ST2.1 Profit versus Revenue Maximization. Presto Products, Inc., manufactures small electrical appliances and has recently introduced an innovative new dessert maker for frozen yogurt and fruit smoothies that has the clear potential to offset the weak pricing and sluggish volume growth experienced during recent periods.

Monthly demand and cost relations for Presto's frozen dessert maker are as follows:

\[ P = 60 - 0.005Q \]

\[ TC = 100,000 + 5Q + 0.0005Q^2 \]

\[ MR = \frac{\Delta TR}{\Delta Q} = 60 - 0.01Q \]

\[ MC = \frac{\Delta TC}{\Delta Q} = 5 + 0.001Q \]

A. Set up a table or spreadsheet for Presto output (Q), price (P), total revenue (TR), marginal revenue (MR), total cost (TC), marginal cost (MC), total profit (π), and marginal profit (Mπ). Establish a range for Q from 0 to 10,000 in increments of 1,000 (i.e., 0, 1,000, 2,000, ..., 10,000).

B. Using the Presto table or spreadsheet, create a graph with TR, TC, and π as dependent variables, and units of output (Q) as the independent variable. At what price/output combination is total profit maximized? Why? At what price/output combination is total revenue maximized? Why?

C. Determine these profit-maximizing and revenue-maximizing price/output combinations analytically. In other words, use Presto's profit and revenue equations to confirm your answers to part B.

D. Compare the profit-maximizing and revenue-maximizing price/output combinations, and discuss any differences. When will short-run revenue maximization lead to long-run profit maximization?
B. The price/output combination at which total profit is maximized is \( P = $35 \) and \( Q = 5,000 \) units. At that point, \( MR = MC \) and total profit is maximized at $37,500. The price/output combination at which total revenue is maximized is \( P = $30 \) and \( Q = 6,000 \) units. At that point, \( MR = 0 \) and total revenue is maximized at $180,000. Using the Presto table or spreadsheet, a graph with \( TR \), \( TC \), and \( \pi \) as dependent variables, and units of output (Q) as the independent variable appears as follows:

\[ \text{Presto Products, Inc.} \\
\text{Profit vs. Revenue Maximization} \]
C. To find the profit-maximizing output level analytically, set MR = MC, or set $M\pi = 0$, and solve for Q. Because

$$\text{MR} = \text{MC}$$

$$60 - 0.01Q = 5 + 0.001Q$$

$$0.011Q = 55$$

$$Q = 5,000$$

At Q = 5,000,

$$P = 60 - 0.005(5,000)$$

$$= 35$$

$$\pi = -$100,000 + 55(5,000) - 0.0055(5,000^2)$$

$$= 37,500$$

(Note: This is a profit maximum because total profit is falling for Q > 5,000.)

To find the revenue-maximizing output level, set MR = 0, and solve for Q. Thus,

$$\text{MR} = 60 - 0.01Q = 0$$

$$0.01Q = 60$$

$$Q = 6,000$$

At Q = 6,000,

$$P = 60 - 0.005(6,000)$$

$$= 30$$

$$\pi = TR - TC$$

$$= (60 - 0.005Q)Q - 100,000 - 5Q - 0.0005Q^2$$

$$= -100,000 + 55Q - 0.0055Q^2$$

$$= -100,000 + 55(6,000) - 0.0055(6,000^2)$$
\[ = \$32,000 \]

(Note: This is a revenue maximum because total revenue is decreasing for output beyond \( Q > 6,000 \).)

D. Given downward sloping demand and marginal revenue curves and positive marginal costs, the profit-maximizing price/output combination is always at a higher price and lower production level than the revenue-maximizing price-output combination. This stems from the fact that profit is maximized when \( MR = MC \), whereas revenue is maximized when \( MR = 0 \). It follows that profits and revenue are only maximized at the same price/output combination in the unlikely event that \( MC = 0 \).

In pursuing a short-run revenue rather than profit-maximizing strategy, Presto can expect to gain a number of important advantages, including enhanced product awareness among consumers, increased customer loyalty, potential economies of scale in marketing and promotion, and possible limitations in competitor entry and growth. To be consistent with long-run profit maximization, these advantages of short-run revenue maximization must be at least worth Presto's short-run sacrifice of \( \$5,500 (= \$37,500 - \$32,000) \) in monthly profits.

**ST2.2 Average Cost-Minimization.** Pharmed Caplets, Inc., is an international manufacturer of bulk antibiotics for the animal feed market. Dr. Indiana Jones, head of marketing and research, seeks your advice on an appropriate pricing strategy for Pharmed Caplets, an antibiotic for sale to the veterinarian and feedlot-operator market. This product has been successfully launched during the past few months in a number of test markets, and reliable data are now available for the first time.

The marketing and accounting departments have provided you with the following monthly total revenue and total cost information:

\[
TR = 900Q - 0.1Q^2 \\
TC = 36,000 + 200Q + 0.4Q^2
\]

\[
MR = \frac{\Delta TR}{\Delta Q} = 900 - 0.2Q \\
MC = \frac{\Delta TC}{\Delta Q} = 200 + 0.8Q
\]

A. Set up a table or spreadsheet for Pharmed Caplets output (\( Q \)), price (\( P \)), total revenue (\( TR \)), marginal revenue (\( MR \)), total cost (\( TC \)), marginal cost (\( MC \)), average cost (\( AC \)), total profit (\( \pi \)), and marginal profit (\( M\pi \)). Establish a range for \( Q \) from 0 to 1,000 in increments of 100 (i.e., 0, 100, 200, ..., 1,000).

