Notes

I. Overview
   a. NCTM expectations
      i. Expectations for number focus on children in prekindergarten through grade two counting with understanding and recognizing “how many?” in sets of objects
      ii. Children are also expected to develop understanding of the relative position and size of whole numbers, ordinal and cardinal numbers, and their connections to each other
      iii. Children also expected to develop a sense of whole numbers and be able to represent and use them in many ways
      iv. These expectations should be achieved through real-world experiences and through using physical materials
   b. Included in the NCTM standards is the standard that children should develop whole number skills that enable them to:
      i. Construct number meanings through real-world experiences and the use of physical materials
      ii. Understand our number system by relating, counting, grouping, and (eventually) place-value concepts
      iii. Develop number sense
         1. Number sense and counting can be integrated in other content areas
   c. Major Focal Point is the development of an understanding of whole numbers
      i. Prekindergarteners need to learn to recognize objects in small groups by counting and without counting
         1. Need to understand that numbers refer to quantities
         2. Begin to use number to solve everyday problems
         3. Need to be prepared to relate numerals and groups when they enter kindergarten
   d. Number sense
      i. Concept of number or understanding number is referred to as number sense
      ii. Makes the connection between quantities and counting
      iii. Underlies the understanding of:
         1. More and less
         2. Relative amounts
         3. Relationship between space and quantity
         4. Parts and wholes of quantities
      iv. Enables children to understand important benchmarks such as 5 and 10 as they relate to other quantities
      v. Helps children estimate quantities and measurements
      vi. Counting assists children in the process of understanding quantity
         1. Understanding that the last number named is the quantity in the group is a critical fundamental concept
   e. Substitizing
i. When shown a group, seeing “how many” instantly is called subitizing

ii. Two types of subitizing:
   1. Perceptual subitizing
      a. Is, when shown a group, one can state how many without counting or grouping
      b. Young children usually learn to subitize up to four items perceptually
   2. Conceptual subitizing
      a. Involves seeing number patterns within a group such as the larger dot patterns on a domino
      b. Viewer may break the eight-dot pattern down into two groups of four, which makes up the whole

iii. Perceptual subitizing is thought to be the basis for counting and cardinality (understanding the last number named is the amount in a group)
   1. Develops from counting and patterning and helps develop number sense and arithmetic skills

iv. Preschoolers can subitize perceptually
v. Conceptual subitizing for small quantities usually begins in first grade
   1. Clements suggests some games that can be played in kindergarten that can bridge into conceptual subitizing
   2. Quantities from one to four or five are the first to be recognized
      a. Concept of number is constructed bit by bit from infancy through the preschool years and gradually becomes a tool that can be used in problem solving

f. Counting
   i. Includes two operations:
      1. Rote counting
         a. Involves reciting the names of the numerals in order from memory
      2. Rational counting
         a. Involves matching each numeral name in order to an object in a group
         b. Builds on children’s understanding of one-to-one correspondence
   ii. Four principles of rational counting
      1. Only one number name may be assigned to each of the objects to be counted
      2. There is a correct order in which the number names may be assigned, that is, one, two, three, and so on
      3. Counting can start with any of the items in the group
      4. The cardinality rule states that the last number name used is the number of objects in the group
   iii. Basic understanding of accurate rote counting and one-to-one correspondence is the foundation of rational counting
      1. Ability to rational count assists children in understanding the
concept of number by enabling them to check their labeling of quantities as being a specific amount

a. Also helps them to compare equal quantities of different things, regardless of what makes up a group

2. In the long term, number, counting, and one-to-one correspondence all serve as the basis for developing the concept of number conservation, which is usually mastered by age six or seven

iv. Too often the preprimary mathematics program centers on counting with repeated teacher-directed drill and practice

1. Children need repeated and frequent practice to develop counting skills, but this practice should be of short duration and center on naturalistic and informal instruction

2. Structured activities should include many applications, such as:
   a. How many children in the class have brothers? Sisters? Neither?
   b. How many days will it be until the first seed sprouts?
      i. Let’s make some guesses, and then we’ll keep track and see who comes close.
   c. How many days did we mark off before the first egg hatched?
   d. How many carrots do we need so that each rabbit will get one?
   e. How many of the small blocks will balance the big block on the balance scale?

