Chapter 14 of the textbook addressed pricing decisions for firms that produce several alternative products that are technically independent in the production process. **Joint products**, in contrast, are interdependent in the production process; that is, a change in the production of one produces a change in the variable cost or the availability of the other. Examples of joint products include the production of liquid oxygen and nitrogen from air, beef and hides from steers, and gasoline and fuel oil from crude oil. In some cases, such as the production of beef and hides from cattle, the outputs are obtained in relatively fixed proportions. In other cases, such as the production of gasoline and fuel oil from crude oil, variable proportions of the outputs can be obtained through pressure and heat changes in the chemical cracking process in a refinery. We will examine each of these cases further in this appendix.

**JOINT PRODUCTS IN FIXED PROPORTIONS**

When outputs are produced in fixed proportions, they should be analyzed as a *product package*. Because the products are jointly produced, all costs are incurred in production of the package, and no conceptually correct method exists for allocating these costs to the individual products. Determination of the optimal output and prices of the products involves a comparison of the total marginal revenue from all the products with the marginal cost. In the following analysis, each unit of the product package consists of the output obtained from one unit of input. For example, the slaughtering of a steer might yield a product package consisting of 500 pounds of beef and one hide.

Figure WE.1(a) shows the demand functions and their respective marginal revenue functions for two products (A and B) that make up a product package, along with the marginal cost function for the production process. The total marginal revenue function \( MR_T \) for the product package is obtained by *vertically* summing the marginal revenue functions for the individual products \( MR_A \) and \( MR_B \). The net revenue gain to the firm of producing one more unit of the product package is the additional (marginal) revenue from Product A plus the inseparable additional (marginal) revenue from Product B. The intersection of the total marginal revenue function \( MR_T \) and the marginal cost function \( MC \) determines the optimal output of the product package \( Q^* \) along with the optimal prices of the two individual products (i.e., \( P_A^* \) and \( P_B^* \)).

**EXAMPLE**

**PRICING OF JOINT PRODUCTS: WILLIAMS COMPANY**

Suppose the Williams Company is faced with the following demand functions for two joint products produced in fixed proportions:

\[
P_1 = 50 - 0.5Q \quad \text{[WE.1]}
\]

\[
P_2 = 60 - 2Q \quad \text{[WE.2]}
\]
Furthermore, suppose that the marginal cost function for the joint products is
\[ MC = 38 + Q \]  
[WE.3]

The two marginal revenue functions are obtained as follows:
\[ TR_1 = P_1Q = (50 - 0.5Q)Q = 50Q - 0.5Q^2 \]
\[ MR_1 = \frac{dTR_1}{dQ} = 50 - Q \]
\[ TR_2 = P_2Q = (60 - 2Q)Q = 60Q - 2Q^2 \]
\[ MR_2 = \frac{dTR_2}{dQ} = 60 - 4Q \]

Summing the two inseparable marginal revenue functions vertically yields
\[ MR_T = MR_1 + MR_2 \]
\[ = (50 - Q) + (60 - 4Q) \]
\[ = 110 - 5Q \]  
[WE.4]

Setting the total marginal revenue function equal to the marginal cost function and solving for \( Q \) yields the optimal output
\[ MR_T = MC \]
\[ 110 - 5Q = 38 + Q \]
\[ 72 = 6Q \]
\[ Q^* = 12 \]
or 12 units of the product package. Substituting this value into the demand functions (Equations WE.1 and WE.2) gives the optimal prices of the two products:

\[ P_A^* = 50 - 0.5(12) \]
\[ = 44 \text{ per unit of Product A} \]
\[ P_B^* = 60 - 2(12) \]
\[ = 36 \text{ per unit of Product B} \]

One complication in the preceding analysis can occur if the marginal cost function (MC) intersects the total marginal revenue function (MRT) at an output in excess of \( Q_1 \) in Figure WE.1(a). Above \( Q_1 \), the marginal revenue of Product B is negative, and the firm would not want to sell more than \( Q_1 \) units of Product B. When this situation occurs, as shown in Figure WE.1(b), the optimal solution is to produce \( Q_A^* \) units of the product package. This quantity is determined at the intersection of the \( MRA \) and MC functions. \( Q_A^* \) units of Product A should be sold at a price of \( P_A^* \). However, only \( Q_B^* = Q_1 \) units of Product B should be sold at a price of \( P_B^* \). The excess output of Product B, namely \( Q_A^* - Q_B^* \), should be destroyed or discarded so as not to depress the market price.

