Factoring Polynomials with Common Factors

Now you will switch from the process of multiplying polynomials to the reverse process—factoring polynomials. This section and the next section deal only with polynomials that have integer coefficients. Remember that in Section P.3 you used the Distributive Property to multiply and remove parentheses, as follows.

\[3x(4 - 5x) = 12x - 15x^2\] \hspace{1cm} \text{Distributive Property}

In this and the next section, you will use the Distributive Property in the reverse direction to factor and create parentheses.

\[12x - 15x^2 = 3x(4 - 5x)\] \hspace{1cm} \text{Distributive Property}

Factoring an expression (by the Distributive Property) changes a sum of terms into a product of factors. Later you will see that this is an important strategy for solving equations and for simplifying algebraic expressions.

To be efficient in factoring, you need to understand the concept of the greatest common factor (or GCF). Recall from arithmetic that every integer can be factored into a product of prime numbers. The greatest common factor of two or more integers is the greatest integer that divides evenly into each integer. To find the greatest common factor of two integers or two expressions, begin by writing each as a product of prime factors. The greatest common factor is the product of the common prime factors. For instance, from the factorizations

\[18 = 2 \cdot 3 \cdot 3 \quad \text{and} \quad 42 = 2 \cdot 3 \cdot 7\]

you can see that the common prime factors of 18 and 42 are 2 and 3. So, it follows that the greatest common factor is 2 \cdot 3 or 6.

**Example 1**  
**Finding the Greatest Common Factor**

Find the greatest common factors of (a) 36, 8x, 64y and (b) 6x^5, 30x^4, 12x^3.

**Solution**

(a) From the factorizations

\[36 = 2 \cdot 2 \cdot 3 \cdot 3 = 4(9)\]

\[8x = 2 \cdot 2 \cdot 2 \cdot x = 4(2x)\]

\[64y = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot y = 4(16y)\]

you can conclude that the greatest common factor is 4.

(b) From the factorizations

\[6x^5 = 2 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot x = (6x^3)(x^2)\]

\[30x^4 = 2 \cdot 3 \cdot 5 \cdot x \cdot x \cdot x \cdot x = (6x^3)(5x)\]

\[12x^3 = 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x = (6x^3)(2)\]

you can conclude that the greatest common factor is 6x^3.

**CHECKPOINT**  
Now try Exercise 5.
Consider the three terms given in Example 1(b) as terms of the polynomial
6x^5 + 30x^4 + 12x^3.
The greatest common factor, 6x^3, of these terms is called the **greatest common monomial factor** of the polynomial. When you use the Distributive Property to remove this factor from each term of the polynomial, you are factoring out the greatest common monomial factor.

\[
6x^5 + 30x^4 + 12x^3 = 6x^3(x^2) + 6x^3(5x) + 6x^3(2) = 6x^3(x^2 + 5x + 2)
\]

If a polynomial in \(x\) (with integer coefficients) has a greatest common monomial factor of the form \(a x^n\), the statements below must be true.
1. The coefficient \(a\) of the greatest common monomial factor must be the greatest integer that divides each of the coefficients in the polynomial.
2. The variable factor \(x^n\) of the greatest common monomial factor has the lowest power of \(x\) of all terms of the polynomial.

**Example 2**  ■ Factoring out a Greatest Common Monomial Factor

Factor out the greatest common monomial factor from \(24x^3 - 32x^2\).

**Solution**

For the terms \(24x^3\) and \(32x^2\), 8 is the greatest integer factor of 24 and 32 and \(x^2\) is the highest-powered variable factor common to \(x^3\) and \(x^2\). So, the greatest common monomial factor of \(24x^3\) and \(32x^2\) is \(8x^2\). You can factor the given polynomial as follows.

\[
24x^3 - 32x^2 = (8x^2)(3x) - (8x^2)(4) = 8x^2(3x - 4)
\]

**CHECKPOINT** Now try Exercise 19.

The greatest common monomial factor of a polynomial is usually considered to have a positive coefficient. However, sometimes it is convenient to factor a negative number out of a polynomial, as shown in the next example.

