26

Product Differentiation and Innovation in Markets

Solutions for *Microeconomics: An Intuitive Approach with Calculus (International Ed.)*

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- Each end-of-chapter exercise begins on a new page. This is to facilitate maximum flexibility for instructors who may wish to share answers to some but not all exercises with their students.
- If you are assigning only the A-parts of exercises in *Microeconomics: An Intuitive Approach with Calculus*, you may wish to instead use the solution set created for the companion book *Microeconomics: An Intuitive Approach*.
- *Solutions to Within-Chapter Exercises are provided in the student Study Guide.*
26.1 We introduced the topic of differentiated products in a simple 2-firm Bertrand price setting model in which each firm's demand increases with the price of the other firm's output. The specific context we investigated was that of imperfect substitutes.

A: Assume throughout that demand for each firm's good is positive at \( p = MC \) even if the other firm sets its price to 0. Suppose further that firms face constant MC and no fixed costs.

(a) Suppose that instead of substitutes, the goods produced by the two firms are complements — i.e. suppose that an increase in firm j's price causes a decrease rather than an increase in the demand for firm i's good. How would Graph 26.3 change assuming both firms end up producing in equilibrium?

Answer: This is illustrated in panel (a) of Graph 26.1. The complementarity of the goods now means that, as \( p_2 \) increases, the "residual" demand for firm 1's good decreases (or shifts inward.) This implies that firm 1 will set a lower price as a result of \( p_2 \) increasing. (You can convince yourself of this by drawing out two parallel demand curves, drawing the corresponding MR curves, and including a constant MC. Then see where the monopolist's solution for \( p^M \) falls in the two cases). As a result, best response functions slope down rather than up — leading to the Nash equilibrium indicated by A in the graph.

Graph 26.1: Price competition under Different Assumptions

(b) What would the in-between case look like in this graph — i.e. what would the best response functions look like if the price of firm j's product had no influence on the demand for firm i's product?

Answer: In this case, the two firms are simply unrelated monopolies — and both will simply solve the monopoly problem — thus setting price at \( p^M \) regardless of what price the other firm sets. The best response functions are then as indicated in panel (b) of Graph 26.1 — with both firms ending up at B where both set their respective monopoly prices.

(c) Suppose our three cases — the case of substitutes (covered in the text), of complements (covered in (a)) and of the in-between case (covered in (b)) — share the following feature in common: When \( p_j = 0 \), it is a best response for firm i to set \( p = \overline{p} > MC \). How does \( \overline{p} \) relate to what we would have called the monopoly price in Chapter 23?

Answer: As already discussed in the answer to the previous part, \( \overline{p} \) is equal to the monopoly price that each firm sets assuming the other operates in an entirely unrelated market.

(d) Compare the equilibrium price (and output) levels in the three cases assuming both firms produce in each case.

Answer: This is done in panel (c) of Graph 26.1 where A and B are the equilibria from panels (a) and (b) while C is the equilibrium for the case where the two firms produce imperfect
Consider identical firms 1 and 2, and suppose that the demand for firm i’s output is given by
\[ x_i(p_i, p_j) = A - ap_i - \beta p_j. \]
Assume marginal cost is a constant c and there are no fixed costs.

(a) What range of values correspond to goods \( x_i \) and \( x_j \) being substitutes, complements and in-between goods as defined in part A of the exercise.

**Answer:** If \( \beta > 0 \), then an increase in the price \( p_j \) causes a decrease in demand for good \( x_i \) — which implies the two goods are complements. If \( \beta < 0 \), an increase in the price \( p_j \) causes an increase in demand for \( x_i \) — thus the two goods are substitutes. When \( \beta = 0 \), a change in \( p_j \) has no effect on \( x_i \) — implying that we are in the in-between case.

(b) Derive the best response functions. What are the intercepts and slopes?

**Answer:** Following the steps in the text (where an almost identical demand equation was used — except that \( \beta p_j \) entered positively), we get
\[ p_i(p_j) = \frac{A + ac - \beta p_j}{2a} \quad \text{and} \quad p_j(p_i) = \frac{A + ac - \beta p_i}{2a}. \] (26.1)

(c) Are the slopes of the best response functions positive or negative? What does your answer depend on?

**Answer:** The slope of \( i \)'s best response function to \( j \) is \(-\beta/(2a)\) — which is positive when \( \beta < 0 \) (and the two goods are complements) and positive when \( \beta > 0 \) (when the two goods are substitutes.)

(d) What is the equilibrium price in terms of \( A \), \( a \), \( \beta \) and \( c \). Confirm your answer to A(d).

**Answer:** Following the same steps as in the text — i.e. substituting \( p_j(p_i) \) into \( p_i(p_j) \) and solving for \( p_i \), we get the equilibrium price
\[ p^* = \frac{A + ac}{2a + \beta}. \] (26.2)

at which the two firms are best responding to one another. Letting \( p^*_n \) denote the equilibrium price when the two goods are substitutes (i.e. \( \beta < 0 \)), \( p^*_0 \) the price when they are neither substitutes nor complements (i.e. \( \beta = 0 \)) and \( p^*_c \) the price when they are complements (i.e. \( \beta > 0 \)), we then get
\[ p^*_c < p^*_n < p^*_0. \] (26.3)

as intuitively derives in part A. (Note that this holds so long as \( a \) is greater than the absolute value of \( \beta \) when \( \beta < 0 \) — which we explained in the text is a natural assumption to make for a model like this. Note also that \( p^*_c \) is equal to what we get for the monopoly price when we have a stand-alone monopoly — which we have two of when \( \beta = 0 \)).

(e) Under what conditions will only one firm produce when the two goods are relatively complementary?

**Answer:** Both firms will produce if \( p^* \geq MC \); i.e. if
\[ p^* = \frac{A + ac}{2a + \beta} \geq c. \] (26.4)
Solving this for $c$, we get $c = A/(\alpha + \beta)$. Thus, as long as marginal costs are not greater than $A/(\alpha + \beta)$, both firms produce in equilibrium.
In Section B of the text, we developed a model of tastes for diversified goods — and then applied a particular functional form for such tastes to derive results, some of which we suggested hold for more general cases.

B: We first introduced a general utility function representing such tastes in equation (26.33) before working with a version that embeds the sub-utility for y goods into a Cobb-Douglas functional form in equation (26.34). Consider now the more general version from equation (26.33).

(a) Begin by substituting the budget constraint into the utility function for the x term (as we did in the Cobb-Douglas case in the text).

Answer: This would give us

\[ u = u \left( \left[ 1 - \sum_{i=1}^{N} p_i y_i \right], \left[ \sum_{i=1}^{N} y_i^p \right]^{1/p} \right) \]  

(26.5)

(b) Derive the first order condition that differentiates utility with respect to \( p_i \).

Answer: This first order condition gives us

\[ \frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} \left( \sum_{i=1}^{N} y_i^p \right)^{-1/\rho} \frac{1}{p_i} \]  

(26.6)

where \( \frac{\partial u}{\partial x} \) is the derivative of \( u \) with respect to the first argument in the utility function and \( \frac{\partial u}{\partial y} \) is the derivative with respect to the second argument. Letting these be denoted \( u_x \) and \( u_y \), we can solve for \( y_i \) as

\[ y_i = \left( \frac{u_x \left[ \sum_{i=1}^{N} y_i^p \right]^{1/\rho} \left( \sum_{i=1}^{N} y_i^p \right)^{-1/\rho}}{u_y \left( \sum_{i=1}^{N} y_i^p \right)} \right)^{1/(1+\rho)} p_i^{-1/(1+\rho)}. \]  

(26.7)

(c) Assume that the number of firms is sufficiently large such that terms in which \( y_i \) plays only a small role can be approximated as constant. Then use your first order condition from (b) to derive an approximate demand function that is just a function of \( p_i \) and a constant. What is the price elasticity of demand of this (approximate) demand function?

Answer: The bracketed term in equation (26.7) is not appreciably affected by \( y_i \) if the number of firms is large. Thus, we can define

\[ \beta = \left( \frac{u_x \left[ \sum_{i=1}^{N} y_i^p \right]^{1/\rho} \left( \sum_{i=1}^{N} y_i^p \right)^{-1/\rho}}{u_y \left( \sum_{i=1}^{N} y_i^p \right)} \right)^{1/(1+\rho)} p_i^{-1/(1+\rho)}. \]  

(26.8)

and express demand for \( y_i \) as

\[ y_i(p_i) = \beta p_i^{-1/(1+\rho)}. \]  

(26.9)

Applying our formula for price elasticity of demand, we then get

\[ \frac{dy_i}{dp_i} p_i y_i(p_i) = \left( \frac{-\beta}{\rho+1} \right) p_i^{-(2+\rho)/(\rho+1)} \left( \frac{p_i}{\beta p_i^{-1/(\rho+1)}} \right) \]  

\[ = \left( \frac{-\beta}{(\rho+1)p_i^{(2+\rho)/(\rho+1)}} \right) \left( \frac{p_i^{(2+\rho)/(\rho+1)}}{\beta} \right) \]  

\[ = \frac{-1}{\rho+1}. \]  

(26.10)

(d) Set up firm i’s profit maximization problem given the demand function you have derived. Then solve for the price \( p_i \) that the firm will charge.

Answer: The profit maximization problem is
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\[
\max_{p_i} \pi_i = (p_i - c)y_i(p_i) = (p_i - c)\beta p_i^{-1/(\rho+1)}.
\]  

(26.11)

Taking first order conditions by setting the partial derivative of \( \pi_i \) (with respect to \( p_i \)) to zero, we can then solve for \( p_i \) charged by firm \( i \) for output \( y_i \) as

\[
p_i = -\frac{c}{\rho}.
\]  

(26.12)

(e) True or False: The equilibrium price \( p^* = -c/\rho \) we derived in the text for the Cobb-Douglas case does not depend on the Cobb-Douglas specification.

**Answer:** This is true. In the text, we derived the same price as the one we have just derived — and thus this equilibrium price in the model does not depend on the precise specification of utility used in the text.

(f) Recalling our Chapter 15 discussion of treating groups of consumers as if they behaved like a “representative consumer”, what form for the utility function might you assume if you were concerned that the Cobb-Douglas version we used in the text might technically not satisfy the conditions for a representative consumer? Would the implied equilibrium price differ from the Cobb-Douglas case?

**Answer:** We would want to assume a utility function that does not give rise to income effects in the market we are analyzing — which would mean a quasilinear specification of the form

\[
u(x, v(y_1, y_2, ..., y_N)) = x + \left( \sum_{i=1}^{N} y_i^\rho \right)^{-1/\rho}.
\]  

(26.13)

Since this is just an example of the more general utility function we have worked with in this exercise, the equilibrium price would still be

\[
p^* = -\frac{c}{\rho}.
\]  

(26.14)
26.3 Everyday Application: Cities and Land Values. Some of the models that we introduced in this chapter are employed in modeling the pattern of land and housing values in an urban area.

A: One way to think about city centers is as places that people need to come to in order to work and shop.

(a) Consider the Hotelling line \([0,1]\) that we used as a product characteristics space. Suppose instead that this line represents physical distance, with a city located at 0 and another city located at 1. Think of households as locating along this line — with a household that locates at \(n \in [0,1]\) having to commute to one of the two cities unless \(n = 0\) or \(n = 1\). What does this imply for the distribution of consumer “ideal points”?

**Answer:** The ideal points for consumers that value city access are then the endpoints of the Hotelling line — i.e. 0 and 1. Locations in the interior of the line become increasingly costly due to the commuting costs to the cities.

(b) If land along the Hotelling line were equally priced, where would everyone wish to locate? If the city at 0 is larger than the city at 1 — and if bigger cities offer greater job and shopping opportunities, how would this affect your answer?

**Answer:** With equally priced land, everyone would want to live at the endpoints of the Hotelling line. If the city at 0 is larger than the city at 1, everyone’s ideal point would be at 0 and everyone would want to live at 0.