When solving a numerical problem, one can check to see whether the marginal cost function intersects the total marginal revenue function at an output greater than \( Q_1 \) by substituting the optimal output (\( Q^* \)) into the \( MRA \) and \( MRB \) functions. If either marginal revenue value is negative, then the marginal cost function should be set equal to the marginal revenue function of the other product in determining the optimal price and output combination.\(^1\) For example, if the \( MR_B \) function is negative, then one would use \( MRA \) (rather than \( MRT \)) to determine the optimal solution.

**JOINT PRODUCTS IN VARIABLE PROPORTIONS**

When the outputs can be produced in variable proportions, the analysis is somewhat more complex than the fixed proportions case.

**EXAMPLE**

**PRICING OF JOINT PRODUCTS: SLUSSER CHEMICAL COMPANY**

The decision facing the Slusser Chemical Company is illustrated in Figure WE.2. The quantities of two chemicals (X and Y) that may be produced are indicated on the vertical and horizontal axes. The isocost or production possibility curves (labeled \( TC \)) indicate the amounts of X and Y that may be produced for the same total cost. For instance, looking at the isocost curve labeled \( TC = 8 \), we see that the firm may produce \( Q_X \) units of X and \( Q_Y \) units of Y, \( Q'_X \) units of X and \( Q'_Y \) units of Y, or any possible combination along that curve at an equivalent total cost of \( TC = 8 \). Hence

\(^1\) Note in the Williams Company example that when \( Q^* = 12 \), \( MR_1 = 50 - 12 = 38 > 0 \) and \( MR_2 = 60 - 4(12) = 12 > 0 \). Hence, no excess output of either product was being produced.
Slusser can increase the output of, say, Product X in two ways. One way is to move along the isocost curve, increasing the output of X at the expense of Y. The other is to increase the amount of the inputs or factors (e.g., capital and/or labor) in the production process; that is, move in a northeast direction to a higher isocost curve. The only requirement of isocost or production possibility curves is that they be concave to the origin, indicating an imperfect adaptability of the firm’s productive resources in producing X and Y.

The isorevenue lines (labeled TR) take into account the prices that Slusser receives for its two outputs. Each line is of equal revenue, indicating that any combination of X and Y along any particular line will yield the same total revenue. The straight isorevenue lines in Figure WE.2 indicate that products X and Y are being sold in purely competitive markets; that is, the prices of X and Y do not change as output changes. (If this relationship were not the case, the isorevenue lines would no longer be straight; nevertheless, the general tangency solution for an optimal output combination does not change.) Line TR = 25 is constructed such that \( Q_{y2} \) times the price of Y \( (P_y) \) equals \( Q_{x2} \) times the price of X \( (P_x) \). The slope of each isorevenue line is equal...
to \( P_y \div P_x \), because the slope of \( TR = 25 \) equals \( Q_{x2} \div Q_{y2} \), and \( P_x(Q_{x2}) = P_y(Q_{y2}) \); therefore

\[
\frac{P_y}{P_x} = \frac{Q_{x2}}{Q_{y2}}
\]

A whole family of isorevenue lines exists that is defined by the prices and levels of output for \( X \) and \( Y \). The further one moves in a northeast direction, the greater the total revenue associated with any isorevenue line.

The solution for an optimum combination of outputs requires a point of tangency between the isocost and isorevenue curves. This solution may be illustrated with the \( TC = 14 \) isocost curve. Under the conditions depicted in Figure WE.2 Slusser should produce \( Q_{x3} \) units of \( X \) and \( Q_{y3} \) units of \( Y \) because total profit, \( \pi \) (the difference between \( TR \) and \( TC \)), is maximized at that point. To produce any other possible output combination along the \( TC = 14 \) isocost curve would result in the same costs (14), but would place the firm on a lower isorevenue curve, thereby reducing profit. Because profits are maximized at the point of tangency (\( \pi = 6 \)), the marginal cost of producing each product must be exactly equal to the marginal revenue each product generates.

The analysis presented here could be expanded considerably by dropping some of the assumptions. For instance, the two-product case could be expanded to a more general \( n \)-product case. One could also assume a far greater number of variable factors of production than the one factor (or bundle of factors) implicitly assumed. In addition, the assumption that the prices of input factors are not a function of their use and the assumption that the prices of outputs are independent of the quantity produced could be dropped. Cases such as these may be analyzed with calculus,\(^2\) but in many instances the simplified model presented provides an adequate framework for analysis. Linear programming also provides an extremely useful tool for examining problems of allocating common productive facilities among two or more products to maximize profits.