**Example 3**  ■ A Negative Common Monomial Factor

Factor the polynomial \(-3x^2 + 12x - 18\) in two ways.

(a) Factor out a 3.  
(b) Factor out a \(-3\).

**Solution**

(a) By factoring out the common monomial factor of 3, you obtain

\[
-3x^2 + 12x - 18 = 3(-x^2) + 3(4x) + 3(-6) = 3(-x^2 + 4x - 6).
\]

(b) By factoring out the common monomial factor of \(-3\), you obtain

\[
-3x^2 + 12x - 18 = -3(x^2) + (-3)(-4x) + (-3)(6) = -3(x^2 - 4x + 6).
\]

**CHECKPOINT** Now try Exercise 39.

---

**Study Tip**

Whenever you are factoring a polynomial, remember that you can check your results by multiplying. That is, if you multiply the factors, you should obtain the original polynomial.
Section P.5 Factoring Polynomials

Factoring by Grouping

Some polynomials have common factors that are not simple monomials. For instance, the polynomial \( x^2(2x - 3) + 4(2x - 3) \) has the common binomial factor \( 2x - 3 \). Factoring out this common factor produces

\[ x^2(2x - 3) + 4(2x - 3) = (2x - 3)(x^2 + 4). \]

This type of factoring is part of a more general procedure called **factoring by grouping**.

**Example 4**  ■  A Common Binomial Factor

Factor the polynomial \( 5x^2(6x - 5) - 2(6x - 5) \).

**Solution**

Each of the terms of this polynomial has a binomial factor of \( 6x - 5 \). Factoring this binomial out of each term produces

\[ 5x^2(6x - 5) - 2(6x - 5) = (6x - 5)(5x^2 - 2). \]

Now try Exercise 47.

In Example 4, the given polynomial was already grouped, and so it was easy to determine the common binomial factor. In practice, you will have to do the grouping as well as the factoring. To see how this works, consider the expression \( x^3 - 3x^2 - 5x + 15 \) and try to factor it. Note first that there is no common monomial factor to take out of all four terms. But suppose you group the first two terms together and the last two terms together. Then you have

\[ x^3 - 3x^2 - 5x + 15 = (x^3 - 3x^2) - (5x - 15) \]

\[ = x^2(x - 3) - 5(x - 3) \]

\[ = (x - 3)(x^2 - 5). \]

**Example 5**  ■  Factoring by Grouping

Factor the polynomial \( x^3 - 5x^2 + x - 5 \).

**Solution**

\[ x^3 - 5x^2 + x - 5 = (x^3 - 5x^2) + (x - 5) \]

\[ = x^2(x - 5) + 1(x - 5) \]

\[ = (x - 5)(x^2 + 1). \]

Now try Exercise 53.

Note that in Example 5 the polynomial is factored by grouping the first and second terms and the third and fourth terms. You could just as easily have grouped the first and third terms and the second and fourth terms, as follows.

\[ x^3 - 5x^2 + x - 5 = (x^3 + x) - (5x^2 + 5) \]

\[ = x(x^2 + 1) - 5(x^2 + 1) \]

\[ = (x^2 + 1)(x - 5). \]
Factoring the Difference of Two Squares

Some polynomials have special forms that you should learn to recognize so that they can be factored easily. Here are some examples of forms that you should be able to recognize by the time you have completed this section.

- Difference of two squares
- Sum of two cubes
- Difference of two cubes

One of the easiest special polynomial forms to recognize and to factor is the form called a difference of two squares. This form arises from the special product in Section P.4.

To recognize perfect squares, look for coefficients that are squares of integers and for variables raised to even powers. Here are some examples.