(c) What do your answers imply for the distribution of land values along the Hotelling line if land at each location is scarce and only one household can locate at each point on the line?

**Answer:** In equilibrium, consumers will have to be indifferent between all the locations on the Hotelling line. Thus, land prices would be highest at 0 and 1 and would decline with distance as we move away from 0 and 1. If the two cities are equally sized and every location contains a household in equilibrium, the lowest land price would be at 0.5.

(d) Suppose instead that more than one household can potentially locate at each point on the line — but if multiple households locate at a point, each consumes less land. (For instance, 100 families might share a high rise apartment building.) Suppose this results in unoccupied farm land toward the middle of the Hotelling line. How would you expect population density to vary along the line?

**Answer:** We would expect population density to increase as one moves closer to the endpoints of the line.

(e) In recent decades, a new phenomenon called “edge cities” has emerged — with smaller cities forming in the vicinity of larger cities — and land values adjusting accordingly. How would the distribution of land values change as edge cities appear on the Hotelling line?

**Answer:** Land prices would now have multiple peaks along the Hotelling line — with the largest peaks at the endpoints and smaller peaks at the edge cities.

(f) What do you think will happen to the distribution of land values along the Hotelling line if commuting costs fall? What would happen to population density along the line?

**Answer:** As commuting costs fall, the cost to consumers of living away from their ideal point would decline. As a result, the decline in price with distance from the cities will have to be less steep in order to insure that households are indifferent between the locations on the Hotelling line.

(g) Could you similarly see how land values are distributed in our “circle” model if cities are located at different points on the circle?

**Answer:** The same logic applies — with land values peaking at city centers and declining with distance away from cities.

B: Now consider the model of tastes for diversified product markets in Section 26B.4.

(a) Can you use the intuitions from this model to explain why larger cities on the Hotelling line (or the circle) in part A of the exercise will have higher land values?

**Answer:** This specification of utility — with tastes for diversity of product offerings — gives one natural way of representing tastes that value access to cities where product offerings are more diversified (as, for instance, the number of available restaurants to choose).
(b) Consider two cities in the same general area (but sufficiently far apart that consumers would rarely commute from one to the other). Suppose the model used to derive Table 26.2 in the text was the appropriate model for representing consumer tastes in this state, and suppose that city A had 100 restaurants and city B had 1,000. If the typical household in this economy has an annual income of $60,000 and a typical apartment in city A rents for $6,000 per year, what would you estimate this same apartment would rent for in city B?

**Answer:** The table tells us that consumers would be willing to forego about 37% of income to live in city A rather than a city with only a single restaurant — and they would be willing to forego about 50% of income to live in a city with 1000 restaurants rather than just 1 restaurant. Thus, they are willing to give up about 13% more income to live in city B rather than city A. At an income of $60,000, 13% of income amounts to $7,800 — implying that the same apartment in city B would rent for $13,800 (all else being equal).

In exercise 26.1, we investigated different ways in which the markets for good \( x_i \) (produced by firm \( i \)) and good \( x_j \) (produced by firm \( j \)) may be related to each other under price competition. We now investigate the incentives for firms to merge into a single firm in such environments — and the level of concern that this might raise among antitrust regulators.

At one way to think about firms that compete in related markets is to think of the externality they each impose on the other as they set price. For instance, if the two firms produce relatively substitutable goods (as described in (a) below), firm 1 provides a positive externality to firm 2 when it raises \( p_1 \) because it raises firm 2's demand when it raises its own price.

(a) Suppose that two firms produce goods that are relatively substitutable in the sense that, when the price of one firm's good goes up, this increases the demand for the other firm's goods. If these two firms merged, would you expect the resulting monopoly firm to charge higher or lower prices for the goods previously produced by the competing firms? (Think of the externality that is not being taken into account by the two firms as they compete.)

**Answer:** As firm 1 raises its price, it only considers its own profit and not firm 2's profit. But as firm 1 raises price, it is raising demand — and thus profit — for firm 2. This is a positive externality that firm 1 is not taking into account — and, as a result, it will set "too low" a price relative to what the firms would do if they could make joint decisions (as they can if they merge into a single monopoly). Thus, if the two firms merge, the price of both goods will increase.

(b) Next, suppose that the two firms produce goods that are relatively complementary in the sense that an increase in the price of one firm's good decreases the demand for the other firm's good. How is the externality now different?

**Answer:** When firm 1 raises price, it now lowers demand (and profit) for firm 2 — thus emitting a negative (rather than a positive) externality that it is not taking into account when it sets its own price.

(c) When the two firms in (b) merge, would you now expect price to increase or decrease?

**Answer:** Since firm 1 is not taking into account the damage it does to firm 2 when it raises price, firm 1 will raise its price "too high" relative to what the two firms would decide jointly. Thus, as the firms merge, I would expect prices to fall.

(d) If you were an antitrust regulator, which merger would you be worried about: The one in (a) or the one in (b)?

**Answer:** You would worry about the merger in (a) — the merger of firms that produce relatively substitutable goods. In those cases, we determined that the merged firm will raise price — whereas in the case of complements we determined the merged firm will lower price. In both cases, the firms exercise market power when they merge into a monopoly — but in the case of complements, they are (by merging) eliminating an externality that resulted in too high a price. Anti-trust regulators worry about mergers resulting in higher prices as firms collude — and would probably not be concerned about mergers that result in a reduction in output price.

(e) Suppose that instead the firms were producing goods in unrelated markets (with the price of one firm not affecting the demand for the goods produced by the other firm). What would you expect to happen to price if the two firms merge?

**Answer:** If the two markets are unrelated and a change in price in one market has no impact on the demand for goods in the other, then the initial firms were already two separate monopolists. Without any relationship between the markets, a merger does not change this — the merged firm would continue to behave as a monopolist in the individual markets. Thus, in this case, we would not expect price to change.

(f) Why are the positive externalities we encountered in this exercise good for society?

**Answer:** The positive externality we encountered was for the case of competing oligopolistic firms in markets that produce relative substitutes. Here, neither firm took into account the "benefit" it creates for the other firm's profits as it raises price and thereby increases demand for the other firm. Because of this, each firm will set price lower than it would if it were colluding with the other firm — and this is good for consumers (which are the rest of society in this example).
B: Suppose we have two firms — firm 1 and 2 — competing on price. The demand for firm $i$ is given by $x_i(p_i, p_j) = 1000 - 10p_i + \beta p_j$, and each firm faces constant marginal cost $c = 20$ (and no fixed costs).

(a) Calculate the equilibrium price $p^*$ as a function of $\beta$.

Answer: Using either the results from the text or referring to results from exercise 26.1,

$$p^* = \frac{1,200}{20 - \beta}.$$  \hspace{1cm} (26.15)

(b) Suppose that the two firms merged into one firm that now maximized overall profit. Derive the prices for the two goods (in terms of $\beta$) that the new monopolist will charge — keeping in mind that the monopolist now solves a single optimization problem to set the two prices. (Given the symmetry of the demands, you should of course get that the monopolist will charge the same price for both goods).

Answer: The monopolist then solves the problem

$$\max_{p_1, p_2} \pi = (p_1 - 20)(1000 - 10p_1 + \beta p_2) + (p_2 - 20)(1000 - 10p_2 + \beta p_1).$$ \hspace{1cm} (26.16)

From the two first order conditions, we get

$$p_1 = \frac{600 - 10\beta + \beta p_2}{10} \quad \text{and} \quad p_2 = \frac{600 - 10\beta + \beta p_1}{10}.$$ \hspace{1cm} (26.17)

Substituting the latter into the former and solving for $p_1$, we get

$$p_1 = \frac{6000 + 500\beta - 10\beta^2}{(100 - \beta^2)} \quad \text{and} \quad p_2 = \frac{600 - 10\beta}{10 - \beta}.$$ \hspace{1cm} (26.18)

Substituting this back into our equation for $p_2$, we get the same for $p_2$. Thus,

$$p^M = p_1 = p_2 = \frac{600 - 10\beta}{10 - \beta}.$$ \hspace{1cm} (26.19)

(c) Create the following table: Let the first row set different values for $\beta$ ranging from minus 7.5 to 7.5 in 2.5 increments. Then, derive the the equilibrium price (for each $\beta$) when the two firms compete and report it in the second row. In a third row, calculate the price charged by the monopoly (that results from the merging of the two firms) for each value of $\beta$.

Answer: This is done in Table 26.1.

(d) Do your results confirm your intuition from part A of the exercise? If so, how?

Answer: In part A, we discussed that fact that price setting oligopolists create externalities for each other as they raise price — a positive externality in the case of substitutes and a negative externality in the case of complements. As a result, they will price “too low” relative to the monopoly outcome in the case of substitutes and “too high” in the case of complements. This implies that price will rise as a result of a merger if the two goods are substitutes — and it will fall when they are complements. This is precisely what the table shows — with $\beta > 0$ representing cases where the two firms produce relative substitutes and with $\beta < 0$ implying they produce relative complements.

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<th>-7.5</th>
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<th>-2.5</th>
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<th>5.0</th>
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</table>

Table 26.1: Change in Pricing and Profit from Merger
(e) Why would firms merge if, as a result, they end up charging a lower price for both goods than they were able to charge individually?

Answer: By merging, they can internalize the externality that keeps them from jointly attaining the maximum profit when they are in the oligopoly setting — and this is true whether the firms are creating positive or negative externalities for one another.

(f) Add two rows to your table — calculating first the profit that the two firms together make in the competitive oligopoly equilibrium and then the profit that the firms make as a monopoly following a merger. Are the results consistent with your answer to (e)?

Answer: This is done in the final two rows of Table 26.1. Note that profit increases as a result of the merger in all cases except for the case where $\beta = 0$ and the two markets are therefore entirely unrelated. In that special case, the two firms are independent monopolies to begin with — and merging simply makes them continue to do what they did before. But in all other cases, there is an externality that can be internalized when the firms merge — allowing the monopoly that owns both firms to make more profit than the firms can independently.
26.5 Business Application: Advertising as Quality Signal. In the text, we have discussed two possible motives for advertising, one focused on providing information (about the availability of goods or the prices of goods) and another focused on shaping the image of the product. Another possible motive might be for high quality firms to signal that they produce high quality goods to consumers who cannot tell the difference prior to consuming a good. Consider the following game that captures this: In each of two periods, firms get to set a price and consumers get to decide whether or not to buy the good. In the first period, consumers do not know if a firm is producing high or low quality goods — all they observe is the prices set by firms and whether or not firms have advertised. But if a consumer buys from a firm in the first period, she experiences the quality of the firm’s product and thus knows whether the firm is a high or low quality firm when she makes a decision of whether to buy from this firm in the second period. Assume throughout that a consumer who does not buy from a firm in the first period exits the game and does not proceed to the second period.

As Notice that firms and consumers play a sequential game in each period, with firms offering a price first and consumers then choosing whether or not to buy. But in the first period, firms also have the option to advertise in an attempt to persuade consumers of the product’s value.

(a) Consider the second period first. Given that the only way a consumer enters the second period is if she bought from the firm in the first period, and given that she then operates with the benefit of having experienced the good’s quality, would any firm choose to advertise in the second period if it could?

Answer: Since everyone has complete information about quality in the second period, there is no reason to advertise. Thus, firms would not choose to advertise given that consumers already know their quality — unless there are other reasons (such as those discussed in the text) for firms to advertise.

(b) Suppose that both firms incur a marginal cost of $MC$ for producing their goods. High quality firms produce goods that are valued at $v_h > MC$ by consumers and low quality firms produce goods that are valued at $v_f > MC$ (with $v_h > v_f$). In any subgame perfect equilibrium, what prices will each firm charge in the second period — and what will consumer strategies be (given they decide whether to buy after observing prices)?

