Graphing Calculator Manual
TI-83

Kathy V. Rodgers
University of Southern Indiana
Evansville, Indiana

to accompany
Trigonometry, 5th Ed.
Charles P. McKeague and Mark Turner
Preface

Technology, used appropriately, enhances the teaching and learning of mathematics. The purpose of this manual is to provide sequences of keystrokes for developing calculator skills, to assist students in interpreting calculator screens, and to relate the capabilities of technology with students’ analytical skills. The ultimate goal is to deepen the students’ understanding of trigonometry and its application to problem solving.

How to use this Manual.

Anytime you are asked to complete a command that is in capital letters, then you are being asked to press a specific calculator key. For example if the directions say ENTER then you should press the ENTER button on your calculator; however, if the directions ask you to enter 4, then you are being asked to enter the number 4--press the 4 key. After you are given a sequence of keys to press, you will be given a calculator screen to compare with the screen of your calculator.

Alert or Note

**ALERT** will be used when caution needs to be exercised when using the calculator. For example, $\frac{1}{2x}$, cannot be entered in the calculator as 1/2x.; the calculator would interpret this as $\frac{1}{2}x$. Hence you would be alerted that you must use parentheses and enter this as 1/(2x). The word **Note** will precede additional information or the interpretation of a calculator screen.

Explanation of Exercises from the Text

Actual problems from the text are worked in this manual. Each section will be identified and the specific problem number will be in bold print. Every time a problem from the text is discussed, the necessary calculator skills are explained as well as the necessary analytical skills. After completing the problem the student is encouraged to interpret and to check the answer.

Keyboard Layout

Study the face of your calculator. Notice how the keys are grouped by color. Also take note that there is yellow and/or green (blue for the TI-86) writing above

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each button; also note there is a yellow key and a green key on the upper left of
the key pad. If you wish to access any of commands or symbols written in yellow
you must first press the yellow key; if you wish to access any of the letters or
symbols written in green (blue), you must first press the green key. The blue
keys down the side are your operation keys, the blue keys(TI-83) across the top
relate to graphing, and the blue arrow keys let you move the cursor on the
calculator screen.

**Screen Brightness**

Turn your calculator on. Is the screen too dark or too light? If you are not
satisfied with the brightness of your calculator, press 2nd and use the up or down
blue (gray) arrows to either make your screen darker or lighter. (Do not hold the
2nd key down; press 2nd, release this key, and then press the up or down blue
arrow.) You will see a number appear briefly in the upper right corner of the
calculator screen, this is the brightness setting which ranges from zero to nine. If
your screen is still dim when the number nine is showing, you may need new
batteries.

The author has written this manual with the specific goal of enhancing
understanding and minimizing calculator magic (pushing buttons until an answer
magically appears). If you have questions or comments, the author may be
reached at the address given below.

Kathy V. Rodgers
Department of Mathematics
University of Southern Indiana
8600 University Boulevard
Evansville, IN 47712

email: krogers@usi.edu
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Chapter 1
The Six Trigonometric Functions

Section 1.1

Calculator skills needed for this section include the exponent key and the square root key. Practice the following examples before attempting to work problem 25.

To raise 6 to the second power complete the following keystrokes.

press 6
press $x^2$ You will find the $x^2$ button in the middle of the left column.
press ENTER

There is a second method for squaring six. Use the following keystrokes. Learning these key strokes is important if you wish to raise a quantity to some power where there is not a special key such as the $x^2$ key.

press 6
press ^
press 2
press ENTER

To find the square root of 36, press the following sequence.

press $\sqrt{}$ To access the $\sqrt{}$, press 2nd and the $x^2$ key.
enter 36 (In this particular case, closing the parentheses does not matter; however, it is best to develop the habit of closing all parentheses.)
close the parenthesis
press ENTER

\[
\sqrt{36} = 6
\]

A second way to find the root of a number is to write the expression in exponential form.

\[
36^{\frac{1}{2}}
\]

Enter this in your calculator and check the screen that follows to verify your work.

\[
36^{\left(\frac{1}{2}\right)} = 6
\]

To find the answer to question 25, complete the following sequence of steps.

enter 3  
press \(x^2\)  
press +  
press 4  
press \(x^2\)  
press ENTER  
press \(\sqrt{}\)  
press ANS  
press ENTER

To access the ANS key press 2nd and (-).

\[
3x^2 + 4 = \frac{25}{5}
\]

You could complete this problem in one step by first solving for c. Your paper-pencil work should resemble the following.
\[ c^2 = 3^2 + 4^2 \]
\[ c = \sqrt{9 + 16} \]
\[ c = \sqrt{25} \]
\[ c = 5 \]

The calculator keystrokes to mirror this work are:

press \[ \sqrt{ } \]
enter \( 3^2 + 4^2 \)
press ENTER

**Section 1.2**

To prepare your calculator for graphing functions

press MODE

The settings on the left should all be dark.

To change settings, move the cursor until it is flashing on the setting that you need and then press ENTER.

Press CLEAR or QUIT and this will return to the home screen.

**Problem 13** asks you to graph \( 3x + 2y = 6 \).

First solve this equation for \( y \) to get \( y = \frac{-3}{2}x + 3 \).

press \( Y = \) One of the five blue menu keys on the top row of your calculator.
press WINDOW
enter -4.7 for $X_{\text{min}}$

There is a reason for selecting a minimum x-value of -4.7 and a maximum x-value of 4.7. The difference between 4.7 and -4.7 is 9.4. The number of pixels is 94 and $\frac{9.4}{94}$ is 0.1. This setting makes the x-values increase by 0.1 as you trace along the graph. You will see these values or some multiple of these values used throughout this manual.

enter 4.7 for $X_{\text{max}}$
enter 1 for $X_{\text{scl}}$
enter -5 for $Y_{\text{min}}$
enter 5 for $Y_{\text{max}}$
enter 1 for $Y_{\text{scl}}$
enter 1 for $X_{\text{res}}$

You will want to leave the x resolution set on one most of the time. You are telling the calculator to light all of the pixels. If you entered a two, you would be telling the calculator to light one pixel out of two.

press GRAPH

The following are calculator screens for $Y=,$ for the window settings, and the graph $Y_1$.

Note: The calculator really does not care if you simplify the expression. You would have gotten the same answer had you entered $y = \frac{(6 - 3x)}{2}$. However, you must use the parentheses around the numerator; otherwise the calculator will only divide the last term by 2.

The calculator permits you to scroll along the graph.

press TRACE
press the right blue arrow
Note. As you trace along the graph, the coordinates of the highlighted point are displayed on the bottom of the calculator screen. In the preceding example, the cursor is flashing on a point with the coordinates of (2.4, -0.6). In the upper left corner of the screen, you will see the function displayed that was entered in \( Y_1 \). After selecting TRACE if you do not see the function displayed in the upper left hand corner, go to FORMAT (above the blue zoom key) and select ExprOn.

The calculator permits you to view a table of values, much like a t-table created manually.

Press TBLSET To access TBLSET, press 2nd and WINDOW
Enter 0 for TblStart
Enter 1 for \( \Delta Tbl \)
\( \Delta Tbl \) (read delta table) is asking by what increment you want the x-values to increase. For example, if you enter 1, each x-value will differ by one, if you enter two, each x-value will differ by two.

Press TABLE To access TABLE press 2nd and GRAPH

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>2.5</td>
</tr>
<tr>
<td>0</td>
<td>1.5</td>
</tr>
<tr>
<td>1</td>
<td>-1.5</td>
</tr>
<tr>
<td>2</td>
<td>-0.5</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
</tr>
</tbody>
</table>

Note. You may use the up and down blue arrow keys to scroll up and down the table. Remember, the values you see in this table are the coordinates of points on the graph of the function you entered in \( Y_1 \).

Alert! Question 16 asks you to graph a vertical line. Remember that a vertical line is not a function. As long as your graphing utility is in function mode, you cannot use the graphing utility to graph this line. However, this is an easy graph to create using paper and pencil. This problem says that \( x \) must be 3 and that \( y \) can be any number. Hence the following ordered pairs are all on this graph: (3,0), (3,-1), (3,2) etc.

Your calculator will draw, but not graph a vertical line for you. Clear \( Y_1 \) and use the following keystrokes.

Press GRAPH
Press DRAW To access DRAW, press 2nd and PRGM.
Select VERTICAL Use the down blue arrow until the cursor is flashing on Vertical and then press ENTER or simply press the number 4.

Press the right blue arrow until the x-value on the bottom of the screen is 3.
Note. If you press ENTER, the vertical line will stay where you have placed it and you may draw a second vertical line by using the right and left blue arrow keys.

Alert! When you use the DRAW feature of the calculator, you cannot use the TRACE or the CALC features of the calculator. If you use the blue arrows, you have a free floating cursor that is not locked to the line.

Alert! To remove the vertical line press DRAW and select ClrDraw and ENTER.

Question 25 asks you to graph the circle defined by the equation $x^2 + y^2 = 25$. Once again, you must first solve for $y$. Do the following paper-pencil work before trying to enter this in your calculator.

\[ y^2 = 25 - x^2 \]
\[ y = \pm \sqrt{25 - x^2} \]

The calculator does not have a $\pm$ key; you will have to enter this in your calculator as two different expressions.

press $Y =$

enter $\sqrt{25 - x^2}$ in $Y_1$

$-\sqrt{25 - x^2}$ in $Y_2$

press Graph

Alert! Does your circle look more like a football than a circle? Do you even have a complete circle? Try the following WINDOW settings.

press WINDOW

Your minimum and maximum values must be greater than $5$ since the radius of the circle is 5. A -9.4 and 9.4 were chosen as the minimum and maximum x-values because they are multiples of 4.7 as was explained earlier.
press ZOOM
press 5 for ZSquare

The middle blue key on the top row.
You will notice that the viewing window is rectangular, the zsquare changes the x and y window settings to an approximate 3 to 2 ratio.

Note. You may have a gap between the two circles. This is due to the resolution. When you sketch this on your paper, be sure to connect the two semicircles.

Instead of entering \(-\sqrt{25-x^2}\) in \(Y_2\) you could do the following.

Enter \(\sqrt{25-x^2}\) in \(Y_1\) as before; however in \(Y_2\) enter \(-Y_1\).

press \(\text{Y}]=\)
select \(Y_2\)

Move the cursor to \(Y_2\) and if you have something in \(Y_2\), press CLEAR.

press \(-\)
press \(\text{VARS}\)
select \(Y\text{-VARS}\)
select \(\text{FUNCTION}\)
select \(Y_1\)
press \(\text{ENTER}\)

Note. The real benefit of this procedure is that you can very easily graph a second circle by only changing the entry in \(Y_1\).
**Question 29** asks you to graph \( x^2 + y^2 = 1 \). If you had used the method described above, then you would only have to enter \( \sqrt{1-x^2} \) in \( Y_1 \). Since \( Y_2 \) is -\( Y_1 \), this will graph the bottom half of your circle. **Question 29** is asking you to use the TRACE feature of the calculator to find all ordered pairs that have an \( x \)-value of \( \frac{1}{2} \).

1. Press **TRACE**.
2. Enter \( \frac{1}{2} \). There is no need to use the blue arrows.

The directions say to write your answer as an ordered pair and round to four places past the decimal point when necessary. Certainly this should not be difficult for you to do; however, the calculator will do this for you.

1. Select **MODE**.
2. Go to **FLOAT** and use the right arrow until the cursor is blinking on the 4.
3. Press **ENTER**.

You will notice that there are only three decimal places instead of four; the fourth number was a zero, hence not a significant digit.

**Problem 39** asks you to find the distance between the points (3,7) and (6,3).

Using the distance formula you have \( d = \sqrt{(3-6)^2 + (7-3)^2} \). After you have simplified this, you could use your calculator to verify your answer.

1. Press \( \sqrt{\) \( (3-6)^2 + (7-3)^2} \)
2. Enter \( (3-6)^2 + (7-3)^2 \) \)
3. Press **ENTER**.
If you want your calculator to draw the triangle described in problem 83, perform the following keystrokes. (It is probably easier to perform this task by hand.)

press DR
select Line(
pres
press 0,0,5,0)     The calculator understands that these
tnumbers represent the two endpoints of the
line segment.
pres
press DRAW
select Line(
pres
press 5,0,5,12)     To access the colon, press ALPHA and the
press :
pres
press DRAW
select Line(
pres
press 5,12,0,0)     Clear all functions.
pres
press Y=    You know the least x-value is zero and the
greatest x-value is 5. Choose a value less
than zero for the minimum x-value and a value
greater than 5 for the maximum x-value. The
least y-value is zero and the greatest y-value is
12. Select a number less than zero for the
minimum y-value and a number greater than
12 for the maximum y-value.
pres
press ENTER

Note. Remember, the TRACE feature does not work with the DRAW feature. You can use the right and left blue arrows to move the floating cursor around the screen; however, this cursor is not locked to the graph.

Alert! The only way to clear this triangle is to go to ClrDraw. You access this by pressing DRAW and selecting number 1 and pressing ENTER.
Section 1.3
Question 25 asks you to find \( \sin \theta \) and \( \cos \theta \) if the point \((9.36, 7.02)\) is on the terminal side of \( \theta \). Recall that the \( \sin \theta = \frac{y}{r} \) and that \( r = \sqrt{x^2 + y^2} \).

Enter 9.36
Press STO
Enter x
Press ENTER
Enter 7.02
Press STO
Enter y
Press ENTER
Enter \( \sqrt{(x^2 + y^2)} \)
Press STO
Enter r
Press ENTER
Press SIN \( \frac{y}{r} \)

Section 1.4
Question 9 asks you to find \( \csc \theta \) given \( \sin \theta = \frac{4}{5} \). Recall that \( \csc \theta = \frac{1}{\sin \theta} \).

Certainly you could find this answer by simplifying \( \frac{1}{\frac{4}{5}} \). You could also use the following keystrokes.

Enter \( \frac{1}{4} \)
Press x\(^{-1}\) This key is above the \( x^2 \) key.
Press ENTER

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Chapter 2

Right Triangle Trigonometry

Before beginning the exercises in Chapter 2, set your calculator in degree mode.

- press MODE
- select Degree Use the down and right blue arrows until the cursor is flashing on degree.
- press ENTER The degree setting is not selected until you push ENTER even though the cursor is flashing on degree.
- press CLEAR CLEAR returns the calculator to the HOME SCREEN.
- QUIT also returns the calculator to the HOME SCREEN.

Example: To find the \( \sin 16^\circ \) perform the following keystrokes.

- press SIN
- enter 16
- press ENTER

**Note.** It is not necessary to use the degree symbol if you are in degree mode. If you had used the degree symbol, your answer would not have changed. Compare your calculator screen with the calculator screen that follows.

If you are not in degree mode then you must use the degree symbol. For the following, the calculator was not in degree mode. The screen on the left gives the answer using the degree symbol and the answer is correct. The screen on the right does not show the degree symbol being used and the answer is incorrect.
Note. You could also omit closing the parentheses since you are at the end of an argument. If you do not close the parentheses, the calculator automatically assumes it to be at the end of the expression. It is best to develop the habit of closing all parentheses.

