Notes

I. Overview
   a. NCTM standard on number and operations includes expectations for the understanding of operations and how they relate to each other and for fluent computation and making reasonable estimates
      i. During the primary grades, students should:
         1. Reach an understanding of a variety of means for adding and subtracting and the relationship between the two operations
         2. Understand the effects of adding and subtracting whole numbers
         3. Understand the situations where multiplication and division are used
      ii. Children should:
         1. Develop skill at whole number computation for addition and subtraction
         2. Be able to use the basic addition and subtraction number combinations
         3. Use many different computation tools such as “objects, mental computation, estimation, paper and pencil, and calculators”
   b. Focal Points for Number and Operations
      i. First grade
         1. Students develop an understanding of whole numbers from 10 to 100 in relation to groups of tens and ones
      ii. Second grade
         1. Students develop an understanding of the base-ten numeration system and place-value concepts
         2. Able to depict equivalent representations of numbers such as 25 can be shown as two tens and one five
      iii. Third grade
         1. Students extend their understanding of place value to numbers up to 10,000
   c. During the latter part of the preoperational period, children who are adept at manipulating quantities up to 10 can move on to working with quantities above 10
      i. Through manipulation of groups of 10 and quantities between zero and 10, children move through the teens and up to 20
         1. Some will pick up the pattern of the 20s, 30s, and so on up through the 90s
      ii. As children enter concrete operations, they perfect their informal knowledge of numbers above 10 and move on to whole number operations with numbers above 10
   d. To fully understand what they are doing when they use whole number operations involving numbers above 10, they must be able to conceptualize place value
      i. Place value pertains to an understanding that the same numeral represents different amounts depending on which position it is in
      ii. An understanding of place value underlies the understanding of certain trading rules that govern place value and enable whole number operations to be accomplished
      iii. Examples of some trading rules;
         1. Ten ones can be traded for one 10
         2. One 10 can be traded for 10 ones
         3. Ten 10s can be traded for 100
4. One hundred ones can be traded for 100

e. The place-value concept enables us to represent any value using only 10 digits (zero to nine)

f. Place value is one of the most difficult concepts for young children to grasp
   i. Being able to rote and rational count above 10 is only a beginning step on the way to an understanding of place value
      1. Children need many counting experiences and many experiences with concrete models to develop the place-value concept
   ii. All too often children are rushed into the place-value operations involved in regrouping (what used to be referred to as borrowing and carrying) as a rote memory activity without the necessary underlying conceptualization
      1. Through an understanding of place value, children will realize that when they take one from the 10s’ column they are actually taking one group of 10 and that when they add numbers in the ones’ column and arrive at a sum above nine that the amount they move to the 10s’ column represents one or more groups of 10
   iii. Understanding place value will also help them to see that the placement of numerals is critical in determining value

II. Assessment
   a. Understanding place value is a difficult task for young children
      i. They will normally flounder for a while, seeming to understand the concept in some situations and not in others
      ii. Teachers should be patient and accepting and give the children time and appropriate experiences
   b. On average:
      i. First graders can learn to read, write, and understand two-digit numbers
      ii. Second graders can learn to read, write, and understand three-digit numbers
      iii. Third graders can learn to read, write, and understand four-digit numbers
   c. There will be a broad range of normal variation within any particular group
      i. Best rule of thumb is to be sure children understand one-digit numbers before going on to two-digit, two before three, etc.

III. Activities
   a. Young children need many experiences in manipulating objects relative to numerals greater than 10 and place value before proceeding to whole number operations with two-digit numbers
   b. Once the children have a good understanding of counting and subdividing groups greater than 10, they are ready for activities that gradually move them into the complexities of place value
   c. Base 10 blocks provide a different kind of model for working with the place-value concept
      i. Important that children work with a variety of types of materials in model construction so that they do not think there is just one way to view place value with concrete materials
         1. Base 10 blocks depict each place with a solid model
         2. Units (or 1s) are individual cube
         3. Rods (or 10s) are the equivalent of 10 unit cubes stuck together in a row
         4. Flats (or 100s) are the equivalent of 10 rods stuck together
         5. Cubes (or 1,000s) are the equivalent of 10 flats stacked and glued together
d. Trading is another procedure for working with place value
   i. Primary children need many experiences:
      1. Counting piles of objects
      2. Trading for groups of 10
      3. Describing the results
   ii. Once they can do these activities with ease, they can move on to
       regrouping and renaming
   iii. Regrouping happens when one or more items are added or taken away so
       that an amount moves to the next 10, next 100, next 1,000, and so on
       1. Renaming of the group has also occurred

e. Reverse trading would take place if units were removed
   i. Primary grade children need to do many trading activities with concrete
      materials before moving on to paper and pencil computations
   ii. These trades can be practiced with concrete items such as cubes and
       chips or the beads on an abacus or with solids such as base 10 blocks

f. When the children practice trading to regroup and rename on their place-value
   boards, they are actually adding and subtracting informally
   i. This type of addition and subtraction may lead children to believe that
      adding or subtracting with two digits is exactly the same operation as with
      one digit
   ii. Repeat this process several times with different pairs of two-digit numbers:
      1. Sometimes the sum contains more than nine units, and sometimes it
         does not
      2. When the children understand the process without symbols, connect
         the symbols by writing them on the board as you go through the
         process
   iii. Subtraction of two-digit numbers can be introduced in parallel fashion
   iv. After doing many examples without written numbers, go through some
       problems in which you connect the quantities to numbers at each step, thus
       gradually introducing the notation
       1. Then have them do mixed sets of problems
       2. Finally, move on to story problems
   v. Problems and activities that apply place-value concepts can be integrated
      across the curriculum