In conclusion, the decision to add (or delete) products to (from) a firm’s product line must consider true (net) marginal revenue and true (net) marginal cost. If a new product is a reasonably close substitute for an existing product, the addition of the new product is likely to cannibalize the sales of the existing product. This sales reduction must be considered in the marginal revenue analysis. In addition, complementarities in demand between two or more products (when a lower price or increased availability of one product stimulates an increase in demand for another) must also be considered in a multiproduct firm’s price and output decisions.

Finally, in deciding whether to add, delete, or change the relative output of any one product, the impact of that action on the cost of producing the firm’s other outputs must be taken into consideration. Only after true marginal costs and benefits have been accounted for may optimal strategies about the makeup of a firm’s product line be adopted.

Associated with the tremendous growth in the size of corporations has been a trend toward decentralized decision making and control within these organizations. Because of the exceedingly complex coordination and communication problems within large multiproduct national or multinational firms, such firms typically are broken up into a group of semiautonomous operating divisions. Each division constitutes a profit center with the responsibility and authority for making operating decisions and an appropriate set of rewards and incentives to motivate profit-maximizing decisions. Because of the complexity of this problem, we limit the following analysis in several ways.

In practice, the external demand functions of two divisions are often interrelated. For example, a degree of dependence presumably exists between the demand functions of the Chevrolet and Pontiac divisions of General Motors. In the analysis of this section, however, it is assumed that the external demand functions of each division are independent.

The production processes of two divisions are also often cost dependent either through technological interdependence or through the effects of output changes on the costs of inputs employed in the production process. An example of the former type of interdependence would be the case of an oil refinery in which the mix of outputs (e.g., gasoline, kerosene, heating oil, and lubricants) is limited by the production process. An example of the latter type would be two divisions that are bidding for a raw material (sheet metal) or labor skill in short supply and that are, as a result, causing the price to rise. In the ensuing analysis, it is assumed that the production processes are cost independent.

A third source of dependence, and the only one considered in this section, occurs whenever one division sells all or part of its output to another division of the same firm. For example, within the Ford Motor Company a multitude of internal transfers of goods and services takes place. The Engine and Foundry Division, Transmission and Chassis Division, Metal Stamping Division, and the Glass Division, among others, transfer products to the Automotive Assembly Division. The Automotive Assembly Division in turn transfers completed cars to the Ford and Lincoln-Mercury Sales and Marketing Divisions.

The price at which each intermediate good or service is transferred from the selling to the buying division affects the revenues of the selling division and the costs of the buying division. Consequently, the price-output decisions and profitability of each division will be affected by the transfer price.

A transfer price serves two sometimes competing functions in the decentralized firm. One function is to act as a measure of the marginal value of resources used in the division when making the price and output decisions that will maximize profits. The other is to serve as a measure of the total value of the resources used in the division when analyzing the performance of the division. It is sometimes possible for these functions to conflict. The emphasis in this section is on determining the correct transfer price to use in making marginal decisions about product price and output of each division.

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Transfer Price

The price at which an intermediate good or service is transferred from the selling division to the buying division within the same firm.

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**EXAMPLE**

**PRICING OF INTERDEPARTMENTAL SERVICES AT BELL ATLANTIC**

Bell Atlantic (now Verizon) has taken the transfer pricing concept and applied it on an experimental basis to the pricing of interdepartmental services, such as information services, business research, medical services, and training and development. Each of 10 client-service departments charges other departments of the company for the services it renders. For example, a manager who uses an in-house speech writer would have to pay for this service out of his or her department’s budget. The speech writer’s department would then be credited with the amount charged for providing the service. From these revenues, each client-service department is expected to pay all its expenses, including salaries and benefits, rent, office equipment, and electricity. A department that fails to cover its costs could be faced with some difficult choices, such as replacing the manager, reducing its staff, or even possible elimination by giving the work to an outside vendor.

One of the most difficult problems in implementing such a transfer pricing system is determining the costs and market value of a department’s services. Most departments at Bell Atlantic ended up pricing their services in line with what outside vendors charged. Some departments billed for their services on an hourly basis, whereas others charged a set amount for each project. To prevent overcharging by the client-service departments, in-house users were allowed to use outside vendors when they could obtain a better price from the vendor.

The benefits from such a pricing system are twofold. First, some client-service departments found that they were overstaffed and were required to reduce the scale of their operations. For example, the communications services group eliminated 11 positions. Second, users of these services were forced to scale back their requests to more realistic levels if the price quote was too high. Under the old system, service requests were sometimes excessive because the costs were being borne by the department doing the work rather than by the clients. Annual savings with the new system of more than $4 million were reported for four of Bell Atlantic’s client-service groups.

