Notes

I. Overview
   a. Ten areas presented
      i. Algebra
      ii. Classification
      iii. Shape
      iv. Spatial relations
      v. Concrete whole number operations
      vi. Graphs
      vii. Symbolic level activities
      viii. Quantities above 10
      ix. Estimation
      x. Design technology

II. Assessment
   a. Assessment determines where the children are in their zones of proximal development (ZPD)
      i. Where they can work independently and where they can complete tasks with support from scaffolding by an adult or a more advanced peer
   b. Teacher looks at the child’s level in each area and then makes a decision as to when to introduce these activities
      i. When introduced to one child, any one activity could capture the interest of another child who might be at a lower developmental level
         1. Not necessary to wait for all the children to be at the highest level to begin
         2. Children at lower levels can participate in these activities as observers and as contributors
         3. The higher-level child can serve as a model for the lower-level child
         4. Lower-level child might be able to do part of the task following the leadership of the higher-level child.
      ii. Children can work in pairs to solve concrete addition, subtraction, multiplication, and division problems
         1. Can move into higher levels of symbol use and work with numerals and quantities greater than 10
         2. Can also work together exploring calculators and computer software
   c. By the end of kindergarten children should have an understanding of number
      i. Number sense should be well established
         1. They need to understand that the concept of number is independent of size, shape, color, etc.
         2. They need to find groups everywhere that are two, three, four, five, etc.
         3. They must understand that arrangement in space (conservation of number) is independent of amount
4. They need to examine and construct groups using different objects
   ii. With number understood, students are ready to move on to more abstract ideas that are based on an understanding of number

III. Algebraic Thinking
   a. Algebra is viewed by many people as a blockade in their progress in understanding mathematics
      i. Is seen as the mindless abstract manipulation of symbols
      ii. NCTM is promoting a new vision of algebra as “a way of thinking, a method of seeing and expressing relationships”
         1. It is seen as a way of thinking that goes beyond numerical reasoning and that can begin in the elementary grades
   b. For preprimary-level children, algebraic thinking is reflected in their discovery of patterns as they sort and group objects, combine groups and count totals, build with blocks, and use objects as symbolic representations
      i. As young children explore these materials, they construct generalizations that reflect an increasing understanding of patterns and relationships
      ii. Their discoveries are the outcome of the beginnings of algebraic thinking
         1. As children move into higher-level activities, it is important to continue to provide them with opportunities for exploration and discovery
         2. Algebraic reasoning is also supported by science inquiry activities

IV. Classification
   a. The higher levels of classification are called multiple classification, class inclusion, and hierarchical classification
      i. Multiple classification requires the child to classify things in more than one way and to solve matrix problems
      ii. The preoperational child cannot see that one class may be included within another (class inclusion)
      iii. Hierarchical classification has to do with classes being within classes
           1. Basic-level concepts are usually learned first
           2. Superordinate-level concepts are learned next
           3. Finally, children learn subordinate categories
   b. Another interesting aspect of young children’s concept learning is their view of which characteristics the members of a class have in common
      i. Although preoperational-level children tend to be perceptually bound when they attempt to solve many types of conceptual problems, they are able to classify on category membership when shown things that are perceptually similar
   c. Another type of characteristic that is interesting to ask young children about is their view of what is inside members of a class
      i. When young children are asked if members of a class all have the same “stuff” inside, preschoolers tend to say that yes, they have; that
is, all dogs, people, chairs, and dolls are the same inside

ii. Children are aware of more than just observable similarities

iii. By second grade, they can discriminate between natural and synthetic items

1. They realize that living things such as dogs, people, or apples are, for the most part, the same inside as other dogs, people, or apples, although the insides of different types of chairs, dolls, or other manufactured items are not necessarily the same

iv. For the younger children, category membership overwhelms other factors

V. Shape

a. Once the child can match, sort, and name shapes she can also reproduce shapes

i. Can be done informally

ii. Materials that can be used:

1. Geoboards

   a. Square board with headed screws or pegs sticking up at equal intervals
   
   b. Can be purchased or made
   
   c. Child is given a supply of rubber bands and can experiment in making shapes by stretching the rubber bands between the nails

2. Container of pipe cleaners or straws can be put out

   a. Children can be asked to make as many different shapes as they can
   
   b. These can be glued onto construction paper
   
   c. Strips of paper, toothpicks, string, and yarn can also be used to make shapes

3. Pattern blocks

   a. Important material for children to use in exploring shape
   
   b. For beginners, puzzle frames are usually provided that indicate the shapes to be used to fill the frame
   
   c. For more advanced students, frames are provided where the pattern block shapes are partially indicated
   
   d. Children who can select pieces to fill in the puzzle without trial and error may be provided puzzle frames with no hints as to which pattern block shapes will fill the frame
   
   e. Children who master these advanced frames can be offered the challenge of filling the frames in more than one way

VI. Spatial Relations

a. Children can learn more about space after playing the treasure hunt game described in Unit 13 by reproducing the space around them as a floor plan or a map

   i. Start with the classroom for the first map
   
   ii. Then move to the whole building, the neighborhood, and the town or
VII. Design Technology
   a. In Unit 13, design technology was described as a natural component of children's play
      i. The more advanced five- and six-year-olds may be challenged by more advanced design technology problems
      ii. In the more advanced design technology projects children go through several steps:
         1. A problem is identified
         2. Ideas are generated for ways of investigating and solving the problem
         3. A plan of action is devised
         4. A product is designed
         5. The product is made and tested
         6. Students reflect on the results of their process and product
   b. Problems may be worked on by individuals or small groups
   c. The youngest children should start with simple projects that focus on one item such as designing an airplane, building a house, or constructing a miniature piece of playground equipment
      i. Supplied with small boxes and other recycled materials, tape, and glue, children's imaginations take off