v. Normal expectation that rote counting develops ahead of rational counting

1. Rational counting is a fairly complex task
   a. To count objects accurately, the child must know the number names in the correct order and be able to coordinate eyes, hands, speech, and memory
      i. Difficult for the two- or three-year-old because she is still in a period of rapid growth in coordination
      ii. She is also limited in her ability to stick to a task
      iii. Teacher should not push a child to count more things than he can count easily and with success
   b. Most rational counting experiences should be naturalistic and informal

2. Rational counting skills should begin to catch up with rote counting skills by age four or five as the rate of physical growth slows and coordination of eyes, hands, and memory matures
   a. At the same time, naturalistic and informal activities should continue

3. During the kindergarten year, children usually become skilled at rote and rational counting
   a. Many are ready to play more complex games with
quantities such as counting backward and counting on from a known quantity
b. Counting backward and counting on lay the foundation for the whole number operations of addition and subtraction
c. Estimation activities can begin with prekindergartners playing simple games like, “Guess how many beans are in a small jar” or “How many paper clips wide is the table?” followed by checking their guesses by counting the beans and paper clips

4. Kamii emphasizes the necessity of being aware of the coordination of one-to-one correspondence and counting in the development of the concept of number

5. Four levels of development in counting have been identified by asking children to put out the same number of items as an adult put out using groups of sizes four to eight
   a. Children cannot understand what they are supposed to do
   b. They do a rough visual estimation or copy (that is, they attempt to make their group look like the model)
   c. Children do a methodical one-to-one correspondence
      i. It is usually past age five and one-half before children reach this stage
   d. Children count
      i. Child counts the items in the model and then counts out another group with the same amount
      ii. Children usually reach this stage at about age seven

   vi. To develop the coordination of the two concepts, it is essential that children count and do one-to-one correspondence using movable objects
      1. Becomes obvious that, among other weaknesses, the use of workbook pages precludes moving the objects to be counted and/or matched
      2. Opportunities should be provided for the children to work together so they can discuss and compare ideas
         a. As the children work individually and/or with others, watch closely and note the thinking process that appears to take place as they solve problems using counting and one-to-one correspondence

II. Assessment
   a. Adult should note the child’s regular activity
   b. Be sure to record naturalistic and informal events

III. Naturalistic Activities
   a. Number sense is a concept and counting is a skill used a great deal by young children in their everyday activities
i. Once these are in the child’s thoughts and activity, he will be observed often in number and counting activities
   1. Practices rote counting often
ii. Number appears often in the child’s activities once he has the idea in mind
iii. Number names are used for an early form of division
   1. Child has three cookies, which he divides equally with his friends

IV. Informal Activities
   a. Alert adult can find a multitude of ways to take advantage of opportunities for informal instruction
      i. Because rote counting is learned through frequent but short periods of practice, informal activities should be used most for teaching
   b. Everyday activities offer many opportunities for informal rational counting and number activities
   c. Table setting offers many chances for rational counting
   d. More challenging problem can be presented by asking an open-ended question such as, “Get enough napkins for everyone at your table” or “Be sure everyone has the same number of carrot sticks.”
      i. In these situations, children don’t have a clue to help them decide how many they need or how many each person should get but have to figure out for themselves how to solve the problem
      ii. Often children will forget to count themselves
         1. Presents an excellent opportunity for group discussion to try to figure out what went wrong
         2. Teacher could follow up such an incident by reading the book Six Foolish Fishermen (Elkin, 1968, 1971), in which the fishermen make the same mistake