Section 2.1

Question 17 asks you to find the \(\sin 10^\circ\) using the Cofunction Theorem. Review the Cofunction Theorem from your text. This theorem says the \(\sin 10^\circ\) is equal to the \(\cos 80^\circ\). To complete this on your calculator use the following keystrokes. (Your calculator should be in degree mode.)

\[
\text{press COS enter 80 press ENTER}
\]

For your own information now find \(\sin 10^\circ\).

\[
\text{press SIN press 10 press ENTER}
\]

\[
\begin{array}{c}
\cos(80) \quad .17 \\
\sin(10) \quad .17 \\\n\end{array}
\]

This is an example of the Cofunction Theorem. The \(\cos 80^\circ\) is equal to the \(\sin 10^\circ\).

Suppose you wished to find the \(\sin 39.8^\circ\) using the Cofunction Theorem. The calculator will find the complement for you.

\[
\text{press COS enter (90 - 39.8) press ENTER}
\]

\[
\begin{array}{c}
\cos(90-39.8) \quad .640186995 \\
\end{array}
\]
**Question 29** asks you to find \( \left(2 \cos 30^\circ\right)^2 \). It is important to know how to correctly use parentheses when entering an expression raised to a power.

```
press (  
press 2  
press COS  
enter 30  
press )  
press ^  
press 2  
press ENTER
```

You could enter the * symbol between 2 and COS; however it is not necessary.

You could also use the \( x^2 \) key.

Compare your screen with the screen that follows.

```
(2 cos (30)) ^ 2  
3
```

**Note.** Compare the following.

\[ \sin^2 30^\circ \]
\[ (\sin 30^\circ)^2 \]
\[ \sin (30^\circ)^2 \]

The first two expressions are equal. You are to find the \( \sin 30^\circ \) and then square the answer. The last problem is asking you to find \( \sin 900^\circ \) (\( 30^2 = 900 \)). The answers to the first two are the same; the answer to the third problem is different. Note the calculator screen that follows.

```
(sin^2 (30))  
.25
sin (30)^2  
.25
sin (30^2)  
0
```

**Note.** The calculator will not let you enter \( \sin^2 30^\circ \); this must be entered as \( (\sin(30))^2 \).
Problem 31 could be entered in the calculator in one step. Try it. Compare your answer with the calculator screen that follows. Did you correctly insert the parentheses?

\[ \frac{\sin(60^\circ) + \cos(60^\circ)}{2} = 1.866025404 \]

Question 50 asks you to find the cotangent of an angle. If you search and search, you will not find a cotangent key on your calculator; however, you can still complete this question by using your knowledge of reciprocal identities. The cotangent is defined as:

\[ \cot \theta = \frac{1}{\tan \theta} \]

To complete question 50, use the keystrokes that follow.

- enter 1
- press ÷
- enter (TAN (30))
- press ENTER

\[ \frac{1}{\tan(30)} = 1.732050808 \]

Section 2.2

Question 1 asks you to add the measures of two angles. First recall that 1° = 60’ and 1’ = 60”. Your paper-pencil work should resemble the following.

- \[ 37^\circ + 26^\circ = 63^\circ \]
- \[ 45’ + 24’ = 69’ \]
- Change 69’ to 1°9’
- \[ 63^\circ + 1^\circ = 64^\circ \]
- Final Answer 64° 9’

To complete this same problem on the calculator, use the following sequence of keystrokes.

- enter 37
Chapter 2

press ANGLE
select the degree symbol
enter 45
press ANGLE
select the symbol for minute
press +
enter 26
press ANGLE
select the degree symbol
enter 24
press ANGLE
select the symbol for minute
press ENTER

Your screen returns 64.15°, but you want your answer in degrees and minutes. You could convert the .15° to minutes by simply multiplying 0.15 times 60 to get 9 minutes. However if you will follow these keystrokes, your calculator will give the answer in degrees, minutes and seconds.

press ANS  To recall your previous answer.
press ANGLE
select > DMS  DMS--degrees, minutes, seconds
press ENTER

You could perform this task in one step by pressing > DMS before you press ENTER.

Question 15 asks you to convert from decimal degrees to degrees and minutes. Use the skills you learned in the previous example to answer this question.

enter 35.4
press ANGLE
select > DMS
press ENTER
Question 23 asks you to convert from degrees and minutes to decimal degrees. Your paper-pencil work should include the following.

\[
45 + \frac{12}{60} = 45.2.
\]

Calculator strokes to mirror this work would be:

- enter 45
- press +
- enter 12
- press ÷
- enter 60
- press ENTER

However, your calculator will perform this for you in one step.

- press 45
- press ANGLE
- select degree
- press 12
- press ANGLE
- select minute
- press ENTER

Note. The directions ask you to round the answer to the nearest hundredth of a degree. Yes, your calculator will also do this.

- press MODE
- Move the cursor down until it is flashing on float and to the right until it is flashing on the 2.
- press ENTER
- press QUIT This will return you to the Home Screen.
All of your answers will be rounded to hundredths place; however, it is probably easier to simply use your own knowledge to perform this task.

**Question 31** asks you to find the $\sin 27.2^\circ$. Before entering this in your calculator, verify that you are in degree mode. Since you set your calculator to two decimal places for the previous problem, all answers will be rounded to hundredths until you reset the calculator to let the decimal point float.

$$
\sin(27.2) \\
\approx 0.46
$$

**Question 51** asks you to complete a table comparing $\sin x$ and $\csc x$. Practice the following keystrokes until you have mastered this calculator skill.

- Press $\boxed{Y=}$. This key is one of the blue keys on the top row.
- Enter $\boxed{\sin}$ in $Y_1$.
- Enter $1/\sin(x)$ in $Y_2$. Recall $\csc x = \frac{1}{\sin x}$.

Press $\text{TBLSET}$ to access the TBLSET press 2nd and WINDOW, a blue button on the top row next to the $\boxed{Y=}$ button.

- Enter 0. The cursor will be flashing on the number following TblStart. Since you want your x-values to start at zero enter a zero for TblStart (table start).

- Select $\Delta$ Tbl. Use the down cursor to select $\Delta$ Tbl, the symbol for delta table. Delta is used in mathematics to mean change.
enter 15  You are telling the calculator that you want the first x-value to be zero and the second x-value to be 15, and the third to be 30, etc. In other words, you want the x-values to increase by 15 each time.

**Note.** Select Auto for both the independent and dependent variables.

press TABLE  To access TABLE use 2nd GRAPH.

Your screen should resemble the following calculator screen.

<table>
<thead>
<tr>
<th>X</th>
<th>Y₁</th>
<th>Y₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>ERROR</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>2.68879756989</td>
</tr>
<tr>
<td>30</td>
<td>0</td>
<td>1.424084658</td>
</tr>
<tr>
<td>45</td>
<td>0</td>
<td>1.047296259</td>
</tr>
<tr>
<td>60</td>
<td>0</td>
<td>0.523598775</td>
</tr>
<tr>
<td>75</td>
<td>0</td>
<td>0.204237281</td>
</tr>
<tr>
<td>90</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Notice the ERROR in Y₂. You asked the calculator to divide one be zero which is impossible. The calculator tells you this by displaying the word ERROR.

Recall that you entered \( \sin x \) in Y₁ and \( 1/\sin(x) \) in Y₂. In previous mathematics classes you learned that a number times its reciprocal is 1. Go to Y = and in Y₃ enter Y₁ * Y₂ to investigate what happens when a trigonometric function is multiplied by its reciprocal. Use the following keystrokes.

press **Y =**
select Y₃
press VARS  This button is found to the left of the CLEAR key.
select Y-VARS Use the right cursor to make this selection.
select function
select Y₁  Symbol for multiplication.
pres *
pres VARS
select Y-VARS
select function
select Y₂
press TABLE Use the right cursor until you can see Y₃. All entries in this column are 1 except when \( x = 0 \).
The ERROR in $Y_1$ is the result of trying to divide by zero which is impossible.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>ERROR</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2.0557</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1.5781</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1.3279</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>1.1072</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>1.0000</td>
<td></td>
</tr>
</tbody>
</table>

Note. By looking at this column, you observe the relationship between $\sin(x)$ and $\csc(x)$.

press QUIT  This will return you to the Home Screen.

Question 55 asks you to find $\theta$ given that the $\cos\theta = 0.9770$. To solve this problem, you will need the inverse cosine key $\theta$, which is the key that will give you the value of $\theta$ when you know the value of the cosine.

press $\cos^{-1}$  This is above the COS button.
enter 0.9770
press ENTER

$\cos^{-1}(0.9770) = 12.31225151$

The directions ask you to round your answer to tenths.

Note. The first answer is in radians; the second answer is in degrees. Check your answer by finding the cosine of each answer.

$\cos(12.31225151) = 0.977$

Section 2.3

Question 1 asks you to find the length of side $a$, given angle $A$ is 42° and side $c$ is 15 feet. (From the directions you know you have a right triangle $ABC$ with a right angle $C$.)

To find $a$, you could use the formula for $\sin A$. 

Rodgers, K. 19
\[
\sin A = \frac{a}{c}
\]
\[
\sin 42^\circ = \frac{a}{15}
\]
\[15 \sin 42^\circ = a\]

Using calculator skills you learned from the previous sections, enter this information on one line in your calculator. Your screen would look like the following.

\[15 \sin (42)\]

Interpret your answer. Side \( a \) is 10.04 feet in length.

After you have worked Question 57, model the problem using your graphing calculator. This sequence of keystrokes graphs a circle to represent the Ferris wheel by using parametric equations. The calculator is plotting a series of points \((x, y)\) where the \(x\)-value is defined as \(\cos x\) and the \(y\)-value is defined as \(\sin x\). To change the size of the circle, multiply by 98.5, the radius of the circle. The center of the Ferris wheel is at the origin, this means the bottom half of your Ferris wheel is below the \(x\)-axis. A positive vertical translation of 98.5 feet would put the bottom of the Ferris wheel on the \(x\)-axis, but this would not make the top of the wheel 209 feet above the ground. We need to translate up another 12 feet or a total of 110.5 feet. Try the following.

press MODE
select PAR PAR stands for parametric
press ENTER
press Degree
press ENTER
press CLEAR
press \( Y = \)
select \( X_1T = \) and enter \( 98.5 \cos (T) \) The \( T \) button is to the right of the alpha key.

select \( Y_1T = \) and enter \( 98.5 \sin (T) + 110.5 \)
press WINDOW
set \( T_{\text{min}} = 270 \) By making the minimum value of \( T = 270 \), you are telling your calculator to start evaluating \( x \) and \( y \) at the point where the Ferris wheel is closest to the ground.

set \( T_{\text{max}} = 630 \) Select 630 because this is the sum of 270 and 360, the number of degrees in a complete circle.
set \ T_{\text{step}} = 5 \ \ \text{This is an arbitrary number. If you choose a very small number the calculator would move very slowly; if you were to choose a very large number, your circle would have sharp edges. When you set the T-step at five, you are telling the calculator to plot a point every five degrees.}

Continue with the following settings.

\begin{align*}
X_{\text{min}} &= -246 \\
X_{\text{max}} &= 246 \\
X_{\text{scel}} &= 0 \\
Y_{\text{min}} &= -25 \\
Y_{\text{max}} &= 300 \\
Y_{\text{scel}} &= 0
\end{align*}

press GRAPH

press TRACE

Compare your graph with the screen that follows.

\begin{itemize}
\item \textbf{Note.} The cursor represents the point \((x,y)\) when \(T = 270^\circ\), the minimum value assigned to \(T\). The \(\cos\left(270^\circ\right) = 0\) and the \(\sin\left(270^\circ\right) = -1\). In \(X_{IT}\) you entered \(98.5\cos \left(T\right)\) and in \(Y_{IT}\) you entered \(98.5\sin \left(T\right) + 110.5\). Do the arithmetic. You will see that the coordinates of the minimum point of the Ferris wheel should be \((0,12)\), exactly what you see on your screen.
\item Now press the right blue arrow once. You have told the calculator to move to the next point which occurs when \(T = 275^\circ\). (Recall you set the \(T_{\text{step}}\) at 5.)
\item Continue to press the right arrow until you have made a complete circle. Modeling applications is one of the benefits of a graphing calculator. You may want to read this explanation more than once.
\end{itemize}
Chapter 3

Radian Measure

Section 3.1

Question 13 asks for the exact value of \( \cos 225^\circ \). From the text, you know the terminal side of this angle is in Quadrant III and the reference angle is 45°. Since the cosine is negative in Quadrants II and III, you know your answer will be negative. You should have also memorized the trigonometric functions for the basic angles. If so you know that the \( \cos(45^\circ) = \frac{\sqrt{2}}{2} \). Your answer is \( -\frac{\sqrt{2}}{2} \). Now use your calculator to convert \( -\frac{\sqrt{2}}{2} \) to a decimal and also to find \( \cos 225^\circ \). Compare your answers. They should be the same. Check your work with the calculator screen that follows.

\[
\cos(225^\circ) \approx -0.7071067812
\]

Question 41 asks you to find \( \cos(-315^\circ) \). Your calculator should be in degree mode.

Alert! You must use the negative symbol and not the subtraction sign. Check your screen with the following.

\[
\cos(-315^\circ) \approx 0.7071067812
\]

Question 49 asks you to find \( \theta \) given \( \sin \theta = -0.3090 \) and \( \theta \) in QIII. Use the following sequence of keystrokes to complete this task. This question is directing you to find a value for \( \theta \) such that the \( \sin \theta = -0.3090 \).

press \( \text{SIN}^{-1} \)
enter \(-0.3090\)
press \( \text{ENTER} \)
You have found an angle, but this is not the angle that your text asked you to find. Your directions said to find $\theta$ such that $0^\circ < \theta < 360^\circ$ with $\theta$ in the third quadrant, this angle is in the fourth quadrant.

You have found a reference angle of 17.99877619, the absolute value of -17.99877619. To find the value of the angle with terminal side in the third quadrant, add 180° to the reference angle. The answer is 197.9989762. or 198.0 when rounded to the nearest tenth.

Use this sequence of keystrokes to complete this question.

press MATH
select NUM from the menus across the top
select abs
press ENTER
press ANS
By using the ANS key you do not have to retype your answer from above. The calculator knows your last answer.

enter +
enter 180
By adding 180°, you are placing the terminal side of the angle in Quadrant III.

press ENTER
You should have gotten 197.9989762; this is your value for $\theta$. Now check your answer by finding the $\sin \theta$.

press SIN
press ANS
press ENTER
Check your screen.
Interpret information on this screen. The original question asked you to find a value for \( \theta \) such that the \( \sin \theta = -0.3090 \). You found \( \theta \) to be 197.9989762. Next you found the \( \sin(197.9989762) \) and when your calculator returned -0.309, you knew you had found the correct value for \( \theta \).