Answer: Consumers will buy the high quality goods for any price $p \leq v_h$, and they will buy low quality goods for any price $p \leq v_f$. Otherwise, they will not buy. Knowing this, high quality firms will set period 2 price at $p^h_2 = v_h$ and low quality firms will set price at $p^l_2 = v_f$.

(c) Now consider period 1. If consumers believe that firms who advertise are high quality firms and firms that don’t advertise are low quality firms, what is their subgame perfect strategy in period 1 (after they observe prices and whether a firm has advertised)?

Answer: With such beliefs, consumers will buy if $p_1 \leq v_h$ whenever they have observed the firm advertising and if $p_1 \leq v_f$ if they do not observe the firm advertising.

(d) What is the highest cost $a_h$ (per output unit) of advertising that a high quality firm would be willing to undertake if it thought that consumers would interpret this as the firm producing a high quality good?

Answer: The firm knows that, if the consumer bought in period 1, she will buy again in period 2 at $p^h_2 = v_h$. Thus, period 2 profit is $\pi^h = (v_h - MC)$ if the consumer buys in period 1 and 0 if she does not. If the firm thinks that the consumer will believe the firm’s product to be of high quality after $a$ is spent on advertising in period 1, the firm will again be able to set price at $v_h$ — making a period 1 profit of $\pi^h = (v_h - MC - a)$. Otherwise, period 1 profit is zero. Thus, the high quality firm will advertise in period 1 so long as $\pi^h + x^h_2 \geq 0$; i.e. so long as

$$2v_h - 2MC - a \geq 0.$$ \hfill (26.20)

The highest level of $a$ that satisfies this is $a^h = 2v_h - 2MC$.

(e) What is the highest cost $a_f$ that a low quality firm would be willing to incur if it thought this would fool consumers into thinking that it produced high quality goods (when in fact it produces low-quality goods)?

Answer: If the low quality firm reaches period 2 with a consumer, the consumer knows the firm produces value of $v_f$. At price $p^f_2 = v_f$, the consumer buys in period 2 and the firm
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makes period 2 profit of \( \pi^\ell_2 = (v_\ell - MC) \). If the firm does not advertise in period 1, it will make the same period 1 profit (as the consumer interprets no advertising as a signal of low quality and therefore buys only at \( p_1 \leq v_\ell \). But if the firm advertises and the consumer interprets this as a high quality signal, the firm will be able to charge \( p_1 = v_h \) and make a first period profit of \( \pi^\ell_1 = (v_h - MC - a) \). Since second period profit is the same whether the firm advertises or not, the firm will therefore choose to advertise so long as

\[
v_h - MC - a \geq v_\ell - MC. \tag{26.21}\]

The highest possible value for \( a \) such that this holds is then \( a^*_\ell = v_h - v_\ell \).

(f) Consider a level of advertising that costs \( a^* \). For what levels of \( a^* \) do you think that it is an equilibrium for high quality firms to advertise and low quality firms to not advertise?

Answer: Given our answers to the two previous parts, this equilibrium can emerge so long as

\[
a^*_\ell = v_h - v_\ell \leq a^* \leq 2v_h - 2MC = a^*_h \tag{26.22}\]

(g) Given the information asymmetry between consumers and firms in period 1, might it be efficient for such advertising to take place?

Answer: Since advertising is costly, the equilibrium will be an efficient way of providing information when \( a^* = a^*_\ell = v_h - v_\ell \) which is the lowest it can be and still have low quality firms choose not to advertise. (In this case, low quality firms are indifferent between advertising and not advertising — so it is an equilibrium for them not to advertise).

(b) We often see firms sponsor sporting events — and it is difficult to explain such sponsorships as “informational advertising” in the way we discussed such advertising in the text. Why? How can the model in this exercise nevertheless be rationalized as informational advertising (rather than simply image marketing)?

Answer: Such advertising conveys neither information about price nor about the availability of a good — and thus is not like what we called informational advertising in the text. Nevertheless, it still results in information being conveyed — because consumers learn about quality. Thus, this may be part of the reason why firms sponsor sporting events — but of course such sponsorships could also be a form of image marketing.

B: Suppose that a firm is a high quality firm \( h \) with probability \( \delta \) and a low quality firm \( \ell \) with probability \( (1 - \delta) \). Firm \( h \) produces an output of quality that is valued by consumers at 4 while firm \( \ell \) produces an output of quality 1 (that is valued by consumers at 1), and both incur a marginal cost equal to 1 per unit of output produced. (Assume no fixed costs.)

(a) Derive the level of \( a^* \) of advertising (as defined in part A) that could take place in equilibrium.

Answer: Given our answers to part A, we can conclude that

\[
a^*_\ell = v_h - v_\ell = 4 - 1 = 3 \leq 2(4) - 2(1) = 6 = a^* \leq 2v_h - 2MC = a^*_h \tag{26.23}\]

— i.e. \( a^* \) has to lie between 3 and 6.

(b) What is the most efficient of the possible equilibria in which high quality firms advertise but low quality firms do not advertise?

Answer: The most efficient such equilibrium is one where \( a^* = 3 \).

(c) Do your answers thus far depend on \( \delta \)?

Answer: None of our answers thus far depend on \( \delta \). This is because firms fully reveal their type through advertising — and consumers therefore know with probability 1 what type of firm they face in period 1 (and 2).

(d) The equilibria you have identified so far are separating equilibria because the two types of firms behave differently in equilibrium — thus allowing consumers to learn from observing advertising whether or not a firm is producing a high or low quality good. Consider now whether both firms choosing \((p, a)\) — and firms thus playing a pooling strategy — could be part of an equilibrium. Why is period 2 large irrelevant for thinking about this?
Answer: Period 2 is irrelevant because consumers are fully informed in period 2 — and thus the subgame perfect equilibrium strategies for period 2 have to hold under the pooling strategies as well. With the parameters that we have set in this example, the high quality firm will then be able to make a second period profit of $(4 - 1) = 3$, the other firm will make a second period profit of $(1 - 1 = 0)$ and consumers will get zero payoff given that firms set period 2 prices at consumer marginal willingness to pay. Firms and consumers can take all that as given when they think about period 1. Since consumers as well as the low quality firm get zero payoff in the second period, nothing that happens in period 1 can have an impact on their overall payoffs. The only one that has a real stake in the game continuing to stage 2 is the high quality firm that will get a 2nd period profit of 3.

(e) If the firms play the pooling strategy $(p, a)$, what is the consumer’s expected payoff from buying in period 1? In terms of $\delta$, what does this imply is the highest price $p$ that could be part of the pooling equilibrium?

Answer: Consumers will get a first period payoff of $(4 - p)$ if they consume from a high quality firm and $(1 - p)$ if they consume from the low quality firm. Given that $\delta$ is the probability they face a high quality firm (and no information is revealed by the firms’ pooling strategies), consumer expected payoff is

\[
\text{Consumer 1st Period Payoff} = \delta(4 - p) + (1 - \delta)(1 - p). \tag{26.24}
\]

This has to be greater than or equal to zero for consumers to buy — otherwise they can get a zero payoff from just exiting the market in period 1. Setting the consumer first period payoff to zero and solving for $p$, we then get

\[
p \leq 3\delta + 1. \tag{26.25}
\]

(f) Suppose consumers believe a firm to be a low quality firm if it deviates from the pooling strategy. If one of the firms has an incentive to deviate from the pooling strategy, which one would it be? What does this imply about the lowest that $p$ can be relative to $a$ in order for $(p, a)$ to be part of a pooling equilibrium?

Answer: If any firm has an incentive to deviate, it is the low quality firm that will deviate if it is not making period 1 profit under the strategy. The high quality firm will only deviate if its period 1 and period 2 profits don’t sum to zero — and thus is less likely to deviate than the low quality firm. The low quality firm will then not deviate so long as its profit in period 1 is non-negative — implying

\[
p - a - 1 \geq 0 \tag{26.26}
\]

which implies $p \geq (a + 1)$.

(g) Using your answers from (b) and (c), determine the range of $p$ in terms of $\delta$ and $a$ such that $(p, a)$ can be part of a Bayesian Nash pooling equilibrium.

Answer: Combining our two previous answers, we have

\[
a + 1 \leq p \leq (3\delta + 1). \tag{26.27}
\]

(h) What equilibrium beliefs do consumers hold in such a pooling equilibrium when they have to decide whether or not to buy in period 1? What out-of-equilibrium beliefs support the equilibrium?

Answer: In equilibrium, consumers will see $(p, a)$ from both types of firms — and will thus not be able to update their beliefs from the initial $\delta$ with which high quality was assigned (by Nature) to the firm. Thus, when the time comes to buy or not to buy in period 1, the consumer believes she is facing a high quality firm with probability $\delta$ and a low quality firm with probability $(1 - \delta)$. Beliefs about what type the firm is if it does not play $(p, a)$ in the first stage of the game are "out-of-equilibrium" in the sense that they do not happen in equilibrium. Bayes rule therefore does not tell us anything about how consumers would update their beliefs — and any beliefs can therefore be equilibrium beliefs. But the equilibrium requires that consumers interpret a deviation from the pooling strategy as the actions of a low quality firm.
(i) Can advertising in a pooling equilibrium ever be efficient?

Answer: Advertising is costly — and in a pooling equilibrium, no information comes to light. It is therefore a social cost without a social benefit in the pooling equilibrium — and can therefore never be efficient.
26.6 Business Application: Price Leadership in Differentiated Product Markets: We have considered how oligopolistic firms in a differentiated product market price output when the firms simultaneously choose price. Suppose now that two firms have maximally differentiated products on the Hotelling line \([0,1]\) and that the choice of product characteristics is no longer a strategic variable. But let’s suppose now that your firm gets to move first — announcing a price that your opponent then observes before setting her own price. This is similar to the Stackelberg quantity-leadership model we discussed in Chapter 25 except that firms now set price rather than quantity.

A: Suppose you are firm 1 and your opponent is firm 2, with both firms facing constant marginal cost (and no fixed costs).

(a) Begin by reviewing the logic behind sequential pricing in the pure Bertrand setting where the two firms produce undifferentiated products. Why does the sequential (subgame perfect) equilibrium price not differ from the simultaneous price setting equilibrium?

**Answer:** In the undifferentiated products case, your firm knows (when it announces price first) that your competitor will price just below you and get the whole market unless you price at marginal cost. Thus, you price at marginal cost, and your competitor does the same, with the two firms splitting the market. This is the same result that emerges in the case of simultaneous pricing decisions.

(b) Now suppose that you are producing maximally differentiated products on the Hotelling line. When firm 2 sees your price \(p_1\), illustrate its best response in a graph with \(p_2\) on the horizontal and \(p_1\) on the vertical axis.

**Answer:** This is illustrated in panel (a) of Graph 26.2, with your competitor’s best response denoted \(BR_2\).

(c) Include in your graph the 45-degree line and indicate where the price equilibrium falls if you and your competitor set prices simultaneously.

**Answer:** The price equilibrium in the simultaneous move case falls at point \(A\) in panel (a) of Graph 26.2 — with both firms setting the same price above \(MC\) (because product differentiation softens the price competition that leads to price equal to marginal cost in the undifferentiated products case).

(d) Let \(\bar{P}\) be the price that results in zero demand for your goods assuming that your competitor observes \(\bar{P}\) before setting her own price. Indicate \(\bar{P}\) in a plausible place on your graph. Then,
on a graph next to it, put \( p_1 \) on the vertical axis and \( x_1 \) — the good produced by your firm — on the horizontal. Where does your demand curve start on the vertical axis given that you take into account your competitor’s response?

Answer: This is illustrated in panels (a) and (b) of Graph 26.2 — with the demand curve for your good starting on the vertical axis at \( \overline{p} \).

(e) Draw a demand curve for \( x_1 \) and let this be the demand for \( x_1 \) given you anticipate your competitor’s response to any price you set. Include MC and MR in your graph and indicate \( p^* \) — the price you will choose given that you anticipate your competitor’s price response once she observes your price.

