Algebraic Expressions

A basic characteristic of algebra is the use of letters (or combinations of letters) to represent numbers. The letters used to represent the numbers are variables, and combinations of letters and numbers are algebraic expressions. Here are a few examples.

The terms of an algebraic expression are those parts that are separated by addition. For example, the algebraic expression has three terms: and . Note that is a term, rather than because Think of subtraction as a form of addition.

The terms and are called the variable terms of the expression, and 6 is called the constant term of the expression. The numerical factor of a variable term is called the coefficient of the variable term. For instance, the coefficient of the variable term is and the coefficient of the variable term is 1. (The constant term of an expression is also considered to be a coefficient.)

Example 1 Identifying Terms and Coefficients

Identify the terms and coefficients in each algebraic expression.

(a) (b) (c)

Solution

<table>
<thead>
<tr>
<th>Terms</th>
<th>Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) $5x, \ -\frac{1}{3}$</td>
<td>$5, \ -\frac{1}{3}$</td>
</tr>
<tr>
<td>(b) $4y, \ 6x, \ -9$</td>
<td>$4, \ 6, \ -9$</td>
</tr>
<tr>
<td>(c) $x^2y, \ -\frac{1}{x}, \ 3y$</td>
<td>$1, \ -1, \ 3$</td>
</tr>
</tbody>
</table>
Properties of Algebra

The properties of real numbers (see Section P.2) can be used to rewrite algebraic expressions. The following list is similar to those given in Section P.2, except that the examples involve algebraic expressions. In other words, the properties are true for variables and algebraic expressions as well as for real numbers.

<table>
<thead>
<tr>
<th>Property</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commutative Property of Addition</td>
<td>$a + b = b + a$</td>
</tr>
<tr>
<td></td>
<td>$5x + x^2 = x^2 + 5x$</td>
</tr>
<tr>
<td>Commutative Property of Multiplication</td>
<td>$ab = ba$</td>
</tr>
<tr>
<td></td>
<td>$(3 + x)x^3 = x^3(3 + x)$</td>
</tr>
<tr>
<td>Associative Property of Addition</td>
<td>$(a + b) + c = a + (b + c)$</td>
</tr>
<tr>
<td></td>
<td>$(-x + 6) + 3x^2 = -x + (6 + 3x^2)$</td>
</tr>
<tr>
<td>Associative Property of Multiplication</td>
<td>$(ab)c = a(bc)$</td>
</tr>
<tr>
<td></td>
<td>$(5x \cdot 4y)(6) = (5x)(4y \cdot 6)$</td>
</tr>
<tr>
<td>Distributive Properties</td>
<td>$a(b + c) = ab + ac$</td>
</tr>
<tr>
<td></td>
<td>$2x(4 + 3x) = 2x \cdot 4 + 2x \cdot 3x$</td>
</tr>
<tr>
<td></td>
<td>$(a + b)c = ac + bc$</td>
</tr>
<tr>
<td></td>
<td>$(y + 6)y = y \cdot y + 6 \cdot y$</td>
</tr>
<tr>
<td>Additive Identity Property</td>
<td>$a + 0 = 0 + a = a$</td>
</tr>
<tr>
<td></td>
<td>$4y^2 + 0 = 0 + 4y^2 = 4y^2$</td>
</tr>
<tr>
<td>Multiplicative Identity Property</td>
<td>$a \cdot 1 = 1 \cdot a = a$</td>
</tr>
<tr>
<td></td>
<td>$(-5x^3)(1) = (1)(-5x^3) = -5x^3$</td>
</tr>
<tr>
<td>Additive Inverse Property</td>
<td>$a + (-a) = 0$</td>
</tr>
<tr>
<td></td>
<td>$4x^2 + (-4x^2) = 0$</td>
</tr>
<tr>
<td>Multiplicative Inverse Property</td>
<td>$a \cdot \frac{1}{a} = 1, a \neq 0$</td>
</tr>
<tr>
<td></td>
<td>$(x^2 + 1)\left(\frac{1}{x^2 + 1}\right) = 1$</td>
</tr>
</tbody>
</table>

Because subtraction is defined as “adding the opposite,” the Distributive Property is also true for subtraction. For instance, the “subtraction form” of $a(b + c) = ab + ac$ is

$$a(b - c) = a[b + (-c)]$$

$$= ab + a(-c)$$

$$= ab - ac.$$
In addition to these properties, the properties of equality, zero, and negation given in Section P.2 are also valid for algebraic expressions. The next example illustrates the use of a variety of these properties.

