As we have seen, bankruptcy, or even the possibility of bankruptcy, can cause significant trauma for a firm’s managers, investors, suppliers, customers, and community. Thus, it would be beneficial to be able to predict the likelihood of bankruptcy so that steps could be taken to avoid it or at least to reduce its impact. One approach to bankruptcy prediction is **multiple discriminant analysis (MDA)**, a statistical technique similar to regression analysis. In this Extension, we discuss MDA in detail, and we illustrate its application to bankruptcy prediction.¹

Suppose a bank loan officer wants to segregate corporate loan applications into those likely to default and those unlikely to default. Assume that data for some past period are available on a group of firms that includes both companies that went bankrupt and companies that did not. For simplicity, we assume that only the current ratio and the debt/assets ratio are analyzed. These ratios for our sample of firms are given in Columns 2 and 3 at the bottom of Figure 25B-1. The Xs in the graph represent firms that went bankrupt, while the dots represent firms that remained solvent. For example, Firm 2, which had a current ratio of 3.0 and a debt ratio of 20%, did not go bankrupt. Therefore, a dot is used to represent the plot of its current ratio versus its debt/assets ratio. This dot is labeled “A” and is shown the upper left section of the graph. Firm 19, which had a current ratio of 1.0 and a debt ratio of 60%, did go bankrupt, so an X is used to represent the plot of its current ratio versus its debt/assets ratio. This dot is labeled “B” and is shown in the lower right section of the graph.

### FIGURE 25B-1 Discriminant Boundary between Bankrupt and Solvent Firms

- **Current Ratio**
- **Debt/Assets Ratio (%)**
- **Discriminant Boundary, Z**
- **Good: Low Probability of Bankruptcy**
- **Bad: High Probability of Bankruptcy**

#### Table: Firm Characteristics and Discriminant Scores

<table>
<thead>
<tr>
<th>Firm Number (1)</th>
<th>Current Ratio (2)</th>
<th>Debt/Assets Ratio (3)</th>
<th>Did Firm Go Bankrupt? (4)</th>
<th>Z Scorea (5)</th>
<th>Probability of Bankruptcy (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.6</td>
<td>60%</td>
<td>No</td>
<td>−0.780</td>
<td>17.2%</td>
</tr>
<tr>
<td>2(A)</td>
<td>3.0</td>
<td>20%</td>
<td>No</td>
<td>−2.451</td>
<td>0.8</td>
</tr>
<tr>
<td>3</td>
<td>3.0</td>
<td>60%</td>
<td>No</td>
<td>−0.135</td>
<td>42.0</td>
</tr>
<tr>
<td>4</td>
<td>3.0</td>
<td>76%</td>
<td>Yes</td>
<td>0.791</td>
<td>81.2</td>
</tr>
<tr>
<td>5</td>
<td>2.8</td>
<td>44%</td>
<td>No</td>
<td>−0.847</td>
<td>15.5</td>
</tr>
<tr>
<td>6</td>
<td>2.6</td>
<td>56%</td>
<td>Yes</td>
<td>0.062</td>
<td>51.5</td>
</tr>
<tr>
<td>7</td>
<td>2.6</td>
<td>68%</td>
<td>Yes</td>
<td>0.757</td>
<td>80.2</td>
</tr>
<tr>
<td>8</td>
<td>2.4</td>
<td>40%</td>
<td>Yesa</td>
<td>−0.649</td>
<td>21.1</td>
</tr>
<tr>
<td>9</td>
<td>2.4</td>
<td>60%</td>
<td>Noa</td>
<td>0.509</td>
<td>71.5</td>
</tr>
<tr>
<td>10</td>
<td>2.2</td>
<td>28%</td>
<td>No</td>
<td>−1.129</td>
<td>9.6</td>
</tr>
<tr>
<td>11</td>
<td>2.0</td>
<td>40%</td>
<td>No</td>
<td>−0.220</td>
<td>38.1</td>
</tr>
<tr>
<td>12</td>
<td>2.0</td>
<td>48%</td>
<td>Noa</td>
<td>0.244</td>
<td>60.1</td>
</tr>
<tr>
<td>13</td>
<td>1.8</td>
<td>60%</td>
<td>Yes</td>
<td>1.153</td>
<td>89.7</td>
</tr>
<tr>
<td>14</td>
<td>1.6</td>
<td>20%</td>
<td>No</td>
<td>−0.948</td>
<td>13.1</td>
</tr>
<tr>
<td>15</td>
<td>1.6</td>
<td>44%</td>
<td>Yes</td>
<td>0.441</td>
<td>68.8</td>
</tr>
<tr>
<td>16</td>
<td>1.2</td>
<td>44%</td>
<td>Yes</td>
<td>0.871</td>
<td>83.5</td>
</tr>
<tr>
<td>17</td>
<td>1.0</td>
<td>24%</td>
<td>No</td>
<td>−0.072</td>
<td>45.0</td>
</tr>
<tr>
<td>18</td>
<td>1.0</td>
<td>32%</td>
<td>Yes</td>
<td>0.391</td>
<td>66.7</td>
</tr>
<tr>
<td>19(B)</td>
<td>1.0</td>
<td>60%</td>
<td>Yes</td>
<td>2.012</td>
<td>97.9</td>
</tr>
</tbody>
</table>