V. Structured Activities
   a. Rote counting is learned mostly through naturalistic and informal activities
      i. There are also short, fun things that can be used to help children learn the number names in the right order
         1. Rhyme songs
         2. Finger plays
      ii. More direct ways of practicing rote counting are also good
         1. Clapping and counting at the same time teaches number order and gives practice in rhythm and coordination
   b. Groups that have zero to four items are special in the development of rational counting skills
      i. Number of items in groups this size can be perceived without counting
         1. These groups are easy for children to understand
      ii. Children should have many experiences and activities with groups of size zero to four before they work with groups of five and more
         1. Wise to start with groups of size two because there are so many that occur naturally
      iii. When the children have the idea of groups from zero to four, they can
go on to groups larger than four
1. Some children are able to perceive five without counting just as
   they perceive zero through four without actually counting
2. Having learned the groups of four and fewer, children can be
   taught five by adding on one more to groups of four
3. When the children understand five as four with one more and
   six as five with one more and so on, then more advanced
   rational counting can begin.
   a. Children can work with groups of objects where they can
      find the number only by actually counting each object
4. Before working with a child on counting groups of six or more,
   the adult must be sure the child can do the following activities:
   a. Recognize groups of zero to four without counting
   b. Rote count to six or more correctly and quickly
   c. Recognize that a group of five is a group of four with one
      more added
iv. Four-to-six-year-olds can play simple group games that require
    them to apply their counting skills
    1. Not only can they count, they can compare amounts to find out
       who has the most and if any of them have the same amount
    2. Older children will be interested in writing numerals and might
       realize that instead of tick marks they can write down the
       numeral that represents the amount to be recorded
v. Students who are skilled at counting enjoy sorting small objects
    1. At first they might be given a small amount such as 10 items
       with which to work
    2. Can compare the amounts in each of the groups that they
       construct as well as compare the amounts in their groups with a
       partner’s
       a. Eventually they can move on to larger groups of objects
          and to more complex activities such as recording data
          with number symbols and constructing graphs
       b. As a special treat, this type of activity can be done with
          small edibles
vi. Another activity that builds number concepts is the Hundred Days
    Celebration
vii. Kindergartners can work with simple problem-solving challenges
    1. Schulman and Eston described a type of problem that
       kindergartners find intriguing
       a. Children were in the second half of kindergarten and had
          previously worked in small groups
       b. Basic situation was demonstrated as the “carrot and
          raisin” problem
          i. Children were told that a person had carrots and
             raisins on her plate, and had seven items in all
          ii. Children had to determine what she had on her
2. Computer software has also been designed to reinforce counting skills and the number concept

VI. Ideas for Children with Special Needs
a. Most young children learn to count and develop number sense through informal everyday experiences and through books and rhymes
   i. Others need additional teacher-directed experiences
      1. Some children will need extra verbalization through finger plays and rhymes
      2. Others will need more multisensory experiences using their gross-motor, fine-motor, tactual, auditory, and visual senses
b. Rational counting follows a prescribed sequence which should take on a rhythmical pattern
   i. Children can be helped to achieve this goal through the following types of activities
      1. Counting objects into containers provides both an auditory, tactile, and verbal experience
      2. Self correcting form boards or peg boards
c. Some children have difficulty organizing their work
   i. May count a group in a random manner
   ii. May need to be taught to methodically pick up each object for counting and put each in a new place
      1. Start with straight lines and move to irregular patterns
      2. Gradually move to asking the child to count out a specific number of objects
      3. Count each object with the child
d. Most young children enjoy learning a new language
   i. While ELL students need to learn to count in English, English-speaking students enjoy counting in a second language

VII. Evaluation
a. Informal evaluation can be done by noting the answers given by the children during direct instruction sessions
   i. Teacher should also observe the children during center time and notice whether they apply what they have learned
   ii. When they choose to explore materials used in the structured lessons during center time, questions can be asked
b. Hundreds collections and graphs can be placed in portfolios
   i. Photos of children working with materials can also be included
   ii. Anecdotes and checklists can be used to record milestones in number concept growth and development
c. Kathy Richardson provides questions to guide evaluation of counting and number sense
   i. To what amount can children work with (i.e., five, 10, 12, etc.)
   ii. What kinds of errors do the children make? Are they consistent or random?
   iii. Do they stick with the idea of one-to-one correspondence as they
count?
iv. Are they accurate? Do they check their results?
v. Do they remember the number they counted to and realize that is the amount in the group (cardinal number)?
d. Some key questions to consider when evaluating number sense include:
i. Can the children subitize perceptually, that is, recognize small groups of four or five without counting?
ii. Knowing the amount in one group, can they use that information to figure out how many are in another group?
iii. Can they make reasonable estimates of the amount in a group and revise their estimate after counting some items?
iv. When items are added to a group that they have already counted, do they count on or begin again?