**Question 59** directs you to find \( \theta \) given the \( \sec \theta = 1.4325 \) with \( \theta \) in QIV. Your calculator does not have a \( \text{SEC}^{-1} \) key. However, from your previous study of reciprocal identities you know the following:

\[
\cos \theta = \frac{1}{\sec \theta} \\
\cos \theta = \frac{1}{1.4325} \\
\theta = \cos^{-1}\left(\frac{1}{1.4325}\right)
\]

You would enter this in your calculator using the following keystrokes.

- press \( \text{COS}^{-1} \)
- press ( enter 1/1.4325 press ) press ENTER

This angle is in the first quadrant and the question asks for an angle in the fourth quadrant. The correct answer is found by subtracting 45.7° from 360°. Use the ANS key on your calculator to find the correct angle. Since your answer is positive, it is not necessary to take the absolute value before performing the addition or subtraction. Finally, check your answer by finding the secant of this angle. If you are correct, your answer will be 1.4325.
Section 3.2

Question 7 asks you to find the radian measure of angle \( \theta \), if \( \theta \) is a central angle of a circle with radius of \( \frac{1}{4} \) cm and an intercepted arc length of \( \frac{1}{2} \) cm. Recall that \( \theta \) (in radians) = \( \frac{s}{r} \). To enter this on the Home Screen of your calculator, use parentheses around each fraction. Check your calculator screen with the screen that follows.

\[
\frac{1}{2} \times \left( \frac{1}{4} \right) \div 2
\]

Interpret your answer. The measure of angle \( \theta \) is 2 radians.

Question 11 asks you to give the reference angle in both degrees and radians. The reference angle for \( 30^\circ \) is \( 30^\circ \). To find the reference angle in radians, you must convert \( 30^\circ \) to radians. Your paper-pencil work should resemble the following:

\[
30^\circ \times \left( \frac{\pi}{180^\circ} \right) = \frac{\pi}{6}
\]

Note. If you need an exact numerical answer, you must use the previous method. Your calculator will only give you a decimal approximation.

The following keystrokes convert an angle from degree measure to radian measure.

press MODE
select Radian
press ENTER
press CLEAR
enter 30
press ANGLE
select #1 for degree
press ENTER

Use the blue arrows to move the cursor to Radian.
Pressing either ENTER or 1 will paste the degree symbol on the Home Screen.
Interpret your answer. $30^\circ$ is approximately $0.5235$ radians. Compare this answer with the exact numerical answer you obtained. Use your calculator to compare your two answers.

\[
\begin{align*}
\text{enter } & \pi \\
\text{press } & \div \\
\text{enter } & 6 \\
\text{press } & \text{ENTER}
\end{align*}
\]

To complete Question 23, you must be able to convert $120^\circ 40'$ to radians.

\[
\begin{align*}
\text{enter } & 120 \\
\text{press } & \text{ANGLE} \\
\text{select } & \text{degree} \\
\text{enter } & 40 \\
\text{press } & \text{ANGLE} \\
\text{select } & \text{minute} \\
\text{press } & \text{ENTER}
\end{align*}
\]

Your calculator has converted $120^\circ 40'$ to decimal degrees; it has not converted the answer to radians.

\[
\begin{align*}
\text{press } & \text{ANS} \\
\text{press } & \text{ANGLE} \\
\text{select } & \text{degree} \\
\text{press } & \text{ENTER}
\end{align*}
\]

You must tell the calculator that you are entering an angle that is in degree measure.

\[
\begin{align*}
\text{press } & \text{ANGLE} \\
\text{select } & \text{degree} \\
\text{press } & \text{ENTER}
\end{align*}
\]

Since you are in Radian mode, the calculator now converts the measure of this angle to radians.

**Note.** You could do all of this in one step by entering $(120^\circ 40')$ in your calculator.

Now interpret your answer. $120^\circ 40'$ is approximately $2.11$ radians. (You can have your calculator round to hundredths; however, it is probably easier for you to do this mentally.)
Question 43 asks you to convert an angle with a measure of one radian to degrees. (You know the 1 represents radian measure; if the 1 represented degree measure, then the degree symbol would have followed the 1.)

Note. If you want to convert to degree measure, then set your calculator to degree mode.

press MODE
select degree
press ENTER
press CLEAR
enter 1
press ANGLE
select radian (#3) Either press 3 or ENTER.
press ENTER

After you have completed question 51, you can use your calculator to check your answer. From previous work, you know

\[
\theta = \frac{4\pi}{3}
\]

\(\hat{\theta}\) is the symbol for reference angle.

\(\hat{\theta} = \frac{\pi}{3}\) and \(\theta\) in QIII

and \(\sin \left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}\).

Since the sine is negative in the third quadrant, you must take the negative of your answer.

\[\sin \left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}\]
On your calculator:

press SIN
enter (4\pi+3) The key for $\pi$ is below the CLEAR key.
press ENTER

Compare your calculator answer with your exact numerical answer.

Alert! Always check the mode setting of your calculator. If you had not been in radian mode, you would have still gotten an answer, a wrong answer.

Question 73 asks you to find the $y$-value given $y = \sin x$, for $x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$.

Certainly you could find this problem one value at a time. For example, after you have checked to make sure your calculator is in radian mode, use the following keystrokes.

press SIN
enter 0
press ENTER

Interpret your answer. $Y$ is zero when $x$ is zero giving the ordered pair $(0,0)$. To complete the problem, you would need to continue this process for each $x$-value.

A second method is to enter all of the $x$-values at one time. Try the following key strokes.

press SIN
press { The bracket tells the calculator you are going to enter a list.
enter $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$
enter })
press ENTER
Note. The three dots to the right tell you there are more answers. Use the right arrow key and scroll to the right to see the rest of the answers. To get fewer decimal places go to MODE and change the decimal setting. The following screen gives the answers rounded to hundredths.

A third procedure for answering this question is to use the Table Feature. Try the following keystrokes.

press Y=
enter sin x
press TBLSET
enter 0 in TblStart
enter \( \frac{\pi}{4} \) in ΔTbl
press TABLE

Interpret your answer. You have the following ordered pairs. You know the x-values are 0, \( \frac{\pi}{4} \), \( \frac{\pi}{2} \), etc. because you told the calculator to start the x-values at 0 and to increase each x-value by \( \frac{\pi}{4} \). You have the following ordered pairs: (0,0), \( \left( \frac{\pi}{4}, 0.70711 \right) \), \( \left( \frac{\pi}{2}, 1 \right) \), \( \left( \frac{3\pi}{4}, 0.70711 \right) \), etc.

Note. You can conveniently use the Table Feature when the independent variable is increasing by the same quantity. In the preceding example, the independent
variable was increasing by $\frac{\pi}{4}$. All three procedures are suitable methods; learn to use the method best suited for the question you are investigating.

Section 3.3

**Question 1** asks you to find the values of all six trigonometric functions given the angle measure of $150^\circ$. Without a doubt, you should memorize the trigonometric functions for the basic angles. You know $150^\circ$ is in the second quadrant and the reference angle is $30^\circ$ ($180^\circ - 150^\circ$). If you have memorized the sine and the cosine for $30^\circ$, you can now find the values for all six trigonometric functions.

To use the unit circle to answer this question, use the following keystrokes.

- press MODE
- select Par
- press ENTER
- select Degree
- press ENTER
- press Y=
- select $X_1T$ and enter cos(T)
- select $Y_1T$ and enter sin(T)
- press TBLSET
- enter 0 in TblStart
- enter 10 in Δ Tbl
- press TABLE

<table>
<thead>
<tr>
<th>T</th>
<th>$X_1T$</th>
<th>$Y_1T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0.86603</td>
<td>0.5</td>
</tr>
<tr>
<td>20</td>
<td>0.64279</td>
<td>0.60622</td>
</tr>
<tr>
<td>30</td>
<td>0</td>
<td>0.86603</td>
</tr>
<tr>
<td>40</td>
<td>-0.30902</td>
<td>-0.95106</td>
</tr>
<tr>
<td>50</td>
<td>-0.76604</td>
<td>-0.64279</td>
</tr>
<tr>
<td>60</td>
<td>-1</td>
<td>-0.86603</td>
</tr>
<tr>
<td>T</td>
<td>$X_1T$</td>
<td>$Y_1T$</td>
</tr>
<tr>
<td>70</td>
<td>-0.93969</td>
<td>-0.66943</td>
</tr>
<tr>
<td>80</td>
<td>-0.76604</td>
<td>-0.30902</td>
</tr>
</tbody>
</table>

Interpret the table. When T (the angle measure) is 0, the cos(0) = 1 and the sin (0) = 0. Skip down to 30--the information from this table tells you that the cos(30) = 0.86603 and the sin (30) = 0.5. By scrolling down the table, you can read the sine and cosine for every angle that is a multiple of 10.

- press WINDOW
- enter the following
Chapter 3

TI-83

\[ T_{\text{min}} = 0 \]
\[ T_{\text{max}} = 360 \quad \text{Choose 360 because there are 360° in a circle.} \]
\[ T_{\text{step}} = 10 \quad \text{Choose 10 because 10 was used in the table.} \]

press ZOOM
select Zsquare
press ENTER
You could have also pressed 5 to make the same selection.

press TRACE

You have a picture of the unit circle. Notice the values displayed on the bottom of your screen. When the angle (T) is zero, then the x-value which you defined as the \( \cos (T) \) is 1 and the y-value which you defined as \( \sin (T) \) is 0.

Press the right blue arrow. You can read the values for the \( \cos (T) \) and \( \sin (T) \) for every multiple of 10 angle on the unit circle. Try it.

Suppose you want to know the x and y-values when T = 12. As you trace around the circle, the T-values are multiples of 10. To find the x and y-values when T is 12, just press 12 and your calculator will return the correct x and y-values. (You must have first pressed TRACE for this to work. If you press 12 without first pressing TRACE, your calculator will return you to the Home Screen.)

You could also use the following keystrokes to find the cosine and sine when T = 12.

press CALC
select value
press ENTER
enter 12
press ENTER

Check your screens with the screens that follow.
By knowing this series of keystrokes, you can enter any value for T.

**Note.** If you want your answer in radians, then repeat the preceding process. First put your calculator in radian mode; change the window setting to the following

\[
\begin{align*}
T_{\text{min}} &= 0 \\
T_{\text{max}} &= 2\pi \\
T_{\text{step}} &= \frac{\pi}{12}
\end{align*}
\]

Choose 2\pi because there are 2\pi radians in a circle. \(\frac{\pi}{12}\) is a convenient \(T_{\text{step}}\) because it will include all of the basic angles.

Press GRAPH (You will see the same unit circle; however when you TRACE, the angles measure (T-values) will be in radians and not degrees.

After you have completed your drawing for question 51 use your calculator to show that \(\sin(180' - \theta) = \sin(\theta)\).

Press Y= enter \(Y_1 = \sin(180 - X)\)

Enter \(Y_2 = \sin(X)\)

Press TBLSET enter any number in TblStart enter any number in \(\Delta Tbl\)

Press TABLE

Look at your values in the columns \(Y_1\) and \(Y_2\). The numerical values are the same in both columns. These are numerical values for the identity that you were showing true in question 51.
Section 4.1

Section 4.1 of your text introduces the graph of the sine function. First you look at ordered pairs \((x,y)\) that make \(y = \sin x\) a true statement. Secondly, you obtained the graph of the sine function by using the unit circle definition. The following sequence of keystrokes will graph the unit circle and the sine function.

1. Press `Mode`.
2. Select `Radian`.
3. Press `ENTER`.
4. Select `Par`.
5. Press `ENTER`.
6. Select `Simul`.
   - `Simul` (simultaneous) tells the calculator to graph the entries simultaneously rather than sequentially.

7. Press `ENTER`.
8. Press `CLEAR`.
10. Enter `\cos (T)` in \(X_\text{IT}\).
11. Enter `\sin (T)` in \(Y_\text{IT}\).
12. Enter `T` in \(X_\text{JT}\).
13. Enter `\sin (T)` in \(Y_\text{JT}\).
15. Enter `0` for \(T_{\text{min}}\).
16. Enter `2\pi` for \(T_{\text{max}}\).
17. Enter \(\frac{\pi}{12}\) for \(T_{\text{step}}\).
18. Enter `-2\pi` for \(X_{\text{min}}\).
19. Enter `2\pi` for \(X_{\text{max}}\).
20. Enter \(\frac{\pi}{2}\) for \(X_{\text{scl}}\).
21. Enter `-4` for \(Y_{\text{min}}\).
22. Enter `4` for \(Y_{\text{max}}\).
23. Enter `1` for \(Y_{\text{scl}}\).
**Note.** Instead of manually setting the window, you could have pressed ZOOM and selected Ztrig; however, when you are first learning to graph trigonometric functions, manually entering the window values enhances your understanding of the processes.

The real value of this model comes when you start to TRACE around the circle and move from one graph to the other. Press TRACE.

Notice the expressions on the top of the screen; this tells you that you are tracing on the graph defined by these two equations. At the point where the cursor is flashing the angle (T) is zero, the cos (T) is one and the sin (T) is 0. Press the down arrow.

Notice that the expressions at the top of the screen have changed. You are now tracing on the graph defined by the second set of equations. The cursor moved to the point (0,0). Interpret this. Remember, you are graphing \( y = \sin (X) \). You defined the x-value to be equal to the angle value which is T. For this particular case, the values for X and T are equal because you defined them as such. The y-value is the \( \sin (T) \).

Press the right arrow six times. (You set the T-step to \( \frac{\pi}{12} \); if you press the arrow six times you have found \( 6 \times \frac{\pi}{12} \) or \( \frac{\pi}{2} \).) Check your knowledge against what the calculator is telling you. You know the \( \sin \left( \frac{\pi}{2} \right) = 1 \) and \( \frac{\pi}{2} \approx 1.5707963 \). Now look at the calculator. Your graph shows that when \( x = \frac{\pi}{2} \), \( y = 1 \). Press the down arrow.
cursor has moved back to the unit circle; however, the sine value did not change. The x-value changed because you defined the x-value for the first graph to be \( \cos(T) \). Continue to trace around the unit circle.

Now look at the cosine function. Leave the parametric equations for the unit circle in place but make the following changes to graph the cosine function.

leave \( T \) in \( X_2T \)
enter \( \cos(T) \) in \( Y_2T \)
press GRAPH
press TRACE

Spend some time interpreting what you see on the calculator screen. When the angle \( T \) is zero you see that the \( \cos(T) = 1 \) and the \( \sin(T) \) is 0. Press the down arrow.

The cursor has moved to the second graph. Again, the X and T values are equal. Remember you defined them that way. Notice now that the y-value is one. You defined the y-value to equal the \( \cos(T) \) and the \( \cos(0) = 1 \). Use the blue arrow keys to continue exploring the relationship between these two graphs.

To answer question 1, first generate a table of values. Your table of values should resemble the following table.
Use the following sequence of keystrokes to generate this table.

