Multiobjective Decision Making

CHAPTER 16

EVALUATING AND PRIORITIZING PROJECTS AT NASA

More public pressure than ever before is on NASA to justify its choice of projects to undertake. There is demand for accountability, pressure to cut costs, and an increasing number of potential projects to choose from. In the past, a committee of 15 members from NASA met once a year to review the 30 to 50 proposals submitted by contractors and divisions with the Kennedy Space Center. The five voting members (the decision makers, or DMs) gave each proposal a score from 1 to 10, the scores were averaged over the five DMs, and the top scoring proposals were selected until the budget was exceeded. Because the process was viewed as intuitive, management expressed concern about its subjectivity and consistency. It wanted to replace this process with a more comprehensive and structured process. Tavana (2003) describes the system he developed to meet these needs. He calls it consensus-ranking organizational-support system (CROSS).

The selection of projects at NASA is clearly a multiobjective decision-making problem. As Tavana describes, there are a number of stakeholders for each project. Essentially, they are the different departments within NASA—including Safety, Systems Engineering, Reliability, and others—and each has its own criteria for a successful project. For example, Safety might be concerned about eliminating the possibility of death or serious injury, Systems Engineering might be concerned about eliminating reliance on identified obsolete technology, and Reliability might be concerned about increasing the mean time between failures. CROSS uses AHP (Analytic Hierarchy Process, discussed later in this chapter) to obtain the information each DM...
needs to obtain a score for each project. It then combines the DMs’ scores to get an overall consensus ranking of projects. Finally, it uses this consensus ranking, along with project costs and the overall budget, to select the projects to be funded.

More specifically, the system first asks each DM to use AHP to evaluate the importance of the various stakeholders. For example, one DM might give Safety an importance weight of 0.5, whereas another might give Safety a weight of 0.4. In the next step, each stakeholder is asked to use AHP to evaluate the importance of its various criteria. This leads to a set of weights for each stakeholder-criterion combination. The stakeholders are also asked to estimate the probability that each potential project will be successful in satisfying each criterion. The system uses these probabilities to adjust the previous weights. Next, all of the weights from AHP are used to calculate a project-success factor for each project, as assessed by each DM, and these factors are used to obtain each DM’s rankings of the projects. Finally, the system attempts to reach consensus in the rankings using another (non-AHP) methodology.

The system is now being used successfully to select NASA projects. As a measure of its perceived quality, 71 projects were submitted during the first two years of implementation of CROSS. Using this system, the DMs chose 21 projects of the 71, and management subsequently approved all 21 choices.

16.1 INTRODUCTION

In many of your classes, you have probably discussed how to make good decisions. Usually, you assume that the correct decision optimizes a single objective, such as profit maximization or cost minimization. In most situations you encounter in business and life, however, more than one relevant objective exists. For example, when you graduate, many of you will receive several job offers. Which should you accept? Before deciding which job offer to accept, you might consider how each job “scores” on several objectives, such as salary, interest in work, quality of life in the city you will live in, and nearness to family. In this situation, combining your multiple objectives into a single objective is difficult. Similarly, in determining an optimal investment portfolio, you want to maximize expected return, but you also want to minimize risk. How do you reconcile these conflicting objectives? In this chapter, we discuss three tools, goal programming, trade-off curves, and the Analytic Hierarchy Process, that decision makers can use to solve multi-objective problems.

**Fundamental Insight**

Optimizing with Multiple Objectives

When there are multiple objectives, you can proceed in several fundamental ways. First, you can prioritize your objectives. This is done in goal programming, where the highest priority objective is optimized first, then the second, and so on. Second, you can optimize one objective while constraining the others to be no worse than specified values. This approach is used to find trade-off curves between the objectives. Finally, you can attempt to weight the objectives to measure their importance relative to one another. This is the approach taken by the Analytic Hierarchy Process. All of these approaches have their critics, but they can all be used to make difficult decision problems manageable.
In many situations, a company wants to achieve several objectives. Given limited resources, it may be impossible to meet all objectives simultaneously. If the company can prioritize its objectives, then goal programming can be used to make good decisions.

To understand goal programming, you need to understand soft constraints and deviations from goals. An example will help to clarify these ideas. Suppose a company wants to change its hiring policy so that at least 20% of all new hires are minorities. This is one of the company’s goals. Rather than adding a “hard” constraint in the usual way, where “hard” means that the constraint must be satisfied, the company adds a “soft” constraint, meaning that they will try to satisfy it, but they will consider solutions where fewer than 20% of new hires are minorities—that is, they will consider solutions where the soft constraint is violated to some extent. The deviation from the goal is the amount by which the soft constraint is violated. For example, if only 18% of new hires are minorities, then the deviation from the goal is the difference, 2%. However, if 22% of new hires are minorities, then the deviation from the goal is the difference, 2%. However, if 22% of new hires are minorities, then the deviation from the goal is the difference, 2%. However, if 22% of new hires are minorities, then the deviation from the goal is the difference, 2%. However, if 22% of new hires are minorities, then the deviation from the goal is the difference, 2%

In goal programming, several goals and their corresponding undesirable deviations are stated. Unfortunately, it is usually impossible to find a solution that makes all of these deviations zero. Therefore, trade-offs are required. This can be done in two ways. First, a method called preemptive goal programming ranks the goals in order of importance. Then it uses a solution procedure where the ranking is followed in a strict sense. For example, suppose the ranking of three goals is 1, 2, 3; goal 1 is considered most important, then goal 2, and then goal 3. A sequence of optimizations is performed.

- In the first optimization, only goal 1 is considered, and its deviation is minimized. It is possible that this minimization leads to a subset $S_1$ of all possible solutions, where each solution in this subset is equally good in terms of goal 1.

- A second optimization is then performed to find the best solution in terms of goal 2, but this optimization is restricted to solutions in $S_1$. By searching only through solutions in $S_1$, the best solution in this second step will be just as good in terms of goal 1, and it might be better in terms of goal 2. Again, this optimization might find a subset $S_2$ of solutions that are equally good in terms of goal 2.

- A third optimization is then performed to find the best solution in terms of goal 3, but it is restricted to solutions in $S_2$. By searching only through solutions in $S_2$, the best solution in this second step will be just as good in terms of goals 1 and 2, but it might be better in terms of goal 3. Any solution in this third step is considered optimal.

The problem with this procedure is that there might not be any “wiggle room” after the first or second optimizations. For example, the first optimization might lead to a single solution that minimizes the deviation from goal 1. In this case, there is no optimization left to perform in the second or third steps—and goals 2 and 3 are ignored. Essentially, preemptive goal programming gives infinitely more weight to goal 1 than to goal 2, and it

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1Preemptive goal programming was the only version discussed in previous editions of the book.
gives infinitely more weight to goal 2 than to goal 3. We see this as too restrictive, so we will not pursue it further here.

The second method assigns weights to the various deviations from goals and then performs a single optimization to minimize the weighted sum of deviations from goals. This is much easier to implement, as we will illustrate soon, but it requires difficult trade-offs. For example, you might assess that goal 1 is more important than goal 2, but how much more important is it? Should the weight for goal 1 be twice as large as the weight for goal 2? Should it be three times as large? Obviously, the weights can have a significant effect on the optimal solution; therefore, at the very least, some sensitivity analysis of the weights should be performed.

The following media selection problem is typical of the situations in which goal programming is useful. This example presents a variation of the advertising model discussed in Chapters 4 and 7.

**Example 16.1 Determining an Advertising Schedule at Burnit**

The Leon Burnit Ad Agency is trying to determine a TV advertising schedule for a client. The client has three goals, listed here in descending order of importance, concerning its target audience:

- Goal 1: at least 65 million high-income men (HIM)
- Goal 2: at least 72 million high-income women (HIW)
- Goal 3: at least 70 million low-income people (LIP)

Burnit can purchase several types of TV ads: ads shown on live sports shows, on game shows, on news shows, on sitcoms, on dramas, and on soap operas. At most $700,000 total can be spent on ads. The advertising costs and potential audiences (in millions of viewers) of a one-minute ad of each type are shown in Table 16.1. As a matter of policy, the client requires that at least two ads each be placed on sports shows, news shows, and dramas. Also, it requires that no more than 10 ads be placed on any single type of show. Burnit wants to find the advertising plan that best meets its client’s goals.

<table>
<thead>
<tr>
<th>Table 16.1 Data for the Advertising Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ad Type</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>Sports show</td>
</tr>
<tr>
<td>Game show</td>
</tr>
<tr>
<td>News</td>
</tr>
<tr>
<td>Sitcom</td>
</tr>
<tr>
<td>Drama</td>
</tr>
<tr>
<td>Soap opera</td>
</tr>
</tbody>
</table>

**Objective** To use goal programming to meet the company’s goals of reaching various target audiences as much as possible, while staying within an advertising budget.