<table>
<thead>
<tr>
<th>Original Polynomial</th>
<th>Think: Difference of Squares</th>
<th>Factored Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^2 - 9 )</td>
<td>( x^2 - 3^2 )</td>
<td>( (x + 3)(x - 3) )</td>
</tr>
<tr>
<td>( x^3 + 8 )</td>
<td>( (x + 2)(x^2 - 2x + 4) )</td>
<td>( )</td>
</tr>
<tr>
<td>( x^3 - 1 )</td>
<td>( (x - 1)(x^2 + x + 1) )</td>
<td>( )</td>
</tr>
</tbody>
</table>

**Example 6**  ■  Factoring the Difference of Two Squares

Factor (a) \( x^2 - 64 \) and (b) \( 4y^2 - 49 \).

**Solution**

(a) Because \( x^2 \) and 64 are both perfect squares, you can recognize this polynomial as the difference of two squares. So, the polynomial factors as follows.

\[
x^2 - 64 = x^2 - 8^2 = (x + 8)(x - 8)
\]

(b) Because both \( 4y^2 \) and 49 are perfect squares, you can recognize the difference of two squares. So, the polynomial factors as follows.

\[
4y^2 - 49 = (2y)^2 - 7^2 = (2y + 7)(2y - 7)
\]

**CHECKPOINT**  Now try Exercise 65.
Example 7 ■ Factoring the Difference of Two Squares

Factor the polynomial $49x^2 - 81y^2$.

**Solution**

Because $49x^2$ and $81y^2$ are both perfect squares, you can recognize this polynomial as the difference of two squares. So, the polynomial factors as follows.

$$49x^2 - 81y^2 = (7x)^2 - (9y)^2$$  
Write as difference of two squares.

$$= (7x + 9y)(7x - 9y)$$  
Factored form

Now try Exercise 71.

Remember that the rule $u^2 - v^2 = (u + v)(u - v)$ applies to polynomials or expressions in which $u$ and $v$ are themselves expressions. The next example illustrates this possibility.

Example 8 ■ Factoring the Difference of Two Squares

Factor the difference of two squares.

(a) $(x + 2)^2 - 9$  
(b) $(x - 5)^2 - 16$

**Solution**

(a) $(x + 2)^2 - 9 = (x + 2)^2 - 3^2$

$$= [(x + 2) + 3][(x + 2) - 3]$$  
Write as difference of two squares.

$$= (x + 5)(x - 1)$$  
Factored form

Simplify.

(b) $(x - 5)^2 - 16 = (x - 5)^2 - 4^2$

$$= [(x - 5) + 4][(x - 5) - 4]$$  
Write as difference of two squares.

$$= (x - 1)(x - 9)$$  
Factored form

Simplify.

To check these results, write the original polynomial in standard form. Then multiply the factored form to see that you obtain the same standard form.

Now try Exercise 79.

Factoring the Sum or Difference of Two Cubes

The last type of special factoring discussed in this section is factoring of the sum or difference of two cubes. The patterns for these two special forms are summarized below. In these patterns, pay particular attention to the signs of the terms.

### Sum or Difference of Two Cubes

Let $u$ and $v$ be real numbers, variables, or algebraic expressions. Then the expressions $u^3 + v^3$ and $u^3 - v^3$ can be factored as follows.

$$u^3 + v^3 = (u + v)(u^2 - uv + v^2)$$  
Like signs

$$u^3 - v^3 = (u - v)(u^2 + uv + v^2)$$  
Like signs

$$u^3 + v^3 = (u + v)(u^2 - uv + v^2)$$  
Unlike signs

$$u^3 - v^3 = (u - v)(u^2 + uv + v^2)$$  
Unlike signs
Example 9  ■ Factoring Sums and Differences of Cubes

Factor each polynomial.
(a) $x^3 - 125$  
(b) $8y^3 + 1$  
(c) $27x^3 - 64y^3$

**Solution**

(a) This polynomial is the difference of two cubes, because $x^3$ is the cube of $x$ and 125 is the cube of 5. So, you can factor the polynomial as follows.