*Note:*

a The firms shown in bold were misclassified.
versus its debt/assets ratio. This X is labeled “B” and is shown in the lower right section of Figure 25B-1.

The objective of discriminant analysis is to construct a boundary line through the graph such that if the firm is to the left of the line it is unlikely to become insolvent, whereas if it falls to the right it is likely to go bankrupt. This boundary line is called the **discriminant function**, and in our example it takes this form:

\[
Z = a + b_1(\text{Current ratio}) + b_2(\text{Debt ratio})
\]

Here Z is called the **Z score**, a is a constant term, and b₁ and b₂ indicate the effects of the current ratio and the debt ratio on the probability of a firm going bankrupt.

Although a full discussion of discriminant analysis would go well beyond the scope of this book, some useful insights may be gained by observing these points:

1. The discriminant function is fitted (that is, the values of a, b₁, and b₂ are obtained) using historical data for a sample of firms that either went bankrupt or did not go bankrupt during some past period. When the data in the lower part of Figure 25B-1 were fed into a “canned” discriminant analysis program (the computing centers of most universities and large corporations have such programs), the following discriminant function was obtained:

\[
Z = -0.3877 - 1.0736(\text{Current ratio}) + 0.0579(\text{Debt ratio})
\]

2. This equation was plotted on Figure 25B-1 as the locus of points for which Z = 0. All combinations of current ratios and debt ratios shown on the line result in Z = 0.² Companies that lie to the left of the line (and also have Z values less than zero) are unlikely to go bankrupt, while those that lie to the right (and have Z greater than zero) are likely to go bankrupt. It can be seen from the graph that one X, indicating a failing company, lies to the left of the line, while two dots, indicating nonbankrupt companies, lie to the right of the line. Thus, the discriminant analysis failed to properly classify three companies:

<table>
<thead>
<tr>
<th>Z Positive: MDA Predicts Bankruptcy</th>
<th>Z Negative: MDA Predicts Solvency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Did subsequently go bankrupt</td>
<td>8</td>
</tr>
<tr>
<td>Remained solvent</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>8</td>
</tr>
</tbody>
</table>

²To plot the boundary line, let D/A = 0% and 80%, and then find the current ratio that forces Z = 0 at those two values. For example, at D/A = 0,

\[
Z = -0.3877 - 1.0736(\text{Current ratio}) + 0.0579(0) = 0
\]

\[
0.3877 = -1.0736(\text{Current ratio})
\]

\[
\text{Current ratio} = 0.3877 / (-1.0736) = -0.3611.
\]

Thus, -0.3611 is the vertical axis intercept. Similarly, the current ratio at D/A = 80% is found to be 3.9533. Plotting these two points on Figure 25B-1 and then connecting them provides the discriminant boundary line, which is the line that best partitions the companies into bankrupt and nonbankrupt. It should be noted that nonlinear discriminant functions may be used, and we could also use more dependent variables.
The model did not perform perfectly, as two predicted bankruptcies remained solvent and one firm that was expected to remain solvent went bankrupt. Thus, the model misclassified 3 out of 19 firms, or 16% of the sample. Its success rate was 84%.

