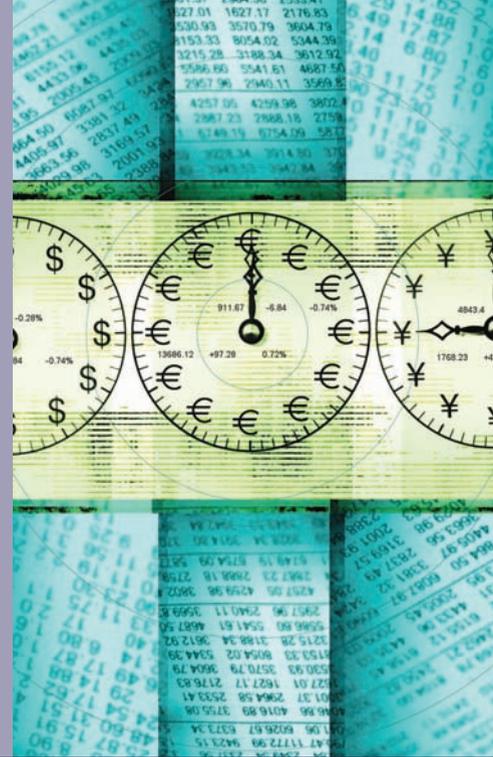


WEB EXTENSION 13B

Certainty Equivalents and Risk-Adjusted Discount Rates



Two alternative methods have been developed for incorporating project risk into the capital budgeting decision process. One is the **certainty equivalent method**, in which the expected cash flows are adjusted to reflect project risk: Risky cash flows are scaled down because the riskier the flows, the lower their certainty equivalent values. The second is the **risk-adjusted discount rate method**, where differential project risk is dealt with by changing the discount rate: Average-risk projects are discounted at the firm's corporate cost of capital, above-average-risk projects are discounted at a higher cost of capital, and below-average-risk projects are discounted at a rate below the corporate cost of capital. The risk-adjusted discount rate method is used by most companies, so we focused on it in earlier chapters. However, the certainty equivalent approach does have some advantages, so financial managers should be familiar with it as well.

As we noted in Chapter 13, it is not always possible to find existing models for financial options that correspond to all real options. In such a situation, the analyst must use financial engineering techniques. One technique, **risk-neutral valuation**, is analogous to the certainty equivalent method, except it is applied using simulation.

THE CERTAINTY EQUIVALENT METHOD

The certainty equivalent (CE) method follows directly from the concept of utility theory. Under the CE approach, the decision maker must first evaluate a cash flow's risk and then specify how much money, to be received with certainty, will make him or her indifferent between the riskless and the risky cash flows. To illustrate, suppose a rich eccentric offered you the following two choices:

1. Flip a fair coin. If heads comes up, you receive \$1 million, but if tails comes up, you get nothing. The expected value of the gamble is $(0.5)(\$1,000,000) + (0.5)(\$0) = \$500,000$, but the actual outcome will be either \$0 or \$1 million, so this choice is risky.
2. Do not flip the coin and simply pocket \$300,000 cash.

If you find yourself indifferent between the two alternatives, then \$300,000 is your certainty equivalent for this particular risky \$500,000 expected cash flow. The certain (or riskless) \$300,000 thus provides you with the same utility as the risky \$500,000 expected return.

Now ask yourself this question: In the preceding example, exactly how much cash-in-hand would it actually take to make you indifferent between a certain sum and the risky \$500,000 expected return? If you are like most people, your certainty equivalent would be significantly less than \$500,000, indicating that you are risk averse. In general, people are risk averse, and the lower the certainty equivalent, the greater the decision maker's risk aversion.

The certainty equivalent concept can be applied to capital budgeting decisions, at least in theory, in the following way:

1. Estimate the certainty equivalent cash flow in each Year t , CE_t , based on the expected cash flow and its riskiness.
2. Given these certainty equivalents, discount by the risk-free rate to obtain the project's NPV.¹

To illustrate, suppose Project A, whose expected net cash flows are shown in Table 13B-1, is to be evaluated using the certainty equivalent method. Assume that the initial net cost, \$2,000, is fixed by contract and hence known with certainty. Further, assume that the capital budgeting analyst estimates that the cash inflows in Years 1 through 4 all have average risk, and that the appropriate certainty equivalent is \$700. The project's NPV, found using a risk-free discount rate of 5%, is \$482.17:

$$NPV_A = -\$2,000 + \frac{\$700}{(1.05)^1} + \frac{\$700}{(1.05)^2} + \frac{\$700}{(1.05)^3} + \frac{\$700}{(1.05)^4} = \$482.17$$

Since the risk-adjusted NPV is positive, the project should be accepted.