**Where Do the Numbers Come From?**

As in previous advertising models, the company needs to estimate the number of viewers reached by each type of ad, and it needs to know the cost of each ad. Beyond this, however, management determines the goals. They can set whatever goals they believe are in the company’s best interests, and they can prioritize these goals.
Most of this problem is like previous optimization models. The decision variables are the numbers of ads placed on the types of shows, and there are hard constraints for the minimum and maximum numbers of ads and the budget. However, the goals are treated as soft constraints. For example, the deviation from the first goal is 0 if the goal is satisfied, and it is the amount short otherwise. These soft constraints could be handled in a spreadsheet model with IF functions, but this would lead to a nonsmooth model. The more common way to implement them, the way used here, is to add new decision variables for each goal, the deviation under and the deviation over. This will be explained in more detail soon, but its advantage is that it keeps the model linear. Then we minimize the weighted sum of the undesirable deviations, in this case, the deviations under.

What weights should be used? The company has only indicated a ranking of the three goals, which implies that the HIM weight should be at least as large as the HIW weight, which in turn should be at least as large as the LIP weight. Beyond this, choosing the weights is a difficult trade-off. However, the trade-off is somewhat easier in this problem than in many other multiobjective problems because all of the goals are in the same units: numbers of exposures. As an example, Burnit might assess that falling 1 million exposures short of the HIM goal is 1.25 times as bad as falling one million exposures short of the HIW goal. Then the weights for these first two goals could be 10 and 8. (Actually, any values in the ratio 1.25 to 1 could be used; only the ratios, not the magnitudes, matter.). For illustration, we will use weights 10, 8, and 5 at first, but we will then investigate other possible weights.

### Developing the Spreadsheet Model

The spreadsheet model appears in Figure 16.1. (See the file Advertising Goals.xlsx.) It can be developed with the following steps.

1. **Inputs.** Enter all inputs in the light blue ranges.
2. **Numbers of ads.** Enter any trial values for the numbers of ads in the Number_purchased range.
3. **Total cost.** Calculate the total amount spent on ads in cell B20 with the formula
   \[=\text{SUMPRODUCT}(B9:G9,\text{Number\_purchased})\]
4. **Exposures obtained.** Calculate the number of people (in millions) in each group that the ads reach in the Exposures range. Specifically, enter the formula
   \[=\text{SUMPRODUCT}(B5:G5,\text{Number\_purchased})\]
   in cell B24 for the HIM group, and copy this to the rest of the Exposures range for the other two groups.
5. **Deviations.** The keys to making this a linear model are the deviations in columns C and D. These should be designated as decision variable cells, so that they can initially have any values. However, the Solver solution (the one shown in Figure 16.1) will make at least one of them 0 for each goal. For example, you can see that there are currently 62 million exposures to the HIM group. This is 3 million below the goal, so the under deviation is 3, and the over deviation is 0.
6. **Goals.** For each goal, calculate the number of exposures plus the under deviation minus the over deviation. That is, enter the formula
   \[=B24+C24-D24\]
in cell E24 for the first goal, and copy this down for the other goals. These values will be constrained to equal the goals in column G. (Make sure you understand why this is logically correct.)

**Figure 16.1** Goal Programming Model

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>LP goal programming model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>2</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>3</td>
<td>Exposures to various groups per unit of advertising</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>High-income men</td>
<td>Sports ad</td>
<td>7</td>
<td>3</td>
<td>6</td>
<td>4</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>High-income women</td>
<td>Game show ad</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>Low-income people</td>
<td>News show ad</td>
<td>8</td>
<td>6</td>
<td>3</td>
<td>7</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>Sitcom ad</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>Drama ad</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>10</td>
<td></td>
<td>Soap opera ad</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Cost/ad</td>
<td>120</td>
<td>40</td>
<td>50</td>
<td>40</td>
<td>60</td>
<td>40</td>
<td></td>
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<tr>
<td>12</td>
<td>Advertising plan</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Minimum ads required</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Number purchased</td>
<td>&lt;=</td>
<td>&lt;=</td>
<td>&lt;=</td>
<td>&lt;=</td>
<td>&lt;=</td>
<td>&lt;=</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Maximum ads allowed</td>
<td>2.000</td>
<td>0.000</td>
<td>2.000</td>
<td>4.000</td>
<td>3.333</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
<td>&lt;=</td>
<td>&lt;=</td>
<td>&lt;=</td>
<td>&lt;=</td>
<td>&lt;=</td>
<td>&lt;=</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Budget constraint</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

**Note:** All monetary values are in $1000s, and all exposures to ads are in millions of exposures.

7 **Weighted deviations.** Enter the weights you want to try in column H, and then calculate the weighted average of under deviations in cell B29 with the formula

\[ =\text{SUMPRODUCT(H24:H26,Deviation\_under)}/\text{SUM(H24:H26)} \]

Note that only the under deviations contribute to this objective. Burnit is perfectly happy with over deviations, so they are not part of the objective to minimize. (In general, the objective is a weighted average of all undesirable deviations.)

**Using Solver**

The completed Solver dialog box is shown in Figure 16.2. Again, note that the deviation cells are designated as decision variables. Also, the first three constraints shown are the usual types of hard constraints, whereas the last equality constraint logically relates the exposures and deviations to the goals.

**Discussion of the Solution**

With the given weights, the optimal advertising plan in Figure 16.1 comes up 3 million exposures short on the HIM goal, 7.333 million short on the HIW, and right on target on the LIP goal. This is despite the fact that the HIM goal gets the most weight and LIP gets the least, but it illustrates that surprises can occur. Given the priorities on these three
goals, this is the best possible solution. Note that all of the hard constraints are satisfied, as they must be. For example, no more than 10 ads of any type are used, and the budget is not exceeded. Note also that the amounts over the goals are all 0. This is not guaranteed to happen, but it did in this example.

**Sensitivity Analysis**

Sensitivity analysis should be a part of goal programming just as it is for previous models we have discussed. One possibility appears in Figure 16.3, where we have

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Budget (cell SD$20) values along side, output cell(s) along top</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Under_deviation_1</td>
<td>Over_deviation_1</td>
<td>Under_deviation_2</td>
<td>Over_deviation_2</td>
<td>Under_deviation_3</td>
<td>Over_deviation_3</td>
<td>Weighted_average_of_under_deviations</td>
</tr>
<tr>
<td>5</td>
<td>$500</td>
<td>23.000</td>
<td>0.000</td>
<td>33.000</td>
<td>0.000</td>
<td>29.000</td>
<td>0.000</td>
</tr>
<tr>
<td>6</td>
<td>$550</td>
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<td>0.000</td>
<td>26.750</td>
<td>0.000</td>
<td>20.250</td>
<td>0.000</td>
</tr>
<tr>
<td>7</td>
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<td>0.000</td>
</tr>
<tr>
<td>8</td>
<td>$650</td>
<td>8.000</td>
<td>0.000</td>
<td>14.250</td>
<td>0.000</td>
<td>7.250</td>
<td>0.000</td>
</tr>
<tr>
<td>9</td>
<td>$700</td>
<td>3.000</td>
<td>0.000</td>
<td>7.333</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>10</td>
<td>$750</td>
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<td>13</td>
<td>$900</td>
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<td>0.000</td>
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</tbody>
</table>
kept the weights the same but have allowed the budget to vary from $500,000 to $1 million in increments of $50,000. As you can see, when the budget decreases, it is more difficult to satisfy the goals, even the more important ones. In the other direction, when the budget is large enough, all of the goals can be satisfied. Somewhat surprisingly, when the budget is $750,000, the over deviation for the HIM goal is positive, even though there is no incentive in the model to make this happen. Evidently, the advertising plan needed to make the HIW and LIP deviations small yields more HIM exposures than are required.

**Effect of Different Weights**

A different kind of sensitivity analysis can be performed on the weights. There is no easy way to do this with SolverTable, but it is easy to change the weights and rerun Solver. The results from several runs appear in Figure 16.4. These results are typical in the sense that there are usually only a few different solutions possible, regardless of the weights. As you can see, when goal 1 has a much larger weight than the other two (row 6), its goal is satisfied completely at the expense of the other goals. As another example, we already saw that goal 3 can be satisfied even when its weight is the lowest (row 5), so nothing changes when its weight is increased (rows 8 and 9). But when the weight for goal 2 is the largest (row 10), a somewhat different solution is optimal. (You can check that this solution remains optimal no matter how large the weight for goal 2 is. Evidently, goal 2 is blocked by the hard constraints.)