<table>
<thead>
<tr>
<th>x</th>
<th>decimal app</th>
<th>y</th>
<th>decimal app</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>π/4</td>
<td>0.785</td>
<td>√2/2</td>
<td>0.707</td>
</tr>
<tr>
<td>π/2</td>
<td>1.571</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3π/4</td>
<td>2.356</td>
<td>√2/2</td>
<td>-0.707</td>
</tr>
<tr>
<td>π</td>
<td>3.142</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>5π/4</td>
<td>3.927</td>
<td>√2/2</td>
<td>-0.707</td>
</tr>
<tr>
<td>3π/2</td>
<td>4.712</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7π/4</td>
<td>5.497</td>
<td>√2/2</td>
<td>0.707</td>
</tr>
<tr>
<td>2π</td>
<td>6.283</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

You should always check to see whether your calculator is in degree or radian mode.

You need to return to function graphing.

You select \( \frac{\pi}{4} \), since the directions said to use x-values that are multiples of \( \frac{\pi}{4} \).

Compare this table with the table you generated without the calculator.
Note. One feature of the calculator-generated table is the ability to scroll up and down the table. By using the blue arrow keys you can see an infinite number of values. Try it!

**Question 1** also asks you to sketch the graph of \( y = \cos x \) from the table of values. Compare your paper-pencil graph with a calculator-generated graph.

press \( Y = \)
enter \( \cos(\text{X}) \) in \( Y_1 \)
pRESS \( \text{WINDOW} \)
enter \(-2\pi\) for \( X_{\text{min}} \)
enter \(2\pi\) for \( X_{\text{max}} \)
enter \(\frac{\pi}{4}\) for \( X_{\text{scl}} \)
enter \(-4\) for \( Y_{\text{min}} \)
enter \(4\) for \( Y_{\text{max}} \)
enter \(1\) for \( Y_{\text{scl}} \)
pRESS \( \text{GRAPH} \)

Press \( \text{TRACE} \).

Compare the \((x,y)\)-values to the corresponding \((x,y)\)-values of your table; they should be the same. Do you want to check a second value in your table with the graph? How about letting \( x = \frac{\pi}{4} \)? Use the following keystrokes.

press \( \text{TRACE} \)
enter \(\frac{\pi}{4}\) for \( X \)
pRESS \( \text{ENTER} \)
Take a moment to relate what you see on the screen to your previous paper-pencil work. You see the cursor flashing on a point on the graph and the coordinates of the point are shown at the bottom of the screen. Compare these values with values in your table. If you were asked to give the domain of the function, you would answer all real numbers; however, you can see very clearly that the range values are from -1 to 1.

Before leaving this question, learn one more feature of the calculator.

press MODE
select Radian
press ENTER
select G-T (G-T stands for graph and table.)
press ENTER
press GRAPH
press TRACE

Interpret the screen! You have the graph of the cosine function on the left and the table of values on the right. Notice that the cursor is flashing on the point at the left and the coordinates of this point are at the bottom of the screen as well as highlighted in the table. If you wish to scroll down the table, press TABLE and use the blue arrows. If you wish to move the cursor around on the graph, press TRACE and use the blue arrows.

If you wish to have your calculator generate a table for question 3, you will have to recall the reciprocal identities from Chapter 1. The calculator does not have a CSC key for cosecant. You are asked to find \( y = cscx \); however, \( cscx = \frac{1}{\sin x} \). Hence enter \( y = \frac{1}{\sin x} \) in your calculator to find \( y = cscx \). Use the same table settings and window settings as you used for question 1.

To compete question 5, you must interpret what you see on the calculator screen.
press MODE
select Radian
press ENTER
press Full
press ENTER
press CLEAR
press Y=
enter TAN (x)
press WINDOW
enter -2π for X_{min}
enter 2π for X_{max}
enter \frac{\pi}{2} for Xscl
enter -6 for Y_{min}
enter 6 for Y_{max}
enter 1 for Y_{scl}

Is this the graph of \( y = \tan(x) \)? Are the straight lines a part of this graph? The answer is no. As long as your graph is in connect mode the calculator will connect two points that are close together. Analytically you know that \( \tan(x) = \frac{\sin(x)}{\cos(x)} \) and whenever the \( \cos(x) = 0 \), the tangent function is undefined. Hence for all \( x \)-values of the form \( k \left( \frac{\pi}{2} \right) \), where \( k \) is an integer, the \( \cos(x) = 0 \) and the tangent function is undefined.

One way to check this graph, or any graph with what appears to be vertical lines, is to change the calculator form connect mode to dot mode.

press MODE
select DOT
press ENTER
press GRAPH

The straight lines have disappeared; the lines are not a part of the graph of \( y = \tan(x) \).
**Note.** DOT mode is very useful; however, you should normally leave your calculator in CONNECTED mode. DOT mode gives the appearance that within each interval the graph is not continuous. That is not true, the graph is continuous within the interval; as the graph is stretched out, there simply is not a sufficient number of pixels on the screen to give the appearance of a connected graph.

Before answering question 13, use the following keystrokes to visualize when the \( \cos(x) = 0 \).

**Note.** Since you have been using many features of the calculator check the MODE prior to working this problem. For this investigation, all of the selections on the left should be black.

press \( Y = \)

enter \( \cos x \) in \( Y_1 \)

press WINDOW

You should start to have a feel for setting the window of your calculator. You want to view the graph from 0 to \( 2\pi \) since the period for the cosine function is \( 2\pi \). This tells you your X settings must include an interval of this length. You could select 0 for the minimum x-value and \( 2\pi \) as the maximum x-value. Sometimes it is helpful if you can view the graph from the left and right of the interval. Maybe \(-\frac{\pi}{4}\) for the minimum value and \(\frac{9\pi}{4}\) for the maximum value would be better. That is a decision that you must make. You should recognize that the amplitude for \( y = \cos x \) is 1 and hence the range is \([-1, 1]\). You could use those exact settings for the minimum and maximum values for \( y \). Again, it is probably better to see above and below the actual graph. Maybe -4 for the minimum y-value and 4 for the maximum y-value would be a better selection. A good choice for the Xscl is either \( \frac{\pi}{4} \) or \( \frac{\pi}{2} \). You could press ZOOM and select ZTRIG; however, unless you understand the window settings, you could misinterpret the graph. Always be hesitant to use calculator magic; that is just pushing a button and viewing the results.

![Graph Image]
Before you just start to TRACE, have a look at the values in the TABLE. Go to TBLSET and enter zero as the starting point and \( \frac{\pi}{12} \) for \( \Delta \text{Tbl} \). (\( \frac{\pi}{12} \) was chosen because 12 is the common denominator for all of the basic angle measures of \( \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3} \) and \( \frac{\pi}{2} \).) Now press TABLE. Compare your calculator screen with the screen that follows.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( Y_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>0.056</td>
<td>0.984</td>
</tr>
<tr>
<td>0.112</td>
<td>0.968</td>
</tr>
<tr>
<td>0.168</td>
<td>0.953</td>
</tr>
<tr>
<td>1.042</td>
<td>0.128</td>
</tr>
<tr>
<td>1.098</td>
<td>0.082</td>
</tr>
<tr>
<td>1.154</td>
<td>0.037</td>
</tr>
<tr>
<td>1.210</td>
<td>0</td>
</tr>
</tbody>
</table>

You are looking for \( x \)-values where the \( Y_1 = 0 \). When \( x \approx 1.5708 \), then \( Y_1 = 0 \). You could find other such values by using the blue up and down arrow keys.

1. Press GRAPH
2. Press CALC
3. Select zero

The calculator is asking you for an \( x \)-value to the left of the zero. (Use the left blue arrow until the cursor is flashing to the left of the zero.) You will see a \( \blacktriangledown \) above the graph. You are defining an interval in which the calculator will find the zero (\( x \)-intercept) in question.

1. Press ENTER
2. Press ENTER
3. Press ENTER

Now the calculator is asking you for an \( x \)-value to the right of the curve. Use the right blue arrow until the flashing cursor is to the right of the zero.

The calculator now asks you to GUESS. Simply trace until you are close to the point of zero.
Interpret what you see on the screen. At the point where the cursor is flashing is a zero of the function in the defined interval, and the coordinates of that point are (1.570796, 0).

Repeat this process to find the second zero.

Before you begin your paper-pencil work do one last thing. Expand the window.

press WINDOW
enter $-4\pi$ for $X_{\text{min}}$
enter $4\pi$ for $X_{\text{max}}$
enter $\frac{\pi}{2}$ for $X_{\text{scl}}$
enter $-4$ for $Y_{\text{min}}$
enter $4$ for $Y_{\text{max}}$
enter $1$ for $Y_{\text{scl}}$

Do you recognize that the zeros are occurring at regular intervals? Now complete your paper-pencil work and compare your answers with the zeros from this graph of the function. After all when you are solving $\cos(x) = 0$, you are searching for $x$-values such that when you find the cosine of that $x$-value the answer is zero. This corresponds to the $x$-intercepts on the graph of the function.

**Problem Set 4.2**

Use your graphing calculator to discover the meaning of amplitude and how it affects the graph of the function.

press MODE
select Radian
press ENTER
press Y=
enter $\cos x$ in $Y_1$
enter $2 \cos(x)$ in $Y_2$
enter $5 \cos(x)$ in $Y_3$
press WINDOW
enter $-2\pi$ for $X_{\text{min}}$
enter $2\pi$ for $X_{\text{max}}$
enter $\pi$ for Xscl
enter $-6$ for $Y_{\text{min}}$
enter $6$ for $Y_{\text{max}}$
enter $1$ for $Y_{\text{scl}}$

Remember the calculator, in sequential mode, graphs in order--$Y_1$ first, $Y_2$ second, etc. This helps you associate the graph with the correct function. However, if it is still difficult for you to know which graph to associate with which function, there are two things that you can do.

press Y=

Use the blue arrows until the cursor is flashing on the \ to the left of $Y_2$.

press ENTER Did the \ become a bold line?

Use the blue arrows until the cursor is flashing on the \ in $Y_3$.

Continue pressing ENTER until you see a dotted line.

This is one method for distinguishing between the graphs. Use the following keystrokes for a second method.
press \ Y=
Use the blue arrows to change all of the \ back to the normal line.
press \ MODE
select \ Radian
press \ ENTER
select \ HORIZ
This provides a split screen—you can view the graph as well as the functions that you have entered.
press \ ENTER
press \ Y=
press \ TRACE
Use the up and down blue arrow keys and note the change in the y-values. The x-value stays the same.
press \ Y=
edit \ Y_2 \ to \ 4 \cos(x) \ and \ Y_3 \ to \ 8 \cos(x)
press \ GRAPH
Does your screen resemble the screen that follows?

You do not have a complete graph. This means you need to change the settings in the WINDOW. It appears that the y-values need to be changed.
press \ WINDOW
The x-values were OK; just change the y-values.
Note. With the split screen you will have to use the blue arrow keys to scroll down to the y-values.

enter -10 for \( Y_{\text{min}} \)  
-10 was selected for the minimum y-value, but you could have selected -11, or -12, etc.

enter 10 for \( Y_{\text{max}} \)
enter 1 for \( Y_{\text{scl}} \)
press GRAPH

Press TRACE and use the down arrow keys as you did previously.

Try one more thing.

press MODE
select G-T
press ENTER
press GRAPH

press TRACE

Use the up and down blue arrows to change between the graphs of the functions. Your screens should look like the following. In each case note that the x-value does not change, but the y-value does.

Can you see that the general shape of the graphs does not change. The graph has been stretched vertically by a factor equal to the amplitude. From Section 4.1, the definition of amplitude is:
The greatest value of $y$ is $M$ and the least value of $y$ is $m$, then the amplitude of the graph of $y$ is defined to be $A = \frac{1}{2}|M - m|$. 

From Section 4.2, the definition is generalized to say:

If $A$ is a positive number, then the graphs of $y = A \sin x$ and $y = A \cos x$ will have amplitude $|A|$.

Now use your graphing calculator to discover the meaning of period and how it affects the graph of the function.

press MODE
select Radian
press ENTER
select Full
press ENTER
press Y=
enter $\cos(x)$ in Y$_1$
Enter $\cos(2x)$ in Y$_2$
Enter $\cos(4x)$ in Y$_3$
press WINDOW
enter $-2\pi$ for $X_{\min}$
Enter $2\pi$ for $X_{\max}$
Enter $\frac{\pi}{2}$ for Xscl
Enter $-2$ for $Y_{\min}$
Enter $2$ for $Y_{\max}$
Enter $1$ for $Y_{scl}$

Does your graph resemble a broken tape cassette?

There are several things that you can do to make the graph easier to interpret. Press $Y =$ and turn off Y$_3$. You do this by using the down error until the cursor is flashing on $\cos(4x)$ and then use the left arrow until the cursor is flashing on the equal sign. Now press ENTER. Did the equal sign change colors? (When you want to turn this graph back on, move the cursor until it is flashing on the equal sign and press ENTER.)
Now move your cursor up to \( \cos (4x) \) and to the left until the cursor is flashing on the symbol to the left of \( Y_2 \) and press ENTER. (You should see the symbol change from a narrow line to a heavy line.) Finally press WINDOW and change the minimum value for \( X \) to 0. Press GRAPH.

![Graph of \( \cos (4x) \) and \( \cos (2x) \)](image)

Study the graph. Look at the screens below. On the first screen is the graph of \( y = \cos x \) and on the second screen is the graph of \( y = \cos (2x) \). The interval is 0 to \( 2\pi \) on both screens.

The screen on the left shows one complete cycle of the cosine curve while the screen on the right shows two complete cycles of the cosine curve. To make room for the curve twice, the curve had to be compressed. The length of the period in the graph on the left is \( 2\pi \). The length of the period for the graph on the right is \( \pi \).

Now compare the graphs of \( y = \cos (x) \) and \( y = \cos (4x) \).

![Graphs of \( \cos (x) \) and \( \cos (4x) \)](image)

Can you see that the graph on the right is actually four repetitions of the graph on the left. The length of the period of \( y = \cos (x) \) is \( 2\pi \) and the length of the period on the right is \( \pi \). \( \left( \frac{\pi}{2} = \frac{2\pi}{4} \right) \)

Now compare the graphs of \( y = \cos (x) \) and \( y = \cos \left( \frac{1}{2}x \right) \) displayed on the following screens.

![Graphs of \( \cos (x) \) and \( \cos \left( \frac{1}{2}x \right) \)](image)
Can you see that the graph on the right is only half of the graph on the left? You would have to extend the maximum x-value to $4\pi$ to see a complete period. The length of the period for $y = \cos(x)$ is $2\pi$ and the length of the period for $y = \cos\left(\frac{1}{2}x\right)$ is $4\pi$. $(2\pi + \frac{1}{2})$ Compare these examples with the definition of period in your text.

For any function $y = f(x)$, the smallest positive number $p$ for which $f(x + p) = f(x)$ for all $x$ is called the period of $f(x)$.

If $B$ is a positive number, the graphs of $Y = A \sin(Bx)$ and $y = A \cos(Bx)$ will each have a period of $\frac{2\pi}{B}$.