3. Once we have determined the parameters of the discriminant function, we can calculate the Z scores for other companies, say, loan applicants at a bank, to predict whether or not they are likely to go bankrupt. The higher the Z score, the worse the company looks from the standpoint of bankruptcy. Here is an interpretation:

- **Z = 0**: 50–50 probability of future bankruptcy (say, within 2 years). The company lies exactly on the boundary line.
- **Z < 0**: If Z is negative, there is a less than 50% probability of bankruptcy. The smaller (more negative) the Z score, the lower the probability of bankruptcy. The computer output from MDA programs gives this probability, and it is shown in Column 6 of Figure 25B-1.
- **Z > 0**: If Z is positive, the probability of bankruptcy is greater than 50%, and the larger Z, the greater the probability of bankruptcy.

4. The mean Z score of the companies that did not go bankrupt is −0.583, while that for the bankrupt firms is +0.648. These means, along with approximations of the Z score probability distributions of the two groups, are shown in Figure 25B-2. We may interpret this graph as indicating that if Z is less than about −0.3, there is a very small probability that the firm will go bankrupt, whereas if Z is greater than +0.3, there is only a small probability that it will remain solvent. If Z is in the range ±0.3, called the **zone of ignorance**, we are uncertain about how the firm should be classified.

5. The signs of the coefficients of the discriminant function are as you might expect. A high current ratio is good, and since its coefficient is negative, the higher the current ratio, the lower the probability of failure. Similarly, high debt...
ratios produce high Z scores, and this is consistent with a higher probability of bankruptcy.

6. Our illustrative discriminant function has only two variables, but other characteristics could be introduced. For example, we could add such variables as the rate of return on assets, the times-interest-earned ratio, the days sales outstanding, the quick ratio, and so forth. Had the rate of return on assets been introduced, it might have turned out that Firm 8 (which failed) had a low ROA, while Firm 9 (which did not fail) had a high ROA. A new discriminant function would be calculated:

\[ Z = a + b_1(\text{Current ratio}) + b_2(D/A) + b_3(\text{ROA}) \]

Firm 8 might now have a positive Z, while Firm 9’s Z might become negative. Thus, it is likely that by adding more characteristics we would improve the accuracy of our bankruptcy forecasts. In terms of Figure 25B-2, this would cause each probability distribution to become tighter, narrow the zone of ignorance, and lead to fewer misclassifications.

In a classic paper (see Footnote 1), Edward Altman applied MDA to a sample of corporations, and he developed a discriminant function that has seen wide use in actual practice. Altman’s function was fitted as follows:

\[ Z = 0.012X_1 + 0.014X_2 + 0.033X_3 + 0.006X_4 + 0.999X_5 \]  \hspace{1cm} (25B-1)

Here,

- \( X_1 = \) net working capital/total assets.
- \( X_2 = \) retained earnings/total assets.\(^4\)
- \( X_3 = \) EBIT/total assets.
- \( X_4 = \) market value of common and preferred stock/book value of debt.\(^5\)
- \( X_5 = \) sales/total assets.

The first four variables in Equation 25B-1 are expressed as percentages rather than as decimals. (For example, if \( X_3 = 14.2\% \), then 14.2, not 0.142, is used as its value.)

Also, Altman’s 50-50 point was 2.675, and not 0.0 as in our hypothetical example; his zone of ignorance was from \( Z = 1.81 \) to \( Z = 2.99 \); and in his model the larger the Z score, the less the probability of bankruptcy.\(^6\)

---

\(^3\)With more than two variables, it is difficult to graph the function, but this presents no problem in actual usage because graphs are used only to explain MDA.