![Figure 16.4](image_url)

**Modeling Issues**

1. The results for the Burnit model are based on allowing the numbers of ads to have noninteger values. They could easily be constrained to integer values, and the solution method would remain exactly the same. However, the goals might not be met as fully as before because of the extra integer constraints.

2. The method of preemptive goal programming discussed earlier can be implemented by choosing weights of totally different magnitudes, such as 10,000, 1000, and 1 for the three goals in their order of priority.

3. All of the deviations in the objective of the Burnit model are under deviations. However, it is certainly possible to include over deviations as objectives. For example, if the budget constraint were treated as a soft constraint, you would try to minimize its over deviation to stay as little over the budget as possible. It is even possible for both the under and over deviations of some goal to be included as objectives. This occurs in situations where you want to come as close as possible to some target value—neither under nor over.
4. The use of decision variable cells for the under and over deviations might not be intuitive, but it serves two purposes. First, it provides exactly the information needed for the goal programming objective. Second, it keeps the model linear. If you used IF functions instead (without the under and over cells) to capture the under deviations, the model would be nonlinear and nonsmooth, and Evolutionary Solver would be necessary.

**PROBLEMS**

Solutions for problems whose numbers appear within a colored box can be found in the Student Solutions Files. Refer to this book’s preface for purchase information.

For each of the following problems, try at least three sets of weights for the goals that are consistent with the priorities listed, and discuss whether the weights chosen make a difference.

**Level A**

1. Gotham City must determine how to allocate ambulances during the next year. It costs $5000 per year to run an ambulance. Each ambulance must be assigned to one of two districts. Let $x_i$ be the number of ambulances assigned to district $i$, $i = 1, 2$. The average time (in minutes) it takes for an ambulance to respond to a call from district 1 is $40 - 3x_1$; for district 2, the time is $50 - 4x_2$. Gotham City has three goals (listed in order of priority):
   - **Goal 1:** At most $100,000 per year should be spent on ambulance service.
   - **Goal 2:** Average response time in district 1 should be at most five minutes.
   - **Goal 3:** Average response time in district 2 should be at most five minutes.

   **a.** Use goal programming to determine how many ambulances to assign to each district.
   **b.** How does your answer change if goal 2 has the highest priority, then goal 3, and then goal 1?

2. Teknix Computer Company is ready to make its annual purchase of computer chips. The company can purchase chips (in lots of 100) from three suppliers. Each chip’s quality is rated as excellent, good, or mediocre. During the coming year, Teknix needs 5 million excellent chips, 3 million good chips, and 1 million mediocre chips. The characteristics of the chips purchased from each supplier are shown in the file [P16_02.xlsx](P16_02.xlsx). Teknix has set the following goals (listed in order of importance):
   - **Goal 1:** A maximum of $60,000 can be spent on purchasing TVs and Blu-ray disc players.
   - **Goal 2:** Highland should earn at least $7000 profit from the sale of TVs and Blu-ray disc players.
   - **Goal 3:** TVs and Blu-ray disc players should not use up more than 200 square yards of storage space.

   Use goal programming to determine how many TVs and Blu-ray disc players Hiland should order. Then modify the model so that goal 3 is to use up exactly 200 square yards of storage space.

3. Hiland Appliance must determine how many TVs and Blu-ray disc players to stock. It costs Hiland $1000 to purchase a TV and $200 to purchase a Blu-ray player. A TV requires three square yards of storage space, and a Blu-ray disc player requires one square yard. The sale of a TV earns Hiland a profit of $150, and each Blu-ray disc player sale earns a profit of $50. Hiland has set the following goals (listed in order of importance):
   - **Goal 1:** A maximum of $60,000 can be spent on purchasing TVs and Blu-ray disc players.
   - **Goal 2:** Highland should earn at least $7000 profit from the sale of TVs and Blu-ray disc players.
   - **Goal 3:** The cost per pound of sausage should not exceed $0.06.

   Use goal programming to determine the company’s optimal blending plan.

4. Based on Steuer (1984). Deancorp produces sausage by blending beef head, pork chuck, mutton, and water. The cost per pound, fat per pound, and protein per pound for these ingredients are listed in the file [P16_04.xlsx](P16_04.xlsx). Deancorp needs to produce 100,000 pounds of sausage and has set the following goals, listed in order of priority:
   - **Goal 1:** Sausage should consist of at least 15% protein.
   - **Goal 2:** Sausage should consist of at most 8% fat.
   - **Goal 3:** The cost per pound of sausage should not exceed $0.06.

   Use goal programming to determine the company’s optimal blending plan.

5. Based on Welling (1977). The Touche Young accounting firm must complete three jobs during the next month. Job 1 will require 500 hours of work, job 2 will require 300 hours, and job 3 will require 100 hours. At present, the firm consists of five partners, five senior employees, and five junior employees, each of whom can work up to 40 hours per week. The dollar amount (per hour) that the company can bill
depends on the type of accountant assigned to each job, as shown in the file P16_05.xlsx. (The “X” indicates that a junior employee does not have enough experience to work on job 1.) All jobs must be completed. Touche Young has also set the following goals, listed in order of priority:

- Goal 1: Monthly billings should exceed $74,000.
- Goal 2: At most one partner should be hired.
- Goal 3: At most three senior employees should be hired.
- Goal 4: At most one junior employee should be hired.

Use goal programming to help Touche solve its problem.

6. The city of Bloomington has 17 neighborhoods. The number of high school students in each neighborhood and the time required to drive from each neighborhood to each of the city’s two high schools (North and South) are listed in the file P16_06.xlsx. The Bloomington Board of Education needs to determine how to assign students to high schools. All students in a given neighborhood must be assigned to the same high school. The Board has set (in order of priority, from highest to lowest) the following goals:

- Goal 1: Ensure that the difference in enrollment at the two high schools differs by at most 50.
- Goal 2: Ensure that average student travel time is at most 13 minutes.
- Goal 3: Ensure that at most 4% of the students must travel at least 25 minutes to school.

a. Determine an optimal assignment of students to high schools.

b. If the enrollment at the two high schools can differ by at most 100 (a change in goal 1), how do the results change?

Level B

7. Based on Lee and Moore (1974). Faber University is admitting students for the class of 2015. Data on its applicants are shown in the file P16_07.xlsx. Each row indicates the number of in-state or out-of-state applicants with a given SAT score who plan to be business or nonbusiness majors. For example, 1900 of its in-state applicants have a 700 SAT score, and 1500 of these applicants plan to major in business. Faber has set four goals for this class, listed in order of priority:

- Goal 1: The entering class should include at least 5000 students.
- Goal 2: The entering class should have an average SAT score of at least 640.
- Goal 3: The entering class should consist of at least 25% out-of-state students.
- Goal 4: At least 2000 members of the entering class should not be business majors.

Use goal programming to determine the number of applicants of each type to admit. Assume that all applicants who are admitted will accept the invitation to attend Faber.

8. Based on Taylor and Keown (1984). The city of Springfield is trying to determine the type and location of recreational facilities to build during the next decade. Four types of facilities are under consideration: golf courses, swimming pools, gymnasiums, and tennis courts. Six sites are under consideration. If a golf course is built, it must be built at either site 1 or site 6. Other facilities can be built at sites 2 through 5. The amounts of available land (in thousands of square feet) at sites 2 through 5 are given in the file P16_08.xlsx. The cost of building each facility (in thousands of dollars), the annual maintenance cost (in thousands of dollars) for each facility, and the land (in thousands of square feet) required for each facility are also given in the same file. The number of user-days (in thousands) for each type of facility, also shown in this file, depends on where it is built.

a. For the following set of priorities, use goal programming to determine the type and location of recreation facilities in Springfield.

- Priority 1: The amount of land used at each site should be no greater than the amount of land available.
- Priority 2: Construction costs should not exceed $1.2 million.
- Priority 3: User-days should exceed 200,000.
- Priority 4: Annual maintenance costs should not exceed $200,000.

b. Solve part a with the following set of priorities:

- Priority 1: The amount of land used at each site should be no greater than the amount of land available.
- Priority 2: User-days should exceed 200,000.
- Priority 3: Construction costs should not exceed $1.2 million.
- Priority 4: Annual maintenance costs should not exceed $200,000.

9. A small aerospace company is considering eight projects:

- Project 1: Develop an automated test facility.
- Project 2: Bar code all inventory and machinery.
- Project 3: Introduce a CAD/CAM system.
- Project 4: Buy a new lathe and deburring system.
- Project 5: Institute an FMS (Flexible Manufacturing System).
- Project 6: Install a LAN (Local Area Network).
10. A new president has just been elected and has set the following economic goals (listed from highest to lowest priority):

- **Goal 1**: Balance the budget (this means revenues are at least as large as costs).
- **Goal 2**: Cut spending by at most $150 billion.
- **Goal 3**: Raise at most $550 billion in taxes from the upper class.
- **Goal 4**: Raise at most $350 billion in taxes from the lower class.

