11.5 Systems of Equations and Multiple Representations

NCTM Standards

- model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions (3–5)
- relate and compare different forms of representation for a relationship (6–8)
- model and solve contextualized problems using various representations such as graphs, tables, and equations (6–8)

Systems of Equations

Some applications can be modeled with two equations. When two or more equations are considered together, they form a system of equations. One sometimes looks for a set of solutions that works with all of the equations.

LE 1 Reasoning

Consider the following problem. “Suvi leaves her house and walks north on a trail at noon. She walks at a speed of 3 miles per hour. Panu leaves at 2 P.M. and wants to catch up, so he runs at 6 miles per hour. At what time will he catch up?”

(a) Make a table of values for each person for different values of $t$ to find out when they will meet.

(b) If $t$ is the number of hours after 12 noon, write a formula for the distance traveled $y$ in terms of $t$, for each person. (Hint: When $t = 3$, how many hours has Panu been walking?)

(c) Graph the two equations from part (b) to find out when they will meet. Use a graphing calculator if it is available.

(d) Solve the system of two equations algebraically to find the solution.

What method did you use to solve part (d) in LE 1? What problems can children solve to prepare them for the algebraic methods of solving a system of two equations?

LE 2 Connection

Consider the following problem. “Suppose two apples and one pear cost 59 cents and three apples cost 54 cents. Find the unit cost of an apple and the unit cost of a pear.”

(a) Describe a method for solving this problem without using any algebra.

(b) Write a system of two equations that represents this problem, where $x$ is the unit cost of an apple and $y$ is the unit cost of a pear.

(c) Solve your system of equations from part (b) with the method you used in part (a).

LE 2 illustrates a situation that could help develop the substitution method of solving a system of equations. The next lesson exercise concerns the addition/subtraction method for solving a system of equations.
LE 3 Connection

Consider the following problem. “Suppose two grapefruits and one orange cost $0.92 and two grapefruits and four oranges cost $1.58. Find the unit cost of an orange and the unit cost of a grapefruit.”

(a) Describe a method for solving this problem without writing equations.
(b) Write a system of two equations that represents this problem.
(c) Solve your system of equations from part (b) with the method you used in part (a).

Did you solve LE 3 by subtracting one equation from the other, or did you come up with a different method?

Which Representation Is Best?

In this chapter, you have solved problems involving words, symbols, tables, and graphs. What are the advantages and drawbacks of each form? The following problem uses all four representations together so you can compare them.

LE 4 Connection

Four companies offer long-distance phone plans. Frayed Fibers charges $6 per month plus $0.05 per minute. The graph (Figure 11–20) shows the monthly cost of different numbers of minutes with Static Lines. The following table shows the monthly cost of different numbers of minutes with Hung Up.

<table>
<thead>
<tr>
<th>Number of minutes</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly cost ($)</td>
<td>0</td>
<td>0.80</td>
<td>1.60</td>
<td>2.40</td>
<td>3.20</td>
<td>4</td>
<td>4.80</td>
</tr>
</tbody>
</table>

Loose Wires’ monthly charge is $8 + 0.03n dollars for n minutes of calls. In parts (a)–(d), for which company is it easiest to answer each of the following questions?
(a) What is the monthly cost for someone who makes 40 minutes worth of calls?
(b) What is the monthly cost for someone who makes 70 minutes worth of calls?
(c) What is the monthly cost for someone who makes 200 minutes worth of calls?
(d) Why might someone find Frayed Fibers’ rate easier to compute than Loose Wires’?
(e) Which two companies are easiest to compare?
(f) Describe the advantages and drawbacks of each of the four representations.

The verbal representation is useful for communicating everyday information about a problem and stating the final results. However, the verbal representation can be complicated and ambiguous.

The graphical representation gives the best visual representation and can make trends easy to recognize. However, the graphical representation lacks precision and makes it more difficult to read specific data values.

The tabular representation gives a set of concrete data that is easy to read and understand. However, it may be difficult to generalize or go much beyond this information.

The symbolic representation is straightforward, general, and precise. However, it is more abstract than the other representations, so its everyday meaning may be less clear. Research suggests that learning numerical and graphical representations will improve a student’s understanding of equations and expressions.
LE 5 Reasoning

You want to compare the cost of two refrigerators. The Ice Queen costs $650 and has an average annual energy cost of $52. The Slush King costs $550 and has an average annual energy cost of $64.