TABLE 13B-1 Project A: Certainty Equivalent Analysis

Year	Expected Net Cash Flow	Degree of Risk	Certainty Equivalent Cash Flow
0	(\$2,000)	Zero	(\$2,000)
1	1,000	Average	700
2	1,000	Average	700
3	1,000	Average	700
4	1,000	Average	700

¹Note that the risk-free rate normally does not reflect the tax advantage of debt as does the WACC. Thus, either the certainty equivalent estimates or the risk-free rate should be constructed so that they capture this benefit.

The certainty equivalent method is simple and neat. Further, it can easily accommodate differential risk among cash flows. For example, if the Year 4 expected net cash flow of \$1,000 included a very risky estimated salvage value, we could simply reduce the certainty equivalent, say, from \$700 to \$500, and recalculate the project's NPV. Unfortunately, there is no practical way to estimate certainty equivalents. Each individual would have his or her own estimate, and these could vary significantly. To further complicate matters, certainty equivalents should reflect shareholders' risk preferences rather than those of management. For these reasons, the certainty equivalent method is not used very often in corporate decision making. However, it is conceptually a powerful tool, and in a later section we will use certainty equivalents to help see some assumptions embodied in constant risk-adjusted discount rates.

The Risk-Adjusted Discount Rate Method

With the risk-adjusted discount rate method, we use the expected cash flow values, CF_t , and the risk adjustment is made to the denominator of the NPV equation (the discount rate) rather than to the numerator. To illustrate, again consider Project A, whose expected cash flows were given in Table 13B-1. Suppose the firm evaluating Project A had a corporate cost of capital of 15%. Thus, all average-risk projects would be evaluated using a discount rate of 15%. Now assume that a risk analysis of Project A indicated that the project had above-average risk, and a project cost of capital of 22% was subjectively assigned to the project. In this situation, the risk-adjusted discount rate method produces an NPV of \$493.64:

$$NPV_A = -\$2,000 + \frac{\$1,000}{(1.22)^1} + \frac{\$1,000}{(1.22)^2} + \frac{\$1,000}{(1.22)^3} + \frac{\$1,000}{(1.22)^4} = \$493.64$$

In theory, if managers were able to estimate precisely both a project's certainty equivalent cash flows and its risk-adjusted discount rate (or rates), the two methods would produce the identical NPV. However, the risk-adjusted discount rate method is easier to use in practice because the discount rate for average-risk projects (the firm's corporate cost of capital) can be estimated from observable market data, but no market data are available to help managers estimate certainty equivalent cash flows.

CERTAINTY EQUIVALENTS VERSUS RISK-ADJUSTED DISCOUNT RATES

As noted above, investment risk can be handled by making adjustments either to the numerator of the present value equation (the certainty equivalent, or CE, method) or to the denominator (the risk-adjusted discount rate, or RADR, method). The RADR method dominates in practice because people find it far easier to estimate suitable discount rates based on current market data than to derive certainty equivalent cash flows. Some financial theorists have suggested that the certainty equivalent approach is theoretically superior, but other theorists have shown that if risk increases with time, then using a risk-adjusted discount rate is a valid procedure.²

Risk-adjusted rates lump together the pure time value of money as represented by the risk-free rate and a risk premium: $RADR = r_{RF} + R_{RP}$. On the other hand,

²See Alexander A. Robichek and Stewart C. Myers, "Conceptual Problems in the Use of Risk-Adjusted Discount Rates," *Journal of Finance*, December 1966, pp. 727-730, and Houngh-Yhi Chen, "Valuation under Uncertainty," *Journal of Financial and Quantitative Analysis*, September 1967, pp. 313-326.

the CE approach keeps risk and the time value of money separate. This separation gives a theoretical advantage to certainty equivalents, because lumping together the time value of money and the risk premium compounds the risk premium over time. By compounding the risk premium over time, the RADR method automatically assigns more risk to cash flows that occur in the distant future, and the farther into the future, the greater the implied risk. Since the CE method assigns risk to each cash flow individually, it does not impose any assumptions regarding the relationship between risk and time.