Answer: This is done in panel (b) of Graph 26.2 where your firm sets the monopoly price given the demand curve it faces.

(f) Finally, find your competitor’s price \( p^*_2 \) on your initial graph. Does it look like \( p^*_1 \) is greater or less than \( p^*_2 \)?

Answer: Bringing \( p^*_1 \) back to panel (a) of the graph, we now read off your competitor’s best response from her best response function \( BR^2 \) — giving us her price response \( p^*_2 \). It looks like \( p^*_1 > p^*_2 \) — which is exactly what we find mathematically in part B.

(g) Who will have greater market share on the Hotelling line — you as the price leader, or your competitor?

Answer: Given that you charge a higher price than your competitor, your competitor will have a larger market share than you.

B: Suppose that the costs (other than price) that consumers incur is quadratic as in the text — i.e. a consumer \( n \) whose ideal point is \( n \in [0, 1] \) incurs a cost \( \alpha(n - y)^2 \) from consuming a product with characteristic \( y \in [0, 1] \). Continue to assume that firm 1 has located its product at 0 and firm 2 has located its product at 1 — i.e. \( y_1 = 0 \) and \( y_2 = 1 \). Firms incur constant marginal cost \( c \) (and no fixed costs).

(a) For what value of \( \alpha \) is this the Bertrand model of Chapter 25? In this case, does the equilibrium price differ depending on whether one firm announces a price first or whether they announce price simultaneously? (Assume subgame perfection in the sequential case.)

Answer: The product differentiation disappears when \( \alpha = 0 \) — in which case the two firms become pure Bertrand competitors that price at \( MC \) in equilibrium. As we point out in Chapter 25, this conclusion does not depend on whether the pricing game is sequential or simultaneous.

(b) Now suppose \( \alpha > 0 \). If the firms set price simultaneously, what is the equilibrium price?

Answer: Using the equations \( p^*_1(y_1, y_2) \) and \( p^*_2(y_1, y_2) \) from the text and substituting \( y_1 = 0 \) and \( y_2 = 1 \), we get

\[
p^*_1 = p^*_2 = c + \alpha.
\]

(26.28)

(c) Next, suppose firm 1 announces its price first, with firm 2 then observing firm 1’s price before setting its own price. Using the same logic we used in the Stackelberg model of quantity competition, derive the price firm 1 will charge (as a function of \( c \) and \( \alpha \).) (Hint: You can use the best response function for firm 2 derived in the text — substituting \( y_1 = 0 \) and \( y_2 = 1 \) — to set up firm 1’s optimization problem.)

Answer: Firm 1 knows firm 2’s best price response function which is given in the text as \( p_2(p_1) \). Substituting \( y_1 = 0 \) and \( y_2 = 1 \) into this equation, we get

\[
p_2(p_1) = \frac{p_1 + c + \alpha}{2}.
\]

(26.29)

Since firm 1 moves first, subgame perfection requires that firm 1 then chooses its price by taking this best response function as given and solving the problem

\[
\max_{p_1} \left( p_1 - c \left( \frac{1}{2} + \frac{p_2(p_1) - p_1}{2\alpha} \right) \right)
\]

(26.30)
where the second term is demand for firm 1’s output (derived from our results in the text by setting $y_1 = 0$ and $y_2 = 1$). Substituting equation (26.29) into this maximization problem, we get that firm 1 will solve

$$
\max_{p_1} (p_1 - c) \left( \frac{1}{2} + \frac{(p_1 + c + \alpha/2 - p_1)}{2\alpha} \right) = (p_1 - c) \left( \frac{3\alpha + c - p_1}{4\alpha} \right).
$$

(26.31)

Solving this in the usual way, we get

$$
p_1 = c + \frac{3\alpha}{2}.
$$

(26.32)

(d) What price does this imply firm 2 will set after it observes $p_1$? Which price is higher?

Answer: Substituting $p_1 = c + (3/2)\alpha$ into equation (26.29), we get

$$
p_2 = c + \frac{5\alpha}{4}.
$$

(26.33)

Thus, $p_1 > p_2$.

(e) Derive the market shares for firms 1 and 2. In the Stackelberg quantity setting game, the firm that moved first had greater market share. Why is that not the case here?

Answer: We can now determine the market share for firm 1 by finding the consumer type $n$ that is indifferent between shopping from the two firms; i.e. we need to find $n$ such that

$$
p_1 + \alpha n^2 = p_2 + \alpha (1-n)^2
$$

(26.34)

which, given our solutions for $p_1$ and $p_2$, is

$$
c + \frac{3\alpha}{2} + \alpha n^2 = c + \frac{5\alpha}{4} + \alpha (1-n)^2.
$$

(26.35)

Solving this for $n$, we get $n = 0.375$. Thus, market share for firm 1 is 0.375 and market share for firm 2 is $(1 - 0.375) = 0.625$. Firm 1 now has lower market share because it sets a higher price in equilibrium — with firm 2 undercutting firm 1’s price when it gets to observe it.

(f) Derive profit for the two firms. Which firm does better — the leader or the follower? True or False: The quantity leader in the Stackelberg model has a first mover advantage while the price leader in the Hotelling model has a first mover disadvantage.

Answer: Profit for firm 1 is

$$
\pi_1 = (p_1 - c) \frac{1}{2} = \left( c + \frac{3\alpha}{2} - c \right) (0.375) = 0.5625\alpha.
$$

(26.36)

Profit for firm 2 is

$$
\pi_2 = (p_2 - c)(1-n) = \left( c + \frac{5\alpha}{4} - c \right) (0.625) = 0.78125\alpha.
$$

(26.37)

The statement in the exercise is therefore true — firm 1 makes less profit than firm 2 because firm 2 has an advantage from knowing the price that firm 1 announces. Thus, firm 1 is at a disadvantage from having to move first.

(g) True or False: Both firms prefer sequential pricing in the Hotelling model over simultaneous pricing (given maximal product differentiation).

Answer: In the simultaneous pricing model, we derived the equilibrium price that both firms charge as $p^* = c + \alpha$ — with both firms then getting equal market shares of 0.5. Using this to calculate profit, we get

$$
\pi = (p^* - c)(0.5) = (c + \alpha - c)(0.5) = 0.5\alpha.
$$

(26.38)

This is less than the profit either of the firms make when one of them is a price leader in the sequential setting. Thus, while firm 1 is at a disadvantage relative to firm 2 when firm 1 moves first, even firm 1 is better off in the sequential setting than when there is no price leadership.
26.7 Business Application: The Evolution of the Fashion Industry. Consider the market for clothes and suppose there exist 100 different “styles” that can be produced and can be arranged (and equally spaced) on a circle. Among the billions of consumers of clothes, each has an ideal style somewhere on that circle (either at one of the 100 styles that can potentially be produced or in between two of those). Styles become less appealing the farther they are from the consumer’s ideal. For simplicity, suppose that the marginal cost of producing clothes of any style is constant (once the fixed cost of starting production has been paid), and suppose that a firm that comes into the industry must pay the fixed entry cost for each style it wants to produce.

As: Suppose first that only a single firm operates in the industry (and produces one of the 100 styles) and that the fixed cost of starting production is sufficiently high for no second firm to wish to enter.

(a) Explain how the firm in the industry can be making positive economic profit but the firms outside would make negative economic profit by entering.

Answer: The firm in the industry has already paid the fixed cost — and thus only faces variable costs. All firms outside the industry face fixed costs of entry in addition to marginal costs. Thus, the monopoly in the industry can make positive profit while all others would make negative profit. Even if the fixed entry cost was low enough to allow a firm to enter and replace the monopoly, entry would actually imply splitting the market — thus achieving profit less than the current monopoly.

(b) Over the decades, the price of the equipment necessary for producing clothes has fallen — thus lowering the fixed entry cost into the clothing industry. When the costs fall to the point where the second firm enters, where on the circle would you expect that firm to locate its clothes?

Answer: We would expect the new firm to maximally differentiate its product and locate on the opposite side of the circle (i.e. 180 degrees from the incumbent firm).

(c) What would happen to the price of clothing assuming the two firms are price competitors?

Answer: We would expect the price of clothing to fall as the two firms engage in price competition over their differentiated products.

(d) Suppose entry costs have fallen sufficiently for 100 different firms to be in the clothing industry. Now suppose entry costs fall further and firms continue to be price competitors. How low would entry costs have to fall for another firm to enter the market (assuming only 100 clothing “styles” can potentially be produced)?

Answer: If only 100 clothing styles can be produced and all 100 are already being produced by 1 firm for each style, then entry of new firms would imply entry of a firm producing the same style that is currently produced by another firm. When two firms produce the same clothing style, they are pure Bertrand competitors producing undifferentiated products. Thus, our Bertrand model implies that price will fall to marginal cost. Thus, a firm will only enter under these conditions if fixed entry costs are zero.

(e) Suppose that an avalanche of new ideas has made all clothing styles on the circle — not just the initial 100 — possible to produce. As entry costs fall, how many new entrants would you expect when the next firm finds it profitable to enter?

Answer: A new firm would enter by choosing the midpoint between two existing clothing styles — but there are 100 different identical such midpoints. Thus, if it becomes profitable for 1 firm to enter, it is in fact profitable for 100 firms to enter — one at each midpoint between two existing firms.

(f) Beginning with the case where the industry first consists of 100 firms, would you expect price to fall as entry costs fall even before any additional competitors enter the industry (assuming that existing firms can credibly announce their price before new firms have to make a decision on whether or not to enter)?

Answer: Yes, you would expect existing firms to engage in strategic entry deterrence — lowering price in order to deter new firms from entering. As fixed costs fall and it would be worthwhile for an entrant to come into the market, existing firms thus lower their prices. They will continue to do so until fixed entry costs have fallen sufficiently such that the entry-deterrence price falls so low that profits for incumbent firms would be higher if they simply permitted entry.

(g) Suppose entry costs disappear altogether. What happens to price?
Answer: If entry costs disappear entirely, a firm would locate at every point on the circle — with each consumer being able to get her ideal point. Since there is now virtually no product differentiation between a firm and its adjacent firms, firms are engaged in pure Bertrand competition with their neighbors — with price falling to MC. This is the perfectly competitive outcome.

R: (Part B of this exercise is not directly related to part A but rather offers you a chance to go through solving the “circle model” with a slight modification from the version used in the text.) In our treatment of the “circle model” in Section 26B.3, we assumed that the cost consumer \( n \in [0, 1] \) incurs from consuming a product with characteristic \( y \in [0, 1] \) (rather than her ideal of \( n \)) increases linearly with the distance between \( n \) and \( y \) — i.e. the cost was \( c(n - y) \). In our treatment of the Hotelling “line” model, we instead assumed that this cost increases with the square of the distance — i.e. the cost was \( c(n - y)^2 \).

(a) Consider the second stage of the “circle model” game — i.e. the stage at which \( N \) firms have entered in the first stage having equally spaced their products on the product characteristic circle (of circumference 1). Assume that every point \( y \) on the circle contains one consumer \( n \) whose ideal point is \( y \). What is the farthest that any consumer \( n \)’s ideal point will lie from the closest firm’s product?

Answer: If there are \( N \) firms equally spaced on the circle, the farthest that any consumer’s ideal point is from any producing firm is at the midpoint between two firms. With \( N \) firms and a circle with circumference normalized to 1, adjacent firms are \( 1/N \) apart — implying the midpoint between them is \( 1/(2N) \) away from each of the closest firms.

(b) Suppose that all firms other than firm \( i \) charge a price \( p \) and suppose firm \( i \)’s product characteristic is \( y_i = 0 \). Denote by \( \overline{\pi} \) the consumer who is indifferent between consuming from firm \( i \) and adjacent firm \( j \) (with firm \( j \) producing \( y_j \)) assuming firm \( i \) charges price \( p_i \). Given that the consumer’s total cost from consuming a particular product includes both the price she has to pay and the cost of consuming away from her ideal, what has to be true about the total cost \( \overline{\pi} \) incurs when shopping at firm \( i \) versus firm \( j \)? Express this in an equation and solve it for \( \overline{\pi} \).