(a) For each model, write an equation for the total cost after $T$ years.
(b) Assume each refrigerator could last up to 20 years. Graph both equations for this time interval. Use a graphing calculator if you have one.
(c) For what length of time is each model likely to cost less money?

A spreadsheet or graphing calculator can be used to generate a table that compares the costs of two or more alternatives.

LE 6 Reasoning

You decide to rent a copier from either Crisp Copy or Repro-Man. Crisp Copy charges $60 per month plus $0.02 per copy. Repro-Man charges $40 per month plus $0.025 per copy.

(a) Set up a computer spreadsheet (or use a calculator and make a table) as follows. Enter 1000, 2000, 3000, ..., 10,000 in column A. The top part of the spreadsheet follows.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Copies</td>
<td>Crisp Copy</td>
</tr>
<tr>
<td>2</td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2000</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3000</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>4000</td>
<td></td>
</tr>
</tbody>
</table>

If you have a graphing calculator, enter equations for the two copier stores in $y_1$ and $y_2$. Then enter 1000 to 10000 for $x$. (Skip ahead to part (c).)

(b) Enter appropriate formulas in cells B2 and C2. Copy these formulas in rows 3 through 11.

(c) Tell how you would decide where to rent a copier.

LE 7 Reasoning

A store sells T-shirts for children and adults. The store orders adult T-shirts for $4 each and children’s T-shirts for $3 each. They sell adult shirts for $16 and children’s shirts for $9. It costs $180/day for salaries, rent, insurance, and utilities. Suppose they sell $x$ children’s shirts and $y$ adult’s shirts one day.

(a) Write an expression for the total cost of running the business for that day.
(b) Write an expression for the total revenue (sales) for that day.
(c) Write an expression for the profit $P$ for that day.
(d) Give a possible number of adult and children’s shirts they could sell in a day to break even.

LE 8 Summary

Tell about the different representations for two or more functions in a system of equations.
Answers to Selected Lesson Exercises

1. (a) 4 P.M. (b) \( y = 3t, y = 6(t - 2) \)
2. (b) \( 2x + y = 59, 3x = 54 \) (c) \( x = 18\varphi, y = 23\varphi \)
3. (b) \( 2x + y = 80.92, 2x + 4y = 1.58 \) (c) \( x = 0.35, y = 0.22 \)
4. (f) Answer follows the exercise.
5. (a) \( C_1 = 650 + 52T, C_2 = 550 + 64T \)
   (c) The Slush King would be cheaper for up to \( 8 \frac{1}{3} \) years. Beyond \( 8 \frac{1}{3} \) years, the Ice Queen would be cheaper.
6. (b) Copies Crisp Copy Repro-Man
   
<table>
<thead>
<tr>
<th></th>
<th>4000</th>
<th>140</th>
<th>140</th>
</tr>
</thead>
<tbody>
<tr>
<td>Repro-Man</td>
<td>for less than 4,000 copies in a month; Crisp Copy for more than 4,000 copies in a month</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
7. (a) \( 3x + 4y + 180 \) (b) \( 9x + 16y \) (c) \( 6x + 12y - 180 \) (d) (possible answer) 15 adult shirts

11.5 Homework Exercises

Basic Exercises

1. Consider the following problem. “Compare two deals for T-shirts: either $7 per shirt or $50 plus $4 per shirt.”
   (a) Write a formula for the total cost \( C \) for \( x \) shirts for each deal.
   (b) Make a table of values for each deal for different values of \( x \), and find out what circumstances make each deal better.
   (c) Graph the two equations from part (a) to find out what circumstances make each deal better. Use a graphing calculator if it is available.
   (d) Solve the system of two equations algebraically to find the solution.
   (e) Which method did you prefer?
2. Consider the following problem. “You want to produce a 500-page book. Company A has a one-time charge of $0.05 per page for typesetting and $2 for binding each book. Company B has a one-time charge of $0.03 per page for typesetting and $2.50 for binding each book.”
   (a) Write a formula for the total cost \( C \) to produce \( x \) books for each company.
   (b) Make a table of values for each company for different values of \( x \) and find out what circumstances make each deal better.
   (c) Graph the two equations from part (a) to find out what circumstances make each deal better. Use a graphing calculator if one is available.
   (d) Solve the system of two equations algebraically to find the solution.
   (e) Which method did you prefer?
3. Consider the following problem. “Suppose two grapefruits and three oranges cost $2.07, and two oranges cost $0.78. Find the unit cost of an orange and the unit cost of a grapefruit.” Describe a method for solving this problem.
   (a) Describe a method for solving this problem without writing equations.
   (b) Write a system of two equations that represents this problem.
   (c) Solve your system of equations from part (b) with the method you used in part (a).
4. Consider the following problem. “Suppose two lemons and four peppers cost $1.98, and three lemons cost $0.75. Find the unit cost of a lemon and the unit cost of a pepper.” Describe a method for solving this problem.
   (a) Describe a method for solving this problem without writing equations.
   (b) Write a system of two equations that represents this problem.
   (c) Solve your system of equations from part (b) with the method you used in part (a).
5. Consider the following problem. “Suppose five apples and two pears cost $2.37, and two apples and two pears cost $1.32.” Find the unit cost of an apple and the unit cost of a pear.”