**Example 2  ■ Identifying the Properties of Algebra**

Identify the property of algebra illustrated in each statement.

(a) \((5x^2)3 = 3(5x^2)\)  
(b) \((3x^2 + x) - (3x^2 + x) = 0\)

(c) \(3x + 3y^2 = 3(x + y^2)\)  
(d) \((5 + x^2) + 4x^2 = 5 + (x^2 + 4x^2)\)

(e) \(5x \cdot \frac{1}{5x} = 1, x \neq 0\)  
(f) \((y - 6)3 + (y - 6)y = (y - 6)(3 + y)\)

**Solution**

(a) This statement illustrates the Commutative Property of Multiplication. In other words, you obtain the same result whether you multiply \(5x^2\) by 3, or 3 by \(5x^2\).

(b) This statement illustrates the Additive Inverse Property. In terms of subtraction, this property simply states that when any expression is subtracted from itself the result is zero.

(c) This statement illustrates the Distributive Property. In other words, multiplication is distributed over addition.

(d) This statement illustrates the Associative Property of Addition. In other words, to form the sum \(5 + x^2 + 4x^2\) it does not matter whether 5 and \(x^2\) are added first or \(x^2\) and \(4x^2\) are added first.

(e) This statement illustrates the Multiplicative Inverse Property. Note that it is important that \(x\) be a nonzero number. If \(x\) were zero, the reciprocal of \(x\) would be undefined.

(f) This statement illustrates the Distributive Property in reverse order. 

\[ ab + ac = a(b + c) \]  
**Distributive Property**

\[ (y - 6)3 + (y - 6)y = (y - 6)(3 + y) \]

Note in this case that \(a = y - 6, b = 3,\) and \(c = y.\)

**EXPLORATION**

**Discovering Properties of Exponents**  Write each of the following as a single power of 2. Explain how you obtained your answer. Then generalize your procedure by completing the statement “When you multiply exponential expressions that have the same base, you . . . .”

a. \(2^3 \cdot 2^3\)  
b. \(2^4 \cdot 2^1\)  
c. \(2^5 \cdot 2^2\)  
d. \(2^3 \cdot 2^4\)  
e. \(2^1 \cdot 2^5\)
Properties of Exponents

Just as multiplication by a positive integer can be described as repeated addition, repeated multiplication can be written in what is called exponential form (see Section P.1). Let \( n \) be a positive integer and let \( a \) be a real number. Then the product of \( n \) factors of \( a \) is given by

\[
a^n = a \cdot a \cdot a \cdot \ldots \cdot a.
\]

\( a \) is the base and \( n \) is the exponent.

When multiplying two exponential expressions that have the same base, you add exponents. To see why this is true, consider the product \( a^3 \cdot a^2 \). Because the first expression represents \( a \cdot a \cdot a \) and the second represents \( a \cdot a \), the product of the two expressions represents \( a \cdot a \cdot a \cdot a \cdot a \cdot a \), as follows.

\[
a^3 \cdot a^2 = (a \cdot a \cdot a) \cdot (a \cdot a) = (a \cdot a \cdot a \cdot a \cdot a) = a^5
\]

3 factors 2 factors 5 factors

### Example 3 □ Illustrating the Properties of Exponents

(a) To multiply exponential expressions that have the same base, add exponents.

\[
x^2 \cdot x^4 = x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x = x^{2+4} = x^6
\]

2 factors 4 factors 6 factors

(b) To raise the product of two factors to the same power, raise each factor to the power and multiply the results.

\[
(3x)^3 = 3x \cdot 3x \cdot 3x = 3 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x = 3^3 \cdot x^3 = 27x^3
\]

3 factors 3 factors 3 factors

(c) To raise an exponential expression to a power, multiply exponents.

\[
(x^3)^2 = (x \cdot x \cdot x) \cdot (x \cdot x \cdot x) = (x \cdot x \cdot x \cdot x \cdot x \cdot x) = x^{3 \cdot 2} = x^6
\]

3 factors 3 factors 6 factors

**CHECKPOINT**  Now try Exercise 29.
Example 4 ■ Illustrating the Properties of Exponents

(a) To divide exponential expressions that have the same base, subtract exponents.
\[
\frac{x^5}{x^2} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x} = x^{5-2} = x^3
\]
(b) To raise the quotient of two expressions to the same power, raise each expression to the power and divide the results.
\[
\left(\frac{x}{3}\right)^3 = \frac{x^3}{3^3} = \frac{x^3}{27}
\]

Example 5 ■ Applying Properties of Exponents

Use the properties of exponents to simplify each expression.