$$x^3 - 125 = x^3 - 5^3$$

$$= (x - 5)(x^2 + 5x + 25)$$

(b) This polynomial is the sum of two cubes, because $8y^3$ is the cube of $2y$ and 1 is its own cube. So, you can factor the polynomial as follows.

$$8y^3 + 1 = (2y)^3 + 1^3$$

$$= (2y + 1)((2y)^2 - (2y)(1) + 1^2)$$

$$= (2y + 1)(4y^2 - 2y + 1)$$

(c) Because $27x^3$ is the cube of $3x$ and $64y^3$ is the cube of $4y$, this polynomial is the difference of two cubes. So, you can factor the polynomial as follows.

$$27x^3 - 64y^3 = (3x)^3 - (4y)^3$$

$$= (3x - 4y)((3x)^2 + (3x)(4y) + (4y)^2)$$

$$= (3x - 4y)(9x^2 + 12xy + 16y^2)$$

**CHECKPOINT**  Now try Exercise 89.

**Factoring Completely**

Sometimes the difference of two squares can be hidden by the presence of a common monomial factor. Remember that with all factoring techniques, you should first factor out any common monomial factors.

Example 10  ■ Factoring out a Common Monomial Factor First

Factor the polynomial $125x^2 - 80$ completely.

**Solution**

The polynomial $125x^2 - 80$ has a common monomial factor of 5. After factoring out this factor, the remaining polynomial is the difference of two squares.

$$125x^2 - 80 = 5(25x^2 - 16)$$

$$= 5[(5x)^2 - 4^2]$$

$$= 5(5x + 4)(5x - 4)$$

**CHECKPOINT**  Now try Exercise 95.

The polynomial in Example 10 is said to be **completely factored** because none of its factors can be factored further using integer coefficients.
Example 11  ■  Factoring out a Common Monomial Factor First

Factor the polynomial $3x^4 + 81x$ completely.

**Solution**

$$3x^4 + 81x = 3x(x^3 + 27)$$  
Factor out common monomial factor.  
$$= 3x(x^3 + 3^3)$$  
Write as sum of two cubes.  
$$= 3x(x + 3)(x^2 - 3x + 9)$$  
Factored form

To factor a polynomial completely, always check to see whether the factors obtained might themselves be factorable. For instance, after factoring the polynomial $x^4 - 16$ once as the difference of two squares

$$x^4 - 16 = (x^2)^2 - 4^2 = (x^2 + 4)(x^2 - 4)$$

you can see that the second factor is itself the difference of two squares. So, to factor the polynomial completely, you must continue factoring, as follows.

$$x^4 - 16 = (x^2 + 4)(x^2 - 4) = (x^2 + 4)(x + 2)(x - 2)$$

Example 12  ■  Factoring Completely

Factor (a) $x^4 - y^4$ and (b) $81m^4 - 1$ completely.

**Solution**

(a) Recognizing $x^4 - y^4$ as a difference of two squares, you can write

$$x^4 - y^4 = (x^2)^2 - (y^2)^2 = (x^2 + y^2)(x^2 - y^2).$$

Note that the second factor $(x^2 - y^2)$ is itself a difference of two squares and you therefore obtain

$$x^4 - y^4 = (x^2 + y^2)(x^2 - y^2) = (x^2 + y^2)(x + y)(x - y).$$

(b) Recognizing $81m^4 - 1$ as a difference of two squares, you can write

$$81m^4 - 1 = (9m^2)^2 - (1)^2 = (9m^2 + 1)(9m^2 - 1).$$

Note that the second factor $(9m^2 - 1)$ is itself a difference of two squares and you therefore obtain

$$81m^4 - 1 = (9m^2 + 1)(9m^2 - 1) = (9m^2 + 1)(3m + 1)(3m - 1).$$

Example 13  ■  Using Factoring to Find the Length of a Rectangle

The area of a rectangle of width $5x$ is given by $15x^2 + 40x$, as shown in Figure P.15. Factor this expression to determine the length of the rectangle.