**Question 1** asks you to graph one complete cycle of $y = \sin(2x)$. Before you start to enter this in your calculator, look at the function analytically and answer the following questions. What is the period of $y = \sin(2x)$? The coefficient of $x$ is two--the $B$ value is two. You also know that the period is $\frac{2\pi}{B}$. The period of $y = \sin(2x)$ is $\frac{2\pi}{2}$ or $\pi$. The amplitude is 1. Now enter $y = \sin 2x$ this in your calculator and verify your analytical calculations. Check your calculator screens with the screens that follow.

**Note.** As you sketch a graph on your paper, be sure to label all of the x and y-intercepts.

**Question 9** asks you to graph one complete cycle of $y = \csc(3x)$.

$$\csc(3x) = \frac{1}{\sin(3x)}$$
Since the cosecant is the reciprocal of the sine function, you know that whenever the sine function is equal to zero, the cosecant function is undefined. The sine function is equal to zero at $k\pi$ where $k$ is an integer. You also know that the period of the cosecant function is equal to the period of the sine function; the cosecant function does not have amplitude.

Before you start to graph $y = \csc(3x)$, determine the period.

$$\text{period} = \frac{2\pi}{3}$$

(Remember that $\frac{2\pi}{B}$ gives the period.)

When $3x$ equals $k\pi$ ($k$ is an integer), the cosecant function will be undefined. In other words when $x = \frac{k\pi}{3}$ ($k$ is an integer), there will be a vertical asymptote. Now enter the function in your calculator.

press MODE
select Radian
press ENTER
press Y=
enter \frac{1}{\sin(3x)}
press WINDOW
enter 0 for $X_{\text{min}}$
enter $\frac{2\pi}{3}$ for $X_{\text{max}}$
enter $\frac{\pi}{3}$ for Xscl
enter -5 for $Y_{\text{min}}$
select y-values for the minimum and the maximum that permits you to see the shape of the graph.
enter 5 for $Y_{\text{max}}$
enter 1 for $Y_{\text{scl}}$
press GRAPH
Is the straight line part of the graph? If you do not know the answer, use the following keystrokes.

press MODE
select Dot
press ENTER
press GRAPH

The answer to the preceding question was no, the line that appear to be vertical is not part of the graph.

**ALERT!** Look at the preceding graph. It is not a totally accurate graph of \( y = \csc(3x) \). The graph of this function has a range of \((-\infty, -1] \cup [1, \infty)\). A more accurate graph of this function is one that you have manually enhanced to indicate that the graph approaches positive infinity as the x-values approach the asymptotes.

The reference curve for this graph is \( y = \sin(3x) \). Enter this function in your calculator and look at the graph.
Note. Every time the \( \sin(3x) = 0 \), the \( \csc 3x \) is undefined. This supports your earlier work. You know that the \( \csc(3x) = \frac{1}{\sin(3x)} \) and when the \( \sin(3x) = 0 \), you have a zero in the denominator causing the function to be undefined. Also note that the two functions are equal at the relative maximum and minimum points of \( y = \sin(3x) \).

Look at question 23. Use your paper-pencil skills to determine the period and the amplitude before you start to graph. This information tells you how to set your WINDOW. By inspection, you know the amplitude is \( \frac{1}{2} \). Your paper-pencil work to find the period should resemble the following.

\[
\text{period} = \frac{2\pi}{\pi} = 2
\]

\[
\text{period} = \frac{2\pi}{\frac{\pi}{2}} = 4
\]

Use the following strokes to graph the function.

\[
\text{press Y=}
\text{enter } \frac{1}{2} \sin \left( \frac{\pi}{2} x \right)
\text{press WINDOW enter 0 for } X_{\text{min}}
\text{enter 4 for } X_{\text{max}}
\]

If you want to see only one period of the graph set the x-values from 0 to 4. If you want to see more than one period or if you want to see to the left and right of the graph then expand the x-values for the window. Finding the length of the period helps you determine the window settings.

\[
\text{enter } \frac{\pi}{2} \text{ for } X_{\text{scl}}
\text{enter } -2 \text{ for } Y_{\text{min}}
\text{enter 2 for } Y_{\text{max}}
\text{enter 1 for } Y_{\text{scl}}
\text{press GRAPH}
\]
Interpret the graph. You know you have the general shape of the sine curve; you know the amplitude is \( \frac{1}{2} \) and the period is 4; the graph supports your analytical calculations. Also note that the graph intersects the x-axis at the midpoint of the period. You will find this always to be true if there has not been a vertical shifting or a rotation of the graph.

Before graphing one period of question 29, find the period and the vertical asymptotes. Your preliminary paper-pencil work should be as follows.

\[
y = 2 \tan(3x)
\]

period = \( \frac{\pi}{B} \)

period = \( \frac{\pi}{3} \)

vertical asymptotes = \( \pm \frac{1}{2} \cdot \frac{\pi}{3} + \frac{\pi}{3}k \), where \( k \) is an integer

examples of vertical asymptotes are \( x = \frac{\pi}{6}, x = \frac{3\pi}{6}, x = \frac{5\pi}{6} \)

**Note.** The tangent function does not have amplitude. The graph of this function is stretched by a factor of 2 when compared to the graph of \( y = \tan(3x) \).

Now you are ready to enter this function in your calculator.

- press **MODE**
- select **Radian**
- press **ENTER**
- press **Y=**
- enter \( 2 \tan(3x) \) in \( Y_1 \)
- press **WINDOW**
- enter \( \frac{\pi}{6} \) for \( X_{min} \) The directions for this problem say to graph one period of the function hence the interval \( \left( \frac{\pi}{6}, \frac{\pi}{2} \right) \) or
any other period of length \( \frac{\pi}{3} \) would be ok. Notice that the endpoints are not included.

Enter \( \frac{\pi}{2} \) for \( X_{\text{max}} \)

Enter \( \frac{\pi}{6} \) for \( X_{\text{scl}} \)

There is no best value for this x-scale; \( \frac{\pi}{6} \) was chosen since the vertical asymptotes will occur at multiples of \( \frac{\pi}{6} \).

Enter \(-5\) for \( Y_{\text{min}} \)

Select y-values for the minimum and the maximum that permits you to see the shape of the graph.

Enter \(5\) for \( Y_{\text{max}} \)

Enter \(1\) for \( Y_{\text{scl}} \)

Press GRAPH

Expand the x-values so as to include more than one period of the graph.

Alert. What appears to be vertical lines are not vertical asymptotes and are not part of the graph of \( y = 2 \tan(3x) \). You can always check to see if the lines are part of the graph by going to Dot Mode.

Use your calculator to investigate the graph.

Press TRACE

Enter \( \frac{\pi}{6} \)

Press ENTER
Interpret the screen. When you asked the calculator to find the y-value for $x = \frac{\pi}{6}$, the calculator returned $y = \_$. This is the calculator's way of saying "the function is undefined at $x = \frac{\pi}{6}$." This supports your earlier work. You had found a vertical asymptote at $x = \frac{\pi}{6}$. (Recall vertical asymptotes occur at x-values where the tangent function is undefined.) If $\frac{\pi}{6}$ is one vertical asymptote and the period is $\frac{\pi}{3}$, the next vertical asymptote should occur at $\frac{3\pi}{6}$ (which simplifies to $\frac{\pi}{2}$). Try it. Press TRACE and enter $\frac{\pi}{2}$. Is there a y-value?

You have supported your paper-pencil work with the graph. You can easily translate the graph on your calculator screen to paper because you know that each unit on the x-axis is equal to $\frac{\pi}{6}$. (You know this because you set the x-scale at $\frac{\pi}{6}$.)

**Question 33** asks you to graph one complete cycle of $y = 4 + 4\sin(2x)$. Again, look at the problem analytically before starting to graph. Given $y = C + A\sin(Bx)$, the C tells you the amount of the vertical shift, the $|A|$ gives the amplitude, and $\frac{2\pi}{B}$ gives the period. Hence the graph of $y = 4 + 4\sin(2x)$ has a vertical shift up of 4 units, an amplitude of 4, and a period of $\pi$. You are now ready to enter this function in your calculator.

press MODE
select Radian
press ENTER
press Y=
enter $4 + 4\sin(2x)$
press WINDOW
enter 0 for $X_{\text{min}}$
press π for $X_{\text{max}}$

Since the period is $\pi$, you could set the window from 0 to $\pi$. You would then see exactly one period.

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When deciding how to set the y-values, recall that you have an amplitude of 4. That would mean the graph would have a minimum y-value of -4 and a maximum y-value of 4; however, you also had a vertical shift up of 4 units. You should then add 4 to both the minimum and maximum y-values. If you used 0 and 8 for your minimum and maximum values, you would have no space above or below the graph. Usually this is not good. Maybe -2 and 10, would be better y-values. Do not think that these are the only acceptable values; a -5 and 13 would work just as well as would numerous other y-values.

Interpret the graph. Do you have a complete cycle? Does the amplitude and vertical shift support your earlier calculations? In this case the answer is yes, if the answer had been no, then you should have checked both your analytical work and your calculator keystrokes.

Continue your exploration of the sine and cosine functions by completing the following keystrokes.

Use the blue right arrow to move the cursor until it is flashing on the symbol to the left of $Y_2$. Press ENTER until the heavy line is flashing. This will let you easily distinguish between the graphs of the two functions.
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Interpret the graph. The heavy line which represents $y = \sin \left( x + \frac{\pi}{2} \right)$ is to the left of the graph of $y = \sin(x)$. There has been a horizontal shift left of $\frac{\pi}{2}$ units. Look at another example.

Let $y = \sin \left( x + \frac{\pi}{3} \right)$.

In relation to the graph of $y = \sin(x)$ the graph of $y = \sin \left( x + \frac{\pi}{3} \right)$ has been shifted to the left $\frac{\pi}{3}$ units. Notice the graph has not moved vertically. You should now repeat the previous examples using the cosine function.

**Section 4.3**

**Question 1** asks you to graph $y = \sin \left( x + \frac{\pi}{4} \right)$. Before you begin to enter this in your calculator compare the equation to the generalized form $y = A \sin(Bx + C)$. You know that in **question 1**, the value for $A$ is one, the value for $B$ is one, and the value for $C$ is...
\[
\frac{\pi}{4}
\].  From this you know the amplitude is one, the period is \(2\pi\), and the phase shift is \(\frac{\pi}{4}\) units to the left.

press \( \text{MODE} \)
select \( \text{Radian} \)
press \( \text{ENTER} \)
press \( y = \)
enter \( \sin(x) \) in \( Y_1 \)
enter \( \sin(x + \frac{\pi}{4}) \) in \( Y_2 \)  
Make this curve darker.
press \( \text{WINDOW} \)
enter \(-\frac{\pi}{4}\) for \( X_{\min} \)  
The phase shift is to the left \(\frac{\pi}{4}\) units.
That is an appropriate choice for the minimum \(x\)-value. Normally you would select zero for the minimum \(x\)-value; however the graph has been shifted to the left \(\frac{\pi}{4}\) units. Hence you subtract \(\frac{\pi}{4}\) from zero.
enter \( 2\pi \) for \( X_{\max} \)  
Add the length of the period, \(2\pi\), to initial value, \(-\frac{\pi}{4}\).
enter \( \frac{\pi}{4} \) for \( X_{\text{scl}} \)  
There is no best value for this \(x\)-scale; \(\frac{\pi}{4}\) was chosen since the phase shift was to the left \(\frac{\pi}{4}\) units.
enter \(-2\) for \( Y_{\min} \)  
Select \(y\)-values for the minimum and the maximum that permit you to see the shape of the graph.

enter \( 2 \) for \( Y_{\max} \)
enter \( 1 \) for \( Y_{\text{scl}} \)
press \( \text{GRAPH} \)
Interpret the graph. Do you see one complete cycle of the function? Is the amplitude one? Is there a phase shift to the left $\frac{\pi}{4}$ units? The answer to all of the questions is yes. (Actually you see more than one complete cycle since you have expanded the window to include the phase shift.) The graph supports your analytical work.

**Question 11** asks you to identify the amplitude, period, and phase shift of $y = \sin(2x - \pi)$ and to sketch the graph. First write the function as $y = \sin\left(2\left(x - \frac{\pi}{2}\right)\right)$.

Now you know the amplitude is one; the period is $\frac{\pi}{2}$ since $2\pi / 2 = \pi$, and the phase shift is $\frac{\pi}{2}$ units to the right. Now enter $y = \sin(2x - \pi)$ in your calculator using the keystrokes listed previously. Before you press GRAPH, set the WINDOW. The phase shift is $\frac{\pi}{2}$ units to the right; make this the minimum x-value. The period is $\pi$, so make the maximum x-value equal to $\frac{3\pi}{2}$. (Recall to find the ending point of this period, you should add the length of the period to the beginning value.)

Now interpret the graph. Make sure the graph supports your analytical work. If you find it difficult to interpret the graph without seeing the y-axis, change the minimum x-value to 0 and change the y = screen to the following. $Y_1 = \sin\left(2x - \pi\right)\left(\frac{\pi}{2} \leq x\right)$. (The $\left(\frac{\pi}{2} \leq x\right)$ is telling the calculator to start at $\frac{\pi}{2}$.)

To verify your paper-pencil work, do the following to find the coordinate of the x-value for the beginning of the period:

press Trace

![Graph of $y = \sin(2x - \pi)$]
**Note.** The \( Y = 4.102E^{-10} \) is scientific notation and is 0.0000004102. When \( x = \frac{\pi}{2} \), the y-value should be zero and if the calculator had sufficient power this would be the case. Try the following keystrokes.

```
press CALC
select value
enter \( \frac{\pi}{2} \) for the x-value
press ENTER
```

Did you get an error message? If you had entered \( \frac{\pi}{2} \) for your minimum x-value in the window setting the calculator had entered a decimal representation for this value, a value when rounded off that was slightly larger than \( \frac{\pi}{2} \). The error message is telling you that the x-value you requested is not in the interval. No problem.

```
press WINDOW
enter 0 for \( X_{\text{min}} \)
press CALC
select value
enter \( \frac{\pi}{2} \)
```

The graph was shifted \( \frac{\pi}{2} \) units to the right. Continue to trace along the function.

The period is \( \pi \); if this is correct the ending point of the cycle that began at \( \frac{\pi}{2} \) should be \( \frac{\pi}{2} + \pi \approx 4.712 \). You will have to change the maximum x-value to a larger value--2\( \pi \) for example.
Your graph supports that the period is $\pi$ and further investigation will support that the amplitude is one.

To get a better picture of the graph, you could have done the following.

Press \texttt{GRAPH}

![Graph Image]

Again, when you entered the $\left(\frac{\pi}{2} \leq x\right)$, you were limiting the permissible $x$-values to values greater than or equal to $\frac{\pi}{2}$.

Press \texttt{TRACE}

Enter $\frac{\pi}{2}$

![Trace Image]

This screen supports your work that the period begins at $\frac{\pi}{2}$. (Recall the decimal representation for $\frac{\pi}{2}$ is approximately 1.571.)