Currently the government spends $1 trillion per year. Revenue can be raised in two ways: through a gas tax and through an income tax. You must determine $G$, the per-gallon tax rate (in cents); $T_1$, the tax rate charged on the first $30,000 of income; $T_2$, the tax rate charged on any income earned over $30,000; and $C$, the cut in spending (in billions). If the government chooses $G$, $T_1$, and $T_2$, then we assume that the revenue given in the file P16_9.xlsx (in billions of dollars) is raised. Of course, the tax rate on income over $30,000 must be at least as large as the tax rate on the first $30,000 of income. Use goal programming to help the president meet his goals. (Of course, we are all dreaming if we think this is how Washington operates!)

11. The HAL computer must determine which of eight research and development (R&D) projects to undertake. For each project, four quantities are of interest: (1) the net present value (NPV, in millions of dollars) of the project; (2) the annual growth rate in sales generated by the project; (3) the probability that the project will succeed; and (4) the cost (in millions of dollars) of the project. The relevant information is given in the file P16_11.xlsx. HAL has set the following four goals:

- **Goal 1**: The total NPV of all chosen projects should be at least $200 million.
- **Goal 2**: The average probability of success for all projects chosen should be at least 0.75.
- **Goal 3**: The average growth rate of all projects chosen should be at least 15%.
- **Goal 4**: The total cost of all chosen projects should be at most $1 billion.

For the following sets of priorities, use goal programming with integer constraints to determine the projects that should be selected.

**a.** Goal 2, Goal 4, Goal 1, Goal 3.

**b.** Goal 1, Goal 3, Goal 4, Goal 2.

12. Based on Klingman and Phillips (1984). The Marines need to fill three types of jobs in two cities (Los Angeles and Chicago). The numbers of jobs of each type that must be filled in each city are shown in the file P16_12.xlsx. The Marines available to fill these jobs have been classified into six groups according to the types of jobs each person is capable of doing, the type of job each person prefers, and the city in which each person prefers to live. The data for each of these six groups are also listed in this file. The Marines have the following three goals, listed from highest priority to lowest priority:

- **Goal 1**: Ensure that all jobs are filled by qualified workers.
- **Goal 2**: Ensure that at least 8000 employees are assigned to the jobs they prefer.
- **Goal 3**: Ensure that at least 8000 employees are assigned to their preferred cities.

Determine how the Marines should assign their workers. (Note: You may allow fractional assignments of workers.)

13. Based on Vasko et al. (1987). Bethlehem Steel can fill orders using five different types of steel molds. Up to three different molds of each type can be purchased. Each individual mold can be used to fill up to 100 orders per year. Six different types of orders must be filled during the coming year. The waste (in tons) incurred if a type of mold is used to fill an order is shown in the file P16_13.xlsx (where an “x” indicates that a type of mold cannot be used to fill an order). The number of each order type that must be filled in each city is shown in this file. Bethlehem Steel has the following two goals, listed in order of priority:

- **Goal 1**: Because molds are very expensive, Bethlehem wants to use at most five molds.
- **Goal 2**: Bethlehem wants to have at most 600 tons of total waste.

Use goal programming to determine how Bethlehem should fill the coming year’s orders.
In a multiobjective problem with no uncertainty, it is common to search for Pareto optimal solutions. We assume that the decision maker has exactly two objectives and that the set of feasible solutions under consideration must satisfy a prescribed set of constraints.

First, we need to define some terms. A solution (call it $A$) to a multiobjective problem is called **Pareto optimal** if no other feasible solution is at least as good as $A$ with respect to every objective and strictly better than $A$ with respect to at least one objective. A related concept is **domination**. A feasible solution $B$ dominates a feasible solution $A$ to a multiobjective problem if $B$ is at least as good as $A$ on every objective and is strictly better than $A$ on at least one objective. From this definition, it follows that Pareto optimal solutions are feasible solutions that are not dominated.

If the “score” of all Pareto optimal solutions is graphed in the $x$–$y$ plane, with the $x$-axis score being the score on objective 1 and the $y$-axis score being the score on objective 2, the graph is called a **trade-off curve**. It is also called the **efficient frontier**. To illustrate, suppose that the set of feasible solutions for a multiobjective problem is the shaded region bounded by the curve $AB$ and the axes in Figure 16.5. If the goal is to maximize both objectives 1 and 2, then the curve $AB$ is the set of Pareto optimal points. All points below the $AB$ curve are dominated by points on the curve.

As another illustration, suppose the set of feasible solutions for a multiobjective problem is all shaded points in the first quadrant bounded from below by the curve $AB$ in Figure 16.6. If the goal is to maximize objective 1 and minimize objective 2, then the curve $AB$ is the set of Pareto optimal points. In this case, all points to the left of the curve are dominated by points on the curve.
Finding a Trade-off Curve

To find a trade-off curve, you can proceed according to the following steps.

1. Choose an objective, say objective 1, and determine its best attainable value $V_1$. For the solution attaining $V_1$, find the value of objective 2 and label it $V_2$. Then $(V_1, V_2)$ is a point on the trade-off curve.

2. For values $V$ of objective 2 that are better than $V_2$, solve the optimization problem in step 1 with the additional constraint that the value of objective 2 is at least as good as $V$. Varying $V$ (over values of $V$ preferred to $V_2$) yields other points on the trade-off curve.

3. Step 1 located one endpoint of the trade-off curve. Now determine the best value of objective 2 that can be attained, to obtain the other endpoint of the trade-off curve.

We illustrate the concept of Pareto optimality (and how to determine Pareto optimal solutions) with the following example.

**Example 16.2 Maximizing Profit and Minimizing Pollution at Chemcon**

Chemcon plans to produce eight products. The profit per unit, the labor and raw material used per unit produced, and the pollution emitted per unit produced are given in Table 16.2. This table also includes lower and upper limits on production that Chemcon has imposed. Currently 1300 labor hours and 1000 units of raw material are available. Chemcon’s two objectives are to maximize profit and minimize pollution produced. Chemcon wants to graph the trade-off curve for this problem.

**Table 16.2 Data for the Chemcon Example**

<table>
<thead>
<tr>
<th>Product</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor hrs/unit</td>
<td>5</td>
<td>5</td>
<td>1</td>
<td>4</td>
<td>3.5</td>
<td>4</td>
<td>2</td>
<td>3.5</td>
</tr>
<tr>
<td>Raw material/unit</td>
<td>3</td>
<td>4.5</td>
<td>5</td>
<td>5</td>
<td>4.5</td>
<td>2</td>
<td>3.5</td>
<td>3</td>
</tr>
<tr>
<td>Pollution/unit</td>
<td>25</td>
<td>29</td>
<td>35</td>
<td>26</td>
<td>17</td>
<td>25</td>
<td>28</td>
<td>6</td>
</tr>
<tr>
<td>Profit/unit</td>
<td>53</td>
<td>69</td>
<td>73</td>
<td>69</td>
<td>51</td>
<td>49</td>
<td>71</td>
<td>40</td>
</tr>
<tr>
<td>Min production</td>
<td>0</td>
<td>30</td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>50</td>
<td>30</td>
<td>0</td>
</tr>
<tr>
<td>Max production</td>
<td>190</td>
<td>110</td>
<td>140</td>
<td>140</td>
<td>190</td>
<td>190</td>
<td>110</td>
<td>150</td>
</tr>
</tbody>
</table>

**Objectives**

To find the trade-off curve between pollution and profit by solving a number of LP problems.

**Where Do the Numbers Come From?**

The required data here is basically the same as in the product mix problems from Chapter 3. Of course, the company also needs to find how much pollution each product is responsible for, which requires some scientific investigation.

**Solution**

The model itself is a straightforward version of the product mix models from Chapter 3. The objective is to find the product mix that stays within the lower and upper production...
limits, uses no more labor or raw material than are available, keeps pollution low, and
keeps profit high. None of the formulas in the spreadsheet model (see Figure 16.7 and the
file Pollution Trade-off.xlsx) presents anything new, so we focus instead on the solution
procedure.

Referring to the general three-step procedure for finding the trade-off curve, let profit
be objective 1 and pollution be objective 2. To obtain one endpoint of the curve (step 1),
you maximize profit and ignore pollution. That is, you maximize the Profit cell and delete
the constraint indicated in row 26 from the Solver dialog box. You can check that the solution
has profit $20,089 and pollution level 9005.2 (This is not the solution shown in the figure.)
At the other end of the spectrum (step 3), you minimize the pollution in cell B26 and
ignore any constraint on profit. You can check that this solution has pollution level 3560
and profit $8360. In other words, profit can get as high as $20,089 by ignoring pollution or
as low as $8360, and pollution can get as low as 3560 or as high as 9005. These establish
the extremes. Now you can search for points in between (step 2).