Answer: The consumer \( \overline{\pi} \) must be indifferent between consuming from firm \( i \) and consuming from firm \( j \) — which implies

\[
p_i + \alpha \overline{\pi}^2 = p + \alpha (y_j - \overline{\pi})^2. \tag{26.39}
\]

Solving this for \( \overline{\pi} \), we get

\[
\overline{\pi} = \frac{(p - p_i)N}{2\alpha} + \frac{y_j}{2}. \tag{26.40}
\]

(c) Given that there are \( N \) (equally spaced) firms in the industry, what is \( y_j \) (when \( y_i = 0 \))? Substitute this into your expression for \( \overline{\pi} \). What is the demand \( D^i(p_i, p) \) that firm \( i \) faces? Explain.

Answer: With \( N \) firms and \( y_i = 0 \) (and a circle of circumference 1), it must be that \( y_j = 1/N \). Substituting this into equation (26.40), we get

\[
\overline{\pi} = \frac{(p - p_i)N}{2\alpha} + \frac{1}{2N}. \tag{26.41}
\]

Firm \( i \) then experiences this much demand from both of its sides — i.e. demand for firm \( i \) is twice \( \overline{\pi} \):

\[
D^i(p_i, p) = 2\overline{\pi} = \frac{(p - p_i)N}{\alpha} + \frac{1}{N}. \tag{26.42}
\]

(d) Using your expression for \( D^i(p_i, p) \), derive firm \( i \)’s best (price) response function to all other firms setting price \( p \) (with all firms facing constant marginal cost \( c \)).

Answer: Firm \( i \) now maximizes profit by solving

\[
\max_{p_i} \pi_i = (p_i - c)D^i(p_i, p) = (p_i - c) \left( \frac{(p - p_i)N}{\alpha} + \frac{1}{N} \right). \tag{26.43}
\]

Setting the derivative of profit with respect to \( p_i \) equal to zero and solving for \( p_i \), we get
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\[ p_i = \frac{p + c}{2} + \frac{\alpha}{2N^2}. \]  

(e) Since all firms end up charging the same price in equilibrium, what is the equilibrium price \( p^* (N) \) in terms of \( c \), \( \alpha \) and \( N \) given that \( N \) firms have entered in stage 1 of the "circle game"?

Answer: Removing the \( i \) subscript in equation (26.44) and solving for \( p \), we get

\[ p^* (N) = c + \frac{\alpha}{N^2}. \]  

(f) Assuming that firms have to pay a fixed cost \( FC \) to enter the circle market in stage 1 of the game, how many will enter (given they forecast \( p^* \) in the second stage)? Denote this as \( N^* \). What is the equilibrium price that will emerge as a result?

Answer: In equilibrium, the profit from entering must be approximately zero. When all firms charge the same price and there are \( N \) firms, each firm gets market share of \( \frac{1}{N} \); i.e.

\[ D(p^* (N), p^* (N)) = \frac{1}{N}. \]

Thus, the zero profit entry condition can be written as

\[ (p^* (N) - c)D(p^* (N), p^* (N)) - FC = \left(c + \frac{\alpha}{N^2} - c\right) \left(\frac{1}{N}\right) - FC = \frac{\alpha}{N^2} - FC = 0. \]  

Solving for \( N \), we get

\[ N^* = \left(\frac{\alpha}{FC}\right)^{1/3}. \]

Substituting this back into the equilibrium price equation \( p^* (N) \) in stage 2, we get the equilibrium price of

\[ p^* = c + \alpha^{1/3}FC^{2/3}. \]

(g) Now consider the problem a social planner who wants to maximize efficiency faces when deciding how many firms to set up on the circle. Suppose the planner sets the number of firms at \( N \). Explain why the cost consumers incur from not consuming at their ideal is

\[ 2N \int_{0}^{1/(2N)} x^2 \, dx. \]

Answer: This is illustrated in Graph 26.3. If there are \( N \) firms on the circle with circumference 1, then the distance between firm 0 and firm 1 is \( 1/N \) — illustrated by the end-points in the graph. The consumer at the midpoint is a consumer with ideal point \( 1/(2N) \). This consumer incurs a cost equal to the square of the distance from the firm from which she consumes — i.e. \( (1/2N)^2 \). The consumer whose ideal point is at 0 does not incur any costs, nor does the consumer whose ideal point lies at \( 1/N \). All other consumers incur a cost that is equal to the square of the distance from the nearest firm — with this illustrated by the curve that reaches its peak at \( 1/(2N) \). The shaded area in the graph is \( \int_{0}^{1/(2N)} x^2 \, dx \), and the area under the whole cost "mountain" in the graph is twice that; i.e. \( 2 \int_{0}^{1/(2N)} x^2 \, dx \). With \( N \) firms, we have \( N \) of these pictures — implying the cost for consumers along the entire circle is \( 2N \int_{0}^{1/(2N)} x^2 \, dx \).

(h) What is the socially optimal number of firms \( N^* \) that the planner would set up? How does it compare to the equilibrium number of firms \( N^* \) — and what has to be true for the two to converge to one another?

Answer: We can solve this integral to give us the total consumer cost from \( N \) firms as

\[ 2N \int_{0}^{1/(2N)} x^2 \, dx = \frac{2Nx^3}{3} \bigg|_{0}^{1/(2N)} = \frac{1}{12N^2}. \]  

The social planner then wants to minimize the sum of all the fixed costs of setting up firms plus the costs incurred by consumers (from not reaching their ideal); i.e. the social planner solves the problem

\[ \text{min} + N (FC) + \frac{1}{12N^2}. \]
Setting the first derivative with respect to $N$ to zero and solving for $N$, we get

$$N_{\text{opt}} = \left( \frac{a}{bF_C} \right)^{1/3}. \tag{26.51}$$

This is clearly less than $N^*$ — i.e. the market results in too many firms. The optimal number of firms $N_{\text{opt}}$ only converges to the market number $N^*$ as $F_C$ approaches zero and both expressions go to infinity — i.e. as all barriers to entry fall and the market becomes perfectly competitive (with a different firm locating at each point on the circle).
26.8 Business Application: Deterring Entry of Another Car Company. Suppose that there are currently two car companies that form an oligopoly in which each faces constant marginal costs. Their strategic variables are price and product characteristics.

At: Use the Hotelling model to frame your approach to this exercise and suppose that the two firms have maximally differentiated their products, with company 1 selecting characteristic 0 and company 2 selecting characteristic 1 from the set of all possible product characteristics \( \{0,1\} \).

(a) Explain why such maximal product differentiation might in fact be the equilibrium outcome in this model.

**Answer:** As we have shown repeatedly, two oligopolists can soften price competition by differentiating their product — thus both pricing above \( MC \) (which would not be possible if they did not differentiate their product).

(b) Next, suppose a new car company plans to enter the market and chooses 0.5 as its product characteristic. If the new company enters in this way and existing companies can no longer vary their product characteristics, what happens to car prices? In what way can we view this as two distinct Hotelling models?

**Answer:** We know that consumers whose ideal point lies to the left of 0.5 will buy either from firms 1 (whose product characteristic is 0) or from the new entrant — and consumers whose ideal point lies above 0.5 will either buy from firm 2 or from the new entrant. We can then consider the interval \([0,0.5]\) as a Hotelling model with firms 1 and 3 located at the extremes of the interval, and we can consider the interval \([0.5,1]\) as a second Hotelling model with firms 3 and 2 located at the extremes. Considering each model separately, we know that the equilibrium will involve each firm in each model to set the same price and thus split the market — with firms 1 and 2 getting one fourth of the overall \([0,1]\) market and firm 3 getting half the market (in the interval \([0.25,0.75]\)). Since products are less differentiated, price competition will imply that price will be lower if the third firm enters the market.

(c) How much profit would the new company make relative to the original two?

**Answer:** Since the new company has twice the market share of the other two, it will make twice the profit of the other two.

(d) Suppose that the existing companies announce their prices prior to the new company making its decision on whether or not to enter. Suppose further that the existing companies agree to announce the same price. If the new company has to pay a fixed cost prior to starting production, do you think there is a range of fixed costs such that companies 1 and 2 can strategically deter entry?

**Answer:** Firms 1 and 2 know that, if firm 3 enters, they will each see their prices fall and their market share reduced — thus experiencing a decrease in profit. They would therefore be willing to lower their prices somewhat if this has the effect of keeping firm 3 out of the market. Suppose the fixed entry cost is such that firm 3 would make a small profit (when fixed costs are included in the profit calculations) by entering. Then, if the firms can credibly commit to a price prior to firm 3 entering, they can lower prices a little bit and turn firm 3’s profit from entering into a loss — thus causing firm 3 not to enter. As fixed costs fall, the amount that firms 1 and 2 would have to lower their price in order to deter entry increases — and eventually fixed entry costs will be sufficiently low such that the two firms would make less of a profit by lowering price and keeping half the market share than by just letting entry happen and restricting themselves to serving only a quarter of the market share. Thus, there is a range of fixed costs for which it would make sense for the two firms to announce a lower price that deters entry (assuming they can do so credibly.)

(e) What determines the range of fixed costs under which the existing companies will successfully deter entry?

**Answer:** We already reasoned through this in the previous part. The key for the incumbent firms is whether they will make more profit by lowering price and deterring entry or by just letting entry happen. In the former case, they keep half the market share each, in the latter they know they will be reduced to only a quarter of the market share.

(f) If the existing companies had foreseen the potential of a new entrant who locates at 0.5, do you think they would have been as likely to engage in maximum product differentiation in order to soften price competition between each other?
Answer: Since the appearance of this entry causes them to lower price — either because of strategic entry deterrence or because of entry, they might instead have chosen to accept stiffer price competition between themselves by not maximally differentiating their products in order to make it less attractive for this potential entrant to come into the market.

(g) We have assumed throughout that the entrant would locate at 0.5. Why might this be the optimal location for the entrant?

Answer: If the entrant locates anywhere else, he differentiating his product less from one of the other firms — thereby engaging in fiercer price competition on one side of the market. By choosing 0.5, he is maximally differentiating his product from both other firms.

B: Consider the version of Hotelling’s model from Section 26B.2 and suppose that two oligopolistic car companies, protected by government regulations on how many firms can be in the car industry, have settled at the equilibrium product characteristics of 0 and 1 on the interval [0, 1]. Suppose further that \( a = 12,000 \) and \( c = 10,000 \) and assume throughout that car companies cannot change their product characteristics once they have chosen them.

(a) What prices are the two companies charging? How much profit are they making given that they do not incur any fixed costs (and given that we have normalized the population size to 1)?

Answer: Using the equations \( p_1^* (y_1, y_2) \) or \( p_2^* (y_1, y_2) \) derived in the text and plugging in \( c = 10,000, a = 12,000, y_1 = 0 \) and \( y_2 = 1 \), we get \( p_1^* = 22,000 \). Profit for each firm is then

\[
\pi^* = 0.5(p_1^* - c) = 0.5(22000 - 10000) = 6,000. \tag{26.52}
\]

Note that this is normalized given we normalized the population to size 1. With population of 1 million, this would imply a profit of $6 billion for each firm.

(b) Now suppose that the government has granted permission to a third company to enter the car market at 0.5. But the company needs to pay a fixed cost \( FC \) to enter. If the third company enters, we can now consider the intervals \([0, 0.5]\) and \([0.5, 1]\) separately — and treat each of these as a separate Hotelling model. Derive \( D^1(p_1, p_3) \). Then derive \( D^3(p_1, p_3) \) (taking care to note that the relevant interval is now \([0, 0.5]\) rather than \([0, 1]\).)