\(^4\)Retained earnings is the balance sheet figure, not the addition to retained earnings for the year.

\(^5\)\(\left(\frac{\text{Shares of common outstanding}}{\text{Price per share}} + \frac{(\text{Shares of preferred})}{\text{Price per share of preferred}}\right) / \text{Balance sheet value of total debt, including all short-term liabilities.}\)

\(^6\)These differences reflect the software package Altman used to generate the discriminant function. Altman’s program did not specify a constant term, and his program simply reversed the sign of Z from ours.
Altman’s function can be used to calculate a Z score for MicroDrive Inc. based on the data presented previously in Chapter 7, Tables 7-1 and 7-2. Here is the calculation, ignoring the small amount of preferred stock, for 2009:

\[ \begin{align*}
X_1 &= \text{net working capital/total assets} \\
&= ($1,000 - $310)/$2,000 = 0.345 = 34.5%
\end{align*} \]

\[ \begin{align*}
X_2 &= \text{retained earnings/total assets} \\
&= $766/$2,000 = 0.383 = 38.3%
\end{align*} \]

\[ \begin{align*}
X_3 &= \text{EBIT/total assets} \\
&= $283.8/$2,000 = 0.142 = 14.2%
\end{align*} \]

\[ \begin{align*}
X_4 &= \text{market value of common and preferred stock/book value of debt} \\
&= [50($23)]/($110 + $754) = 1.331 = 133.1%
\end{align*} \]

\[ X_5 = \text{sales/total assets} \\
= $3,000/$2,000 = 1.5. \]

\[ \begin{align*}
Z &= 0.012X_1 + 0.014X_2 + 0.033X_3 + 0.006X_4 + 0.999X_5 \\
&= 0.012(34.5) + 0.014(38.3) + 0.033(14.2) + 0.006(133.1) + 0.999(1.5) \\
&= 3.00
\end{align*} \]

Because MicroDrive’s Z score of 3.00 at the 2.99 upper limit of Altman’s zone of ignorance, the data indicate that there is a borderline chance that MicroDrive will go bankrupt within the next 2 years. (Altman’s model predicts bankruptcy reasonably well for about 2 years into the future.)

Altman and his colleagues’ later work updated and improved his original study. In their more recent work, they explicitly considered such factors as capitalized lease obligations, and they applied smoothing techniques to level out random fluctuations in the data. The new model was able to predict bankruptcy with a high degree of accuracy for 2 years into the future and with a slightly lower but still reasonable degree of accuracy (70%) for about 5 years.

MDA is used with success by credit analysts to establish default probabilities for both consumer and corporate loan applicants, and by portfolio managers considering both stock and bond investments. It can also be used to evaluate a set of pro forma ratios, or to gain insights into the feasibility of a reorganization plan filed under the Bankruptcy Act. Altman’s model is used by investment banking houses to appraise the quality of junk bonds used to finance takeovers and leveraged buyouts. The technique is described in detail in many statistics texts, while several articles cited at the end of this chapter discuss financial applications of MDA. The interested reader is urged to study this literature, for MDA has many potentially valuable applications in finance.

When using MDA in practice it is best to create your own discriminant data using a recent sample from the industry in question. For example, it is not reasonable to assume that the financial ratios of a steel company facing imminent bankruptcy are the same as for a retail grocery chain in equally dire straits. If both firms were analyzed using Z scores calculated with the same equation, it might turn out that the grocery chain had a relatively high score, signifying (incorrectly) a low probability of bankruptcy, while the steel company had a relatively low score, indicating (correctly) a high probability of bankruptcy. The misclassification of the
grocery company could result from the fact that it has very high sales for the amount of its book assets; hence its $X_5$, which has the largest coefficient, is much higher than for an average firm in an average industry facing potential bankruptcy. To remove any such industry bias, the MDA analysis should be based on a sample with characteristics similar to those of the firm being analyzed. Unfortunately, though, it is often not possible to find enough firms that have recently gone bankrupt to conduct an industry MDA.