Figure 16.7 The Chemcon Model

| A   | B       | C   | D   | E   | F   | G   | H   | I   | J   | K   | L   |
|-----|---------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1   | Chemcon profit versus pollution model |       |     |     |     |     |     |     |     |     |     |     |
| 2   |         |     |     |     |     |     |     |     |     |     |     |     |
| 3   | Input data |     |     |     |     |     |     |     |     |     |     |     |
| 4   | Product | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | Actual_pollution =Model!$B$26 |
| 5   | Labor hours/unit | 3   | 5   | 3   | 4   | 3.5 | 4.2 | 4.3 | 3.5 | Max_production =Model!$B$17:$I$17 |
| 6   | Raw material/unit | 3   | 4.5 | 5   | 5   | 4.5 | 4   | 2   | 3.5 | Min_production =Model!$B$13:$I$13 |
| 7   | Pollution/unit | 25  | 29  | 35  | 26  | 17  | 25  | 28  | 6   | Pollution_upper_bound =Model!$B$26 |
| 8   | Profit/unit | $53 | $69 | $73 | $69 | $52 | $49 | $73 | $40 | Resources_available =Model!$B$21:$I$22 |
| 9   | Units produced | 0   | 30  | 0   | 10  | 20  | 50  | 30  | 0   | Units_produced =Model!$B$15:$I$15 |
| 10  | Constraints on resources |       |     |     |     |     |     |     |     |     |     |     |
| 11  | Labor hours | 1086.0 <= 1300 |
| 12  | Raw material | 1000.0 <= 1000 |
| 13  | Units produced | 0.0 <= 30.0 |
| 14  | Max production | 190 <= 190 |
| 15  | Objective to maximize | $15,738 |

Get other points on the trade-off curve by maximizing profit, constraining pollution with varying upper bounds.

Fortunately, SolverTable is the perfect tool. According to step 2, you need to con-
strain pollution to various degrees and see how large profit can be. This is indicated in
Figure 16.7, where the objective is to maximize profit with an upper limit on pollution.
(You could get the same effect by minimizing pollution and putting a lower limit on profit.)
The only upper limits on pollution you need to consider are those between the extremes,
3560 and 9005. Therefore, you can use SolverTable with the setup shown in Figure 16.8.
Note that we have used the option to enter nonequally spaced inputs: 3560, 4000, 4500,

2Actually, this is not quite true, as one user pointed out. If you maximize profit and ignore pollution, the result-
ing pollution level is 8980. To find the maximum possible pollution level, you need to maximize pollution. The
resulting pollution level is 9005. Surprisingly, the profit from this solution is less than the maximum profit,$20,089.
and so on, ending with 9005. Alternatively, equally spaced inputs could be used. All that is required is a representative set of values between the extremes. The results appear in Figure 16.9.

**Discussion of the Solution**

These results show that as you allow more pollution, profit increases. Also, the product mix shifts considerably. Product 8, a low polluter with a low profit margin, eventually...
leaves the mix when pollution is allowed to increase, which makes sense. It is less clear why the level of product 6 increases so dramatically. Product 6 is only a moderate polluter and has a moderate profit margin, so the key is evidently that it requires low levels of labor and raw materials. The trade-off curve is created as a scatter chart (with the points connected) directly from columns J and K of the table. This curve appears in Figure 16.10. It indicates that profit indeed increases as Chemcon allows more pollution, but at a decreasing rate. For example, when pollution is allowed to increase from 4000 to 4500, Chemcon can make an extra $3187 in profit. However, when pollution is allowed to increase from 8000 to 8500, the extra profit is only $532. All points below the curve are dominated—for a given level of pollution, the company can achieve a larger profit—and all points above the curve are unattainable.

Trade-off curves are not limited to linear models. The following example illustrates a trade-off curve in a situation where the objective is a nonlinear function of the changing cells.

Figure 16.10
Trade-off Curve for Profit versus Pollution

ExAMPlE 16.3 TRADE-OFFS BETWEEN EXPOSURES TO MEN AND WOMEN AT LEON BURNIT

This example is a modification of the Burnit advertising example in Example 16.1. Now we assume that Burnit’s client is concerned only with two groups of people, men and women. Also, the number of exposures to these groups is now a nonlinear square root function of the number of ads placed of any particular type. This implies a marginal decreasing effect of ads—each extra ad of a particular type reaches fewer extra people than the previous ad of this type.3

The data for this problem appear in Tables 16.3 and 16.4. The first of these specifies the proportionality constants for the square root exposure functions. For example, if five ads are placed in sports shows, this will achieve $15\sqrt{5} = 33.541$ million exposures to men, but only $5\sqrt{5} = 11.180$ million exposures to women. Evidently, what works well for men does not work so well for women, and vice versa. Given a budget of $1.5$ million, find the trade-off curve for exposures to men versus exposures to women.

3The square root function is an alternative to the exponential advertising response function we used in Examples 7.5 and 7.6 of Chapter 7. Each increases at a decreasing rate.
### Table 16.3 Proportionality Constants for Square Root Exposure Functions

<table>
<thead>
<tr>
<th>Sports Show</th>
<th>Game Show</th>
<th>News Show</th>
<th>Sitcom</th>
<th>Drama</th>
<th>Soap Opera</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>15</td>
<td>3</td>
<td>7</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>Women</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>10</td>
<td>9</td>
</tr>
</tbody>
</table>

### Table 16.4 Data on Ads for the Burnit Example

<table>
<thead>
<tr>
<th>Sports Show</th>
<th>Game Show</th>
<th>News Show</th>
<th>Sitcom</th>
<th>Drama</th>
<th>Soap Opera</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost/ad ($1000s)</td>
<td>120</td>
<td>40</td>
<td>50</td>
<td>40</td>
<td>60</td>
</tr>
<tr>
<td>Lower limit</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Upper limit</td>
<td>10</td>
<td>5</td>
<td>10</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

**Objective** To find the trade-off curve for exposures to men versus exposures to women by solving a number of NLP problems.

**Where Do the Numbers Come From?**

We have discussed these same types of numbers in previous examples. Specifically, the parameters in Table 16.3 can be estimated from historical data, exactly as described in Example 7.5 of Chapter 7.

**Solution**

Again, the model itself is straightforward, as shown in Figure 16.11. (See the file *Advertising Trade-off.xlsx*. You calculate the exposures achieved in rows 22 and 23 by entering the formula

\[=\text{IF}(B$14>0,B5*\sqrt{B$14},0)\]

in cell B19 and copying it to the range B19:G20. (The reason for the IF function is that Solver’s GRG algorithm allows the values of the decision variable cells to become negative, even though the nonnegativity option is checked. If this happens, the square root isn’t defined and Solver reports an error. The IF function prevents this.) You then sum these in cells B30 and B33, and calculate the total cost in the usual way with the SUMPRODUCT function.

For the three-step trade-off curve procedure, let exposures to men be objective 1 and exposures to women be objective 2. For step 1, you maximize exposures to men and ignore women. That is, you do not include the constraint in row 27 in the Solver dialog box. You can check that the corresponding solution achieves 89.515 million exposures to men and 79.392 million exposures to women. Reversing the roles of men and women (step 3), you can check that if you maximize exposures to women and ignore men, the solution achieves 89.220 million exposures to women and only 84.899 million exposures to men. (Because a nonlinear algorithm is used, these numbers could vary slightly from one Solver run to the next, depending on the starting values in the decision variables cells.)

All other points on the trade-off curve are between these two extremes, and they can again be found easily with SolverTable. You now set up Solver to maximize exposures to men, and you include the lower limit constraint on exposures to women in the Solver dialog box. (Do you see why it is a lower limit constraint in this example, whereas it was an upper limit constraint in the previous example? There the objective was to make pollution low. Here the objective is to make exposures to women high.) The lower limit cell (D27)
Assumption: The number of exposures (in millions) to each group is proportional to the square root of the number of ads of a particular type shown.