IMPLICATIONS

A firm using the RADR approach for its capital budgeting decisions will have a corporate cost of capital that reflects its overall market-determined riskiness. This rate should be used for “average” projects—that is, for projects that have the same risk as the firm’s existing assets. Lower rates should be used for less risky projects, and higher rates should be used for riskier projects. To facilitate the decision process, corporate headquarters generally prescribes rates for different classes of investments (for example, replacement, expansion of existing lines, and expansion into new lines). Then, investments of a given class within a given division are analyzed in terms of the prescribed rate. For example, replacement decisions in the retailing division of an oil company might all be evaluated with a 10% discount rate, while exploratory drilling projects might be evaluated at a 20% rate.

However, a constant RADR implies that risk increases with time, and it therefore imposes a relatively severe burden on long-term projects. This means that short-payoff projects will tend to be selected over those with longer payoffs when, for example, there are alternative ways of performing a given task. This is often appropriate, but there are projects for which distant returns are *not* more risky than near-term returns. For example, the estimated returns on a water pipeline serving a developing community may be highly uncertain in the short run, because the rate of growth of the community is uncertain. However, the water company may be quite sure that, in time, the community will be fully developed and will utilize the full capacity of the pipeline. Similar situations could exist in many public projects—water projects, highway programs, schools, and so forth, and when industrial firms are building plants or retailers are building stores to serve growing geographic markets.

To the extent that the implicit assumption of rising risk over time reflects the facts, then a constant discount rate may be appropriate. Indeed, in the vast majority of business situations, risk undoubtedly is an increasing function of time, so a constant risk-adjusted discount rate is generally reasonable. However, one should be aware of the relationships described in this section and avoid the pitfall of unwittingly penalizing long-term projects when they are not, in fact, more risky than short-term projects.

RISKY CASH OUTFLOWS: AN APPLICATION OF THE CE AND RADR METHODS

Projects often have negative cash flows during their lives, and when this occurs, special problems may arise. To illustrate, suppose Duke Power Company has concluded that it needs a new generating plant, and it is choosing between a nuclear plant and a coal-fired plant. Both plants will produce the same amount of electricity; hence they will have the same revenues. However, the coal plant will have a smaller investment requirement but higher operating costs; hence it will have smaller annual cash

flows. Also, the nuclear plant will have to be closed down at the end of its 30-year life. At the present time, the company knows that the costs of tearing down the plant and removing the radioactive material will be high, but the exact cost is highly uncertain.

Table 13B-2 shows the projected cash flows from the two plants. The coal plant has a cost of \$2 billion, and it is expected to produce net cash flows of \$250 million per year for 30 years. The nuclear plant has a cost of \$4 billion, it is expected to produce cash flows of \$474 million per year for 30 years, and then the company expects to have to spend \$2 billion to decommission the radioactive plant. All of the cash flows except the decommissioning costs have the same risk as Duke Power's other cash flows; hence they are to be discounted at the company's WACC, 10%. However, the decommissioning costs are much more uncertain; hence they should be discounted at a rate that reflects their higher risks.

TABLE 13B-2 Expected Cash Flows, Coal versus Nuclear Power Plants (Millions of Dollars)

Panel A: Operating Cash Flows			
	Year	Cash Flows from	
		Coal Plant	Nuclear Plant
	0	(\$2,000)	(\$4,000)
	1	250	474
	2	250	474
	.	.	.
	.	.	.
	.	.	.
	28	250	474
	29	250	474
	30	250	474
	NPV of operating cash flows @ 10%	\$ 357	\$ 468
Panel B: Present Value of Nuclear Plant's Shutdown Costs			
	Year	Nuclear Plant's Shutdown Costs	
	31	(\$2,000)	
If Cost of Capital = 10%			
	PV of shutdown costs	(\$ 104)	
	Total NPV of operations and shutdown	\$ 364	
If Cost of Capital = 15%			
	PV of shutdown costs	(\$ 26)	
	Total NPV of operations and shutdown	\$ 442	
If Cost of Capital = 5.38%			
	PV of shutdown costs	(\$ 394)	
	Total NPV of operations and shutdown	\$ 74	

In Panel A of Table 13B-2 we show the operating cash flows' NPVs. The NPV of the coal plant, discounting at the 10% WACC, is \$357 million. At the 10% WACC, the nuclear plant's operating cash flows have a higher NPV, \$468 million, but this value does not reflect the nuclear plant's decommissioning costs. We might discount the shutdown costs at $WACC = 10\%$. Panel B shows this gives a $-\$104$ PV of shutdown costs, which gives a total NPV of the nuclear plant of $\$468 - \$104 = \$364$ million. This is higher than the NPV of the coal plant, but doesn't reflect the higher risk of the shutdown costs. Our first inclination is to discount the decommissioning costs at a higher rate to reflect their greater risk. Using a discount rate of 15% for the decommissioning costs, we get a PV of shutdown costs of $-\$26$ million and a total NPV of $\$468 - \$26 = \$442$ million. But note—something is amiss. We applied a higher risk-adjusted discount rate to penalize the nuclear plant for its higher risk, but this *improved* its relative position!