In the following analysis, assume that a decentralized firm consists of two separate divisions that form a two-stage process to manufacture and market a single product. The production division manufactures an intermediate product, which is sold internally to the marketing division at the transfer price. The marketing division converts the intermediate product into a final product, which it then sells in an imperfectly competitive (that is, monopolistic) external market.

Under the assumptions of demand and cost independence already discussed, three possible cases can be considered:

- **No external market for the intermediate product.**
- **Perfectly competitive** external market for the intermediate product.
- **Imperfectly competitive** external market for the intermediate product.

---

The first two cases are examined in the remainder of this section. The third case of an imperfectly competitive external market can be analyzed using the third-degree price discrimination model discussed in Chapter 14 of the textbook. It leads to the counterintuitive result that optimal transfer prices may exceed external prices at which the manufacturer will sell to outside buyers. This price discrimination case is not reexamined here.

**NO EXTERNAL MARKET FOR THE INTERMEDIATE PRODUCT**

With no external market for the intermediate product, the production division would be unable to dispose of any excess units over and above the amount desired by the marketing division. Likewise, if demand for the final product should exceed the capacity of the production division, the marketing division would be unable to obtain additional units of the intermediate product externally. Therefore, the quantity of the product manufactured by the production division must necessarily be equal to the amount sold by the marketing division.\(^5\) The determination of the profit-maximizing price-output combination and the resulting transfer price are shown in Figure WE.3. The marginal cost per unit to the firm, \(MC\), of any level of

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\(^5\) This analysis assumes that all units produced during the period must be sold during the period; that is, no inventories of the intermediate product can be carried over into the next period.
output is the sum of the marginal costs per unit of production, \( MC_p \), and marketing, \( MC_m \). By equating marginal cost \( MC \) to external marginal revenue \( MR_m \) (Point A), one obtains the firm's profit-maximizing decisions—\( P_m^* \) as the optimal price and \( Q_m^* \) as the optimal quantity of the final product to be sold by the marketing division in the external market. Therefore, the optimal transfer price \( P_t^* \) is set equal to the marginal production cost per unit \( MC_p \) at the optimum output level \( Q_p^* \) (Point B). At this point, each division, when seeking to maximize its own division profit, will maximize the overall profit of the firm. This result can be demonstrated in the following manner.

Once the transfer price is established, the production division will face a horizontal demand curve (and corresponding marginal revenue curve) at the given transfer price for the intermediate product. The profits of the production division will be maximized at the point where its divisional marginal cost equals divisional marginal revenue—in this case where the \( P_t \) line intersects the \( MC_p \) curve. This condition yields \( Q_p^* \) as the optimum quantity of the intermediate product, which is identical to the optimum quantity of the final product \( Q_m^* \) determined previously. Similarly, once the transfer price is established, the marketing division is faced with a marginal cost curve \( MC_t \), which is the sum of the marginal marketing cost per unit \( MC_m \) and the given transfer price \( P_t \). The profits of the marketing division will be maximized at the point where its divisional cost is equal to its divisional marginal revenue—where the \( MC_t \) and \( MR_m \) curves intersect. This condition results in the same optimal price and output decision (i.e., \( P_m^* \) and \( Q_m^* \)) as was obtained previously in maximizing the overall profits of the firm.

**EXAMPLE**

**Determining the Optimal Transfer Price: Portland Electronics**

The production division (\( p \)) of the Portland Electronics Company manufactures a component that it sells internally to the marketing division (\( m \)), which promotes and distributes the product through its own domestic retail outlets. Assume that this component has no external market (i.e., the production division cannot sell any excess production of the component to outside buyers and the marketing division cannot obtain additional components from outside suppliers). The marketing division’s demand function for the component is

\[
P_m = 100 - 0.001Q_m
\]  
**[WE.5]**

where \( P_m \) is the selling price (in dollars per unit) and \( Q_m \) is the quantity sold (in units). The marketing division’s total cost function in dollars (excluding the cost of the component) is

\[
C_m = 300,000 + 10Q_m
\]  
**[WE.6]**

The production division’s total cost function (in dollars) is

\[
C_p = 500,000 + 15Q_p + 0.0005Q_p^2
\]  
**[WE.7]**

where \( Q_p \) is the quantity produced and sold.