**Solution**

Factoring out the common monomial factor of $5x$ from the area polynomial $15x^2 + 40x$ results in the polynomial $3x + 8$. So, the length of the rectangle is $3x + 8$. To check this result, multiply $5x(3x + 8)$ to see that you obtain the area polynomial $15x^2 + 40x$.

**CHECKPOINT** Now try Exercise 113.
Writing About Mathematics

Factoring Higher-Degree Polynomials

Have each person in your group create a cubic polynomial by multiplying three polynomials of the form 
\[(x - a)(x + a)(x + b).

Then write the result in standard form on a piece of paper and describe the steps used to create the polynomials and the steps used to write the polynomial in standard form. Give it to another person in the group to factor. When each person has factored his or her polynomial, compare the results with the original factors used to create the polynomials.

P.5  Exercises

VOCABULARY CHECK: Fill in the blanks.
1. The ________ of a group of expressions is the product of the common prime factors.
2. In the expression $5x^2 + 10x$, $5x$ is the ________.
3. The process of writing a polynomial as a product is called ________.
4. A polynomial of the form $u^2 - v^2$ is called a ________ of ________.
5. A polynomial of the form $u^3 + v^3$ is called a ________ of ________.
6. A polynomial is considered ________ if each of its factors cannot be factored further using integer coefficients.

In Exercises 1–12, find the greatest common factor of the expressions.

1. 48, 90
2. 36, 150, 100
3. $3x^2, 12x$
4. $27x^4, 18x^3$
5. $30z^2, 12z^3$
6. $45y, 150y^3$
7. $28ab^2, 14a^2b^3, 42a^2b^5$
8. $16x^2y, 84xy^2, 36x^2y^2$
9. $42(x + 8)^2, 63(x + 8)^3$
10. $66(3 - y), 44(3 - y)^2$
11. $4x(1 - z)^2, x^2(1 - z)^3$
12. $2(x + 5), 8(x + 5)$

In Exercises 13–34, factor out the greatest common monomial factor. (Some of the polynomials have no common monomial factor other than 1 or −1.)

13. $8z - 8$
14. $5x + 5$
15. $24x^2 - 18$
16. $14z^2 + 21$
17. $2x^2 + x$
18. $-a^3 - 4a$
19. $21u^2 - 14u$
20. $36y^4 + 24y^2$
21. $11u^2 + 9$
22. $16x^2 - 3y^3$
23. $3x^2y^2 - 15y$
24. $4uv + 6u^2v^2$
25. $28x^2 + 16x - 8$
26. $9 - 27y - 15y^2$
27. $14x^4 + 21x^3 + 9x^2$
28. $25x^6 - 15z^4 + 35z^3$
29. $17x^3y^3 - xy^2 + 34y^2$
30. $8y^3 + 6y^2z - 2y^2z^2$
31. $9x^3y + 6xy^2$
32. $24y^4z^2 + 12y^2z^2$
33. $3x^3y^3 - 2x^2y^2 + 5xy$
34. $18a^2b^4 + 24a^2b^2 - 12a^2b$

In Exercises 35–42, factor a negative real number from the polynomial and then write the polynomial factor in standard form.

35. $10 - 5x$
36. $32 - 4x$
37. $7 - 14x$
38. $15 - 5x$
39. $8 + 4x - 2x^2$
40. $12x - 6x^2 - 18$
41. $2t - 15 - 4t^2$
42. $16 + 32s^3 - 5s^4$

In Exercises 43–46, fill in the missing factor.