**Question 33** asks you to graph $y = 4\cos\left(2x - \frac{\pi}{2}\right)$, $-\frac{\pi}{4} \leq x \leq \frac{3\pi}{2}$ and to label the axes so that the amplitude, period, and phase shift are easy to read.
Before entering this function in your calculator, look at the question analytically. First write the problem in the form \( y = A \cos(B(x + C)) \). You do this by factoring.

\[
y = 4 \cos \left( 2 \left( x - \frac{\pi}{4} \right) \right)
\]  
(Mentally multiply \( 2 \left( x - \frac{\pi}{4} \right) \); did you get the original expression of \( \cos \left( 2x - \frac{\pi}{2} \right) \)?

Once you have rewritten the function, you know the value of \( A \) is one, the value of \( B \) is two, and the value of \( C \) is \( -\frac{\pi}{4} \). Hence the amplitude is 4; the period is \( \pi \), and the phase shift is \( \frac{\pi}{4} \) units to the right. Now you are ready to use your calculator. As always, check to be sure you are in radian mode.

press \( y = \)  
Enter \( 4 \left( \cos \left( 2x - \frac{\pi}{2} \right) \right) \) in \( Y_1 \)  
Enter the function as written in your text. If you have made an arithmetic error in your paper-pencil work, you do not want to graph your error.

press WINDOW  
Enter \( -\frac{\pi}{4} \) for \( X_{\text{min}} \)  
The directions give the interval \( -\frac{\pi}{4} \leq x \leq \frac{3\pi}{2} \), so enter \( -\frac{\pi}{4} \) for the minimum x-value.

enter \( \frac{3\pi}{2} \) for \( X_{\text{max}} \)  
enter \( \frac{\pi}{4} \) for \( X_{\text{scl}} \)  
Since there is no best value for this x-scale; \( \frac{\pi}{4} \) was chosen since the phase shift was to the left \( \frac{\pi}{4} \) units.

Enter \( -5 \) for \( Y_{\text{min}} \)  
Since the amplitude is 4, you need a number smaller than -4.

enter \( 5 \) for \( Y_{\text{max}} \)  
enter \( 1 \) for \( Y_{\text{scl}} \)  
press GRAPH
On the right is the graph of $y = \cos(x)$. Use this to interpret your graph of $y = 4\cos\left(2\left(x - \frac{\pi}{4}\right)\right)$ on the left. The function has amplitude of 4; the graph supports that. The function has a period of $\pi$; the graph shows one complete cycle from $\frac{\pi}{4}$ to $\frac{5\pi}{4}$. The phase shift is $\frac{\pi}{4}$ units to the right; look at the graph on the right and note that the cycle begins at 0; now look at the graph on the left; one cycle begins at $\frac{\pi}{4}$, a shift of $\frac{\pi}{4}$ units to the right. Your graph supports your analytical work.

**Question 41** asks you to graph $y = \csc\left(x + \frac{\pi}{4}\right)$ by first graphing the reference curve. The reference curve for the cosecant function is the sine curve. First analyze the function $y = \sin\left(x + \frac{\pi}{4}\right)$. From your previous work, you know that the amplitude is one, the period is $2\pi$, and the phase shift is $\frac{\pi}{4}$ units to the left. Recall that the sine function and the cosecant function are inverse functions. From this you know $\csc\left(x + \frac{\pi}{4}\right) = \frac{1}{\sin\left(x + \frac{\pi}{4}\right)}$; hence when $\sin\left(x + \frac{\pi}{4}\right) = 0$, the function is undefined.

Before graphing $y = \csc\left(x + \frac{\pi}{4}\right)$, graph the reference curve $y = \sin\left(x + \frac{\pi}{4}\right)$.

![Graph of y = cos(x)](image1)

![Graph of y = 4cos(2x - \pi/4)](image2)

**Press**

$y = \sin\left(x + \frac{\pi}{4}\right)$ in Y₁

**Enter**

$-\frac{9\pi}{4}$ for Xmin

$\frac{9\pi}{4}$ for Xmax

$\frac{\pi}{4}$ for Xscl

-3 for Ymin

3 for Ymax

1 for Yscl
press GRAPH

The x-values where \( \sin\left(x + \frac{\pi}{4}\right) = 0 \) will be the x-values of the vertical asymptotes.

Now enter \( \frac{1}{\sin\left(x + \frac{\pi}{4}\right)} \) in \( Y_2 \). Press GRAPH.

The graph of \( y = \csc\left(x + \frac{\pi}{4}\right) \) is the same as the graph of \( y = \frac{1}{\sin\left(x + \frac{\pi}{4}\right)} \). Turn off the graph of \( y = \sin\left(x + \frac{\pi}{4}\right) \). Press GRAPH.

Interpret the graph. The straight lines are not part of the graph and if you put your calculator in Dot MODE, you will see the lines disappear. The lines appear to be vertical asymptotes; they are not, but they are very close to vertical lines.
Section 4.4

The directions for question 7 state that one complete cycle of the graph of an equation containing a trigonometric function is given and asks you to find an equation to match the graph. The shape of the curve in question 7 suggests you use the general cosine equation \( y = k + A \cos(Bx + C) \). By inspection, the curve has amplitude of 3; this tells you that \( A \) equals 3. The period is \( 2\pi \); this tells you that the value for \( B \) is one. There is no phase shift and not vertical translation; hence both \( C \) and \( k \) are zero. Replacing the letters with the correct values gives you the equation for the function as \( y = 3 \cos(x) \). Now use your calculator to check your work.

\[
\text{press } y = \\
\text{enter } 3 \cos(x) \text{ in } Y_1 \\
\text{press WINDOW} \\
\text{enter } 0 \text{ for } X_{\text{min}} \\
\text{enter } 2\pi \text{ for } X_{\text{max}} \\
\text{enter } \frac{\pi}{2} \text{ for } X_{\text{scl}} \\
\text{enter } -3 \text{ for } Y_{\text{min}} \\
\text{enter } 3 \text{ for } Y_{\text{max}} \\
\text{enter } 1 \text{ for } Y_{\text{scl}} \\
\text{press GRAPH}
\]

Compare your calculator screen with the graph in your text. If your equation is correct, your graph should match the graph in your text. Press TRACE.

\[
\text{By using the TRACE feature, you can check major points of the graph to see if they are correct. For example, the preceding calculator screen showed the coordinates of the point where the x-value is } \frac{\pi}{2} \text{ (1.5707963). The calculator returned a y-value of zero which matches with the y-value of } \frac{\pi}{2} \text{ in the text. Check several key points.}
\]
before deciding that your equation is correct. Remember that once you have pressed TRACE, you may enter the x-value you wish to check. You do not have to use the CALC feature even though that is an acceptable procedure for finding the y-value when a specific x-value is given.

Try question 19. The curve appears to be the sine curve turned upside down. Mathematically speaking the sine curve has been reflected about the x-axis and the A-value will be negative. From inspection you see that amplitude is 3; the period is 2; and there is no phase shift or vertical translation. To find the value for B recall that the period equals \( \frac{2\pi}{B} \). Your paper-pencil work should resemble the following.

\[
\begin{align*}
2 &= \frac{2\pi}{B} \\
2B &= 2\pi \\
B &= \pi 
\end{align*}
\]

The general equation for the sine function is \( y = k + A\sin(Bx + C) \). Inserting numerical values for the letters gives the equation \( y = -3\sin(\pi x + 0) \) or \( y = -3\sin(\pi x) \). Check your work by entering this equation in your calculator. Set the calculator window to match the settings used in the text. Your \( y = \) screen; WINDOW settings; and graph should resemble the following.

Now use the TRACE feature to verify key points of the graph. For example find the y-value when x is 1. You know from looking at the graph in the text, that the answer is zero. Now check your graph.

Continue checking key points until you are sure you have the correct equation.
**Problem Set 4.5**

Before using your calculator for question 3, analyze the function analytically. Given \( y = 2 - \cos(x) \), you note that the A-value is -1; the B-value is 1; the C-value is 0; and the k-value is 2. Because A is negative, the graph is reflected about the x-axis; since B is one, the period is \( 2\pi \); because C is zero there is no phase shift; and since k is 2, each y-value is increased by 2 (a vertical shift up 2 units). Compare your graph with the following calculator screens. The x-minimum and maximum values for the WINDOW setting are given in the text; the y-minimum and maximum values are determined by the amplitude and vertical shift. (You could use 1 and 3; however it is helpful to have space above and below the curve.) Negative one and four are suitable values for the minimum and maximum y-values, but certainly not the only acceptable values.

![Graph of \( y = 2 - \cos(x) \)]

**Question 25** asks you to generate a table of values for \( y = x \sin(x) \) using multiples of \( \frac{\pi}{2} \) from 0 to \( 4\pi \). You should first generate this table by hand and then use the following keystrokes to verify your table. (Your calculator should be in radian mode.)

1. Press \( \text{Y=} \)
2. Enter \( x \sin(x) \) in \( Y_1 \)
3. Press \( \text{TBLSET} \)
4. Enter \( 0 \) in TblStart
5. Enter \( \frac{\pi}{2} \) in \( \Delta \text{Tbl} \)
6. Press \( \text{TABLE} \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1.5708</td>
<td>1.5708</td>
</tr>
<tr>
<td>3.1416</td>
<td>3.1416</td>
</tr>
<tr>
<td>4.7124</td>
<td>4.7124</td>
</tr>
<tr>
<td>6.2832</td>
<td>6.2832</td>
</tr>
<tr>
<td>7.8540</td>
<td>7.8540</td>
</tr>
</tbody>
</table>

From the table, you know the coordinates of some of the points on the graph of the function. For example, \((0,0), (1.5708,1.5708), (3.1416,0)\), etc. are coordinates of points on the graph of the function. Now graph the function and verify your findings.
Section 4.6

Question 5 asks you to find $\sin^{-1} \left( \frac{\sqrt{3}}{2} \right)$ and to write the answer in radians.

- Press MODE
- Select Radians
- Press ENTER
- Press SIN^{-1}
- Enter $\frac{\sqrt{3}}{2}$
- Press ENTER

Interpret your answer of 1.047197551. Your calculator is telling you if the sine of some angle is $\frac{\sqrt{3}}{2}$, the measure of the angle is 1.047197551 radians. Check your answer.

- Press SIN
- Press ANS
- Press ENTER

The calculator gave the answer as a decimal so you need to convert $\frac{\sqrt{3}}{2}$ to a decimal so you can compare your answers. From the preceding screen, you see that your answer is correct.
Chapter 5

Identities and Formulas

Problem Set 5.1

The directions for this problem set ask you to prove that the identities are true. Basically this is a paper-pencil exercise; however, there are times the calculator can let you know if you are correct. For example, look at the identity given in question 17.

\[
\frac{\cos^4 t - \sin^4 t}{\sin^2 t} = \cot^2 t - 1
\]

Your text suggests that you start with what appears to be the more complicated side and through a series of substitutions and simplifications make it identical to the other side. The following is one way to prove this identity. The calculator is going to be used after each step to support the paper-pencil work.

press \text{Y=}

enter \(\cot^2 t - 1\) in \(Y_1\)  

Use \(x\) instead of \(t\), when entering this in your calculator.

enter \(\frac{\cos^4 t - \sin^4 t}{\sin^2 t}\) in \(Y_2\)

press \text{ZOOM}

select \text{ZTrig}  

This will give you a general trigonometric window which is sufficient for checking this identity. (Either press \# 7 or ENTER after you have moved the cursor to ZTrig.)

\[
\begin{align*}
Y_1 &= (\tan(x))^2 \\
Y_2 &= (\cos(x))^4 - (\sin(x))^4) / (\sin(x))^2 \\
Y_3 &= \\
Y_4 &= \\
\text{WINDOW} \\
X_{\text{min}} &= -6.152285... \\
X_{\text{max}} &= 6.1522856... \\
X_{\text{scl}} &= 1.5707963... \\
Y_{\text{min}} &= -4 \\
Y_{\text{max}} &= 4 \\
Y_{\text{scl}} &= 1 \\
\text{Arres} &= 1
\end{align*}
\]

Note. You noticed that the calculator continued to graph even though you could not see a new graph appearing. Actually the calculator graphed what was in \(Y_i\) which you could see. Then the calculator graphed what was in \(Y_2\), but you could not see this because the graph was on top of the first graph. You knew the calculator was still working because there was a short line moving up and down in the upper right corner. This is not a proof that the two expressions are
identities; however, it is a strong indication. Use the up and down arrows to see if the y-values change. Notice that the y-values do not change.

Now work with simplifying the left side of the given identity. Remember that your goal is to simplify this to match the right side of the identity.

\[
\frac{\cos^4 t - \sin^4 t}{\sin^2 t} = \frac{(\cos^2 t - \sin^2 t)(\cos^2 t + \sin^2 t)}{\sin^2 t}
\]

Since \( \cos^2 t + \sin^2 t = 1 \), make this substitution.

\[
\frac{(\cos^2 t - \sin^2 t)}{\sin^2 t}
\]

Now enter this in expression in \( Y_2 \). You are comparing the graph of \( \frac{(\cos^2 t - \sin^2 t)}{\sin^2 t} \) to the graph of the original expression, \( \frac{\cos^4 t - \sin^4 t}{\sin^2 t} \).

If the graphs are identical, you have the reassurance that your paper-pencil work is correct.

It appears you still have not made a mistake. Continue with your paper-pencil work.

\[
\frac{(\cos^2 t - \sin^2 t)}{\sin^2 t}
\]

You might want to factor the numerator again.

\[
\frac{(\cos t - \sin t)(\cos t + \sin t)}{\sin^2 t}
\]

You are not wrong; however it does not help you.

\[
\frac{(\cos^2 t - \sin^2 t)}{\sin^2 t}
\]

Try writing as the difference of two fractions.

\[
\frac{(\cos^2 t)}{\sin^2 t} - \frac{(\sin^2 t)}{\sin^2 t}
\]

Now enter this in expression in \( Y_2 \) and compare the two graphs.
You work appears to still be correct. Continue!

\[
\frac{\cos^2 t}{\sin^2 t} - \frac{\sin^2 t}{\sin^2 t} \quad \text{Substitute } \cot^2(t) \text{ for } \frac{\cos^2 t}{\sin^2 t} \text{ and } 1 \text{ for } \frac{\sin^2 t}{\sin^2 t}.
\]

\[\cot^2(t) - 1\]

You are finished. Through a series of substitutions you have converted the left side to the same form as the right side. The benefit of the graphing calculator was to support your work and to let you know if you had made a mistake along the way.

**Question 75** asks you to find a value of \( \theta \) that makes the following statement false.

\[\sin \theta = \sqrt{(1 - \cos^2 \theta)}\]

press \( Y = \)

enter \( \sin \theta \) in \( Y_1 \)

enter \( \sqrt{(1 - \cos^2 \theta)} \) in \( Y_2 \)

press \( \text{GRAPH} \)

Interpret your graph. Any \( x \)-value where the two graphs are not identical is an \( x \)-value that would make \( \sin \theta = \sqrt{(1 - \cos^2 \theta)} \) false. For example, when \( x = -\frac{\pi}{2} \), the \( \sin(x) \) is -1 and \( \sqrt{(1 - \cos^2 x)} \) is +1.