(a) Describe a method for solving this problem without writing equations.
(b) Write a system of two equations that represents this problem.
(c) Solve your system of equations from part (b) with the method you used in part (a).

6. Consider the following problem. “Suppose four onions and three tomatoes cost $2.48, and four onions and five tomatoes cost $3.36.” Find the unit cost of an onion and the unit cost of a tomato.

(a) Describe a method for solving this problem without writing equations.
(b) Write a system of two equations that represents this problem.
(c) Solve your system of equations from part (b) with the method you used in part (a).

7. The three scales give information about the weight of three different-shaped blocks.

(a) Find the weight of each shaped block.
(b) Tell how you solved the problem.

8. The three scales give information about the weight of three different-shaped blocks.

(a) Find the weight of each shaped block.
(b) Tell how you solved the problem.

9. You are choosing between two health insurance plans that cost the same amount. One pays 80% of all expenses beyond the first $500 (a $500 deductible). The second pays 60% of all expenses, with no deductible.

(a) Write a formula for the amount $A$ that each policy pays on a claim of $C$ dollars.
(b) Graph both equations on the same graph. (You may use a graphing calculator for parts (b) and (c).)
(c) Find the intersection point.
(d) Use the intersection point and state the conditions that make one plan more economical than the other.
(e) What factors besides price might you consider in choosing a plan?

10. You are planning to buy a new air conditioner. Choice A sells for $400 and costs $30 per month to run. Choice B sells for $550 and is more energy-efficient, so it costs $20 per month to run.

(a) Write a formula for the cost $C$ of purchasing and operating each air conditioner for $M$ months.
(b) Graph both equations on the same graph. (You may use a graphing calculator in parts (b) and (c).)
(c) Find the intersection point on the graph.
(d) Find the intersection point algebraically.
(e) Use the intersection point and state the conditions that make one air conditioner a better buy than the other.
(f) What factors besides price might you consider in deciding which air conditioner to buy?

11. The Screeching Tires have been offered two different recording contracts. Contract A includes $20,000 for signing and $1 royalty per CD. Contract B includes $5,000 for signing and $2 royalty per CD.

(a) Write a formula for the total pay ($y$) for each contract in terms of the number of CDs sold ($x$).
(b) Graph both equations on the same graph. (You may use a graphing calculator in parts (b) and (c).)
(c) Find the intersection point.
(d) Use the intersection point and state the conditions that make each contract the higher-paying one.
12. You are offered two sales jobs. Job A pays $800 per week plus 5% commission. Job B pays $300 per week plus 15% commission.
(a) Write a formula for the weekly pay for each job.
(b) Graph both equations on the same graph. (You may use a graphing calculator in parts (b) and (c).)
(c) Find the intersection point.
(d) Use the intersection point and state the conditions under which each job pays more than the other.
(e) What factors besides pay might you consider in choosing a sales job?

13. Tickets for a college basketball game were $10 for adults and $4 for children. The paid attendance was 2,460, and total ticket sales were $21,060. How many adult tickets and how many children’s tickets were sold?

14. A sample of 1,400 mg of automobile exhaust contains 36% carbon (from CO and CO₂). If CO is 43% carbon and CO₂ is 27% carbon, how many milligrams of exhaust are CO, and how many milligrams are CO₂?

15. You must choose between two different rental car companies for a 3-day trip. Drive-U-Crazy charges $39.95/day with unlimited mileage. Used-a-Bit charges $24.95/day and gives 100 free miles/day; mileage beyond that costs $0.10/mile. The sales tax rate at both places is 8%.
(a) Set up a computer spreadsheet (or use a calculator and make a table) as follows. Enter 500, 700, 900, 1100, 1300, and 1500 in column A. The top part of the spreadsheet follows.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Miles</td>
<td>Drive-U-Crazy</td>
</tr>
<tr>
<td>2</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>700</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>900</td>
<td></td>
</tr>
</tbody>
</table>
(b) Enter appropriate formulas in cells B2 and C2. Copy these formulas in rows 3 through 7.
(c) Tell how you would decide which car to rent.

