{5}$, respectively. What fractional part of the task has been completed?

FINANCE In Exercises 99 and 100, the formula that approximates the annual interest rate $r$ of a monthly installment loan is given by

$$r = \frac{\frac{24(NM - P)}{N}}{\left(P + \frac{NM}{12}\right)}$$

where $N$ is the total number of payments, $M$ is the monthly payment, and $P$ is the amount financed.

99. (a) Approximate the annual interest rate for a four-year car loan of $20,000 that has monthly payments of $475.
(b) Simplify the expression for the annual interest rate $r$, and then rework part (a).

100. (a) Approximate the annual interest rate for a five-year car loan of $28,000 that has monthly payments of $525.
(b) Simplify the expression for the annual interest rate $r$, and then rework part (a).

101. REFRIGERATION When food (at room temperature) is placed in a refrigerator, the time required for the food to cool depends on the amount of food, the air circulation in the refrigerator, the original temperature of the food, and the temperature of the refrigerator. The model that gives the temperature of food that has an original temperature of $75^\circ F$ and is placed in a $40^\circ F$ refrigerator is

$$T = 10\left(\frac{4t^2 + 16t + 75}{t^2 + 4t + 10}\right)$$

where $T$ is the temperature (in degrees Fahrenheit) and $t$ is the time (in hours).

(a) Complete the table.

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<th>4</th>
<th>6</th>
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<table>
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</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) What value of $T$ does the mathematical model appear to be approaching?

102. INTERACTIVE MONEY MANAGEMENT The table shows the projected numbers of U.S. households (in millions) banking online and paying bills online from 2002 through 2007. (Source: eMarketer; Forrester Research)

<table>
<thead>
<tr>
<th>Year</th>
<th>Banking</th>
<th>Paying Bills</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002</td>
<td>21.9</td>
<td>13.7</td>
</tr>
<tr>
<td>2003</td>
<td>26.8</td>
<td>17.4</td>
</tr>
<tr>
<td>2004</td>
<td>31.5</td>
<td>20.9</td>
</tr>
<tr>
<td>2005</td>
<td>35.0</td>
<td>23.9</td>
</tr>
<tr>
<td>2006</td>
<td>40.0</td>
<td>26.7</td>
</tr>
<tr>
<td>2007</td>
<td>45.0</td>
<td>29.1</td>
</tr>
</tbody>
</table>

Mathematical models for these data are

Number banking online $= \frac{-0.728t^2 + 23.81t - 0.3}{-0.049t^2 + 0.61t + 1.0}$

and

Number paying bills online $= \frac{4.39t + 5.5}{0.002t^2 + 0.01t + 1.0}$

where $t$ represents the year, with $t = 2$ corresponding to 2002.

(a) Using the models, create a table to estimate the projected numbers of households banking online and the projected numbers of households paying bills online for the given years.
(b) Compare the values given by the models with the actual data.
(c) Determine a model for the ratio of the projected number of households paying bills online to the projected number of households banking online.
(d) Use the model from part (c) to find the ratios for the given years. Interpret your results.

EXPLORATION

TRUE OR FALSE? In Exercises 103 and 104, determine whether the statement is true or false. Justify your answer.

103. $\frac{x^{2n} - 1^{2n}}{x^n - 1^n} = x^n + 1^n$

104. $\frac{x^2 - 3x + 2}{x - 1} = x - 2$, for all values of $x$

105. THINK ABOUT IT How do you determine whether a rational expression is in simplest form?

106. CAPSTONE In your own words, explain how to divide rational expressions.