IV. Kamii’s Approach
   a. Constance Kamii and her colleagues have been working with primary children
      using open-ended activities that provide for more child trial and error and self-
      sequencing
      i. Interviewing primary students who had been through conventional
         workbook/textbook instruction, they discovered that students were able to
         do regrouping and renaming as a rote process without really knowing the
         meaning of the numbers they were using
   b. When asked what the 1 in 13 means, they said it meant one rather than 10
      i. Kamii has had greater success in getting this concept over to primary
         children using games and letting them discover the relationship of the digits
         on their own
      ii. No workbooks or worksheets are used and neither are the kinds of
          concrete activities described in this unit
      iii. Problems are written on the board, children contribute answers, and every
          answer is listed
          1. Then the children give their rationales for their answers
2. When working with double-digit addition, their natural inclination is to start on the left:
   a. They add the 10s; write the answer; add the 1s; and, if necessary, move any 10s over, erasing the original answer in the 10s' column.
iv. Through trial and error and discussion, they develop their own method and construct their own place-value concept:
   1. Place value is not taught as a separate skill needed prior to doing double-column addition.
   2. This method sounds most intriguing and is more fully described in Kamii.

V. Calculators
   a. When children explore calculators for counting, they notice that the number on the right changes every time, whereas the other numbers change less frequently:
      i. Calculators provide a graphic look at place value and the relation of each place to those adjacent.
      ii. Suggest that the children try 10 and note what happens; that is, which place changes, and how much each time?
         1. This activity will assist children in seeing what the 10s' place means.
   iii. Concepts constructed using manipulatives can be reinforced with calculator activities:
      iv. By adding one to numbers that end in nine and subtracting one from numbers that end in zero, students can see immediately what happens:
         1. Suggest that they guess which number follows nine, 29, and 59.
         2. Then have them use their calculators to check their predictions:
            a. Use the same procedure, except have them predict what will happen if they subtract one from 20, 40, and 70.

VI. Ideas for Children with Special Needs
   a. Children with motor disabilities such as spina bifida (SB), cerebral palsy (CP), or developmental coordination disorder (DCD) are challenged to accomplish any type of motor skills task:
      i. They may not be able to interact with objects in any conventional way depending on the degree of their disability.
      ii. However, they can develop cognitive understanding with the right kind of support:
      iii. Ways have to be found for them to act on their environment:
      iv. Technology can be designed which allows children with motor disabilities to use whatever movement they have available to assist them in learning and communicating:
      v. These children need close attention from adults who provide prompting and cueing as they work:
      vi. In class teachers need to emphasize the children's strengths:
   b. Children with perceptual problems may have difficulties perceiving the commonalities among numbers above nine:
      i. May need to have the teens lined up vertically so the common element of one can be seen:
         1. Color-coding can be helpful:
      ii. These children commonly make reversals, e.g. confusing 13 and 31.
      iii. With numbers of two or more symbols it is important for children to understand spatial terms such as left, right, middle, first, last and beginning:
      iv. Some children may need extra help with auditory memory.
1. The teacher may say 52 but the child may only perceive the two
c. Children may find it interesting to learn how we arrived at base ten for our number system and the systems other cultures use for recording and calculating
   i. Zaslavsky explains how finger counting is common practice in many cultures
      1. In others, fingers and toes are used so the base is 20
   ii. As large amounts are involved each culture has had to invent a method of recording and calculating
      1. Tally marks on bone or wood have been use for thousands of years
      2. The bar codes used so commonly today are based on groups of tally marks of two heights
      3. Knots on string are another method used to record amounts
      4. The abacus, first developed in China, allows for calculation of large numbers using rows of beads

VII. Evaluation
   a. An evaluation technique suggested by Kamii shows if children really understand place value in two-digit numbers
      i. Show the child a three-by-five-inch card with 16 written on it
      ii. Ask, “What does this say?”
      iii. After the child says 16, count out 16 chips
      iv. With the top of a pen circle the 6 of the 16
      v. “What does this part (the 6) mean? Show me with the chips what this part (the 6) means.”
      vi. Circle the 1 of the 16
      vii. “What does this part (the 1) mean? show me with the chips what this part (the 1) means.”
   b. Kamii says that after conventional instruction with workbooks and possibly some manipulatives, all first and second graders can answer correctly regarding the 6, but of the primary children Kamii interviewed, none of the end-of-first-graders, 33% of end-of-third-graders, and only 50% of end-of-fourth-graders said that the 1 means 10
      i. Children who learned about place value through constructing it themselves using Kamii’s method did considerably better
      ii. At the end of the second grade, 66% said that the 1 means 10 and 74% said that the 5 in 54 means 50
   c. Whichever instructional method you use, be sure to observe very carefully the process each child uses
      i. Be sure to question children frequently about what they are doing to be sure they really understand the concepts and are not just answering in a rote manner