(a) \((x^2y^4)(3x)\)  
(b) \(-2(y^2)^3\)  
(c) \((-2y^2)^3\)  
(d) \((3x^2)(-5x^3)\)

Solution

(a) \((x^2y^4)(3x) = 3(x^2 \cdot x)(y^4) = 3(x^{2+1})(y^4) = 3x^3y^4\)
(b) \(-2(y^2)^3 = (-2)(y^{2 \cdot 3}) = -2y^6\)
(c) \((-2y^2)^3 = (-2)^3(y^2)^3 = -8(y^{2 \cdot 3}) = -8y^6\)
(d) \((3x^2)(-5x^3) = 3(-5)(x^2 \cdot x^3) = 3(-125)(x^{2+3}) = -375x^5\)

Example 6 ■ Applying Properties of Exponents

Use the properties of exponents to simplify each expression.

(a) \(\frac{14a^5b^3}{7a^2b^2}\)  
(b) \(\left(\frac{x^2}{2y}\right)^3\)  
(c) \(\frac{x^ny^m}{x^2y^4}\)  
(d) \(\frac{(2a^2b^3)^2}{a^3b^2}\)

Solution

(a) \(\frac{14a^5b^3}{7a^2b^2} = 2(a^{5-2})(b^{3-2}) = 2a^3b\)
(b) \(\left(\frac{x^2}{2y}\right)^3 = \frac{(x^2)^3}{(2y)^3} = \frac{x^{2 \cdot 3}}{2^3y^3} = \frac{x^6}{8y^3}\)
(c) \(\frac{x^ny^m}{x^2y^4} = x^{n-2}y^{m-4}\)
(d) \(\frac{(2a^2b^3)^2}{a^3b^2} = \frac{2^2(a^{2-2})(b^{3-2})}{a^3b^2} = \frac{4a^0b^1}{a^3b^2} = 4(a^{4-3})(b^{3-2}) = 4ab^1\)

\[\text{Now try Exercise 35.}\]

\[\text{Now try Exercise 63.}\]

\[\text{Now try Exercise 65.}\]
Simplifying Algebraic Expressions

One common use of the basic properties of algebra is to rewrite an algebraic expression in a simpler form. To simplify an algebraic expression generally means to remove symbols of grouping such as parentheses or brackets and combine like terms.

Two or more terms of an algebraic expression can be combined only if they are like terms. In an algebraic expression, two terms are said to be like terms if they are both constant terms or if they have the same variable factor(s). For example, the terms $4x$ and $-2x$ are like terms because they have the same variable factor, $x$. Similarly, $2x^2y$, $-x^2y$, and $\frac{1}{2}(x^2y)$ are like terms because they have the same variable factor, $x^2y$. Note that $4x^2y$ and $-x^2y^2$ are not like terms because their variable factors are different.

To combine like terms in an algebraic expression, simply add their respective coefficients and attach the common variable factor. This is actually an application of the Distributive Property, as shown in Example 7.

**Example 7 ■ Combining Like Terms**

Simplify each expression by combining like terms.

(a) $2x + 3x - 4$
(b) $-3 + 5 + 2y - 7y$
(c) $5x + 3y - 4x$

**Solution**

(a) $2x + 3x - 4 = (2 + 3)x - 4$
$$= 5x - 4$$

(b) $-3 + 5 + 2y - 7y = (-3 + 5) + (2 - 7)y$
$$= 2 - 5y$$

(c) $5x + 3y - 4x = 5x - 4x + 3y$
$$= (5x - 4x) + 3y$$
$$= (5 - 4)x + 3y$$
$$= x + 3y$$

**CHECKPOINT** Now try Exercise 93.

**Example 8 ■ Combining Like Terms**

(a) $7x + 7y - 4x - y = (7x - 4x) + (7y - y)$
$$= 3x + 6y$$

(b) $2x^2 + 3x - 5x^2 - x = (2x^2 - 5x^2) + (3x - x)$
$$= -3x^2 + 2x$$

(c) $3xy^2 - 4x^2y + 2xy^2 + (xy)^2$
$$= (3xy^2 + 2xy^2) + (-4x^2y^2 + x^2y^2)$
$$= 5xy^2 - 3x^2y^2$$

