III.  **Topics in Geometry: Introduction to Tessellations**  
*By Bud Maher*

**Background:** This lesson is designed to introduce the students of a regular eighth-grade math class to the concepts of symmetry and tessellation. Included in the activity is a three-level study guide to assist the student with comprehension of the text material they will encounter during the lesson. Examples of tessellations abound in nature, art, and architecture. Exploration of this topic provides tangible examples of symmetry through which the student can develop considerable geometric understanding. The assigned text material for this lesson is web-based, and the sites I have selected explain the material more clearly than any middle-school math text I have yet seen.

I made use of the following texts in the creation and design of this lesson:

Wadsworth/Thomson Learning

Longman Press

These are the Websites containing the text material that the students will refer to:

1. What is a tessellation?
   [http://mathforum.org/sum95/suzanne/whattess.html](http://mathforum.org/sum95/suzanne/whattess.html)

2. The four types of symmetry in the plane
   [http://mathforum.org/sum95/suzanne/symsusan.html](http://mathforum.org/sum95/suzanne/symsusan.html)

3. Historical and geographic connections
   [http://mathforum.org.edu/sum95/suzanne/historytess.html](http://mathforum.org.edu/sum95/suzanne/historytess.html)

**Purpose:** The purpose of the lesson is to prepare students for an in-depth study of the topics. This activity would be followed with an assignment tasking the student to use *The Geometers Sketchpad*, a geometric software program, to create their own original tessellation, and requiring them to explain the processes involved in doing so. **The purpose of the accompanying three-level guide is to assist the student in comprehension of text they will encounter in their visits to the assigned Websites.**

**Objectives:** TSW:

- Understand and explain what tessellations are.
- Understand and explain the requirements for creating regular and semi-regular tessellations in the plane.
- Understand and explain the four symmetries
- Gain an appreciation of the copious nature and aesthetic qualities of various tessellations found in nature, art, and architecture.

**Activity:** Bookmark the URLs provided into the classroom computers, or copy and paste them into a text document to give to the students. As an in-class or flextime activity, or as a take-home assignment (depending on accessibility of computers at home,) tell the students to visit the
Websites in order, from top to bottom, as listed above. Give them each a copy of the three-level guide found on the last page of this lesson plan.

Tell them to visit the first two Websites and to read the material found there. (The third Website is a collection if images of tessellations organized geographically.) Tell them to then complete the three-level guide to check their own understanding of what they have read. After they have done this they should re-visit the two sites to clear up any confusion or misunderstandings that may have been illuminated in the process of completing the three-level guide.

For homework, or as an in-class activity during another instructional block, give the students the task of writing an essay, a few paragraphs in length, answering the following questions:

1. What is the difference between regular and semi-regular tessellations?
2. How are they similar? Hint: What are the requirements for a polygon or group of polygons to tessellate infinitely?
3. Can you find one example of each of the types of symmetry in the collection of tessellation images in the third Website? Give the URL of the example you select, and state which type of symmetry is illustrated.

**Key**

1. Regular tessellations are composed of only one type of polygon. Only regular triangles, regular hexagons, and rectangles can be tessellated infinitely in a regular or “pure” tessellation.
2. Whether regular or semi-regular, infinite tessellations require that the sum of adjacent angles in each vertex total to 360 degrees.
3. Students should provide at least one example of each of the four symmetries, rotation, translation, reflection, and glide/reflection, from the images found in the third Website. There are too many examples of tessellations in the collection of images to bother creating a key. You might have students copy and paste each image they select into a text document with labels for each. If you are concerned about possible copyright infringement issues, then visit the site of each example to confirm that the examples provided are correct. I am not providing a key for this segment. If you cannot recognize examples of the four symmetries, then I advise you to do the activity yourself first!

**Follow-up:** This activity should be followed by an assignment tasking the students to create their own original tessellations. This could be done by drawing or cutting out polygons and gluing them to a piece of cardboard. Alternatively, the students could use The Geometer’s Sketchpad or a similar learning software to create their tessellations. A very nice tutorial in how to use Sketchpad to create tessellations can be found at:

http://mathforum.org/sum95/suzanne/tess.gsp.tutorial.html

**Study Guide**

After visiting the first two Websites, complete this study guide to check your own understanding of the material. If needed, go back to the sites and re-read to clear up any misunderstandings you might have. This step is very important, and will help you make sure you have understood the big ideas!
1. Check what the author said. Check three.

______ The dictionary defines tessellations as formed by arranging small square tiles in a checkered or mosaic pattern.
______ A regular tessellation is composed of regular, congruent polygons.
______ Almost any regular polygon will tessellate infinitely.
______ Semi-regular tessellations are composed of more than one type of polygon.
______ Tessellations originated in Greek Architecture in the second century A.D.

2. What did the author mean? Check three.

______ Regular and semi-regular tessellations can “tile” the plane infinitely only if the sum of the angles at each vertex totals 360 degrees.
______ Only triangles, squares, and hexagons will create regular tessellations.
______ Rotation and glide/reflection alter the pattern of symmetry in angles at the vertices.
______ Translation is like sliding a shape or polygon in a straight line, and then flipping it over.
______ For every reflection there must be a line of symmetry called a “mirror line.”

3. What examples of tessellation can be found in our own lives? Check any.

______ Tiles in our bathrooms and on our floors
______ Shapes of flowers and leaves on plants
______ Works of art
______ Architectural design of buildings and houses
______ The “Create your own tessellations” project my teacher is going to assign me next week