Example 5-7: Compression of an ideal gas

An ideal gas has $C_P^* = (7/2)R$. We are designing a steady state process to compress the gas from an initial state of $P = 1$ bar and $T = 300$ K to a final state of $P = 64$ bar and $T = 135$ K. Find the work added and heat removed per mole of gas for each of the following processes (summarized in Figure 5-16).

The gas is compressed to $P = 64$ bar in an adiabatic reversible compressor, and then enters a heat exchanger in which it is cooled to $T = 135$ K.

A) The gas is cooled to $T = 135$ K in a heat exchanger, is then compressed to $P = 64$ bar in an adiabatic reversible compressor, and then, in a second heat exchanger, is cooled to $T = 135$ K.

B) The gas is compressed in two separate adiabatic reversible compressors; the first has inlet $P = 1$ bar and outlet $P = 8$ bar, and the second compresses the gas from $P = 8$ bar to $P = 64$ bar. Each compressor is preceded by a heat exchanger that cools the gas to $T = 135$ K, and a third heat exchanger cools the final product to $T = 135$ K.

It isn’t realistic that a gas would behave like an ideal gas at $P = 64$ bar, but this example is primarily intended to illustrate the differences between three approaches to designing the compression process.
In Steps 1-3 the system is the compressor. We cannot find $Q$ in the heat exchanger without knowing the temperature of the stream entering it, which we will learn by modeling the compressor first.

Figure 5-16 Three methods of compressing and cooling an ideal gas to $P=64$ bar and $T=135$ K.
**Solution: A)**

**Step 1- Apply energy and entropy balances to compressor**

This is an adiabatic, steady state compressor, so the energy balance we first saw in section 3.6.3 applies. For this problem it is most convenient to write it on a molar basis:

\[
\frac{\dot{W}_s}{n} = \dot{H}_{\text{out}} - \dot{H}_{\text{in}}
\]  

(5.80)

The entropy balance for a steady state, adiabatic, reversible compressor, established in Example 4-8, is:

\[
S_{\text{out}} - S_{\text{in}} = 0
\]

(5.81)

**Step 2- Relate entropy to temperature and pressure**

The gas in this problem is an ideal gas with constant heat capacity, which is the exact situation for which Equation 4.59 was derived. Applying that equation to this compressor:

\[
S_{\text{out}} - S_{\text{in}} = \frac{C_p}{R} \ln \left( \frac{T_{\text{out}}}{T_{\text{in}}} \right) + R \ln \left( \frac{P_{\text{in}}}{P_{\text{out}}} \right)
\]

(5.82)

\[
0 = \left( \frac{7}{2} R \right) \ln \left( \frac{T_{\text{out}}}{300 \text{ K}} \right) + R \ln \left( \frac{1 \text{ bar}}{64 \text{ bar}} \right)
\]

(5.83)

\[
T_{\text{out}} = 984 \text{ K}
\]
Step 3 - Relate enthalpy to temperature

**Margin Note:** The compressors in this problem are assumed to be reversible because this is a simplified example intended to illustrate specific physical phenomena. The compressors in Example 5-8 and Example 5-9 are modeled more realistically.

For any ideal gas, \( d\overline{H} = C_p \* dT \). Because \( C_p \) is constant, the change in enthalpy in equation 5.80 becomes:

\[
\frac{\dot{W}_{s,A}}{\dot{n}} = \overline{H}_{out} - \overline{H}_{in} = C_p(T_{out} - T_{in}) \tag{5.84}
\]

\[
\frac{\dot{W}_{s,A}}{\dot{n}} = \frac{7}{2}(8.314 \frac{J}{mol \cdot K})(984 K - 300 K)
\]

\[
\frac{\dot{W}_{s,A}}{\dot{n}} = 19,915 \frac{J}{mol}
\]

Step 4 - Apply energy balance to heat exchanger

In steps 4-5, the system is the heat exchanger that follows the compressor.
The energy balance for one side of a heat exchanger, as derived in section 3.6.4, is:

\[
\dot{Q} = H_{\text{out}} - H_{\text{in}} \quad (5.85)
\]