We are interested in determining the profit-maximizing outputs for the production and marketing divisions and the optimal transfer price for intracompany sales.
The marginal cost per unit to the firm, $MC$, is equal to the sum of the marginal costs of production, $MC_p$, and marketing, $MC_m$:

$$MC = MC_p + MC_m \quad \text{[WE.8]}$$

The marginal cost of the production division is equal to the first derivative of $C_p$ (Equation WE.7):

$$MC_p = \frac{dC_p}{dQ_p} = 15 + 0.0010Q_p \quad \text{[WE.9]}$$

The marginal cost of the marketing division is equal to the first derivative of $C_m$ (Equation WE.6):

$$MC_m = \frac{dC_m}{dQ} = 10 \quad \text{[WE.10]}$$

Substituting Equations WE.9 and WE.10 into Equation WE.8 and recognizing that $Q_m = Q_p$ we obtain

$$MC = 15 + 0.0010Q_m + 10$$

$$= 25 + 0.0010Q_m \quad \text{[WE.11]}$$

The marketing division’s total revenue function is equal to

$$TR_m = P_mQ_m$$

$$= (100 - 0.001Q_m)Q_m$$

$$= 100Q_m - 0.001Q_m^2 \quad \text{[WE.12]}$$

Taking the first derivative of $TR_m$ (Equation WE.12) gives

$$MR_m = \frac{d(TR_m)}{dQ_m}$$

$$= 100 - 0.002Q_m \quad \text{[WE.13]}$$

Setting Equation WE.11 equal to Equation WE.13 gives the optimal output for the marketing division:

$$MC = MR_m$$

$$25 + 0.0010Q_m = 100 - 0.002Q_m$$

$$Q_m^* = 25,000 \text{ units}$$
Because \( Q_p = Q_m \), the optimal output for the production division is

\[ Q_p^* = 25,000 \text{ units} \]

Therefore the optimal transfer price for intracompany sales of the component is equal to the marginal production cost per unit at the optimal output level of \( Q_p^* = 25,000 \) units, or

\[ P_t^* = MC_p \]
\[ = 15 + 0.0010(25,000) \]
\[ = $40 \text{ per unit} \]

Thus, to maximize profits, Portland’s production division should produce and sell 25,000 units of the component to the marketing division. The marketing division should distribute 25,000 units of the component through its retail outlets. The optimal transfer price for intracompany sales is $40—the production division’s marginal cost per unit at an output of 25,000 units.

**Perfectly Competitive External Market for the Intermediate Product**

With an external market for the intermediate product, the outputs of the production and marketing divisions are no longer required to be equal. In the following analysis, assume that the external market for the intermediate product is perfectly competitive. Two different situations involving supply and demand for the intermediate product are examined here:

- **Excess internal supply.** The production division has the capacity to produce more of the intermediate product than is desired by the marketing division and sells the excess output externally in the competitive market.

- **Excess internal demand.** The marketing division requires more of the intermediate product than can be supplied internally by the production division and buys additional units externally in the competitive market.

**Excess Internal Supply** The derivation of the optimal price-output decisions for the firm is shown in Figure WE.4. With a perfectly competitive market for the intermediate product, the production division is faced with a horizontal external demand curve \( D_p \) for its output at the existing market price \( P_t \). Setting divisional marginal revenue \( MR_p \) equal to the divisional marginal cost \( MC_p \) (Point C) yields a profit-maximizing output of \( Q_p^* \) units of the intermediate product. The marketing division, which must purchase the intermediate product either internally or externally at a price of \( P_m \) will have a marginal cost curve \( MC_m \), which is the sum of the marginal marketing cost per unit \( MC_m \) and the given transfer price \( P_t \). Again, equating divisional marginal revenue \( MR_m \) to divisional marginal cost \( MC_m \) (Point D) shows that profits will be maximized when \( Q_m^* \) units of the final product are sold externally at
a price of $P^*_m$ per unit. The solution indicates that the production division should produce $Q^*_m$ units of the intermediate product, sell $Q^*_m$ units of its output to the marketing division, and sell the difference, $Q^*_p - Q^*_m$, externally, in the intermediate product market.

A clear-cut transfer price emerges from this analysis. The competitive market price $P^*_t$ becomes the optimal transfer price ($P^*_t$) for intracompany sales of the intermediate product. The production division can sell as much output as it wishes externally at this price and therefore would have no incentive to sell internally to the marketing division at a price less than $P^*_t$.