**CHECKPOINT** Now try Exercise 99.
A set of parentheses preceded by a minus sign can be removed by changing the sign of each term inside the parentheses. For instance,

$$8x - (5x - 4) = 8x - 5x + 4 = 3x + 4.$$  

A set of parentheses preceded by a plus sign can be removed without changing the signs of the terms inside the parentheses. For instance,

$$8x + (5x - 4) = 8x + 5x - 4 = 13x - 4.$$  

**Example 9**  ■  **Removing Symbols of Grouping**

Simplify $3(x - 5) - (2x - 7)$.

**Solution**

$$3(x - 5) - (2x - 7) = 3x - 15 - 2x + 7$$  

Distributive Property

$$= (3x - 2x) + (-15 + 7)$$  

Group like terms.

$$= x - 8$$  

Combine like terms.

Now try Exercise 105.

**Example 10**  ■  **Geometry: Perimeter and Area of a Region**

Write and simplify an expression for the perimeter and for the area of each region shown in Figure P.10.

**Solution**

(a) Perimeter of a rectangle = 2 · length + 2 · width

$$= 2(3x - 5) + 2x$$  

Substitute.

$$= 6x - 10 + 2x$$  

Distributive Property

$$= (6x + 2x) - 10$$  

Group like terms.

$$= 8x - 10$$  

Combine like terms.

Area of a rectangle = length · width

$$= (3x - 5)x$$  

Substitute.

$$= 3x^2 - 5x$$  

(b) Perimeter of a triangle = sum of the three sides

$$= 2x + (2x + 4) + (x + 5)$$  

Substitute.

$$= (2x + 2x + x) + (4 + 5)$$  

Group like terms.

$$= 5x + 9$$  

Combine like terms.

Area of a triangle = $\frac{1}{2} \cdot$ base · height

$$= \frac{1}{2}(x + 5)(2x)$$  

Substitute.

$$= \frac{1}{2}(2x)(x + 5)$$  

Commutative Property of Multiplication

$$= x(x + 5)$$  

Multiply.

$$= x^2 + 5x$$  

Distributive Property

Now try Exercise 137.

When removing symbols of grouping, combine like terms within the innermost symbols of grouping first, as shown in the next example.
Example 11  ■  Removing Symbols of Grouping

Simplify each expression by combining like terms.

(a) \(5x - 2[x + 2(x - 7)]\)  
(b) \(-3x(5x^4) + (2x)^5\)

\textbf{Solution}

(a) \(5x - 2[x + 2(x - 7)] = 5x - 2[2x - 11]\)  
\(= 5x - 4x^2 + 22x\)  
\(= -4x^2 + 27x\)

(b) \(-3x(5x^4) + (2x)^5 = -15x^5 + (2^5)(x^5)\)  
\(= -15x^5 + 32x^5\)  
\(= 17x^5\)

\underline{CHECKPOINT}  Now try Exercise 113.

Evaluating Algebraic Expressions

To evaluate an algebraic expression, substitute numerical values for each of the variables in the expression. Here are some examples.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Value of Variable</th>
<th>Substitute</th>
<th>Value of Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>3x + 2</td>
<td>(x = 2)</td>
<td>3(2) + 2</td>
<td>6 + 2 = 8</td>
</tr>
<tr>
<td>4x^2 + 2x - 1</td>
<td>(x = -1)</td>
<td>4(-1)^2 + 2(-1) - 1</td>
<td>4 - 2 - 1 = 1</td>
</tr>
<tr>
<td>2x(x + 4)</td>
<td>(x = -2)</td>
<td>2(-2)(-2 + 4)</td>
<td>2(-2)(2) = -8</td>
</tr>
</tbody>
</table>

Example 12  ■  Evaluating Algebraic Expressions

Evaluate each algebraic expression when \(x = -2\) and \(y = 5\).