Table 16.12
SolverTable Results for the Advertising Trade-off Model

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Women lower bound (cell D$27) values along side, output cell(s) along top</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>79.393</td>
<td>4.839</td>
<td>1.743</td>
<td>6.072</td>
<td>6.000</td>
<td>5.008</td>
<td>0.0776</td>
<td>79.393</td>
<td>89.515</td>
</tr>
<tr>
<td>6</td>
<td>80.000</td>
<td>4.715</td>
<td>1.835</td>
<td>6.100</td>
<td>5.000</td>
<td>5.621</td>
<td>0.927</td>
<td>80.000</td>
<td>89.506</td>
</tr>
<tr>
<td>7</td>
<td>81.000</td>
<td>4.504</td>
<td>1.991</td>
<td>6.146</td>
<td>5.000</td>
<td>5.803</td>
<td>1.219</td>
<td>81.000</td>
<td>89.449</td>
</tr>
<tr>
<td>8</td>
<td>82.000</td>
<td>4.280</td>
<td>2.166</td>
<td>6.180</td>
<td>5.000</td>
<td>5.992</td>
<td>1.554</td>
<td>82.000</td>
<td>89.336</td>
</tr>
<tr>
<td>9</td>
<td>83.000</td>
<td>4.049</td>
<td>2.349</td>
<td>6.201</td>
<td>5.000</td>
<td>6.185</td>
<td>1.954</td>
<td>82.999</td>
<td>89.156</td>
</tr>
<tr>
<td>10</td>
<td>84.000</td>
<td>3.800</td>
<td>2.530</td>
<td>6.219</td>
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<td>8.185</td>
<td>5.000</td>
<td>89.219</td>
<td>84.941</td>
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</table>
A trade-off curve is useful because it gives the ultimate decision maker many undominated solutions to choose from. However, it does not specify a “best” solution. The decision maker still has to make the difficult decision of which solution from the trade-off curve to implement. This can be done subjectively or with the help of a multiattribute utility function. However, estimating these types of functions is difficult, so their use in real-world applications has been limited.

These trade-off models can be generalized to a situation where there are more than two objectives by constructing trade-off curves between each pair of objectives.

**Problems**

**Level A**

14. A company produces two types of widgets. Each widget is made of steel and aluminum and is assembled with skilled labor. The resources used and the per-unit profit contribution (ignoring cost of overtime labor purchased) for each type of widget are given in the file P16_14.xlsx. At present, 200 pounds of steel, 300 pounds of aluminum, and 300 hours of labor are available. Extra overtime labor can be purchased for $10 per hour. Construct a trade-off curve between the objectives of maximizing profit and minimizing overtime labor.

15. A company produces three products. The company employs three workers, and it must determine which product(s) each worker should produce. The number of units each worker would produce if he or she spent the whole day producing each type of product is given in the file P16_15.xlsx. The company is also interested in maximizing the happiness of its workers. The amount of happiness “earned” by a worker who
spends the entire day producing a given product is also given in this file. Construct a trade-off curve between the objectives of maximizing total units produced daily and total worker happiness.

16. If a company spends $a$ on advertising (measured in thousands of dollars) and charges a price of $p$ dollars per unit, then it can sell $1000 - 10p + 20a^{1/2}$ units of the product. The cost per unit of producing the product is $6$. Construct a trade-off curve between the objectives of maximizing profit and maximizing the number of units sold.

17. GMCO produces three types of cars: compact, medium, and large. The variable cost per car and production capacity (per year) for each type of car are given in the file P16_17.xlsx. The annual demand for each type of car depends on the prices of the three types of cars, also given in this file. In this latter table, $P_C$ is the price charged for a compact car (in thousands of dollars). The variables $P_M$ and $P_L$ are defined similarly for medium and large cars. Suppose that each compact car gets 30 mpg, each medium car gets 25 mpg, and each large car gets 18 mpg. GMCO wants to keep the planet pollution free, so in addition to maximizing profit, it wants to maximize the average miles per gallon attained by the cars it sells. Construct a trade-off curve between these two objectives.

18. In the capital budgeting example from Chapter 6 (see Example 6.1), we maximized NPV for a given budget. Now find a trade-off curve for NPV versus budget. Specifically, minimize the amount invested, with a lower bound constraint on the NPV obtained. What lower bounds should you use?

19. The portfolio optimization example from Chapter 7 (see Example 7.9) found the efficient frontier by minimizing portfolio variance, with a lower bound constraint on the expected return. Do it the opposite way. That is, calculate the efficient frontier by maximizing the expected return, with an upper bound on the portfolio standard deviation. Do you get the same efficient frontier as in Example 7.9?

16.4 THE ANALYTIC HIERARCHY PROCESS (AHP)

When multiple objectives are important to a decision maker, choosing between alternatives can be difficult. For example, if you are choosing a job, one job might offer the highest starting salary but rate poorly on other objectives such as quality of life in the city where the job is located and the nearness of the job to your family. Another job offer might rate highly on these latter objectives but have a relatively low starting salary. In this case, it can be difficult for you to choose between job offers. The Analytic Hierarchy Process (AHP), developed originally by Thomas Saaty, is a powerful tool that can be used to make decisions in situations where multiple objectives are present.

The ideas behind AHP are simple and intuitive. However, the numerical implementation is fairly complex and sheds little light on how the process works. Therefore, we focus only on the ideas behind AHP, and we then illustrate how AHP can be used to make decisions with a macro-based Excel application. Admittedly, if you are going to use AHP for making your own decisions, Excel isn’t the best choice of software. We highly recommend the (non-Excel-based) Expert Choice software developed by Expert Choice Inc. (The company offers a three-month, full-featured version for $25 if you’d like to try it.) This software works very much like the macro-based Excel application we will present soon.

AHP assumes that when you make a decision, you base it on multiple criteria. (These are essentially the goals or objectives from previous sections, but AHP uses the term criteria.) For example, if you are deciding which of several jobs to take upon graduation from college, your criteria might be salary, nearness to family, job benefits, interest of work, quality of life, and possibly others. Once you list these criteria, AHP asks you to make pairwise comparisons between them. For example, it asks you which is more important to you, nearness to family or interest of work, and it also asks you how much more important one is than the other. (This is essentially like finding weights for your goals in goal programming.) Note that this can require quite a few pairwise comparisons. For example, if you have five criteria, you need to make 10 pairwise comparisons (the number of ways two things can be chosen from five things).
Next, for each possible decision—in this case, each potential job—AHP asks you to make the same types of pairwise comparisons between jobs on each criterion. For example, it asks you whether you prefer job 1 or job 2 in terms of nearness to family, and it also asks you how much more you prefer one job to the other on this criterion. Again, this can require quite a few pairwise comparisons. For example, if you have five criteria and four possible jobs, you need to make 30 pairwise comparisons (six for each of the five criteria).

AHP keeps track of your choices from all of these pairwise comparisons to calculate weights for the different criteria and weights for the jobs on each of the criteria. It then combines these weights to provide an overall score for each job. For example, if you prefer job 1 to job 2 by a considerable margin on the nearness to family criterion, but you rate nearness to family quite low relative to the other criteria, then job 1 won’t benefit much in its overall score from this comparison to job 2. Finally, AHP presents you with these overall scores, and presumably you will choose the job with the highest score.

As you will discover if you ever try AHP for your own decisions, it is difficult to be consistent when making many pairwise comparisons. Therefore, AHP calculates consistency indexes and informs you if an index is beyond a “normal” range. If it is, you can redo the process and try to be more consistent.

To allow you to see AHP in action, we have provided the file AHP for Choosing Jobs.xlsm. As the .xlsm extension indicates, this file includes Excel macros. These macros not only provide dialog boxes for entering criteria and jobs and making pairwise comparisons but also perform all of the numerical calculations to generate the overall job scores. (Of course, you need to enable the macros when you open the file.)

The opening screen is shown in Figure 16.14. You see an explanation of the application and, by double-clicking the blue area at the top, you see a general explanation of AHP. This is in the bottom textbox in the figure. You can make this bottom textbox disappear by double-clicking the blue area again.

When you click the button to run the application, you first see the dialog box in Figure 16.15, where you can choose your criteria. As you can see, there is a built-in list of criteria you can choose from, or you can add new criteria of your own. For this illustration, we chose salary, location, nearness to family, and interest of work.

When you click the No More button, you then see the dialog box in Figure 16.16, where you can enter the jobs you have been offered. For this illustration, we entered three jobs: Dell, IBM, and Cisco.

When you click the No More button, you see a series of dialog boxes, as shown in Figure 16.17, where you must make a pairwise comparison between two of the criteria. (With four criteria, there are six such comparisons to make.) In the comparison shown, location is chosen as more important than salary. How much more important? It’s about 3 on a scale of 1 to 9, with 1 indicating that they’re considered equally important and 9 indicating that one is “extremely” more important than the other. AHP software typically uses a 1 to 9 scale (or possibly a 1 to 5 or a 1 to 7 scale), with verbal descriptions to “anchor” these numbers.

Once you make all comparisons between criteria, you see another series of dialog boxes, as in Figure 16.18, where you must make a pairwise comparison between two jobs on each of the criteria. (With four criteria and three jobs, there are 12 such comparisons to make, three for each of the four criteria.) For the one shown, Dell is slightly more important than IBM on the salary criterion.