**EXAMPLE**

**DETERMINING THE OPTIMAL TRANSFER PRICE:**

**PORTLAND ELECTRONICS (continued)**

Consider again the Portland Electronics Company discussed earlier. Suppose that the production division ($p$) of Portland Electronics Company manufactures a component that it can sell either internally to the marketing division ($m$), which promotes and distributes the product through its own domestic retail outlets, or externally in a perfectly competitive wholesale market to foreign distributors. The production division can sell the component externally to these distributors at $\$50$ per unit.
The task is to determine the profit-maximizing outputs for the production and marketing divisions and the optimal transfer price for intracompany sales. The production division’s optimal output occurs at the point where divisional marginal revenue equals divisional marginal cost. Because the production division can sell as much output as it wishes (externally) at the competitive market price of $50, its marginal revenue is equal to:

\[ MR_p = 50 \]

As we saw earlier, the production division’s marginal cost relationship is (from Equation WE.9): \[ MC_p = 15 + 0.0010Q_p \]

Setting \( MC_p = MR_p \) yields the optimal output for the production division:

\[ 15 + 0.0010Q_p = 50 \]
\[ Q^*_p = 35,000 \text{ units} \]

The marketing division’s optimal output occurs where divisional marginal revenue equals divisional marginal cost. Marginal cost for the marketing division \((MC_t)\) is equal to the sum of its own marginal marketing costs \((MC_m)\) plus the cost per unit of the components purchased from the production division \((P_t)\) or:

\[ MC_t = MC_m + P_t \quad [WE.14] \]

Because the external wholesale market for the component is perfectly competitive, the production division would not be willing to sell components to the marketing division for less than the market price of $50 per unit. Therefore, the optimal transfer price \((P^*_t)\) is the competitive market price of $50 per unit.

\[ P^*_t = 50 \text{ per unit} \]

As was shown earlier, marginal marketing costs \((MC_m)\) were:

\[ MC_m = 10 \]

Hence, by Equation WE.14, \( MC_t \) is given by:

\[ MC_t = 10 + 50 = 60 \]

The marketing division’s marginal revenue function \((MR_m)\) was given earlier as (from Equation WE.13):

\[ MR_m = 100 - 0.002Q_m \]

Setting \( MR_m = MC_t \) yields the optimal output for the marketing division:

\[ 100 - 0.002Q_m = 60 \]
\[ Q^*_m = 20,000 \text{ units} \]

Thus to maximize profits, Portland’s production division should produce 35,000 units of the component, sell 20,000 units internally to the marketing division,
and sell the remaining 15,000 units (35,000 – 20,000) externally to other (foreign) distributors. The marketing division should distribute 20,000 units of the component through its retail outlets. The optimal transfer price for the intracompany sales is the competitive market price of $50 per unit.

**EXCESS INTERNAL DEMAND** The derivation of the optimal price-output decisions for the firm under excess internal demand is shown in Figure WE.5. Similar to the excess internal supply situation discussed previously, the production division will attempt to maximize its profits by setting divisional marginal revenue \( MR_p \) equal to divisional marginal cost \( MC_p \) (Point E), which results in an optimal solution of \( Q^*_p \) units of the intermediate product. The marketing division, with a marginal cost curve \( MC_t \) equal to the sum of the marginal marketing costs per unit \( MC_m \) and the given transfer price \( P_t \), will attempt to maximize profits by equating divisional marginal revenue \( MR_m \) to divisional marginal cost \( MC_t \) (Point F). This approach yields an optimal solution of \( Q^*_m \) units of the final product being sold externally at a price of \( P^*_m \) per unit. The solution indicates that the production division should produce and sell its entire output of \( Q^*_p \) units of the intermediate product to the marketing division. The marketing division should purchase an additional \( Q^*_m – Q^*_p \) units of the intermediate product externally in the intermediate product market.

**Figure WE.5** Determination of the Transfer Price with a Perfectly Competitive External Market for the Intermediate Products—Excess Internal Demand

![Figure WE.5](image-url)
Multinational corporations have a great deal of flexibility in setting transfer prices, because often no external market standards dictate the level of these intracompany prices. In the absence of differential tax rates between the various countries in which a firm does business, the establishment of appropriate transfer prices involves application of microeconomic decision rules and cost accounting principles. However, because large multinational firms operate in several different countries, each with its own system of taxation and its own unique corporate income tax rates and policies, the use of transfer pricing to aggressively manage and reduce tax liabilities is common and profitable. For example, in 1991 the IRS charged Toyota with systematically overcharging its U.S. subsidiary for most of the vehicles and parts sold in the United States. The effect of these actions was to transfer profits that would have been booked (and taxed at high rates) in the United States to Japan, where tax rates are much lower. Toyota denied any wrongdoing but agreed to pay the IRS $1 billion in a settlement of these claims.