VIII. Graphs
   a. The fourth level of graphs introduces the use of squared paper
      i. Child may graph the same kind of things as discussed in Unit 20
      ii. Will now use squared paper with squares that can be cooled in or paper squares glued in
         1. These should be introduced only after the child has had many experiences of the kinds described in Unit 20
         2. Squares should be large

IX. Concrete Whole Number Operations
   a. Once children have a basic understanding of one-to-one correspondence, number and counting, and comparing, they can sharpen their problem-solving skills with concrete whole number operations
      i. They can solve simple addition, subtraction, division, and multiplication problems using concrete materials
      ii. You can devise some simple problems to use as models
      iii. Children will gradually catch on and begin devising their own problems, such as those described by Skinner
   b. As children grow and develop and have more experiences with whole number operations, they learn more strategies for solving problems
      i. Gradually stop using the less efficient strategies and retain the more efficient ones
      ii. When observing children working with division, note whether or not they make use of their concept of one-to-one correspondence
   c. One More Child by Sharon Young takes the emergent reader through the
process of adding on one from one to four
i. Affords an opportunity to see the logic that when you add one more, the total is the next number in order
ii. Students can use objects or a calculator to continue adding ones
iii. Suggest they add several ones and discover how many they obtain
d. *Six Pieces of Cake* takes the emergent reader through the subtraction process in which one-by-one pieces of cake are taken off a plate

X. The Symbolic Level
a. Children who can connect groups (or sets) and symbols, identify numerals 0 to 9, and do concrete addition and subtraction problems can move to the next step, which is connecting groups (or sets) and symbols in addition and subtraction
b. As children continue to create their own problems and work on teacher-created problems, they can be encouraged to communicate their findings
   i. Suggest that they draw, write, and use numerals to show their results
   ii. They can use cards with numerals written on them and gradually write the numerals themselves
   iii. Children can work in pairs on problems and trade problems with other students
c. As the children work with these concrete symbol/set addition and subtraction problems, they will begin to store the basic facts in their memories and retrieve them without counting
   i. They can then do problems without objects
   ii. Just have the objects on hand in case they are needed to check the answers
d. Excellent resources for games and materials are:
   i. *Workjobs II* (Baratta-Lorton, 1979)
   iii. *The Young Child and Mathematics* (Copley, 2000)
   v. *Developing Number Concepts Using Unifix Cubes®* (Richardson, 1984)
   vi. *Navigating Through Problem Solving and Reasoning in Prekindergarten-Kindergarten* (Greens et al, 2003),
   viii. *Hands-on Standards*, 2007)
   ix. *A Head-start on Science*, (Ritz, 2007)
   x. In *What’s Your Problem?* (Skinner, 1990) a means is provided for encouraging children to develop their own problems.
e. Calculators are also useful as tools for experimentation
   i. Children will learn to make connections between problem solving and the signs on the calculator

XI. Quantities Above Ten
a. Children are ready to move on to counting quantities above ten once they can:
i. Count 10 objects correctly
ii. Identify the numerals 1 to 10
iii. Have a good grasp of one-to-one correspondence
iv. Accurately count by rote past 10

b. They can acquire an understanding of quantities above 10 through an exploration of the relationship of groups of 10 with additional amounts
c. Once the children understand through 19, they can move on to 20, 30, 40, and so on
   i. By exploring the number of 10s and 1s represented by each numeral, they will discover the common pattern from 20 to 99

XII. Estimation
a. Estimation for young children involves making a sensible and reasonable response to the problem of how many are in a quantity or how much in a measurement something is
   i. It is the “process of thinking about a ‘how many’ or ‘how much’ problem and possible solutions” (Lang, 2001, p. 463)
   ii. Children might estimate how many objects (e.g., candies, teddy bears, screws, etc.) are in a jar or how many shoes tall the bookcase is
   iii. In order to come up with a reasonable response children need to have developed number, spatial, and measurement sense
   iv. Without these prerequisite concepts they will make wild guesses rather than reasonable estimates
b. Lang suggests several ways to assist children so they can make reasonable estimates
   i. A referent can be used such as, “If I know how tall John is, I can estimate how tall the bookcase is if he stands next to it.”
   ii. Chunking involves taking a known measurement and using it as a guide for estimating a larger measurement
   iii. Unitizing is another type of chunking where, if one part is known, then the whole can be estimated

c. It is important that children understand the language of comparison in order to give and receive communications regarding their estimates

XIII. Ideas for Children with Special Needs
a. For some children, a useful step for beginning arithmetic is to play a thinking game
   i. Using numbers 2 to 9 the children are asked a series of questions:
      1. “What number comes after 6?”
      2. “What number comes before 6?”
      3. “What is 6 and one more?”
   ii. Patterns are also useful in helping children move into arithmetic
      1. For example, when making a bead pattern the child must think, “How many more beads will I need to complete my pattern?”