(a) \(2y - 3x\)  
(b) \(5 + x^2\)  
(c) \(5 - x^2\)

\textbf{Solution}

(a) When \(x = -2\) and \(y = 5\), the expression \(2y - 3x\) has a value of

\(2y - 3x = 2(5) - 3(-2)\)  
\(= 10 + 6\)  
\(= 16.\)

(b) When \(x = -2\), the expression \(5 + x^2\) has a value of

\(5 + x^2 = 5 + (-2)^2\)  
\(= 5 + 4\)  
\(= 9.\)

(c) When \(x = -2\), the expression \(5 - x^2\) has a value of

\(5 - x^2 = 5 - (-2)^2\)  
\(= 5 - 4\)  
\(= 1.\)

\underline{CHECKPOINT}  Now try Exercise 121.
**Technology**

Most graphing utilities can be used to evaluate an algebraic expression for several values of x and display the results in a table. For instance, to evaluate \(2x^2 - 3x + 2\) when \(x\) is 0, 1, 2, 3, 4, 5, and 6, you can use the steps below.

1. Enter the expression into the graphing utility.
2. Using the table feature, set the minimum value of the table to 0.
3. Set the table step or table increment to 1.
4. Display the table.

The results are shown below. Consult the user’s guide for your graphing utility for specific instructions.

---

**Example 13**  ■  **Evaluating Algebraic Expressions**

Evaluate each algebraic expression when \(x = 2\) and \(y = -1\).

(a) \(x^2 - 2xy + y^2\)  
(b) \(|y - x|\)  
(c) \(\frac{2xy}{5x + y}\)

**Solution**

(a) When \(x = 2\) and \(y = -1\), the expression \(x^2 - 2xy + y^2\) has a value of

\[x^2 - 2xy + y^2 = 2^2 - 2(2)(-1) + (-1)^2\]

\[= 4 + 4 + 1 = 9.\]

Substitute for \(x\) and \(y\). Simplify.

(b) When \(x = 2\) and \(y = -1\), the expression \(|y - x|\) has a value of

\[|y - x| = |-1 - 2|\]

\[= |-3| = 3.\]

Substitute for \(x\) and \(y\). Simplify.

(c) When \(x = 2\) and \(y = -1\), the expression \(\frac{2xy}{5x + y}\) has a value of

\[\frac{2xy}{5x + y} = \frac{2(2)(-1)}{5(2) + (-1)}\]

\[= \frac{-4}{10 - 1} = \frac{-4}{9}.\]

Substitute for \(x\) and \(y\). Simplify.

**CHECKPOINT**  Now try Exercise 131.

---

**Example 14**  ■  **Using a Mathematical Model**

From 1997 to 2004, the average hourly wage for construction workers in the United States can be modeled by the expression

\[-0.0231t^2 + 1.011t + 9.66,\quad 7 \leq t \leq 14\]

where \(t\) represents the year, with \(t = 7\) corresponding to 1997. Create a table that shows the average hourly wages for these years.  (Source: U.S. Bureau of Labor Statistics)

**Solution**

To create a table of values that shows the average hourly wages for the years 1997 to 2004, evaluate the expression

\[-0.0231t^2 + 1.011t + 9.66\]

for each integer value of \(t\) from \(t = 7\) to \(t = 14\).

<table>
<thead>
<tr>
<th>Year</th>
<th>1997</th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t)</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Hourly wage</td>
<td>$15.61</td>
<td>$16.27</td>
<td>$16.89</td>
<td>$17.46</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t)</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>Hourly wage</td>
<td>$17.99</td>
<td>$18.47</td>
<td>$18.90</td>
<td>$19.29</td>
</tr>
</tbody>
</table>

**CHECKPOINT**  Now try Exercise 145.
VOCABULARY CHECK: Fill in the blanks.

1. A collection of ________ and ________ combined using the operations of addition, subtraction, multiplication, division, and exponentiation is called an ________.

2. The ________ of an algebraic expression are those parts that are separated by addition.

3. The numerical factor of a variable term is called the ________ of the variable term.

4. Repeated multiplication can be written in ________.

5. You ________ an algebraic expression by combining like terms and removing symbols of grouping.

6. To ________ an algebraic expression, substitute numerical values for each of the variables in the expression.

In Exercises 1–8, identify the terms of the algebraic expression.

1. 10x + 5
2. −16t^2 + 48
3. −3y^2 + 2y − 8
4. 25z^3 − 4.8z^2
5. 4x^2 − 3y^2 − 5x + 2y
6. 14u^2 + 25uv − 3v^2
7. x^2 − 2.5x − \frac{1}{x}
8. \frac{3}{t^2} − \frac{4}{t} + 6

In Exercises 9–12, identify the coefficient of the term.