Answer: We can now consider a Hotelling model with two firms located at the extremes of the \([0,0.5]\) interval — i.e. \( y_1 = 0 \) and \( y_3 = 0.5 \). The consumer \( \bar{y} \) that is indifferent between shopping at these two companies would satisfy the equation

\[
p_1 + a(\bar{y} - y_1)^2 = p_3 + a(\bar{y} - y_3)^2. \tag{26.53}
\]

Substituting \( y_1 = 0 \) and \( y_3 = 0.5 \) and solving for \( \bar{y} \), we get

\[
\bar{y} = \frac{1}{4} + \frac{(p_3 - p_1)}{a} = \frac{1}{4} \cdot \frac{(p_3 - p_1)}{12,000}. \tag{26.54}
\]

This is then demand for firm 1’s cars — i.e.

\[
D^1(p_1, p_3) = \frac{1}{4} \cdot \frac{(p_3 - p_1)}{12,000}. \tag{26.55}
\]

Demand (from the interval \([0,0.5]\)) for firm 3’s cars is then \((0.5 - \bar{y})\) or simply

\[
D^3(p_1, p_3) = \frac{1}{4} \cdot \frac{(p_3 - p_1)}{12,000}. \tag{26.56}
\]

(c) Determine the best response functions \( p_1(p_3) \) and \( p_3(p_1) \). Then calculate the equilibrium price.

Answer: To determine firm 1’s best response function, we solve

\[
\max_{p_1} (p_1 - c) D^1(p_1, p_3) = (p_1 - 10,000) \left( \frac{1}{4} \cdot \frac{(p_3 - p_1)}{12,000} \right) \tag{26.57}
\]

which gives us
\[ p_1(p_3) = 6,500 + \frac{P_3}{2}. \] (26.58)

For firm 3, we similarly solve
\[ \max_{p_3} (p_3 - c)D(p_1, p_3) = (p_3 - 10,000) \left( \frac{1}{4} + \frac{(p_1 - p_3)}{12,000} \right) \] (26.59)
which gives us
\[ p_3(p_1) = 6,500 + \frac{P_1}{2}. \] (26.60)

Substituting \( p_3(p_1) \) into \( p_1(p_3) \) and solving, we get \( p_1 = 13,000 \), and substituting that back into \( p_3(p_1) \) we get the same for \( p_3 \). Thus, the equilibrium price is \( p^* = 13,000 \). (The same analysis using firms 2 and 3 leads to the same result.)

(d) How much profit will the 3 companies make (not counting the FC that any of them had to pay to get into the market)?

Answer: Firms 1 and 2 will make profit \( 0.25(p^* - c) = 0.25(13,000 - 10,000) = 750 \) since they each now have market share of \( 1/4 \) — while firm 3 will make twice that (since it has market share of 0.5). Thus,
\[ \pi_1 = \pi_2 = 750 \quad \text{and} \quad \pi_3 = 1,500. \] (26.61)

(e) If company 3 makes its decision of whether to enter and what price to set at the same time as companies 1 and 2 make their pricing decisions, what is the highest FC that will still be consistent with the new car company entering?

Answer: Since firm 3 knows that it will make a profit of 1,500 if it enters, the highest fixed cost consistent with entry when everyone moves simultaneously is \( FC = 1,500 \).

(f) Suppose instead that companies 1 and 2 can commit to a price before company 3 decides whether to enter. Suppose further that companies 1 and 2 collude to deter entry — and agree to announce the same price prior to company 3’s decision. What is the most that companies 1 and 2 would be willing to lower price in order to prevent entry?

Answer: The incumbent firms know that they will make a profit of 750 if they allow entry. The most they are willing to lower price is therefore an amount that will give them 750 in profit assuming firm 3 does not enter; i.e. the lowest \( \bar{P} \) they would be willing to set to keep firm 3 out has to satisfy
\[ 0.5(\bar{P} - 10,000) = 750. \] (26.62)
Solving this for \( \bar{P} \), we get \( \bar{P} = 11,500 \).

(g) What is the lowest FC that would now be consistent with company 3 not entering? (Be careful to consider firm 3’s best price response and the implications for market share.)

Answer: Were the incumbent firms to set their price to $11,500, firm 3’s best price response is given by \( p_3(p_1) \) as
\[ p_3(11500) = 6,500 + \frac{11,500}{2} = 12,250. \] (26.63)
But now firm 3 would not get half the market share because it is charging a price higher than the other two firms. To determine what market share it would get, we can determine what \( \pi \) is indifferent between firm 1 and firm 3; i.e. for what \( \pi \) does the following hold:
\[ 11,500 + 12,000(\pi^2) = 12,250 + 12,000(0.5 - \pi^2). \] (26.64)
Solving this, we get \( \pi = 0.3125 \) — which is firm 1’s market share. Firm 2’s market share is the same — leaving firm 3 with market share of 0.375 and profit (absent fixed costs) of
\[ \pi_3 = 0.375(12,250 - 10,000) = 843.75. \] (26.65)
Thus, firms 1 and 2 cannot deter entry unless \( FC > 843.75 \). If incumbent firms can collude to set an entry-deterrent price prior to firm 3 deciding whether to enter, they will therefore be able to do so for fixed costs between 843.75 and 1,500.
Policy Application: Lobbying for Car Import Taxes: In exercise 26.8, we investigated the incentives of existing car companies to deter entry of new companies through lowering of car prices. When the potential new car company is a foreign producer that wants to enter the domestic car market, an alternative way in which such entry might be prevented or softened is through government import fees and/or import tariffs.

A: Suppose throughout that the foreign car company has product characteristic 0.5 while the domestic companies are committed to the maximally differentiated product characteristics of 0 and 1 in the Hotelling model.

(a) Suppose first that the government requires the foreign car company to pay a large fee for the right to import (as many cars as it would like) into the domestic market. If the government makes any revenue from this policy, will it have any impact on the car market when all decisions are made simultaneously?

Answer: The foreign firm will enter so long as the fixed fee is smaller than the profit it will make from entering and competing. If the government does not make any revenue from the fee, it means that the fee was larger than the profit that the foreign firm could have made had it entered — and the fee is therefore large enough to completely protect the domestic industry from foreign competition. If the government does make revenue, it is because the fee was smaller than the profit the foreign firm can make — but since the fee becomes a sunk cost after entry, it does not matter how large the fee is (conditional on the foreign firm still entering). The equilibrium will be the same whether there is a fee or not (as long as the fee is low enough to result in entry). Thus, if the government makes revenue on the fee, the equilibrium is the same as if there were no fee.

(b) For a given fee $F$, why might the domestic car industry expend zero lobbying effort on behalf of this policy? Why might it expend a lot?

Answer: If $F$ is below the threshold level that keeps the foreign firm out of the market, then the fee offers no protection to the domestic car industry — and the industry would therefore not be willing to expend resources to get the fee implemented. If, on the other hand, the fee lies above the threshold that keeps the foreign firm out, the industry gets protected from foreign competition and is thus willing expend resources to get such protection.

(c) Suppose domestic firms can collude on setting a price in anticipation of entry (and can credibly commit to that price). True or False: There is now a range of $F$ under which the foreign company does not enter when it would have entered given conditions in (a). (Assume that if entry occurs, the industry plays the simultaneous Nash pricing equilibrium.)

Answer: This is true. If the firms can move sequentially in this way, the domestic industry can lower price in order to strategically deter entry. It will be willing to do so as long as the resulting profit from splitting the market (in the absence of the foreign firm) is greater than the profit from allowing foreign competition. For fees above what is required for the foreign firm to remain outside the domestic industry in the simultaneous move case, domestic firms would not need to engage in strategic entry deterrence. But for fees just below this, domestic firms can lower their price just a bit and thus make entry less profitable — sufficiently less for entry not to occur at lower fixed entry costs.

(d) Under the conditions in (c), does your answer to (a) change? Is there now a range of fees under which the foreign company does not enter the market but domestic companies lobby for higher fees?

Answer: The answer to (a) does not change. It is still the case that, if entry happens and a fee is paid, the simultaneous Nash pricing game is played and the fee that was paid by the foreign firm becomes a sunk cost. Thus, if the government collects revenue — which is to say, if the fee was low enough to result in entry of the foreign firm — then the equilibrium outcome is the same as if there were no fee. But now there is a range of fees under which the foreign firm is not entering because domestic firms engage in strategic entry deterrence by lowering their prices. When fees are in this range, domestic firms have an incentive to lobby for higher fees even though there is no foreign entry at the current fees — because this will allow domestic firms to raise prices.

---

1We are therefore not considering the case where domestic firms become price leaders, a case we analyze separately in exercise 26.6.
(e) Suppose that instead the government imposes a per-unit tax $t$ on all imported cars. Compared to what would happen in the absence of any government interference, how do you think domestic and foreign car prices will be affected?
Answer: Foreign cars will be more expensive because of the higher marginal costs incurred by the foreign firm — and domestic cars will be more expensive as each of the domestic firms now competes with a competitor that is charging higher prices than before.

(f) How will market share of domestic versus foreign cars differ under the tariff?
Answer: Given the competitive disadvantage for foreign cars, market share for foreign cars should shrink while market share for domestic cars expands.

(g) Suppose the government imposes the lowest tariff that results in no foreign cars being sold. Do you think that domestic car companies can now charge the same price they would if foreign cars were prohibited from the domestic market outright?
Answer: When the lowest tariff that results in no imports is imposed, the three firms — i.e. the two domestic firms and the foreign firm — are still playing a simultaneous game and are best responding to one another. This implies that the foreign firm chooses to import nothing as a result of the fact that, at the optimal foreign car price, consumer $n = 0.5$ is indifferent between consuming with the foreign firm and consuming from either of the domestic firms. The domestic firms in turn best respond to the foreign car price that results in no foreign car sales. But the domestic firms are still having to respond to the foreign price! Were they to raise price, the foreign firm would best respond in a way that would result in sales of foreign cars — and less profit for domestic car firms. Even though the foreign firm is not selling any cars domestically, its presence still affects the equilibrium and constrains domestic car prices. This would not be the case if foreign imports were simply prohibited.

(h) Based on your answer to (g), might domestic firms lobby for higher import tariffs even if no cars are imported at current tariff levels?
Answer: Yes, given our answer to (g), there is a range of tariffs under which the foreign firm does not import in equilibrium — but domestic firms are constrained by the fact that the foreign firm would import if the domestic firms raised their prices back to what they were before there was any threat of entry.

B: Consider again, as in exercise 26.8, the version of the Hotelling model from Section 26B.2 with the domestic car companies having settled at the equilibrium product characteristics of 0 and 1 on the interval $[0,1]$. Suppose again that $a = 12,000$ and $c = 10,000$. Assume throughout that domestic companies cannot change their product characteristics.

(a) If you have not already done so, do parts (a) through (e) of exercise 26.8.
Answer: These are solved in the preceding exercise.

(b) Suppose that the government required the foreign company to pay a fee $F$ in order to access the domestic market (without placing any restrictions on how many cars can be imported). Suppose there is no way for domestic firms to credibly commit to prices prior to the foreign firm deciding whether or not to enter. What is the lowest $F$ that the domestic industry would lobby for assuming there are no other fixed entry costs? Would lobbying efforts be more intense for imposition of a higher fee?
Answer: We know that, if the foreign firm enters, it will make a profit of 1,500 (not counting fixed costs). Thus, if there are no other entry costs, the lowest fee $F$ that will keep the foreign company out of the domestic market is $F = 1,500$. For any fee lower than this, the foreign firm would enter and produce as if there were no fee — leaving the domestic firms just as affected as if there were no fee. For any fee higher than this, the foreign firm would also not enter. Thus, domestic firms would lobby for $F = 1500$ and would have no reason to lobby for a fee higher than this.