43. $\frac{1}{2}x + \frac{5}{6} = \frac{1}{3}(\fbox{\phantom{000}})$
44. $\frac{1}{4}x - \frac{5}{6} = \frac{1}{12}(\fbox{\phantom{000}})$
45. $\frac{5}{8}x + \frac{5}{7}y = \frac{5}{13}(\fbox{\phantom{000}})$
46. $\frac{7}{12}u - \frac{21}{8}v = \frac{7}{24}(\fbox{\phantom{000}})$
In Exercises 47–52, factor the polynomial by factoring out the common binomial factor.

47. \(2y(y - 3) + 5(y - 3)\)
48. \(7(x + 9) - 6(x + 9)\)
49. \(5(r^2 + 1) - 4(r^2 + 1)\)
50. \(3(a^2 - 3) + 10(a^2 - 3)\)
51. \(a(a + 6) + (2a - 5)(a + 6)\)
52. \((5x + y)(x - y) - 5x(x - y)\)

In Exercises 53–64, factor the polynomial by grouping.

53. \(y^3 - 6y^2 + 2y - 12\)
54. \(4x^3 - 2x^2 + 6x - 3\)
55. \(14z^3 + 21z^2 - 6z - 9\)
56. \(7r^3 + 5r^2 - 35r - 25\)
57. \(x^3 + 2x^2 + x + 2\)
58. \(r^3 - 11r^2 + r - 11\)
59. \(a^3 - 4a^2 + 2a - 8\)
60. \(3x^3 + 6x^2 + 5x + 10\)
61. \(z^4 + 3z^3 - 2z - 6\)
62. \(4u^3 - 2u^2 - 6u + 3\)
63. \(cd + 3c - 3d - 9\)
64. \(u^2 + uv - 4u - 4v\)

In Exercises 65–82, factor the difference of two squares.

65. \(x^2 - 25\)
66. \(y^2 - 144\)
67. \(100 - 9y^2\)
68. \(625 - 49x^2\)
69. \(121 - y^2\)
70. \(81 - z^2\)
71. \(16y^2 - 9z^2\)
72. \(9z^2 - 25w^2\)
73. \(x^2 - 4y^2\)
74. \(81a^2 - b^6\)
75. \(a^8 - b^8\)
76. \(y^{10} - 64\)
77. \(a^2b^2 - 16\)
78. \(u^2v^2 - 25\)
79. \((a + 4)^2 - 49\)
80. \((x - 3)^2 - 4\)
81. \(81 - (z + 5)^2\)
82. \(100 - (y - 3)^2\)

In Exercises 83–94, factor the sum or difference of two cubes.

83. \(x^3 - 8\)
84. \(r^3 - 27\)
85. \(y^3 + 125\)
86. \(z^3 + 216\)
87. \(8r^3 - 27\)
88. \(64a^3 - 1\)
89. \(27s^3 + 64\)
90. \(125x^3 + 343\)
91. \(8x^3 - y^3\)
92. \(a^3 - 216b^3\)
93. \(y^3 + 64z^3\)
94. \(z^3 + 125w^3\)

In Exercises 95–106, factor the polynomial completely.

95. \(50x^2 - 8\)
96. \(3z^2 - 192\)
97. \(x^3 - 144x\)
98. \(a^3 - 16a\)
99. \(b^4 - 16\)
100. \(a^4 - 625\)
101. \(y^4 - 81x^4\)
102. \(u^4 - 256u^4\)
103. \(2x^3 - 54\)
104. \(5y^3 - 625\)
105. \(3a^3 + 192\)
106. \(7b^3 + 56\)

In Exercises 107 and 108, factor the expression. (Assume \(n > 0\).)

107. \(4x^{2n} - 25\)
108. \(81 - 16y^{4n}\)

In Exercises 109 and 110, show all the different groupings that can be used to factor the polynomial completely. Carry out the various factorizations to show that they yield the same result.

109. \(3x^3 + 4x^2 - 3x - 4\)
110. \(6x^3 - 8x^2 + 9x - 12\)

111. **Simple Interest** The total amount of money from a principal of \(P\) invested at \(r\%\) simple interest for \(t\) years is given by \(P + Prt\). Factor this expression.