9. 5y^3
10. 4x^6
11. −\frac{3}{t^2}
12. −8.4x

In Exercises 13–22, identify the property of algebra that is illustrated by the statement.

13. 4 − 3x = −3x + 4
14. (10 + x) − y = 10 + (x − y)
15. −5(2x) = (−5 ∙ 2)x
16. (x − 2)(3) = 3(x − 2)
17. (x + 5) \cdot \frac{1}{x + 5} = 1, \quad x \neq −5
18. (x^2 + 1) − (x^2 + 1) = 0
19. 5(y^3 + 3) = 5y^3 + 5 \cdot 3
20. 10x^3y + 0 = 10x^3y
21. (16t^4) \cdot 1 = 16t^4
22. −32(u^2 − 3u) = −32u^2 + 96u

In Exercises 23–28, use the property to rewrite the expression.

23. (a) Distributive Property
   5(x + 6) = ________
   (b) Commutative Property of Multiplication
   5(x + 6) = ________

24. (a) Distributive Property
   6x + 6 = ________
   (b) Commutative Property of Addition
   6x + 6 = ________

25. (a) Commutative Property of Multiplication
   6(xy) = ________
   (b) Associative Property of Multiplication
   6(xy) = ________

26. (a) Additive Identity Property
   3ab + 0 = ________
   (b) Commutative Property of Addition
   3ab + 0 = ________

27. (a) Additive Inverse Property
   4t^2 + (−4t^2) = ________
   (b) Commutative Property of Addition
   4t^2 + (−4t^2) = ________

28. (a) Associative Property of Addition
   (3 + 6) + (−9) = ________
   (b) Additive Inverse Property
   9 + (−9) = ________

In Exercises 29–38, write the expression as a repeated multiplication.

29. x^3 \cdot x^4
30. 2 \cdot 2^5
31. (2y)^3
32. (4t)^4
33. (−2x)^3
34. (−5y)^4
35. \frac{y^4}{5}
36. \frac{z^3}{2}
37. \frac{6^4}{x}
38. \frac{3^5}{t}
In Exercises 39–44, write the expression using exponential notation.
39. \((5x)(5x)(5x)(5x)\)
40. \((2y)(2y)(2y)(2y)\)
41. \((y \cdot y \cdot y)(y \cdot y \cdot y)\)
42. \((x \cdot x \cdot x)(y \cdot y \cdot y)\)
43. \((-z)(-z)(-z)(-z)(-z)(-z)(-z)\)
44. \((-9t)(-9t)(-9t)(-9t)(-9t)(-9t)\)

In Exercises 45–90, use the properties of exponents to simplify the expression.
45. \(x^5 \cdot x^2\)
46. \(y^3 \cdot y^4\)
47. \((a^2)^4\)
48. \((x^3)^3\)
49. \(\frac{y^7}{x^3}\)
50. \(\frac{x^8}{z^3}\)
51. \(\left(\frac{a^2}{b}\right)^2\)
52. \(\left(\frac{x^3}{y}\right)^3\)
53. \(3^y y^4 \cdot y^2\)
54. \(6^x x^3 \cdot x^5\)
55. \((-4x)^2\)
56. \((-4x)^3\)
57. \((-5z^3)^3\)
58. \((-5z^3)^2\)
59. \((a^2b^4)^2\)
60. \((y^2z^6)^3\)
61. \((x^3)(-x)\)
62. \((-x^2)(z^3)\)
63. \((2xy)(3x^2y^3)\)
64. \((-5a^2b^3)(2ab^4)\)
65. \(\frac{3^3x^5}{3^3x^3}\)
66. \(\frac{2^4y^5}{2^3y^3}\)
67. \(\frac{(2xy)^5}{6(xy)^3}\)
68. \(\frac{4^3(ab)^6}{4(ab)^2}\)
69. \((5y)^2(-y^4)\)
70. \((3y)^3(2y^2)\)
71. \((-5z^4)(-5z)^4\)
72. \((-6a)(-3n^2)\)
73. \((-2a^2)(-2a)^2\)
74. \((-2a^2)(-8a)\)
75. \(\frac{(2x)^4}{2x^2}\)
76. \(\frac{3x^6b^7}{a^3(3b)^2}\)
77. \(\frac{6(a^2b)^3}{(3ab)^2}\)
78. \(\frac{(-3c^3d)^2}{(-2cd)^3}\)
79. \(-x^3y^3\)
80. \(-x^2z^2\)
81. \(-\left(\frac{2x^4}{5y}\right)^2\)
82. \(-\left(\frac{3a^2}{2b^3}\right)^3\)
83. \(\frac{x^{n+1}}{x^n}\)
84. \(\frac{a^{m+3}}{a^3}\)
85. \((x^3)^4\)
86. \((a^3)^4\)
87. \(x^n \cdot x^3\)
88. \(y^m \cdot y^2\)
89. \(\frac{y^m z^m}{x^3}\)
90. \(\left(\frac{2^n y^m}{x^3}\right)^2\)

