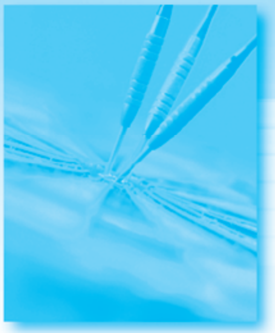

# Appendix C

## Exercises C

*In Exercises 1–14, evaluate the given expression and write your answer in the form $a + bi$.*

**1.** $(3 + 2i) + (5 - 6i)$

**2.** $(1 + i) - (2 - 3i)$

**3.** $(5 + 2i)(3 + i)$

**4.** $(\frac{1}{2} + i)^2$

**5.** $\overline{7 + 4i}$

**6.** $\overline{3i(1 - 2i)}$

**7.** $\dfrac{1}{1 + i}$

**8.** $\dfrac{2}{3 - 4i}$

**9.** $\dfrac{4 - i}{1 + 3i}$

**10.** $\dfrac{\sqrt{3} + i}{1 + \sqrt{3}i}$

**11.** $i^3$

**12.** $i^{2012}$

**13.** $\sqrt{-100}$

**14.** $\sqrt{-2}\sqrt{-18}$

*In Exercises 15–18, find the absolute value of the given complex number.*

**15.** $4 + 3i$

**16.** $1 - i$

**17.** $1 + 2\sqrt{2}i$

**18.** $\frac{3}{2} + 2i$

*In Exercises 19–22, write the given complex number in polar form using its principal argument.*

**19.** $2 - 2i$

**20.** $5i$

**21.** $\sqrt{3} + i$

**22.** $-3 - 4i$

*In Exercises 23–26, find the polar form of $zw$, $z/w$, and $1/z$ by first putting $z$ and $w$ in polar form.*

**23.** $z = -1 + i, w = \sqrt{3} + i$

**24.** $z = 1 + \sqrt{3}i, w = 2\sqrt{3} - 2i$

**25.** $z = 4 + 4i, w = 2i$

**26.** $z = 3(\sqrt{3} + i), w = -1 - i$

*In Exercises 27–30, find the indicated power using De Moivre's Theorem.*

**27.** $(1 - i)^8$

**28.** $(\frac{1}{2} + \frac{1}{2}i)^{10}$

**29.** $(1 + \sqrt{3}i)^5$

**30.** $(2\sqrt{3} - 2i)^3$

*In Exercises 31–34, find the indicated roots and sketch them in the complex plane.*

**31.** The eighth roots of 1

**32.** The sixth roots of $-64$

**33.** The cube roots of $i$

**34.** The cube roots of $4\sqrt{2} + 4\sqrt{2}i$

*In Exercises 35–38, write the given number in the form $a + bi$.*

**35.** $e^{-i\pi/2}$

**36.** $2e^{i\pi/3}$

**37.** $-e^{1 - i\pi}$

**38.** $e^{(1 + i\pi)/2}$

**39.** Prove the following properties of the complex conjugate:

**(a)** $\overline{\overline{z}} = z$

**(b)** $\overline{z + w} = \overline{z} + \overline{w}$

**(c)** $\overline{zw} = \overline{z}\,\overline{w}$

**(d)** If $z \neq 0$, then $\overline{(w/z)} = \overline{w}/\overline{z}$.

**(e)** $z$ is real if and only if $\overline{z} = z$.

**(f)** $z + \overline{z} = 2\text{Re}\, z$ and $z - \overline{z} = 2i\,\text{Im}\, z$.

**40.** Prove the following properties of absolute value:

**(a)** $|\bar{z}| = |z|$

**(b)** $|zw| = |z||w|$

**(c)** If $z \neq 0$, then $|1/z| = 1/|z|$.

**(d)** $|z| = 0$ if and only if $z = 0$.

**(e)** $\text{Re}\, z \leq |z|$

**(f)** $|z + w| \leq |z| + |w|$ [*Hint*: Square the left hand side and expand using the identity $|z|^2 = z\bar{z}$. Exercises 39(f) and 40(e) are useful.]

**41. (a)** Derive the double angle formulas $\cos 2\theta = \cos^2\theta - \sin^2\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$ by expanding $(\cos\theta + i\sin\theta)^2$ and comparing the result with the answer given by De Moivre's Theorem.

**(b)** Imitate the method of part (a) to derive formulas for $\cos 3\theta$ and $\sin 3\theta$.

**(c)** Imitate the method of part (a) to derive formulas for $\cos 4\theta$ and $\sin 4\theta$.

**42.** Prove De Moivre's Theorem using mathematical induction.