Use the TRACE and CALC features of the calculator to find these values. Remember that it only takes one counterexample to prove that these two expressions are not identities.
Problem Set 5.2

Question 1 asks you to find an exact value for $\sin 15^\circ$. Your calculator will only give you a decimal approximation; however, you can use it to check your answer. After you have completed your paper pencil work, convert your answer to a decimal and compare it with the decimal approximation from your calculator.

press MODE
select Degree
press ENTER
press CLEAR
press SIN
enter 15
press ENTER

\[
\sin(15^\circ) = 0.2588190451
\]

Problem Set 5.3

Question 17 asks you to graph $y = 4 - 8\sin^2 x$ from $x = 0$ to $x = 2\pi$.

press MODE
select radian
press ENTER
press Y=
enter $4 - 8(\sin(x))^2$ in $Y_1$ Note the use of parenthesis.
press WINDOW
enter 0 for $X_{\text{min}}$ This value was given to you in the directions.
enter $2\pi$ for $X_{\text{max}}$ This value was given to you in the directions.
enter $\frac{\pi}{2}$ for Xscl
enter -8 for $Y_{\text{min}}$
enter 8 for $Y_{\text{max}}$
enter 1 for Yscl
press GRAPH
Note. You can easily translate your calculator graph to a paper-pencil graph if you know the window settings. For example, in the above graph since you know the x-scale is $\frac{\pi}{2}$, you can identify the x-intercepts very quickly and translate these to your paper. You also know the y-scale is one so you can very quickly identify the y-intercept and the amplitude.

Problem Set 5.4

Question 27 asks you to graph $y = 4\sin^2 \frac{x}{2}$ from $x = 0$ to $x = 4\pi$.

Note. The important thing for you to remember when entering this problem in the calculator is the correct use of parentheses. You are given the minimum and maximum x-values. You should probably start with a minimum y-value of -4 and a maximum y-value of 4 since the amplitude is 4. Recognize you are squaring the sine values which will yield only positive results.

Your calculator screens should resemble the following.

Does your calculator screen appear to be blank when you pressed GRAPH? Are you in degree mode? If so change to radian mode to get the following screen.

The graph supports your analytical work that the graph of this function is never negative.
Question 37 asks you to prove an identity. Remember the calculator will not prove the identity for you, but the graphing capabilities will help you determine if you have an identity. Refer back to the beginning of Chapter 5 for an example of using the graphing calculator to assist you in proving identities.
Chapter 6

Equations

Problem Set 6.1

Question 13 asks you to solve \(4 \sin \theta - 3 = 0\) in the interval \(0^\circ \leq \theta < 360^\circ\) to the nearest tenth of a degree.

You should first do the paper-pencil work to isolate \(\theta\). Your work should resemble the following.

\[
\begin{align*}
4 \sin \theta - 3 & = 0 \\
4 \sin \theta & = 3 \\
\sin \theta & = \frac{3}{4} \\
\sin^{-1}\left(\frac{3}{4}\right) & = \theta
\end{align*}
\]

After you have done the preceding work, use the following keystrokes to complete the task of finding the correct value for \(\theta\).

press \ MODE
select \ Degree
press \ ENTER
press \ CLEAR
press \ SIN^{-1}
enter \ \frac{3}{4}
press \ ENTER

The directions asked you to round your answer to the nearest tenth of a degree. Certainly you can do that; however, if you wish the calculator to give the answer to you to the nearest tenth of a degree use the following keystrokes.
press MODE
select 1 decimal place

press ENTER press CLEAR press 2nd ENTER This will recall your preceding work so you will not have to reenter the expression.

press ENTER

\[
\sin^{-1}\left(\frac{3}{4}\right) = 48.6^\circ
\]

Interpret your answer. You have found a value for \( \theta \) such that \( 4 \sin(48.6^\circ) - 3 \approx 0 \). (The approximately equal symbol was used instead of the equal sign since the value for \( \theta \) was rounded to tenths.) Is this the only value for \( \theta \) in the interval \( 0^\circ \leq \theta < 360^\circ \) that will make \( 4 \sin \theta - 3 = 0 \). One way to answer this question is to look at the graph.

press \( Y= \)
enter \( 4 \sin x - 3 \) in \( Y_1 \) When you enter the expression in the calculator, use the variable \( x \) instead of \( \theta \) to represent the angle.

press WINDOW
enter 0 for \( X_{\text{min}} \) Remember you are in degrees and the interval for the solution is \( 0^\circ \leq \theta < 360^\circ \).

enter 360 for \( X_{\text{max}} \)
enter 10 for \( X_{\text{scl}} \)
enter -8 for \( Y_{\text{min}} \) An estimate for the minimum \( y \)-value is amplitude plus the vertical shift. For this problem that would be -7; however, it is usually wise to make the window just a little larger.

enter 4 for \( Y_{\text{max}} \)
press ENTER

The graph clearly is equal to zero in two places. Earlier you had only found one value. There are two answers to the equation because the sine function is positive in both the first and second quadrants. You found the reference angle to be Rodgers,K. 75
48.6°, which is in the first quadrant. The angle in the second quadrant would be 180° - 48.6° or 131.4°.

You could have found this value from the graph by using the following keystrokes.

press GRAPH
press CALC
select zero

Your screen should resemble the one above. If not use the arrows until the cursor is to the left of the x intercept.

press ENTER

Use the right blue arrow until the cursor is to the right of the x-intercept. You do not have to have the exact x-value for the right bound that is shown on the preceding screen.

press ENTER

The calculator is asking you to guess. What the calculator needs is for you to trace as close to the x-intercept as possible and then press ENTER.
Interpret the screen. You have found a second value in this interval for $\theta$ that makes $4 \sin \theta - 3 = 0$ a true statement.

You are almost finished; however, whenever time permits check your answers. Return to the Home Screen and use the following keystrokes.

```
enter 4 \sin(48.6) -3
press ENTER
```

interpret the 4.4 E-4. It means -0.0004. You wanted the answer to be zero. Even though this is a very small number it is not zero. Recall you rounded $\theta$ to the nearest tenth of a degree. That is why your answer is not exactly equal to zero.

Now check the second value of $\theta$.

```
4\sin(48.6)-3  4.4E-4
4\sin(131.4)-3  4.4E-4
```

You got the same answer. Again, if you had not rounded $\theta$, you would have gotten a zero for the answer. Look at the following screen. The angle $\theta$ was not rounded. This $\theta$-value gives the exact value of zero when checking.

```
sin(3\pi/4)  48.59937789
4\sin(A) -3  0
```

**Note.** There are many key concepts illustrated in this problem. You should understand how to find all of the angle values both analytically and graphically within a given interval.

**Question 23** asks you to give exact values. As you know, the calculator cannot always give you exact answers; however, the calculator can tell you how many x-values will make the equation a true statement in the given interval.
Chapter 6

press MODE
select Radian
press ENTER
press CLEAR
press Y=
enter $\sin(x) + 2\sin(x)\cos(x)$ in Y1
press WINDOW
enter 0 for $X_{\text{min}}$

Remember you are in radians and the interval for the solution is $0 \leq x < 2\pi$.

enter $2\pi$ for $X_{\text{max}}$
enter $\frac{\pi}{6}$ for Xscl
enter -4 for $Y_{\text{min}}$
enter 4 for $Y_{\text{max}}$
press GRAPH

In the interval from 0 to $2\pi$, the graph crosses the x-axis five times. That tells you that there are five x-values that will satisfy the equation, $\sin(x) + 2\sin(x)\cos(x) = 0$. You should find those x-values analytically and then use your graph to support your answers or to check your answers on the Home Screen.

**Question 33** asks you to use the quadratic formula to find all solutions in the interval $0^\circ \leq \theta < 360^\circ$. If you have the quadratic program in your calculator, use the following sequence of strokes to solve this problem. If you do not have the quadratic program in your calculator, go to Appendix A for a copy of the program.

press PRGM PRGM is for program
select QUAD Select the quadratic program. You may have used a different name for the program.
press ENTER

78 Rodgers,K.
The program is asking you for the A-value. Question 33 is $2\sin^2\theta - 2\sin\theta - 1 = 0$.
Hence the A-value is 2, the B-value is -2, and the C-value is -1.

```
enter 2
press ENTER
enter -2
press ENTER
enter -1
press ENTER
```

You are not finished! The two x-values given are values for the $\sin\theta$, not just $\theta$. Is it possible to find $\sin^{-1}(1.366025404)$? Recall the range for the $\sin^{-1}$ function is $-1 \leq x \leq 1$. Hence you disregard any values greater than 1. Now go to the second value for x.

```
\sin^{-1}(-0.3660254038)
```

Use your calculator to find the answer to this expression.
Remember you are looking for the answer in degrees. Check your MODE.

You have found the reference angle. You also know the sine function is negative in the third quadrant and the fourth quadrant. To get the answer in the third quadrant, add $180^\circ$ to the absolute value of the reference angle, and to get the answer in the fourth quadrant add $360^\circ$ to the reference angle.
Your two answers are 201.5° and 338.5°.

Support your answer graphically.

The graph shows two answers. Use the CALC feature to see if your answers match the x-intercepts on the graph.

Press CALC
Select zero

Use the left blue arrow and make sure the cursor is on the left side of the x-intercept.

Press ENTER

Use the right blue arrow and move the cursor to the right of the x-intercept.

Press ENTER
Use the arrows and move the cursor close to the x-intercept and press ENTER.

![Graph showing the x-intercept]

This supports your first answer. Repeat the process and check your second answer.

**Question 47** asks you to find all degree solution(s) for the equation

\[
\cos(2A - 50°) = \frac{\sqrt{3}}{2}.
\]

Your preliminary paper-pencil work should resemble the following.

\[
\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 2A - 50°
\]

Use your calculator to find a value for \(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\). Check the MODE to make sure your calculator is in degree mode.

- Press \(\cos^{-1}\)
- Enter \(\frac{\sqrt{3}}{2}\)
- Press ENTER

\[
\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \approx 30°
\]

The angle measuring 30° is in the first quadrant. The cosine function is also positive in the fourth quadrant; the angle measure would be 330°. Now use these two values to complete your paper-pencil calculations.
\[
\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 2A - 50^\circ
\]

Using \(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30^\circ\)

\[30^\circ + 360^\circ k = 2A - 50^\circ \quad (k \text{ is an integer})\]
\[80^\circ + 360^\circ k = 2A\]
\[80^\circ + \frac{360^\circ}{2} k = A\]
\[40^\circ + 180^\circ k = A\]

Using \(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 330^\circ\)

\[330^\circ + 360^\circ k = 2A - 50^\circ\]
\[380^\circ + 360^\circ k = 2A\]
\[\frac{380^\circ}{2} + \frac{360^\circ}{2} k = A\]
\[190^\circ + 180^\circ k = A\]

Simplified
\[10^\circ + 180^\circ k = A\]

Now use your calculator to support your answer.

Press \(Y=\) and enter \(\cos(2X - 50)\) in \(Y_1\) You will have to use the letter \(X\) instead of \(A\). There is no need to use the degree symbol; however, make sure your calculator is in degree mode.

Enter \(\frac{\sqrt{3}}{2}\) in \(Y_2\)

Press WINDOW
Enter 0 for \(X_{\text{min}}\)
Enter 360 for \(X_{\text{max}}\)
Enter 10 for \(X_{\text{scl}}\)
Enter -2 for \(Y_{\text{min}}\)
Enter 2 for \(Y_{\text{max}}\)
Enter 1 for \(Y_{\text{scl}}\)
Press GRAPH
Interpret the graph. You can do this by viewing the TABLE.

```
<table>
<thead>
<tr>
<th>X</th>
<th>Y₁</th>
<th>Y₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.86603</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>.86603</td>
<td>1</td>
</tr>
<tr>
<td>20</td>
<td>.86603</td>
<td>1</td>
</tr>
<tr>
<td>30</td>
<td>.86603</td>
<td>1</td>
</tr>
<tr>
<td>40</td>
<td>.86603</td>
<td>1</td>
</tr>
<tr>
<td>50</td>
<td>.86603</td>
<td>1</td>
</tr>
<tr>
<td>60</td>
<td>.86603</td>
<td>1</td>
</tr>
</tbody>
</table>
```

From the TABLE you can see when $x = 10°$ and $x = 40°$ the answer is .86603, the decimal representation for $\frac{\sqrt{3}}{2}$. Your graph and TABLE-values support your analytical work. There were four points of intersection on the graph. How do you explain the other two points? You found one solution to be $10°$, the next solution is $10° + 180°k$ where $k$ is an integer. Let $k = 1$. The next $x$-value should be $190°$. Check your graph.

```
press GRAPH
press TRACE
enter 190
press ENTER
```

$190°$ does check. The other point of intersection should occur at $40° + 180°k$; let $k = 1$. The next value would be $40° + 180°$ or $220°$.

```
prompt GRAPH
press TRACE
enter 220
press ENTER
```
Chapter 6

This answer also checks.

**Problem Set 6.2**

**Question 13** asks you to solve \( \cos(2x) - 3\sin(x) - 2 = 0 \) for \( 0 \leq x < 2\pi \) giving only exact values for \( x \). Your preliminary paper-pencil work should resemble the following.

\[
\cos(2x) = 1 - 2\sin^2(x)
\]

Recall the double angle identities.

\[
1 - 2\sin^2(x) - 3\sin(x) - 2 = 0
\]

\[
-2\sin^2(x) - 3\sin(x) - 1 = 0
\]

\[
2\sin^2(x) + 3\sin(x) + 1 = 0
\]

\[
(2\sin(x) + 1)(\sin(x) + 1) = 0
\]

Set each factor equal to zero.

\[
2\sin(x) + 1 = 0\quad \text{or} \quad \sin(x) + 1 = 0
\]

\[
\sin(x) = -\frac{1}{2}\quad \text{or} \quad \sin(x) = -1
\]

\[
\text{SIN}^{-1}\left(-\frac{1}{2}\right) = x\quad \text{or} \quad \text{SIN}^{-1}(-1) = x
\]

\[
x = \frac{7\pi}{6}\quad \text{or} \quad \frac{11\pi}{6}\quad \text{or} \quad x = \frac{3\pi}{2}
\]

press \( \text{MODE} \)
select \( \text{Radian} \)
press \( \text{ENTER} \)
press \( \text{CLEAR} \)
press \( Y= \)
enter \( \cos(2x) - 3\sin(x) - 2 \) in \( Y_1 \)
press \( \text{WINDOW} \)
enter 0 for \( X_{\text{min}} \)
enter \( 2\pi \) for \( X_{\text{max}} \)
enter \( \frac{\pi}{6} \) for \( X_{\text{scl}} \)

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You are looking for the x-values that make the equation equal zero. It is difficult to see that from this graph. For this problem it is not necessary to see a complete graph; hence change the minimum and maximum y-values in the WINDOW.

Again, you do not see a complete graph; however, you only need to see the interval where the graph is crossing the x-axis.

Convert $\frac{7\pi}{6}$, one of the x-values you found analytically, to a decimal and compare this answer with the zero you found graphically. You see that both answers are the same. Use the same sequence of keystrokes and check the other two x-values that you found analytically.