**Step 5- Relate enthalpy to temperature**

As in Step 3, the change in enthalpy can be related to temperature:

\[
\frac{\dot{Q}}{\dot{n}} = C_p(T_{\text{out}} - T_{\text{in}}) \quad (5.86)
\]

The temperature entering the heat exchanger is the same as the temperature leaving the compressor, and the heat exchanger is designed to reduce the temperature to 135 K.

\[
\frac{\dot{Q}_A}{\dot{n}} = \left(\frac{7}{2}\right) \left(8.314 \frac{J}{\text{mol} \cdot \text{K}}\right)(135 \text{ K} - 984 \text{ K}) = -24,717 \frac{J}{\text{mol}} \quad (5.87)
\]

*B) Now we turn to the case with one compressor and two heat exchangers. Since we know exactly what is entering and leaving the first heat exchanger, that is a simple place to start.*

**Step 6- Model heat exchanger before compressor**

The ideal gas is cooled from 300 K to 135 K before entering the compressor. Applying equation 5.86 to this cooling process:

\[
\frac{\dot{Q}_{B1}}{\dot{n}} = C_p(T_{\text{out}} - T_{\text{in}}) = \left(\frac{7}{2}\right) \left(8.314 \frac{J}{\text{mol} \cdot \text{K}}\right)(135 \text{ K} - 300 \text{ K}) \quad (5.88)
\]

\[
= -4801 \frac{J}{\text{mol}}
\]
Step 7- Model compressor for part B

The compressor in variant B can be modeled in the same way as the one in part A, with the only difference being that $T_{in}$ is now 135 K instead of 300 K. Applying equation 5.82:

$$0 = \left(\frac{T}{2} R\right) \ln \left(\frac{T_{out}}{135 \text{ K}}\right) + R \times \ln \left(\frac{1 \text{ bar}}{64 \text{ bar}}\right)$$

$$T_{out} = 443 \text{ K}$$

And applying equation 5.84 to the new inlet and outlet temperature:

$$\dot{W}_{A,B} = \frac{7}{2} \left(8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}}\right) (443 \text{ K} - 135 \text{ K}) = 8962 \frac{\text{J}}{\text{mol}}$$  \hspace{1cm} (5.90)

Step 8- Model heat exchanger that follows compressor

Margin Note: Pitfall Prevention: $W$ in step 7 and $Q$ in step 8 are equal in magnitude and opposite in sign because enthalpy is only a function of temperature for an ideal gas, and the temperature changes in steps 7 and 8 are equal in magnitude. For a real gas, $W$ and $Q$ would not be equal in magnitude, since the pressure change occurring in the compressor would also affect enthalpy.

Applying equation 5.86 to the second heat exchanger gives:

$$\dot{Q}_{B2} = \frac{7}{2} \left(8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}}\right) (135 \text{ K} - 443 \text{ K}) = -8962 \frac{\text{J}}{\text{mol}}$$  \hspace{1cm} (5.91)

Compare parts A and B

The results reveal that far less work was required to compress the colder ($T = 135$ K) gas than the warmer gas ($T = 300$ K). Physically, the result can be understood because low-temperature
gases are denser than high-temperature: work is related to the change in volume, and is therefore smaller when the initial and final volumes are both made smaller. Mathematically, the outcome can be understood by examination of equation 5.84. In both processes, \( T_{\text{out}}/T_{\text{in}} \) is identical and approximately equal to 3.3, but when the values of \( T_{\text{in}} \) and \( T_{\text{out}} \) are decreased in magnitude, the difference \( T_{\text{out}} - T_{\text{in}} \) also decreases.

Work is further decreased when the compression is accomplished in stages, as shown in part C.

**Part C: Apply equations 5.84 and 5.86 to two compressors and three heat exchangers.**

Equation 5.82 reveals that the work required in a compressor is primarily a function of the compression ratio \( P_{\text{out}}/P_{\text{in}} \), rather than the absolute values of \( P_{\text{out}} \) and \( P_{\text{in}} \). Consequently, staged compressors are normally designed with identical compression ratios: here, 8/1 and 64/8.