(c) How would your answer change if the domestic firms could credibly commit to a price prior to the foreign firm deciding on whether or not to enter? (Assume that the domestic firms agree to announce the same price.) For what range of $F$ will domestic firms push to increase $F$? (Note: It is helpful to reason through (f) and (g) of exercise 26.8 prior to attempting this part.)
Answer: We calculated in the previous exercise that domestic firms will make a profit of 750 each if there is an entrant at product characteristic 0.5. The most they would therefore be
willing to lower their price in order to prevent entry is an amount that results in profit of 750. Under any price that successfully deters entry, the domestic firms continue to split the market — so their profit is \((p - 10,000)(0.5)\). Setting this to 750 and solving for \(p\), we get the lowest price the domestic firms would be willing to go to in order to prevent entry. This gives us \(p = \$11,500\). The foreign firm’s best price response to a domestic price of \(\$11,500\) is then \(p_2 = 6,500 + (11,500/2) = \$12,250\). With domestic firms charging \(\$11,500\) and the foreign firm charging \$12,250 per car, the consumer \(\overline{\theta}\) who is indifferent between domestic firm 1 (with \(y_1 = 0\)) and the foreign firm satisfies the equation

\[
11,500 + 12,000\overline{\theta}^2 = 12,250 + 12,000(0.5 - \overline{\theta})^2
\]

which solves to \(\overline{\theta} = 0.3125\). This, then, would be the market shares for each of the two domestic firms — leaving the foreign firm with market share \((1 - 2(0.3125)) = 0.375\) and profit (before fixed costs) of \(0.375(12,250 - 10,000) = 843.75\). Thus, the fee \(F\) can now go as low as 843.75 and still keep the foreign company out. Thus, for fees between 843.75 and 1,500, firms would now lobby for an increase in the fee to keep them from having to lower price as much. (For fees greater than 1,500, firms do not have to lower price in order to prevent entry — so they would be indifferent as to what level the fees reach once they are above 1,500.)

(d) Next, suppose that instead the government imposed a per-unit tariff of \(t\) on all car imports. Treat this as an increase in the marginal cost for importing firms — from \(c\) to \((c + t)\). Derive the equilibrium prices charged by domestic firms and importing firms as a function of \(t\). (Follow the same steps as in B(c) and (d) of exercise 26.8.) What can you say about the tax incidence of this tariff?

Answer: We again consider a Hotelling model with two firms located at the extremes of the \([0,0.5]\) interval — i.e. \(y_1 = 0\) and \(y_3 = 0.5\). The consumer \(\overline{\theta}\) that is indifferent between shopping at these two companies would satisfy the equation

\[
p_1 + a(\overline{\theta} - y_1)^2 = p_3 + a(\overline{\theta} - y_3)^2.
\]

Substituting \(y_1 = 0\) and \(y_3 = 0.5\) and solving for \(\overline{\theta}\), we get

\[
\overline{\theta} = \frac{1}{4} \frac{(p_3 - p_1)}{a} = \frac{1}{4} \frac{(p_3 - p_1)}{12,000}.
\]

This is then demand for firm 1’s cars — i.e.

\[
D^1(p_1, p_3) = \frac{1}{4} \frac{(p_3 - p_1)}{12,000}.
\]

Demand (from the interval \((0.5, 1]\)) for firm 3’s cars is then \((0.5 - \overline{\theta})\) or simply

\[
D^3(p_1, p_3) = \frac{1}{4} \frac{(p_3 - p_1)}{12,000}.
\]

So far, this derivation is identical to that without \(t\) since we have not yet considered the cost side for the firm. To determine firm 1’s best response function, we still do not have to consider \(t\) (since firm 1 does not incur this additional cost) and therefore again solve

\[
\max_{p_1} (p_1 - c)D^1(p_1, p_3) = (p_1 - 10,000) \left( \frac{1}{4} + \frac{(p_3 - p_1)}{12,000} \right)
\]

which gives us

\[
p_1(p_3) = 6,500 + \frac{p_3}{2}.
\]

For firm 3, however, we have to consider the new marginal cost of \((c + t)\) and therefore solve

\[
\max_{p_3} (p_3 - (c + t))D^3(p_1, p_3) = (p_3 - 10,000 - t) \left( \frac{1}{4} + \frac{(p_1 - p_3)}{12,000} \right)
\]

(26.73)
which gives us

\[ p_3(p_1) = 6,500 + \frac{p_1 + t}{2}. \]  

(26.74)

Substituting \( p_3(p_1) \) into \( p_1(p_3) \) and solving, we get \( p_1 = 13,000 + (t/3) \), and substituting that back into \( p_3(p_1) \) we get \( p_3 = 13,000 + (2t/3) \). Thus, the firms charge different prices in equilibrium, with

\[ p_1^* = p_2^* = 13,000 + \frac{t}{3} \quad \text{and} \quad p_3^* = 13,000 + \frac{2t}{3}. \]  

(26.75)

As far as tax incidence, we see from these prices that a third of the tax is being passed to domestic consumers of cars.

(e) Derive the market share for firm 1 (and thus also firm 2) as a function of \( t \). What level of \( t \) will restrict foreign imports to the same level as an import quota that limits foreign cars to one third of the market (assuming no fixed entry costs)?

Answer: To find firm 1’s market share, we find the consumer \( \overline{m}(t) \) that is indifferent between shopping at firm 1 and shopping at firm 3. This consumer is defined by

\[ p_1^* + \alpha \overline{m}^2 = p_3^* + \alpha(\overline{m} - 0.5)^2 \]  

(26.76)

which becomes

\[ 13,000 + \frac{t}{3} + 12,000\overline{m}^2 = 13,000 + \frac{2t}{3} + 12,000(\overline{m} - 0.5)^2 \]  

(26.77)

when we substitute for the equilibrium prices and for \( \alpha = 12,000 \). Solving for \( \overline{m} \), we get

\[ \overline{m}(t) = \frac{1}{4} + \frac{t}{36,000}. \]  

(26.78)

Firms 1 and 2 then have market share of \( 2\overline{m}(t) \), leaving firm 3 with market share

\[ 1 - 2\overline{m}(t) = \frac{1}{2} - \frac{t}{18,000}. \]  

(26.79)

Setting this equal to \( 1/3 \) and solving for \( t \), we can then derive \( t = 3,000 \) as the per unit tariff that will limit the foreign car market share to one third of domestic sales.

(f) What is the lowest level of \( t \) that guarantees no foreign cars will be sold in the domestic market (assuming no fixed entry costs)?

Answer: Zero market share for foreign cars will arise when \( t \) is sufficiently high such that \( \overline{m}(t) = 1/2 \). Thus, we solve

\[ \overline{m}(t) = \frac{1}{4} + \frac{t}{36,000} = \frac{1}{2} \]  

(26.80)

for \( t \) to get \( t = 9,000 \).

(g) What prices will domestic car companies charge if \( t \) is set to \( T \)?

Answer: Plugging \( T = 9,000 \) into our equations for \( p_1^* \) and \( p_2^* \) from part (d), we get

\[ p_1^* = p_2^* = 13,000 + \frac{9,000}{3} = 13,000 + \frac{9,000}{3} = 16,000. \]  

(26.81)

(b) Explain why setting \( T \) differs from the case where the import of foreign cars is prohibited.

Answer: In the absence of any threat from a third firm, we concluded that \( p^* = 22,000 \). The same holds if firms simultaneously choose price — and there is a fee \( F \) high enough to keep firm 3 from paying it. Under \( T = 9,000 \), however, we conclude that firm 3 will not sell any foreign cars and the domestic firms will set price \( p^* (T) = 16,000 \). The reason for this is that, were firms 1 and 2 to set a higher price, firm 3 would indeed enter and sell cars despite the tariff \( T \). Firms 1 and 2 are best responding to firm 3’s best response function which tells
firm 3 to sell cars in the domestic market if firms 1 and 2 charge a price higher than $16,000. This is different from the case where firm 3 does not pay a fixed fee because it knows that the equilibrium results in a profit less than the fixed fee. Entry in this case is a discrete decision that sets a simultaneous move game. In the tariff case, there is no such discrete decision — and saying that firm 3 does not sell in the domestic market when a tariff of \( t \) is set is not the same as saying it has not "entered."

(i) **What level of \( t > T \) is equivalent to prohibiting the entry of the foreign firm?**

**Answer:** It would be a level of \( t \) such that firms 1 and 2 can charge what they would charge in the absence of any threat of entry — which we calculated earlier to be $22,000. In order for \( n = 0.5 \) to be indifferent between buying from firm 1 at \( p_1 = 22,000 \) and buying from 3, then \( p_3 \) is set such that

\[
p_3 = p_1 + \alpha(0.5^2) = 22,000 + 12,000(0.5^2) = 25,000.
\]

Equation (26.74) gives us firm 3's best response to \( p_1 \) as

\[
p_3(p_1) = 6,500 + \frac{p_1 + t}{2}.
\]

Plugging in \( p_3 = 25,000 \) and \( p_1 = 22,000 \) (so that firm 1 charges the price it would in the absence of a threat and firm 3 charges a price such that even \( n = 0.5 \) is indifferent between the two firms), we get

\[
25,000 = 6,500 + \frac{22,000 + t}{2}
\]

which solves to \( t = 15,000 \). This is then the lowest level of tariff at which firms 1 and 2 behave the same as if there were no firm 3 to worry about.
26.10 Business and Policy Application: The Software Industry. When personal computers first came onto the scene, the task of writing software was considerably more difficult than it is today. Over the following decades, consumer demand for software has increased as personal computers became prevalent in more and more homes and businesses at the same time as it has become easier to write software. Thus, the industry has been one of expanding demand and decreasing fixed entry costs.

A: In this part of the exercise, analyze the evolution of the software industry using both the monopolistic competition model from Section 26A.4 as well as insights from our earlier oligopoly models.

(a) Begin with the case where the first firm enters as a monopoly — i.e. the case where it has just become barely profitable to produce software. Illustrate this in a graph with a linear downward sloping demand curve, a constant MC curve and a fixed entry cost.

Answer: This is done in panel (a) of Graph 26.4 where the usual monopoly picture (of price being set at the quantity where \( MR = MC \)) is supplemented by the inclusion of the \( AC \) curve that includes the fixed entry cost. If it has just become profitable for a single firm to enter, this \( AC \) curve must be tangent at \( A \) on the demand curve where profit maximizing production occurs. This then implies that \( AC = p^M \) which in turn implies a profit from entering of zero (even as the firm makes positive profits once it has entered and fixed costs are sunk).

Graph 26.4: Evolution of software industry

(b) Suppose that marginal costs remain constant throughout the problem. In a separate graph, illustrate how an increase in demand impacts the profits of the monopoly and how a simultaneous decrease in fixed entry costs alters the potential profit from entering the industry.

Answer: This is done in panel (b) of Graph 26.4 where demand increases from the (dashed) \( D \) to the (solid) \( D' \). As a result, the monopoly price increases — thus raising monopoly profits for the firm that is in the market. This by itself already increases to profit for any potential firm that is considering entry — and this is further exacerbated by a drop in \( AC \) due to falling fixed entry costs.

(c) Given the possibility of strategic entry deterrence, what might the monopolist do to forestall entry of new firms?

Answer: The monopolist might decide to charge a price below \( p^M \) in order to make it less profitable for entrants to come into the market. In order to do this, the monopolist has to be able to credibly commit to a price even if entry of new firms happens.

(d) Suppose the time comes when a strategic entry deterrence is no longer profitable and a second firm enters. Would you expect the entering firm to produce the same software as the existing firm? Would you expect both firms to make a profit at this point?

Answer: The second firm has an incentive to differentiate its product so as not to set of the fiercest form of Bertrand competition. As a result, both firms will make a positive profit.
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when the second firm enters — with entry costs still sufficiently high to forestall the entry of a third firm.

(e) As the industry expands, would you expect strategic entry deterrence to play a larger or smaller role? In what sense is the industry never in equilibrium?

Answer: Strategic entry deterrence with more than one incumbent firm would imply coordination of pricing by the incumbent firms — which becomes increasingly difficult as there are more incumbent firms. One would therefore expect a decline in strategic entry deterrence as the industry grows. The industry is never in equilibrium as long as demand continues to rise and fixed entry costs continue to fall — with new entry of firms becoming profitable as time passes.