In Exercises 91–102, simplify the expression by combining like terms.
91. \(3x + 4x\)
92. \(-2x^2 + 4x^2\)
93. \(9y - 5y + 4y\)
94. \(8y + 7y - y\)
95. \(3x - 2y + 5x + 20y\)
96. \(-2a + \frac{1}{3}b - 7a - b\)
97. \(8z^2 + \frac{1}{2}z - \frac{1}{2}z^2 + 10\)
98. \(-5y^3 + 3y - 6y^2 + 8y^3 + y - 4\)
99. \(2uv + 5u^2v^2 - uv - (uv)^2\)
100. \(3m^2n^2 - 4mn - n(5m) + 2(mn)^2\)
101. \(5(ab)^2 + 2ab - 4ab\)
102. \(3xy - xy + 8\)

In Exercises 103–120, simplify the algebraic expression.
103. \(10(x - 3) + 2x - 5\)
104. \(3(x^2 + 1) + x^2 - 6\)
105. \((4x + 1) - (2x + 2)\)
106. \((7y^2 + 5) - (8y^2 + 4)\)
107. \(-(3x^2 - 2z + 4) + (z^2 - z - 2)\)
108. \((r^2 + 10r - 3) - (5r - 8)\)
109. \(-3(y^2 + 3y - 1) + 2(y - 5)\)
110. \(5(a + 6) - 4(a^2 - 2a - 1)\)
111. \(4[5 - 3(x^2 + 10)]\)
112. \(2[5x^2 - (x^3 + 5)]\)
113. \(2[3(b - 5) - (b^2 + b + 3)]\)
114. \(-[4(t + 1) - (t^2 - 2t - 5)]\)
115. \(y^2(y + 1) + y(y^2 + 1)\)
116. \(z^2(z^2 - z^2) + 4z^4(z + 1)\)
117. \(x(xy^2 + y) - 2xy(xy + 1)\)
118. \(2ab(b^2 - 3) - ab(b^2 + 2)\)
119. \(2a(3a^2)^3 + \frac{9a^8}{3a}\)
120. \(5y^3 + \frac{4y^5}{2y^2} - (7y)^2\)
In Exercises 121–136, evaluate the algebraic expression for each specified value of the variable(s). If not possible, state the reason.

**Expression** | **Values**
---|---
121. $5 - 3x$ | (a) $x = 5$ (b) $x = \frac{2}{3}$
122. $\frac{3x}{2} - 2$ | (a) $x = 6$ (b) $x = -3$
123. $10 - |x|$ | (a) $x = 3$ (b) $x = -3$
124. $-|x| + 6$ | (a) $x = -4$ (b) $x = 5$
125. $3x^2 - x + 7$ | (a) $x = -1$ (b) $x = \frac{1}{3}$
126. $2x^2 + 5x - 3$ | (a) $x = -3$ (b) $x = \frac{1}{2}$

**Expression** | **Values**
---|---
127. $\frac{x}{x^2 + 1}$ | (a) $x = 0$ (b) $x = 3$
128. $5 - \frac{3}{x}$ | (a) $x = 0$ (b) $x = -6$
129. $3x + 2y$ | (a) $x = 1$, $y = 5$ (b) $x = -6$, $y = -9$
130. $4x - y$ | (a) $x = 2$, $y = 0$ (b) $x = -2$, $y = -5$
131. $x^2 + 3xy - y^2$ | (a) $x = -1$, $y = 2$ (b) $x = -6$, $y = -3$
132. $x^2 - xy + y^2$ | (a) $x = 2$, $y = -1$ (b) $x = -3$, $y = -2$
133. $|y - x|$ | (a) $x = 2$, $y = 5$ (b) $x = -2$, $y = -2$
134. $|x - y|$ | (a) $x = 0$, $y = 10$ (b) $x = 4$, $y = 4$
135. $rt$ | (a) $r = 40$, $t = 5\frac{1}{3}$ (b) $r = 35$, $t = 4$
136. $Prt$ | (a) $P = $5000, $r = 0.085$, $t = 10$ (b) $P = $750, $r = 0.07$, $t = 3$

**Geometry** In Exercises 137–140, (a) write and simplify an expression for the perimeter of the region and (b) write and simplify an expression for the area of the region.