In this system, the first compressor has an inlet temperature of 135 K. The first compressor has an inlet pressure of 1 bar and an outlet pressure of 8 bar.

The outlet temperature can be determined from another application of Equation 5.82:

\[
S_{\text{out}} - S_{\text{in}} = 0 = \frac{C_P}{R} \ln \left( \frac{T_{\text{out}}}{T_{\text{in}}} \right) + R \ln \left( \frac{P_{\text{in}}}{P_{\text{out}}} \right)
\]

\[
0 = \left( \frac{7}{2} \right) R \ln \left( \frac{T_{\text{out}}}{135 \text{ K}} \right) + R \ln \left( \frac{1 \text{ bar}}{8 \text{ bar}} \right)
\]

\[ T_{\text{out}} = 245 \text{ K} \]
The second compressor also has an inlet temperature of 135 K. While the inlet and outlet pressures are higher, their ratio is the same as in the first compressor, so the outlet temperature is the same:

\[
S_{\text{out}} - S_{\text{in}} = 0 = C_p \ln \left( \frac{T_{\text{out}}}{T_{\text{in}}} \right) + R \ln \left( \frac{P_{\text{in}}}{P_{\text{out}}} \right)
\]

\[
0 = \left( \frac{7}{2} R \right) \ln \left( \frac{135 \text{ K}}{135 \text{ K}} \right) + R \ln \left( \frac{8 \text{ bar}}{64 \text{ bar}} \right)
\]

\[
T_{\text{out}} = 244.6 \text{ K}
\]

The energy balance for each compressor is again identical to equation 5.84:

\[
\frac{W_s}{\dot{n}} = H_{\text{out}} - H_{\text{in}} = C_p (T_{\text{out}} - T_{\text{in}})
\]

For an ideal gas, enthalpy is not a function of pressure. Since both compressors have the same inlet and outlet temperature they have the same rate of work added:

\[
\frac{W_{s,C}}{\dot{n}} = \frac{7}{2} \left( 8.314 \frac{1}{\text{mol} \cdot \text{K}} \right) (244.6 \text{ K} - 135 \text{ K}) = 3188 \frac{\text{J}}{\text{mol}}
\]

The energy balance for each heat exchanger is again identical to equation 5.86:

\[
\frac{\dot{Q}}{\dot{n}} = C_p (T_{\text{out}} - T_{\text{in}})
\]
The first heat exchanger has an inlet temperature of 300 K. The heat exchangers are all designed to provide an outlet temperature of 135 K.

\[
\frac{\dot{Q}_{c,1}}{\dot{n}} = C_P(T_{out} - T_{in})
\]

\[
\frac{\dot{Q}_{c,1}}{\dot{n}} = \left(\frac{7}{2}\right) \left(8.314 \frac{J}{mol \cdot K}\right) (135 K - 300 K) = -4801 \frac{J}{mol}
\]

The other two heat exchangers receive the gases leaving the compressors, so they each have an inlet temperature of 244.6 K:

\[
\frac{\dot{Q}_{c,2}}{\dot{n}} = \frac{\dot{Q}_{c,3}}{\dot{n}} = \left(\frac{7}{2}\right) \left(8.314 \frac{J}{mol \cdot K}\right) (135 K - 244.6 K) = -3188 \frac{J}{mol}
\]

The results for parts A, B and C are compared in Table 5-4:

**Table 5-4: Comparison of three different approaches to compression.**

<table>
<thead>
<tr>
<th>PART</th>
<th>SHAFT WORK REQUIRED (J/mol)</th>
<th>HEAT REMOVAL REQUIRED (J/mol)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>19,915</td>
<td>-24,717</td>
</tr>
<tr>
<td>B</td>
<td>8962</td>
<td>-4801 - 8962 = -13,763</td>
</tr>
<tr>
<td>C</td>
<td>3188 + 3188 = 6376</td>
<td>-4801 - 3188 - 3188 = -11,177</td>
</tr>
</tbody>
</table>