When you have made all of the pairwise comparisons, you are finished. The application then shows you the results in Figure 16.19. For the comparisons made, IBM has the largest overall score, 0.545, so this is presumably the job you should prefer. Why did IBM get such a high score? The reason is that your two most important criteria (see the Weights for Criteria section) are nearness to family and interest of work, and IBM scored the highest on these two criteria (see the Scores for jobs on various criteria section). However,
there is some cause for concern, as indicated in the bottom section. There were evidently some inconsistencies in the pairwise comparisons, so you might want to click the middle button and try to be more consistent this time.

**Figure 16.14** Opening Screen of AHP Application

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Double-click here for help. Double-click again to make help disappear.</td>
<td>Selecting Jobs with AHP</td>
<td>Run the application</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>32</td>
<td>33</td>
<td>34</td>
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</tbody>
</table>

This application automates the AHP method for selecting jobs based on several criteria. You can specify any criteria and any jobs. After you make all of the pairwise comparisons, you will see the calculated weights for the criteria, the scores for the jobs on each criterion, the total scores for each job, and consistency measures for each pairwise comparison matrix. You can also view a chart that shows the total scores for the jobs.

AHP (Analytical Hierarchy Process) is a method for making decisions when there are several objectives (or criteria). The key is making pairwise comparisons.

First, you make pairwise comparisons between criteria in order to specify the relative importance of the various criteria to you.

Then for each criterion, you make pairwise comparisons between decisions (in this case jobs) to specify how they rate on that criterion.

AHP then calculates “weights” for the criteria, “scores” for the jobs on each criterion, and “total scores” for the jobs. Presumably, you will prefer the job with the highest total score.

Because it is possible to be inconsistent when making pairwise comparisons, AHP calculates a consistency value (CI/RI) for each set of comparisons. If this value is less than .10, you are being fairly consistent. Otherwise, you might want to reassess your pairwise comparisons.

**Figure 16.15** Choosing Criteria

Choose a criterion from the dropdown list or type in a criterion of your choice. Then click on the Add button to add this to your criteria list. Click the No More button if you have added all you want. Click the Cancel button to exit the application altogether.
16.4 The Analytic Hierarchy Process (AHP)

**Figure 16.16**
Listing Potential Jobs

**Figure 16.17**
Pairwise Comparisons between Criteria

**Figure 16.18**
Pairwise Comparisons between Jobs on a Criterion
Results from AHP

Weights for Criteria

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Salary</th>
<th>Location</th>
<th>Nearness to family</th>
<th>Interest of work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salary</td>
<td>0.088</td>
<td>0.275</td>
<td>0.305</td>
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<td>IBM</td>
<td>0.241</td>
<td>0.194</td>
<td>0.539</td>
<td>0.503</td>
</tr>
<tr>
<td>Cisco</td>
<td>0.211</td>
<td>0.107</td>
<td>0.297</td>
<td>0.148</td>
</tr>
</tbody>
</table>

Scores for jobs on various criteria

<table>
<thead>
<tr>
<th>Job</th>
<th>Salary</th>
<th>Location</th>
<th>Nearness to family</th>
<th>Interest of work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dell</td>
<td>0.548</td>
<td>0.194</td>
<td>0.539</td>
<td>0.503</td>
</tr>
<tr>
<td>IBM</td>
<td>0.241</td>
<td>0.7</td>
<td>0.348</td>
<td></td>
</tr>
<tr>
<td>Cisco</td>
<td>0.211</td>
<td>0.107</td>
<td>0.297</td>
<td>0.148</td>
</tr>
</tbody>
</table>

Overall job scores (best score highlighted)

<table>
<thead>
<tr>
<th>Job</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dell</td>
<td>0.267</td>
</tr>
<tr>
<td>IBM</td>
<td>0.545</td>
</tr>
<tr>
<td>Cisco</td>
<td>0.188</td>
</tr>
</tbody>
</table>

Relative Consistency Indexes

Pairwise comparisons among criteria

- 0.318 (consistency not adequate)

Pairwise comparisons among jobs on various criteria

- On Salary: 0.016 (adequate consistency)
- On Location: 0.008 (adequate consistency)
- On Nearness to family: 0.008 (adequate consistency)
- On Interest of work: 0.415 (consistency not adequate)

Modeling Issues

In this job selection example, suppose that quality of life depends on two subcriteria: recreational facilities and educational facilities. Our macro-based application can’t accommodate such subcriteria, but software such as Expert Choice can. It can handle a hierarchy of criteria and subcriteria—hence the term “hierarchy” in the name of the procedure.

Additional Applications

Automated Manufacturing Decisions Using AHP

Weber (1993) reports the successful use of AHP in deciding which of several technologies to purchase for automated manufacturing. As he discusses, these decisions can have several types of impacts: quantitative financial (such as purchase cost), quantitative nonfinancial (such as throughput, cycle time, and scrap, which are difficult to translate directly into dollars), and qualitative (such as product quality and manufacturing flexibility, which are also difficult to translate into dollars). When the decision maker is trying to rate the different technologies along nonmonetary criteria, then he or she should use the method discussed in this section. (For example, how much more do you prefer technology 1 to technology 2 in the area of product quality?) However, he advises that when quantitative financial data are available (for example,
Whenever you face a problem with multiple competing objectives, as is the case in many real-world problems, you are forced to make trade-offs among these objectives. This is usually a very difficult task, and not all management scientists agree on the best way to proceed. When the objectives are very different in nature, no method can disguise the inherent complexity of comparing “apples to oranges.” Although one method, finding Pareto optimal solutions and drawing the resulting trade-off curve, locates solutions that are not dominated by any others, you still face the problem of choosing one of the (many) Pareto optimal solutions to implement. The other two methods discussed in this chapter, goal programming and AHP, make trade-offs and ultimately locate an “optimal” solution. These methods have their critics, but when they are used carefully, they have the potential to help solve some difficult and important real-world problems.

### Summary of Key Management Science Terms

<table>
<thead>
<tr>
<th>Term</th>
<th>Explanation</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goal programming</td>
<td>Optimization method that optimizes a weighted average of multiple objectives (goals)</td>
<td>16-3</td>
</tr>
<tr>
<td>Hard constraint</td>
<td>A constraint that <strong>must</strong> be satisfied</td>
<td>16-3</td>
</tr>
<tr>
<td>Soft constraint</td>
<td>A constraint you would like to satisfy but don’t absolutely have to satisfy</td>
<td>16-3</td>
</tr>
<tr>
<td>Pareto optimal solution</td>
<td>Solution that is not <strong>dominated,</strong> that is, no other solution is at least as good on all objectives and better on at least one objective</td>
<td>16-12</td>
</tr>
<tr>
<td>Trade-off curve (or efficient frontier)</td>
<td>Curve showing Pareto optimal solutions, used primarily to show the trade-offs between two competing objectives</td>
<td>16-12</td>
</tr>
<tr>
<td>Analytical Hierarchy Process (AHP)</td>
<td>Method used to find best decision when a decision maker faces multiple criteria; requires a series of pairwise comparisons between criteria and between alternative decisions for each criterion</td>
<td>16-20</td>
</tr>
</tbody>
</table>

**16.5 CONCLUSION**

16-25
Level A

20. The Pine Valley Board of Education must hire teachers for the coming school year. The types of teachers and the salaries that must be paid are given in the file P16_20.xlsx. For example, 20 teachers who are qualified to teach history and science have applied for jobs, and each of these teachers must be paid an annual salary of $21,000. Each teacher who is hired teaches the two subjects he or she is qualified to teach. Pine Valley needs to hire 35 teachers qualified to teach history, 30 teachers qualified to teach science, 40 teachers qualified to teach math, and 32 teachers qualified to teach English. The board has $1.4 million to spend on teachers’ salaries. A penalty cost of $1 is incurred for each dollar the board goes over budget. For each teacher by which Pine Valley’s goals are unmet, the following costs are incurred (because of the lower quality of education): science, $30,000; math, $28,000; history, $26,000; and English, $24,000. Determine how the board can minimize its total cost due to unmet goals.

21. Productco produces three products. Each product requires labor, lumber, and paint. The resource requirements, unit price, and variable cost (exclusive of labor, lumber, and paint) for each product are given in the file P16_21.xlsx. At present, 900 labor hours, 1550 gallons of paint, and 1600 board feet of lumber are available. Additional labor can be purchased at $6 per hour. Additional paint can be purchased at $2 per gallon. Additional lumber can be purchased at $3 per board foot. For the following two sets of priorities, use goal programming to determine an optimal production schedule. For set 1:
   - Priority 1: Obtain profit of at least $10,500.
   - Priority 2: Purchase no additional labor.
   - Priority 3: Purchase no additional paint.
   - Priority 4: Purchase no additional lumber.