137.  
138.  

139. 

140. 

**Geometry** In Exercises 131–144, write an expression for the area of the region. Then evaluate the expression for the given value of the variable.

141. $b = 15$
142. $x = 3$
143. $h = 12$
144. $y = 20$

**Using a Model** In Exercises 145 and 146, use the model which approximates the per capita consumption (in pounds per person) of yogurt in the United States from 1996 through 2003 (see figure). In the model, $t$ represents the year, with $t = 6$ corresponding to 1996. **(Source: USDA/Economic Research Service)**

$$0.058t^2 - 0.77t + 8.4, \ 6 \leq t \leq 13$$

145. Use the graph to determine the per capita consumption of yogurt in 1999. Then use the model to approximate the per capita consumption of yogurt in 1999. Compare your results.
146. Use the graph to determine the per capita consumption of yogurt in 2002. Then use the model to approximate the per capita consumption of yogurt in 2002. Compare your results.
147. *Geometry* The area of a trapezoid with parallel bases of lengths \(b_1\) and \(b_2\), and height \(h\), is \(\frac{1}{2}(b_1 + b_2)h\). Use the Distributive Property to show that the area can also be written as \(b_1h + \frac{1}{2}(b_2 - b_1)h\).

148. *Geometry* Use both formulas given in Exercise 147 to find the area of a trapezoid with \(b_1 = 7\), \(b_2 = 12\), and \(h = 3\).

149. *Area of a Rectangle* The figure shows two adjoining rectangles. Demonstrate the Distributive Property by filling in the blanks to write the total area of the two rectangles in two ways.

\[
\boxed{\quad \text{area} \quad} = \quad \boxed{\quad \text{area} \quad} + \quad \boxed{\quad \text{area} \quad}
\]

150. *Area of a Rectangle* The figure shows two adjoining rectangles. Demonstrate the “subtraction version” of the Distributive Property by filling in the blanks to write the area of the left rectangle in two ways.

\[
\boxed{\quad \text{area} \quad} = \quad \boxed{\quad \text{area} \quad} - \quad \boxed{\quad \text{area} \quad}
\]

**Synthesis**

*True or False?* In Exercises 151 and 152, determine whether the statement is true or false. Justify your answer.

151. \((3a)b = 3(ab)\)

152. \(2x(5 - y) = 2x(5) + 2x(-y)\)

153. *Writing* Explain the difference between constants and variables in an algebraic expression.

154. Write, from memory, the properties of exponents.

155. *Writing* Explain the difference between \((2x)^3\) and \(2x^3\).

156. (a) Complete the table by evaluating \(2x - 5\).

\[
\begin{array}{c|c|c|c|c|c}
\hline
x & -1 & 0 & 1 & 2 & 3 & 4 \\
\hline
2x - 5 & & & & & & \\
\hline
\end{array}
\]

(b) From the table in part (a), determine the increase in the value of the expression for each one-unit increase in \(x\).

(c) Complete the table by using the table feature of a graphing utility to evaluate \(3x + 2\).

\[
\begin{array}{c|c|c|c|c|c}
\hline
x & -1 & 0 & 1 & 2 & 3 & 4 \\
\hline
3x + 2 & & & & & & \\
\hline
\end{array}
\]

(d) From the table in part (c), determine the increase in the value of the expression for each one-unit increase in \(x\).

(e) Use the results in parts (a) through (d) to make a conjecture about the increase in the value of the algebraic expression \(7x + 4\) for each one-unit increase in \(x\). Use the table feature of a graphing utility to confirm your result.

(f) Use the results in parts (a) through (d) to make a conjecture about the increase (or decrease) in the value of the algebraic expression \(-3x + 1\) for each one-unit increase in \(x\). Use the table feature of a graphing utility to confirm your result.

(g) In general, what does the coefficient of the \(x\)-term represent in the expression \(ax + b\)?