Westinghouse Electric booked 27 percent of its 1986 domestic profit in Puerto Rico, where it has very few sales. The corporate tax rate in Puerto Rico is set at 0 percent to stimulate the economy. Yamaha Motor Corporation’s U.S. subsidiary paid only $5,272 in taxes in the early 1980s, whereas IRS accountants claim that proper accounting of transfer prices would have resulted in $127 million in taxes.

The issue of setting proper transfer prices is extremely complex. Many differences between company policies and IRS regulations arise because of the complexity of the issue. However, the IRS has become increasingly aggressive in prosecuting blatant cases of abuse. Financial managers of multinational firms will have to give this issue greater attention in coming years if they expect to achieve the goal of maximizing shareholder wealth within the bounds of legal and ethical standards of business practice.

As in the situation of excess internal supply discussed earlier, the optimal transfer price \( P^* \) for intracompany transfers of the intermediate product is equal to the competitive market price \( P_c \). The marketing division can purchase as much of the intermediate product as it wishes externally at this price and therefore would be unwilling to make purchases from the production division at a price higher than \( P^* \).

**Summary**

- **Joint products** are products that are technically interdependent in the production process; that is, a change in the production of one produces a change in the cost or availability of another. When the joint products are produced in **fixed proportions**, the optimal output of the product package (consisting of the individual products) and optimal prices of the individual products are found at the intersection of the total marginal revenue function and the marginal cost function of producing the product package. When joint products are produced in **variable proportions**, the optimal output occurs where the marginal cost of producing each product is equal to the marginal revenue of each product. This occurs at the point of tangency between the isocost and isorevenue curves for the products.

- **Transfer pricing** analysis is important when a firm is faced with the problem of pricing items that are produced and used internally in the firm. When the external market for the intermediate product is perfectly competitive, the firm should use the market-determined price on intracompany sales. In other cases an appropriate profit-maximizing transfer price is a function of the marginal costs and revenues of the respective divisions in the firm.
Exercises

1. Explain why you feel that the interdependency terms between each of the following pairs of products would tend to be either positive (complements), negative (substitutes), or zero (independent):
   a. Polaroid: Instant cameras and film
   b. Nabisco: Fleischmann’s and Blue Bonnet margarine
   c. Nabisco: Ritz crackers and Oreo cookies
   d. Nabisco: Oreo cookies (regular size) and Mini Oreos
   e. Nabisco: Camel and Winston cigarettes
   f. General Motors: Saturn compact cars and Chevrolet compact cars
   g. General Motors: Buick full-size cars and Chevrolet compact cars

2. A company produces both oil and natural gas from a well in the panhandle of West Texas. If these products are produced from the well in fixed proportions, what would one expect the impact of an increase in the price of oil to be on the rate of gas production?

3. Refer to the Williams Company joint products example (Equations WE.1–WE.4):
   a. On a graph with quantity on the horizontal axis and price (and cost) on the vertical axis, plot the demand and marginal revenue functions for the two products and the marginal cost function for the product package.
   b. From the graph in Part (a), determine the optimal output and price for each of the two products. Compare the graphical solution with the algebraic solution in Chapter 14 of the textbook.

4. Refer again to the Williams Company joint products example (Equations WE.1–E.4) and assume that the marginal cost function (Equation WE.3) is replaced with the following one:

   \[ MC = 22 + 0.5Q \]

   Determine the optimal output and selling price for each of the two products.

5. Refer to the Portland Electronics Company transfer pricing example, discussed earlier in this appendix, which dealt with a competitive external market for the intermediate product. Complete the following table (based on the optimal solution):

<table>
<thead>
<tr>
<th>Production Division</th>
<th>Marketing Division</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total revenue</td>
<td></td>
</tr>
<tr>
<td>Total cost</td>
<td></td>
</tr>
<tr>
<td>Total profit</td>
<td></td>
</tr>
</tbody>
</table>

6. Refer again to the Portland Electronics Company transfer pricing example discussed earlier in this appendix, which dealt with a perfectly competitive external market for the intermediate product. Assume that the company can buy (or sell) additional units of the component at $30 per unit. Determine the optimal price and output decisions for the production and marketing divisions and compare them with the solution obtained in the chapter.