This is where the certainty equivalent method can help. Discussions with the company's managers revealed that they would be willing to commit to a certain $-\$2.4$ billion cash flow rather than the uncertain $-\$2$ billion to decommission the nuclear power plant in 31 years. Because the $-\$2.4$ billion is a certainty equivalent, we can find the PV of the shutdown costs by discounting the $-\$2.4$ billion at the risk-free rate of 6%: $PV = -\$2.4/(1 + 0.06)^{31} = -\394 million. This produces a total NPV for the nuclear plant of $\$468 - \$394 = \$74$ million, which is much less than the NPV of the coal plant. Alternatively, we could have discounted the risky $-\$2$ billion shutdown cost at a lower discount rate of 5.38%. Panel B of Table 13B-2 shows that this produces a PV of $-\$394$ million, the same as the certainty equivalent method. Not only is the appropriate RADR for this cash outflow less than the cost of capital, but it is even less than the risk-free rate.

The key to understanding why a lower RADR might be used for a risky outflow is to ask why the managers were willing to commit to a certain payment of $\$2.4$ billion rather than the risky payment of $\$2$ billion. In terms of total corporate risk, the managers must have feared that the shutdown costs would be especially negative when the other cash flows of the firm were low. In other words, they were worried that the shutdown costs would be worse at a time when the company could ill afford them. Therefore, they were willing to pay more than the expected $\$2$ billion to avoid that risk. Putting this into the framework for corporate risk, the managers believed the shutdown cash flows were positively correlated with the firm's other cash flows. For example, if the firm's other cash flows were low, the shutdown cash flows would be even worse than $-\$2$ billion; if the firm's other cash flows were high, the shutdown costs would be better than $-\$2$ billion. The proof is too complicated for a financial management course, but it can be shown that if a negative expected cash flow has positive correlation with the firm's other cash flows, then the discount rate for the negative cash flow has a negative beta.³ In the CAPM, a negative beta leads to a discount rate less than the risk-free rate, which explains the very low discount rate in Panel B of Table 13B-2.

The managers in the example subjectively estimated the certainty equivalent for the shutdown costs. However, it is possible to add some discipline to their judgment. If the managers can estimate the standard deviation of the expected cash flow and its correlation with the firm's other cash flows, then it is possible to calculate the certainty equivalent and the appropriate discount rate. See the paper cited in footnote 3 for details.

³See Phillip R. Daves and Michael C. Ehrhardt, "Capital Budgeting: The Valuation of Unusual, Irregular, or Extraordinary Cash Flows," *Financial Practice and Education*, Fall/Winter 2000, pp. 106–114.

In summary, some projects have cash flows with different risk than the firm's normal operating cash flows, so the firm's cost of capital is not the appropriate rate for the different cash flow. The appropriate rate might be either higher or lower than the firm's cost of capital, depending on the nature of the risk. If the cash flow has a negative expected value and is positively correlated with the firm's other cash flows, then the appropriate discount rate has a negative beta, and the cash flow's cost of capital is less than the risk-free rate. This is the case with our power plant example. On the other hand, if the negative expected cash flow is negatively correlated with the firm's other projects, then the discount rate has a positive beta, and the cash flow's cost of capital is greater than the risk-free rate and is possibly greater than the firm's overall cost of capital.

The same analysis can be applied to a project with an unusual positive expected cash flow, except the sign on the resulting beta is reversed. If the positive expected cash flow is positively correlated with the firm's other cash flows, then the beta used to obtain the discount rate is positive. If the positive expected cash flow is negatively correlated with the firm's other cash flows, then the beta used to obtain the discount rate is negative.

Finally, some negative cash flows are so large that they might put the firm in jeopardy of bankruptcy. In this case, it is better to use real option techniques. A bankruptcy usually includes some very large direct costs, such as legal fees and the sale of assets at low prices. Therefore, accepting a project with a very large risky negative cash flow is like writing a put option on the bankruptcy costs. If the negative cash flow doesn't turn out to be too negative, then the firm doesn't go bankrupt and doesn't have to pay the bankruptcy costs. However, if the negative cash flow does turn out to be very negative, then the firm must pay the direct bankruptcy costs. We can then use the techniques described in the chapter to estimate the cost of this real option.