16. You are offered two sales jobs. Fast Talkers pays $180/week plus 5% commission on sales. Wheeler Dealers pays $225/week plus 3% commission on sales.
(a) Set up a computer spreadsheet (or use a calculator and make a table) as follows. Enter 1000, 2000, 3000, 4000, and 5000 in column A. The top part of the spreadsheet follows.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Total</td>
<td>Pay</td>
</tr>
<tr>
<td>2</td>
<td>Weekly sales</td>
<td>Fast Talkers</td>
</tr>
<tr>
<td>3</td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2000</td>
<td></td>
</tr>
</tbody>
</table>
(b) Enter appropriate formulas in cells B3 and C3. Copy these formulas in rows 4 through 7.
(c) Tell how you would decide which job to take.

17. A manufacturer of television sets has weekly expenses \( E = 1,500 + 100T \) dollars for manufacturing \( T \) televisions. The total revenue (money received) from selling \( T \) televisions is 180\( T \).
(a) What is the profit or loss on manufacturing and selling 100 sets in a week?
(b) What is the profit or loss on manufacturing and selling 20 sets in a week?
(c) What number of sets is the break-even point?

18. A manufacturer of shoes has weekly expenses \( E = 800 + 30x \) for manufacturing \( x \) pairs of shoes. The shoes sell for $50 a pair.
(a) Write a formula for the revenue \( R \) from selling \( x \) pairs of shoes.
(b) Write a formula for the profit \( P \) made from selling \( x \) pairs of shoes in a week.
(c) How many pairs of shoes must be sold to make a profit? Tell how you figured this out.

Extension Exercises

19. Kari and Lisa are both north of the center of town after \( t \) hours. Kari’s distance north (in miles) is \( d_1 = 4t + 3 \), and Lisa’s is \( d_2 = 3t + 9 \).
(a) What do these equations tell you about Kari and Lisa’s locations?
(b) When will they be at the same spot?

20. The equations \( cx - ky = 13 \) and \( cx + ky = -31 \) have a common solution of \((3, -2)\). What are the values of \( c \) and \( k \)?
21. In deciding between a National Motors Integer Maxima with a gas engine and one with a diesel engine, you collect the following information. First, the gas car costs $17,800 and the diesel car costs $18,500. Second, the gas car gets 32 miles per gallon (mpg) and the diesel car gets 48 mpg. Third, gas costs $1.60 per gallon and diesel fuel costs $1.68 per gallon. Under what conditions would each car be a better buy?

11.5 Answers to Selected Homework Exercises

1. (a) \( c_1 = 7x, c_2 = 50 + 4x \)
   (b) \( c_1 \) is lower when \( x \leq 16; c_2 \) is lower when \( x \geq 17 \)

3. (a) 1 orange is \$0.39 \rightarrow 3 oranges cost \$1.17 \rightarrow 2 grapefruits cost \$2.07 - \$1.17 = \$0.90 \rightarrow 1 grapefruit costs \$0.45 \)
   (b) \( 2x + 3y = 2.07; 2y = 0.78 \)

5. (a) 3 apples cost \$2.37 - \$1.32 = \$1.05 \rightarrow 1 apple costs \$0.35 \rightarrow 1 pear cost \$0.31 \)
   (b) \( 5x + 2y = 2.37; 2x + 2y = 1.32 \)

7. (a) Cube weighs 4; cylinder weighs 8; cone weighs 2

9. (a) \( A_1 = 0.8(C - 500) \) and \( A_2 = 0.6C \)
   (b) Ray A1 starts at (500, 0) and passes through (1000, 400). Ray A2 starts at (0, 0) and passes through (1000, 600).
   (c) (2000, 1200)
   (d) The first plan is better for claims less than \$2,000; the second is better for claims of more than \$2,000.
   (e) Service, choice of doctor

11. (a) \( A: y = 20,000 + x \quad B: y = 5,000 + 2x \)
   (b) \( B \)

22. Use a graphing calculator to find all intersection points of \( y = 2^x \) and \( y = x^2 \).

Enrichment Topics

23. Read Chapters 1–7 of *The Education of T. C. Mits* by Lillian Lieber, and write a report that includes a summary of the main ideas and your reactions to them.