**Question 33** asks you to solve $6\cos(\theta) + 7\tan(\theta) = \sec(\theta)$ in the interval $0^\circ \leq \theta < 360^\circ$. To find the answer analytically, use trig identities to convert the tangent and secant functions to sine and cosine functions. To solve this equation graphically, enter the left side of the equation in $Y_1$ and the right side of the equation in $Y_2$. (Use the original
equation. If you have made an error in the conversion to sines and cosines, you want to find it.) Make sure your calculator is in degree mode. Compare your calculator screens with the following screens.

It is difficult to interpret the graph. Try cleaning up the graph by going to Dot MODE.

There appears to be two solutions for $x$ in this interval. Remember you are trying to find $x$-values where $6 \cos(x) + 7 \tan(x) = \sec(x)$; that is you are trying to find $x$-values where the two curves intersect. Find the points of intersection using the CALC feature of your calculator and then compare this with your analytical work.

Continue to use your calculator to support your analytical work.

**Problem Set 6.3**

**Question 7** asks you to find all exact numerical solutions for $\sin(2x) = \frac{1}{\sqrt{2}}$ if $0 \leq x < 2\pi$. After you have solved this problem analytically, use your calculator to support your work.

- press MODE
- select Radian
- press ENTER
- select Connected
- press ENTER

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press $Y =$
enter $\sin(2x)$ in $Y_1$
enter $\frac{1}{\sqrt{2}}$ in $Y_2$

Set your window and then graph.

From the graph, you see four points of intersection. This tells you that there are four $x$-values that will make $\sin(2x) = \frac{1}{\sqrt{2}}$ a true statement in the interval $0 \leq x < 2\pi$.

**ALERT.** This problem set is asking for some answers in degrees and others in radians. Continually check to make sure your calculator is in the correct MODE.

**Problem Set 6.4**

**Question 1** asks you to eliminate the parameter and to sketch the graph, given

$$x = \sin(t)$$

$$y = \cos(t)$$

Recall $\sin^2(t) + \cos^2(t) = 1$. Now use substitution.

$$x^2 + y^2 = 1$$

Support your answer graphically. First enter the parametric equations.

press **MODE**
select Radian
press **ENTER**
select **Par**
press **ENTER**
press **Y=**
enter $\sin(t)$ in $X_{TT}$
enter $\cos(t)$ in $Y_{TT}$
press **WINDOW**
enter 0 in $T_{min}$
enter $2\pi$ in $T_{max}$
enter $\frac{\pi}{12}$ in $T_{\text{step}}$
enter -3 in $X_{\text{min}}$
enter 3 in $X_{\text{max}}$
enter 1 in $X_{\text{scl}}$
enter -2 in $Y_{\text{min}}$
enter 2 in $Y_{\text{max}}$
enter 1 in $Y_{\text{scl}}$
press GRAPH

Is this graph identical to the graph of $x^2 + y^2 = 1$?

Use the following keystrokes. Recall from your algebra studies that $x^2 + y^2 = 1$ is a circle with the center at (0,0) and has a radius of 1.

press $Y=$
turn off the graphs
(Use the blue arrows until the cursor is flashing on the equal marks and press ENTER. The equal marks should change color. Use the same keystrokes later to turn the graph on again.)

press QUIT
press DRAW
2nd and PRGM
select circle
press ENTER
The information is entered as the x-coordinate of the center, the y-coordinate of the center, and the radius.

enter (0,0,1)

press ENTER
Press ENTER and not GRAPH, since you are using the DRAW feature of the calculator.

press $Y=$
turn the graphs on again
press  GRAPH

Your graph supports your analytical work.
Chapter 7

Triangles

You will use your calculator often to complete the exercises in Chapter 7; however, very few new keystrokes will be needed. It is important that you learn to use parentheses so you can enter the entire expression at one time. This practice will minimize rounding errors.

Problem Set 7.1

Question 1 asks you to find side $b$, given $A = 40^\circ$, $B = 60^\circ$, and $a = 12$ cm. Your preliminary paper-pencil work should resemble the following.

Using the Law of Sines

$$\frac{12}{\sin(40)} = \frac{b}{\sin(60)}$$

$$\sin(60) \left( \frac{12}{\sin(40)} \right) = b$$

Your calculator screen should mirror the last line of the preceding paper-pencil work. Check to be sure that you are in degree mode. Compare your screen to the screen that follows.

How can you prove to yourself that the above answer is correct? Substitute this back in the original expression.

$$\frac{12}{\sin(40)} = \frac{16.16755626}{\sin(60)}$$

Enter this expression in your calculator. The calculator uses Boolean logic and will return a one if this is a true statement and a zero if this statement is false.
Chapter 7

Problem Set 7.2

Question 1 asks you to solve for B, and explain why there is no solution to this triangle.

Given \( A = 30^\circ; \ b = 40 \text{ ft}; \) and \( a = 10 \text{ ft} \)

Use the Law of Sines

\[
\frac{\sin(30)}{10} = \frac{\sin(B)}{40}
\]

\[
40 \left( \frac{\sin(30)}{10} \right) = \sin(B)
\]

\[
2 = \sin(B)
\]

The last statement is false; \( \sin(B) \) cannot be greater than one. This tells you that there is no solution; the given conditions do not form a triangle. If you entered the expression in your calculator, your screens would resemble the following. The screen on the left is where the expression is entered. The screen on the right is the message the calculator returns when you press ENTER.

Note. The error message on the top of the screen says DOMAIN. The calculator is reminding you that you have tried to perform an operation on a value that is not in the domain.

Problem SET 7.3

Question 1 asks you to find c, given \( a = 120 \text{ inches}, \ b = 66 \text{ inches}, \) and \( C = 60^\circ \).

Your preliminary paper-pencil work should resemble the following.
Use the Law of Cosines.
\[c^2 = a^2 + b^2 - 2ab \cos(C)\]
\[c^2 = 120^2 + 66^2 - 2(120)(66)\cos(60)\]
\[c = \sqrt{(120^2 + 66^2 - 2(120)(66)\cos(60))}\]

Use parentheses and enter the above expression in your calculator.

![Calculator expression]

Entering the expression all at once is much better than entering the expression in steps. You have found the length of side c, 104.096 inches.

**Problem Set 7.4**

You will need to use Heron’s Formula to complete question 1.

**Heron’s Formula:**

\[S = \sqrt{s(s-a)(s-b)(s-c)}\]

where S stands for the area of the triangle and s is half of the perimeter of the triangle.

Given \(a = 50\) cm, \(b = 70\) cm, \(C = 60^\circ\).

You need to know the value of c.

Use the Law of Cosines.
\[c = \sqrt{(50^2 + 70^2 - 2(50)(70)\cos(60^\circ))}\]
\[c = 62.44997998\]
Now use Heron's Formula.

\[ s = \frac{1}{2}(a + b + c) \]

\[ s = \frac{1}{2}(50 + 70 + 62.4498) \]

\[ s = 91.22499 \]

\[ S = \sqrt{(91.22499(91.22499 - 50)(91.22499 - 70)(91.22499 - 62.4498))} \]

Since you will need to use the value of \( s \) several times, store this value in your calculator.

```
enter  91.22499
press  STO
press  S
press  ENTER
```

Interpret the calculator screen. You are saying the area of \( \triangle ABC \) is 1515.54457 \( \text{cm}^2 \).
Chapter 8
Complex Numbers and Polar Coordinates

Problem Set 8.1

Question 1 asks you to simplify $\sqrt{-16}$. You should definitely master the paper-pencil skills necessary to complete this task. You can use your calculator to check your work.

1. Press MODE
2. Select $a + bi$
3. Press ENTER
4. Press CLEAR
5. Enter $\sqrt{-16}$
6. Press ENTER

Note. If you are not in $a + bi$ mode the calculator returns the following screen.

Interpret this screen. The error message is telling you there is no real solution.

Question 9 asks you to simplify $\sqrt{-4} \cdot \sqrt{-9}$. Again, you should master the paper-pencil skills required to work this problem and only use your calculator to check your answer.

1. Press MODE
2. Select $a + bi$
3. Press ENTER
4. Press CLEAR
5. Enter $\sqrt{-4} \cdot \sqrt{-9}$
6. Press ENTER
Question 23 asks you to combine \((7 + 2i) + (3 - 4i)\).

press MODE
select \(a + bi\)
press ENTER
press CLEAR
enter \((7 + 2i) + (3 - 4i)\)
press ENTER

The \(i\) key is above the decimal point on the bottom row of keys.

Section 8.2

Question 27 asks you to write \(10(\cos 12^\circ + i \sin 12^\circ)\) in standard form rounding to the nearest hundredth. Your paper-pencil work should resemble the following.

\[
10(\cos 12^\circ + i \sin 12^\circ) = 10(0.978 + i 0.208) = 9.78 + 2.08i
\]

Use the following calculator keystrokes to check your work.

press MODE
select \(a + bi\)
press ENTER
select Degree
press ENTER
press CLEAR
enter \(10(\cos 12^\circ + i \sin 12^\circ)\)
press ENTER
Interpret the screen. The dots to the right of the answer indicate there is more. Use the right arrow key and scroll to the right. You will see the full answer.

However, since you were asked to round to hundredths, you could simply tell your calculator to round the answer before you start.

press MODE
select 2 (This is the 2 to the right of Float.)
press ENTER
press CLEAR
press 2nd and ENTER
press ENTER

This answer matches the answer from the paper-pencil work.

**Question 47** asks you to write the complex number $3 + 4i$ in trigonometric form. To complete this task, you must know the value of the modulus ($r$) and the argument ($\theta$). After you have found these values, the trigonometric form of the complex number is $r(\cos \theta + i \sin \theta)$.

press MODE
select Float

You will need to select Float if you still have your calculator set to two decimal places from the previous problem.

press ENTER
select $a + bi$
press ENTER
select Degree
press ENTER
press CLEAR
press ANGLE
select \( R \cdot Pr( \)
enter \( 3,4 \)
press ENTER
press ANGLE
select \( R \cdot P \theta( \)
enter \( 3,4 \)
press ENTER

\[
R \cdot Pr(3,4) \quad 5
\]
\[
R \cdot P \theta(3,4) \quad 53.13010235
\]

Now translate your screen to give the trigonometric form for the complex number \( 3 + 4i \). The \( r \)-value is 5 and the angle measure \( (\theta) \) is 53.13°. The trigonometric form is \( 5(\cos 53.13^\circ + i \sin 53.13^\circ) \). Do you want to check your answer? Use the following keystrokes.

press ANGLE
select \( P \cdot Rx( \)
Enter \( 5,53.130235 \)
press ENTER
press ANGLE
select \( P \cdot Ry( \)
Enter \( 5,53.130235 \)
press ENTER

\[
P \cdot Rx(5,\text{Ans}) \quad 3
\]
\[
P \cdot Ry(5,53.130182) \quad 4
\]

Your answer was correct. You got three for the \( x \)-value and four for the \( y \)-value.
Section 8.3

Question 1 asks you to multiply two complex numbers in trigonometric form and to leave the answer in trigonometric form. Your calculator will multiply the two complex numbers; however, it will return the answer in complex standard form. If you wanted to check an answer, you could multiply using the calculator and then convert the answer to trigonometric form.

You have found the answer in $a + bi$ form. Now change this back to trigonometric form. That is write the complex number $7.713 + 9.193i$ in trigonometric form. To complete this task, you must know the value of the modulus $(r)$ and the argument $(\theta)$. After you have found these values, the trigonometric form of the complex number is $r(\cos \theta + i \sin \theta)$.

Before starting make sure your calculator is in $a + bi$ and degree form.

```
press ANGLE
select R►Pr( Returns r, given x and y.
enter 7.713, 9.193)
p press ENTER
press ANGLE
select R►Pθ( Returns $\theta$, given x and y.
enter 7.713, 9.193))
p press ENTER
```

Note. Your answer is an approximate answer since the x and y values were rounded to three decimal places.

Section 8.4

Question 1 asks you to find two square roots for the complex number $4(\cos 30° + i \sin 30°)$. Your paper-pencil work should resemble the following.
\[ w_k = 4^\frac{1}{3} \left[ \cos \left( \frac{30 + 360k}{2} \right) + i \sin \left( \frac{30 + 360k}{2} \right) \right] \]
\[ = 2 \left[ \cos (15 + 180k) + i \sin (15 + 180k) \right] \]

let \( k = 0 \)
\[ w_1 = 2 \left[ \cos (15) + i \sin (15) \right] \]

let \( k = 1 \)
\[ w_2 = 2 \left[ \cos 195 + i \sin 195 \right] \]

Use your calculator to check your work. To get the concept straight, think of a simple problem. For example, you know \( \sqrt{9} = 3 \) since \( 3^2 = 9 \). Use this same idea to check one of the roots you found for the previous problem. Square the root and you should have the original problem.

press MODE
select Degree
press ENTER
select Pol
press ENTER
select \( a + bi \)
pres ENTER
press CLEAR
enter \( (2(\cos(15) + i \sin(15)))^2 \)

Alert! You must use parentheses correctly. You want the quantity to be multiplied by two and that answer squared.

The answer the calculator gave you is in standard form and the original problem was in trigonometric form. No problem, let the calculator convert the answer to trigonometric form. You know from the original problem the \( r \)-value was 4 and the angle value was 30°.

press ANGLE
select R \( \text{Pr(} \)
pres MATH
select CPX
select real(
pres ENTER or #2
press ANS
enter )

\[ \text{Returns } r \text{ given } x \text{ and } y. \]

Use the right arrow to scroll over to CPX.
enter ,
enter 2
press ENTER

The r-value returned is 4, the same as the r-value in the original problem. Now check the angle value.

press 2nd and ENTER

Repeat these keystrokes until your calculator recalls

\[(2\cos(15) + i \sin(15))^2\]

press ENTER Since you want to use the ANS feature of the calculator, you must have that expression entered preceding the time it is to be used. When you use ANS, the calculator always recall the last answer that it gave.

press ANGLE
select R\_P\_θ (Returns θ given x and y.
press MATH
select CPX
select real(
press ANS
enter )
enter , enter 2
press ENTER

The angle measurement returned is 30°, the same angle-value as in the original problem.

Note. This is just one of several ways you could use your calculator to check this problem.
APPENDIX A

Quadratic Program

PROGRAM: QUADFORM
:Disp “QUADRATIC”
:Disp “FORMULA”
:Disp “AX^2 + BX + C = 0”
:Disp: “A”
:Input A
:Disp “B”
:Input B
:Disp “C”
:Input C
:Ans → D
:If D < 0
:Goto 1
:Ans → P
:Ans → Q
:Goto 2
:Lbl 1
:Disp “REAL ROOTS”
:Disp “ROOT 1”
:Disp P
:Disp “ROOT 2”
:Disp Q
:Goto 2
:Lbl 2
:Disp “COMPLEX ROOTS”
:Ans → W
:Ans → Z
:Disp “REAL PART”
:Disp W
:Disp “IMG PART”
:Disp Z
:GOTO 2
:LBL 2