   For set 2:
   - Priority 1: Purchase no additional labor.
   - Priority 2: Obtain profit of at least $10,500.
   - Priority 3: Purchase no additional paint.
   - Priority 4: Purchase no additional lumber.

22. A hospital outpatient clinic performs four types of operations. The profit per operation, as well as the minutes of X-ray time and laboratory time used, are given in the file P16_22.xlsx. The clinic has 500 private rooms and 500 intensive care rooms. Type 1 and type 2 operations require a patient to stay in an intensive care room for one day, whereas type 3 and type 4 operations require a patient to stay in a private room for one day. Each day, the hospital is required to perform at least 100 operations of each type. The hospital has set the following goals (listed in order of priority):
   - Goal 1: Earn a daily profit of at least $100,000.
   - Goal 2: Use at most 50 hours daily of X-ray time.
   - Goal 3: Use at most 40 hours daily of laboratory time.

   Use goal programming to determine the types of operations that should be performed.

23. Jobs at Indiana University are rated on three factors:
   - Factor 1: Complexity of duties
   - Factor 2: Education required
   - Factor 3: Mental and/or visual demands

   For each job at IU, the requirement for each factor has been rated on a scale of 1 to 4, with a 4 in factor 1 representing high complexity of duty, a 4 in factor 2 representing high educational requirement, and a 4 in factor 3 representing high mental and/or visual demands. IU wants to determine a formula for grading each job. To do this, it will assign a point value to the score for each factor that a job requires. For example, suppose that level 2 of factor 1 yields a point total of 10, level 3 of factor 2 yields a point total of 20, and level 3 of factor 3 yields a point total of 30. Then a job with these requirements has a point total of $10 + 20 + 30 = 60$. A job’s hourly salary equals half its point total. IU has two goals (listed in order of priority) in setting up the points given to each level of each job factor.
   - Goal 1: When increasing the level of a factor by 1, the points should increase by at least 10. For example, level 2 of factor 1 should earn at least 10 more points than level 1 of factor 1. Goal 1 is to minimize the sum of deviations from this requirement.
   - Goal 2: For the benchmark jobs referred to in the file P16_23.xlsx, the actual point total for each job should come as close as possible to the point total listed in the table. Goal 2 is to minimize the sum of the absolute deviations of the point totals from the desired scores.

   Use goal programming to find appropriate point totals. What salary should a job with skill levels of 3 for each factor be paid?

24. At Lummins Engine Corporation, production employees work 10 hours per day, four days per week. Each day of the week, at least the following number of employees must be working: Monday through Friday, seven employees; Saturday and Sunday, three employees. Lummins has set the following goals, listed in order of priority:
   - Goal 1: Meet employee requirements with 11 workers.
   - Goal 2: The average number of weekend days off per employee should be at least 1.5 days.
Goal 3: The average number of consecutive days off an employee gets during the week should not exceed 2.8 days.

Use goal programming to determine how to schedule Lummins employees.

Level B

25. You are the mayor of Gotham City and you must determine a tax policy for the city. Five types of taxes are used to raise money:

- Property taxes. Let \( p \) be the property tax rate.
- A sales tax on all items except food, drugs, and durable goods. Let \( s \) be the sales tax rate.
- A sales tax on durable goods. Let \( d \) be the durable goods sales tax rate.
- A gasoline sales tax. Let \( g \) be the gasoline sales tax rate.
- A sales tax on food and drugs. Let \( f \) be the sales tax on food and drugs.

The city consists of three groups of people: low income (LI), middle income (MI), and high income (HI). The amount of revenue (in millions of dollars) raised from each group by setting a particular tax at a 1% level is given in the file P16_25.xlsx. For example, a 3% tax on durable goods sales will raise 360 million dollars from low-income people. Your tax policy must satisfy the following restrictions:

- Restriction 1: The tax burden on MI people cannot exceed $2.8 billion.
- Restriction 2: The tax burden on HI people cannot exceed $2.4 billion.
- Restriction 3: The total revenue raised must exceed the current level of $6.5 billion.
- Restriction 4: \( s \) must be between 1% and 3%.

Given these restrictions, the city council has set the following three goals (listed in order of priority):

- Goal 1: Limit the tax burden on LI people to $2 billion.
- Goal 2: Keep the property tax rate under 3%.
- Goal 3: If their tax burden becomes too high, 20% of the LI people, 20% of the MI people, and 40% of the HI people may consider moving to the suburbs. Suppose that this will happen if their total tax burden exceeds $1.5 billion. To discourage this exodus, goal 3 is to keep the total tax burden on these people below $1.5 billion.

Use goal programming to determine an optimal tax policy.

26. Based on Sartoris and Spruill (1974). Wivco produces two products, which it sells for both cash and credit. Revenues from credit sales will not have been received but are included in determining profit earned during the current six-month period. Sales during the next six months can be made either from units produced during the next six months or from beginning inventory. Relevant information about products 1 and 2 is as follows:

- During the next six months, at most 150 units of product 1 can be sold on a cash basis, and at most 100 units of product 1 can be sold on a credit basis. It costs $35 to produce each unit of product 1, and each sells for $40. A credit sale of a unit of product 1 yields $0.50 less profit than a cash sale (because of delays in receiving payment). Two hours of production time are needed to produce each unit of product 1. At the beginning of the six-month period, 60 units of product 1 are in inventory.
- During the next six months, at most 175 units of product 2 can be sold on a cash basis, and at most 250 units of product 2 can be sold on a credit basis. It costs $45 to produce each unit of product 2, and each sells for $52.50. A credit sale of a unit of product 2 yields $1.00 less profit than a cash sale. Four hours of production time are needed to produce each unit of product 2. At the beginning of the six-month period, 30 units of product 2 are in inventory.
- During the next six months, Wivco has 1000 hours for production available. At the end of the next six months, Wivco incurs a 10% holding cost on the value of ending inventory (measured relative to production cost). An opportunity cost of 5% is also assessed against any cash on hand at the end of the six-month period.

a. Develop and solve an LP model that yields Wivco’s maximum profit during the next six months. What is Wivco’s ending inventory position? Assuming an initial cash balance of $0, what is Wivco’s ending cash balance?

b. Because an ending inventory and cash position of $0 is undesirable (for ongoing operations), Wivco is considering other options. At the beginning of the six-month period, Wivco can obtain a loan (secured by ending inventory) that incurs an interest cost equal to 5% of the value of the loan. The maximum value of the loan is 75% of the value of the ending inventory. The loan will be repaid one year from now. Wivco has the following goals (listed in order of priority):

- Goal 1: Make the ending cash balance of Wivco come as close as possible to $75.
- Goal 2: Make profit come as close as possible to the profit level obtained in part a.
- Goal 3: At any time, Wivco’s current ratio is defined to be

\[
\text{Current ratio} = \frac{\text{Wivco’s assets}}{\text{Wivco’s liabilities}}
\]
Assuming initially that current liabilities equal $150, six months from now Wivco’s current ratio will equal

\[
\text{Current ratio} = \frac{\text{CR} + \text{AR} + \text{EI}}{150 + \text{Size of loan}}
\]

where CB is the ending cash balance, AR is the value of accounts receivable, and EI is the value of the ending inventory. Six months from now, Wivco wants the current ratio to be as close as possible to 2.

Use goal programming to determine Wivco’s production and financial strategy.

**Modeling Problems**

27. How might you use goal programming to help Congress balance the budget?

28. A company is considering buying up to five other businesses. Given knowledge of the company’s view of the trade-off between risk and return, how could trade-off curves be used to determine the companies that should be purchased?

29. How would you use AHP to determine the greatest sports record of all time? (Many believe it is Joe DiMaggio’s 56-game hitting streak.)

30. You are planning to renovate a hospital. How would you use AHP to help determine what improvements to include in the renovation?

31. You are planning to overhaul a hospital computer system. How would you use AHP to determine the type of computer system to install?

32. You have been commissioned to assign 100 remedial education teachers to the 40 schools in the St. Louis School System. What are some objectives you might consider in assigning the teachers to schools?

33. You have been hired as a consultant to help design a new airport in northern Indiana that will supplant O’Hare as Chicago’s major airport. Discuss the objectives you consider important in designing and locating the airport.

34. In the Indiana MBA program we need to divide a class of 60 students into 10 six-person teams. In the interest of diversity, we have the following goals (listed in descending order of importance):

   ■ At least one woman per team
   ■ At least one member of a minority per team
   ■ At least one student with a financial or accounting background per team
   ■ At least one engineer per team

   Explain how you could use the material in this chapter to develop a model to assign students to teams.