112. **Revenue** The revenue from selling \(x\) units of a product at a price of \(p\) dollars per unit is given by \(xp\). For a particular commodity, the revenue \(R\) is \(R = 800x - 0.25x^2\). Factor the expression for the revenue and determine an expression for the price in terms of \(x\).

113. **Geometry** A rectangle has area and width as indicated in the figure. Factor the expression for area to find the length of the rectangle.

![Figure for 113](image)

\[A = w^2 + 8w\]

\[A = 27t^2 - 18t\]

\[9t\]

114. **Geometry** A rectangle has area and length as indicated in the figure. Find the width of the rectangle.

115. **Geometry** The area of a rectangle of width \(w\) is given by \(32w - w^2\). Factor this expression to determine the length of the rectangle. Draw a diagram to illustrate your answer.

116. **Geometry** The area of a rectangle of length \(l\) is given by \(45l - l^2\). Factor this expression to determine the width of the rectangle. Draw a diagram to illustrate your answer.
117. **Geometry** The area of a rectangle of width $2x^2$ is given by $10x^3 + 4x^2$. Factor this expression to determine the length of the rectangle. Draw a diagram to illustrate your answer.

118. **Geometry** The area of a rectangle of length $y + 5$ is given by $3y^2 + 15y$. Factor this expression to determine the width of the rectangle. Draw a diagram to illustrate your answer.

119. **Geometry** The surface area $S$ of a right circular cylinder is $S = 2\pi r^2 + 2\pi rh$ (see figure). Factor the expression for the surface area.

[Diagram of a cylinder with radius $r$ and height $h$.]

120. **Manufacturing** A washer on the drive train of a car has an inside radius of $r$ centimeters and an outside radius of $R$ centimeters (see figure). Find the area of one of the flat surfaces of the washer and write the area in factored form.

[Diagram of a washer with inner radius $r$ and outer radius $R$.]

121. **Chemistry** The rate of change of an autocatalytic chemical reaction is $kQx - kx^2$, where $Q$ is the amount of the original substance, $x$ is the amount of substance formed, and $k$ is a constant of proportionality. Factor the expression for this rate of change.

**Synthesis**

122. **Writing** Explain the relationship between using the Distributive Property to multiply polynomials and using the Distributive Property to factor algebraic expressions.

123. **Writing** Explain what is meant when it is said that a polynomial is in factored form.

124. **Writing** Explain how the word *factor* can be used as a noun and as a verb.

**Writing** In Exercises 125 and 126, explain how the product could be obtained mentally using the sample as a model.

$48 \cdot 52 = (50 - 2)(50 + 2) = 50^2 - 2^2 = 2496$

125. $79 \cdot 81 = 6399$

126. $18 \cdot 22 = 396$

127. **Exploration** The cube shown in the figure is formed by four solids: I, II, III, and IV.

(a) Explain how you could determine each expression for volume.

<table>
<thead>
<tr>
<th>Volume</th>
<th>$a^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entire cube</td>
<td>$a^3$</td>
</tr>
<tr>
<td>Solid I</td>
<td>$(a - b)a^2$</td>
</tr>
<tr>
<td>Solid II</td>
<td>$(a - b)ab$</td>
</tr>
<tr>
<td>Solid III</td>
<td>$(a - b)b^2$</td>
</tr>
<tr>
<td>Solid IV</td>
<td>$b^3$</td>
</tr>
</tbody>
</table>

(b) Add the volumes of solids I, II, and III. Factor the result to show that the total volume can be written as $(a - b)(a^2 + ab + b^2)$.

(c) Explain why the total volume of solids I, II, and III can also be written as $a^3 - b^3$. Then explain how the figure can be used as a geometric model for the difference of two cubes factoring pattern.

[Diagram of a cube with a layer removed at each corner to reveal the difference of two cubes.]