(f) What happens to profit for firms in the software market as the industry expands? What would the graph look like for each firm in the industry if the industry reaches equilibrium?

Answer: As the market expands, competition becomes more intense — implying that firm profits will fall with time. If the industry were to reach equilibrium, it would imply that all firms in the industry are making positive profit but entry costs are such that potential entrants make non-positive profits. This implies that profits including fixed entry costs are zero. This is illustrated for a monopolistically competitive firm in an industry that has reached equilibrium in panel (c) of Graph 26.4.

(g) If you were an antitrust regulator charged with either looking out for consumers or maximizing efficiency, why might you not want to interfere in this industry despite the presence of market power? What dangers would you worry about if policymakers suggested price regulation to mute market power?

Answer: Your biggest fear might be that regulating this market will dampen innovation and will thus lead to a less diverse set of offerings of software for consumers. While it is true that, at any given moment, it is possible to raise overall surplus through price regulation (because of the presence of market power), it is also true that regulation to address this would dampen the incentives for new firms to emerge — and thus would cause future deadweight losses to be larger.

(h) In what sense does the emergence of open-source software further weaken the case for regulation of the software industry? In what sense does this undermine the case for long-lasting copyrights on software?

Answer: The emergence of open-source software provides another disciplining force on firms in the software industry. The threat of open source software supplanting a firm’s software product provides incentives for software firms to continue to innovate and keep price relatively low — exactly what one would want in order to maximize both short run and long run efficiency and consumer welfare. The fact that the software industry continues to thrive despite open source software competition suggests that innovation in this market may not be primarily due to the existence of patents and copyrights.

B: In this part of the exercise, use the model of monopolistic competition from Section 26A.4. Let disposable income I be $100 billion, $\rho = -0.5$ and marginal cost $c = 10$.

(a) What is the assumed elasticity of substitution between software products?

Answer: The elasticity of substitution $\sigma$ is

$$\sigma = \frac{1}{1 + \rho} = \frac{1}{1 - 0.5} = 2.$$  \hfill (26.85)

(b) Explain how increasing demand in the model can be viewed as either increasing $I$ or decreasing $\alpha$. Will either of these change the price that is charged in the market? Explain.

Answer: The Cobb-Douglas functional form into which the $y$ goods are incorporated implies that overall spending on the software goods is a fraction $(1 - \alpha)$ of income. Thus, the overall share of income devoted to software increases in income and decreases in $\alpha$. The equilibrium price in this model, however, is simply

$$p^* = \frac{c}{\rho} = \frac{10}{(-0.5)} = 20.$$  \hfill (26.86)
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which implies neither $\alpha$ nor $I$ has an impact on price. (Price remains constant so long as $c$ and $\rho$ are unchanged because more firms will simply enter the industry to meet increased demand.)

(c) We noted in part A of the exercise that fixed entry costs in the software industry have been declining. Can that explain falling software prices within this model?

Answer: Since $FC$ does not appear in the equilibrium equation for $p^*$, a decline in fixed entry costs cannot explain a change in price.

(d) True or False: As long as the elasticity of substitution between software products remains unchanged, the only factor that could explain declining software prices in this model is declining marginal cost. (Can you think of real world changes in the software industry that might be consistent with this?)

Answer: If the elasticity of substitution does not change, then $\rho$ does not change — which only leaves $c$ in our equation for $p^*$ to alter the equilibrium price. Falling software prices can thus only be explained (as long as $\rho$ stays constant) by decreasing marginal costs. As the software industry has developed, for instance, distribution has increasingly shifted to downloads from the internet rather than distribution through brick-and-mortar stores — which could be viewed as a decrease in marginal cost.

(e) Now consider how increases in demand and decreases in costs translate to the equilibrium number of software firms. Suppose $\alpha = 0.998$ initially. What fraction of income does this imply is spent on software products? How many firms does this model predict will exist in equilibrium under the parameters of this model assuming fixed entry costs are $100 million? What happens to the number of firms as $FC$ falls to $10 million, $1 million and $100,000? 

Answer: With $\alpha = 0.998$, the fraction of income spent on software is 0.002 or 0.2% of $I$. The equilibrium number of firms in this model is

$$N^* = \frac{(1 - \alpha)(1 + \rho)I}{FC}.$$  \hspace{1cm} (26.87)

With income of $100 billion, $\alpha = 0.998$, and $\rho = -0.5$, we then get 1 firm when $FC$ is equal to $100 million, 10 software firms when $FC$ is equal to $10 million, 100 firms when $FC$ is $1$ million and 1,000 firms when $FC$ is $100,000.$

(f) Suppose that $FC$ is $1,000,000. What happens as $\alpha$ falls from 0.998 to 0.99 in 0.002 increments as demand for software expands through changes in representative tastes when more consumers have computers?

Answer: Using the same formula for $N^*$, the model predicts 100 firms when $\alpha = 0.998$ — increasing to 200 firms when $\alpha = 0.996$, 300 firms when $\alpha = 0.994$, 400 firms when $\alpha = 0.992$ and 500 firms when $\alpha = 0.99.$

(g) Suppose $FC$ is $1,000,000 and $\alpha = 0.99.$ What happens if demand increases because income increases by 10%?

Answer: The number of firms would increase from 500 to 550.
26.11 Policy Application: To Tax or Not to Tax Advertising. In the text, we discussed two different views of advertising — one of which we said arises primarily from an economist’s perspective, the other primarily from a psychologist’s. The nature of public policy toward the advertising industry will depend on which view of advertising one takes.

A: Consider the two views — informational advertising and image marketing.

(a) In what sense does information advertising potentially address a market condition that represents a violation of the first welfare theorem?

Answer: Information advertising is aimed at providing information to consumers who otherwise lack sufficient information to make informed choices. As such, there is an asymmetric information problem in the market — firms know something that consumers do not. Thus, information advertising addresses an asymmetric information problem and can thus be efficiency enhancing — and may even be the efficient way for information to be conveyed. At the same time, the market may end up providing too little information — or it may provide too much. Still, it is an example of how a violation of the first welfare theorem may be addressed by the formation of a new market.

(b) In what sense does image marketing result in potentially negative externalities? Might it result in positive externalities?

Answer: If image marketing creates artificial preferences for products that are otherwise identical, then firms are simply using it to artificially create the appearance of product differentiation in order to soften price competition. In this case, image marketing creates a negative externality for consumers because it imposes costs on consumers who are not able to consume at their artificially created ideal points — and it creates market power that raises prices. Both parts of consumer costs — the prices they pay and the costs they incur from not consuming at their ideal points — are therefore raised without any meaningful underlying product differentiation. At the same time, it may be that image marketing actually changes products in a way that is not only meaningful to consumers (in the sense that they respond to it) but that also makes consumers better off (because they value both the product and the image of the product in an absolute sense). In that case — the case where, for instance, children get added utility from seeing cartoon characters on cereal boxes — it may be that image marketing creates positive externalities. If the welfare gain from increased utility on the part of consumers is sufficiently large, this may result in the benefits outweighing the costs from higher prices (that result from softened price competition).

(c) If you wanted to make an efficiency case for taxing advertising, how would you do it? What if you wanted to make an efficiency case for subsidizing it?

Answer: Within the model of information advertising, you would have to argue that the advertising market provides too much information — with a tax on advertising reducing the amount to a more efficient level. Alternatively, within the model of information advertising, you might find that firms provide inefficiently low levels of information — and a subsidy can get them closer to the efficient level. Within the model of image marketing, a tax can be efficiency enhancing if image marketing simply causes consumers to develop greater preferences for one good over another without changing their absolute utility from consuming the good. On the other hand, if image marketing also creates an increase in the absolute level of enjoyment of goods, one might even argue that advertising should be subsidized.

(d) Suppose a public interest group lobbies for regulatory limits on the amount of advertising that can be conducted. Explain how this might serve the interests of firms?

Answer: This is most easily seen in a model of information advertising where firms increase price competition by informing consumers of the presence of consumer goods. The Nash equilibrium may well be an equilibrium to a game that is a prisoners’ dilemma for firms: All of them could make greater profit under less intense price competition in the absence of information advertising, but, regardless of what all other firms do, it is in the best interest of each firm to advertise its own products. The result is fiercer price competition which is good for consumers and bad for firms. If firms could then have the government enforce a collusive agreement to advertise less, they can essentially use the government to escape the prisoners’ dilemma — thus leaving consumers worse off but themselves better off.
Consider the three-stage image marketing model in Section 26B.6 but assume that $f(a_1, a_2) = a_1^{1/2} + a_2^{1/2}$. Suppose further that the cost for consumer $n$ from consuming $y$ is $a(n - y)^2 - y\alpha$, with $\gamma = 0$ unless otherwise stated.

(a) Solving the game backwards (in order to find subgame perfect equilibria), does anything change in stages 2 and 3 of the game?

Answer: No, nothing changes. In stage 3, firms will set prices given the level of product differentiation $y_1$ and $y_2$ in stage 2 — and these best response price functions take $\alpha$ as given (just as derived earlier in the text). In stage 2, firms choose their product characteristics $y_1$ and $y_2$ — and we have shown that they will choose maximal differentiation $y_1 = 0$ and $y_2 = 1$ for $\alpha > 0$ given the other assumptions of the model.

(b) What would be the advertising levels chosen by each firm.

Answer: Each firm $i$ in stage 1 then maximizes
\[
\max_a (p(a_1, a_2) - c)a - \frac{1}{2}c_\alpha a_i = (a_1^{1/2} + a_2^{1/2} - c) \frac{1}{2} - c_\alpha a_i
\]
which results in equilibrium levels of advertising of
\[
a_i^* = \frac{1}{4c_\alpha}
\]
(c) Suppose the two firms can collude on the amount of advertising each undertakes (but the rest of the game remains the same). Would they choose different levels of $a_1$ and $a_2$?

Answer: If the two firms maximized joint profit in the first stage (with the rest of the game remaining the same — i.e. no collusion in the rest of the game), then they maximize
\[
\max_{a_1, a_2} \pi = (p(a_1, a_2) - c)a - \frac{1}{2}c_\alpha a_1 + (p(a_1, a_2) - c) \frac{1}{2} - c_\alpha a_2 =
\]
\[
= (p(a_1, a_2) - c) - c_\alpha(a_1 + a_2)
\]
or, substituting $p(a_1, a_2) = a_1 + a_2$,
\[
\max_{a_1, a_2} \pi = (a_1^{1/2} + a_2^{1/2} - c) - c_\alpha(a_1 + a_2).
\]
The first order conditions to this optimization problem then imply
\[
a_i^* = a_i^* = \frac{1}{4c_\alpha}
\]
just as in the case where they optimize separately. Thus, there are no gains from colluding in advertising.

(d) For what level of $\gamma = \frac{7}{12}$ is there no efficiency case for either subsidizing or taxing advertising?

Answer: For consumer costs of $a(n - y)^2$, we concluded in Section B.2.3 that the cost incurred by consumers (when firms maximally differentiate their products) is $a/12$. (The reasoning here is identical to the text’s). To offset this cost given the decrease in consumer costs from $\gamma a$, we therefore have to have $\gamma = 1/12$ in order for image advertising to have no efficiency consequences. (While firms will be able to charge higher prices as a result of the product differentiation — and thus consumers will be worse off from these higher prices, the model assumes consumers always buy 1 unit of the good, making the increased price a simple transfer from consumers to producers.)

If $\gamma > 1/12$, image marketing is efficient, and if $\gamma < 1/12$ is it inefficient.

(e) Is there any way to come to a conclusion about the level of $\gamma$ from observing consumer and firm behavior?

Answer: No — the behavioral predictions are the same regardless of the value of $\gamma$. This is because $\gamma$ does not enter the firms’ optimization problems — and consumers are always assumed to buy one unit of the good.