7. Consolidated Sugar Company has two divisions: a farming-preprocessing (p) division and a processing-marketing (m) division. The farming-preprocessing division grows sugar cane and crushes it into juice, which it sells internally to the processing-marketing division or externally in the perfectly competitive open market. The processing-marketing division buys cane juice, either internally from the farming-preprocessing division or externally in the open market, and then evaporates and purifies it and sells it as processed sugar.
The processing-marketing division’s demand function for processed sugar is

\[ P_m = 24 - Q_m \]

where \( P_m \) is the price, in dollars per unit, and \( Q_m \) is the quantity sold, in units, and its cost function (excluding cane juice) is

\[ C_m = 8 + 2Q_m \]

The farming-preprocessing division’s total cost function for cane juice is

\[ C_p = 10 + 2Q_p + Q_p^2 \]

where \( Q_p \) is the quantity produced, in units. Assume that one unit of cane juice is converted into one unit of processed sugar. Furthermore, assume that the open market price for cane juice is $14.

a. What is the profit-maximizing price and output level for the farming-preprocessing division?

b. What is the profit-maximizing price and output level for the processing-marketing division?

c. How much of its output (cane juice) should the farming-preprocessing division sell (i) internally to the processing-marketing division and (ii) externally on the open market?

d. How much of its input (cane juice) should the processing-marketing division buy (i) internally from the farming-preprocessing division and (ii) externally on the open market?

e. What is the minimum price at which the farming-preprocessing division would be willing to sell cane juice to the processing-marketing division? Explain.

f. What is the maximum price that the processing-marketing division would be willing to pay to buy cane juice from the farming-preprocessing division? Explain.

g. To maximize the overall profits of Consolidated Sugar, what price should the company use for intracompany transfers of cane juice from the farming-preprocessing division to the processing-marketing division?

### Case Exercise

**Case Exercise**

DeSoto Engine, a division of International Motors, produces automobile engines. It sells these engines to the automobile assembly division within the corporation. A dispute has arisen between the managers of the DeSoto division and the assembly division concerning the appropriate transfer price for intracompany sales of engines. The current transfer price of $385 per unit was arrived at by taking the standard cost of the engine ($350) and adding a 10 percent profit margin ($35), based on an estimated volume of 450,000 engines per year. The manager of the DeSoto division argues that the transfer price should be raised because the division’s average profit margin on other products is 18 percent. The manager of the assembly division claims that the transfer price should be lowered because an assembly division manager at a competing automobile company indicated that engines cost his division only $325 per unit. The corporation’s chief economist has been asked to solve this intracompany pricing problem.
The economist collected the following demand and cost information. Demand for automobiles is given by the following function:

\[ P_m = 10,000 - 0.01Q_m \]

where \( P_m \) is the selling price (in dollars) per automobile and \( Q_m \) is the number of vehicles sold. (Assume for simplicity that price is the only variable that affects demand.) The total cost function for the assembly division (excluding the cost of the engines) is

\[ C_m = 1,150,000,000 + 2,500Q_m \]

where \( C_m \) is the cost (in dollars). The DeSoto division’s total cost function is

\[ C_p = 30,000,000 + 275Q_p + 0.000125Q_p^2 \]

where \( Q_p \) is the number of engines produced and \( C_p \) is the cost (in dollars).

**Questions**

Assume that no external market exists for these engines. (In other words, the DeSoto division cannot sell any excess engines to outside buyers and the assembly division cannot obtain additional engines from outside suppliers).

1. Determine the profit-maximizing output (vehicles) for the assembly division.
2. Determine the profit-maximizing output (engines) for the DeSoto division.
3. Determine the optimal transfer price for intracompny sales of engines.
4. Calculate (a) total revenue, (b) total cost, and (c) total profits for each division at the optimal solution found in Questions 1, 2, and 3.

The manager of the DeSoto division is dissatisfied with the solution to the transfer-pricing problem. On further investigation, he finds that a perfectly competitive external market exists for automobile engines, with many automobile manufacturers and suppliers willing to sell or purchase engines at the going market price. Specifically, a large German automobile company (BW Motors) has offered to purchase all of DeSoto’s engine output (up to 700,000 engines per year) at a price of $425 per unit.

5. Determine the profit-maximizing output for the assembly division.
6. Determine the profit-maximizing output for the DeSoto division.
7. Determine the optimal transfer price for intracompny sales of engines.
8. Determine how many engines the DeSoto division should sell (a) internally to the assembly division and (b) externally to BW Motors.
9. Calculate (a) total revenue, (b) total costs, and (c) total profits for each division at the optimal solution found in Questions 5, 6, 7, and 8.