B. Using the Pharmed Caplets table or spreadsheet, create a graph with AC and MC as dependent variables and units of output (\( Q \)) as the independent variable. At what price/output combination is total profit maximized? Why? At what price/output combination is average cost minimized? Why?

C. Determine these profit-maximizing and average-cost minimizing price/output combinations analytically. In other words, use Pharmed Caplets' revenue and
cost equations to confirm your answers to part B.

D. Compare the profit-maximizing and average-cost minimizing price/output combinations, and discuss any differences. When will average-cost minimization lead to long-run profit maximization?

ST2.2 SOLUTION

A. A table or spreadsheet for Pharmed Caplets output (Q), price (P), total revenue (TR), marginal revenue (MR), total cost (TC), marginal cost (MC), average cost (AC), total profit (π), and marginal profit (Mπ) appears as follows:

<table>
<thead>
<tr>
<th>Units</th>
<th>Price</th>
<th>Total Revenue</th>
<th>Marginal Revenue</th>
<th>Total Cost</th>
<th>Marginal Cost</th>
<th>Average Cost</th>
<th>Total Profit</th>
<th>Marginal Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$900</td>
<td>0</td>
<td>$900</td>
<td>$36,000</td>
<td>$200</td>
<td>---</td>
<td>$(36,000)</td>
<td>$700</td>
</tr>
<tr>
<td>100</td>
<td>$890</td>
<td>89,000</td>
<td>$880</td>
<td>$60,000</td>
<td>$280</td>
<td>600.00</td>
<td>29,000</td>
<td>600</td>
</tr>
<tr>
<td>200</td>
<td>$880</td>
<td>176,000</td>
<td>$860</td>
<td>$92,000</td>
<td>$360</td>
<td>460.00</td>
<td>84,000</td>
<td>500</td>
</tr>
<tr>
<td>300</td>
<td>$870</td>
<td>261,000</td>
<td>$840</td>
<td>$132,000</td>
<td>$440</td>
<td>440.00</td>
<td>129,000</td>
<td>400</td>
</tr>
<tr>
<td>400</td>
<td>$860</td>
<td>344,000</td>
<td>$820</td>
<td>$180,000</td>
<td>$520</td>
<td>450.00</td>
<td>164,000</td>
<td>300</td>
</tr>
<tr>
<td>500</td>
<td>$850</td>
<td>425,000</td>
<td>$800</td>
<td>$236,000</td>
<td>$600</td>
<td>472.00</td>
<td>189,000</td>
<td>200</td>
</tr>
<tr>
<td>600</td>
<td>$840</td>
<td>504,000</td>
<td>$780</td>
<td>$300,000</td>
<td>$680</td>
<td>500.00</td>
<td>204,000</td>
<td>100</td>
</tr>
<tr>
<td>700</td>
<td>$830</td>
<td>581,000</td>
<td>$760</td>
<td>$372,000</td>
<td>$760</td>
<td>531.43</td>
<td>209,000</td>
<td>0</td>
</tr>
<tr>
<td>800</td>
<td>$820</td>
<td>656,000</td>
<td>$740</td>
<td>$452,000</td>
<td>$840</td>
<td>565.00</td>
<td>204,000</td>
<td>(100)</td>
</tr>
<tr>
<td>900</td>
<td>$810</td>
<td>729,000</td>
<td>$720</td>
<td>$540,000</td>
<td>$920</td>
<td>600.00</td>
<td>189,000</td>
<td>(200)</td>
</tr>
<tr>
<td>1,000</td>
<td>$800</td>
<td>800,000</td>
<td>$700</td>
<td>$636,000</td>
<td>$1,000</td>
<td>636.00</td>
<td>164,000</td>
<td>(300)</td>
</tr>
</tbody>
</table>

B. The price/output combination at which total profit is maximized is P = $830 and Q = 700 units. At that point, MR = MC and total profit is maximized at $209,000. The price/output combination at which average cost is minimized is P = $870 and Q = 300 units. At that point, MC = AC = $440.

Using the Pharmed Caplets table or spreadsheet, a graph with AC, and MC as dependent variables and units of output (Q) as the independent variable appears as follows:
C. To find the profit-maximizing output level analytically, set MR = MC, or set \( \pi = 0 \), and solve for \( Q \). Because

\[
MR = MC
\]

\[
$900 - 0.2Q = 200 + 0.8Q
\]

\[
Q = 700
\]

At \( Q = 700 \),

\[
P = \frac{TR}{Q}
\]

\[
= \frac{(900Q - 0.1Q^2)}{Q}
\]

\[
= 900 - 0.1(700)
\]

\[
= 830
\]

\[
\pi = TR - TC
\]
= $900Q - $0.1Q^2 - $36,000 - $200Q - $0.4Q^2

= -$36,000 + $700(700) - $0.5(700^2)

= $209,000

(Note: This is a profit maximum because profits are falling for Q > 700.)

To find the average-cost minimizing output level, set MC = AC, and solve for Q.

Because

AC = TC/Q

= ($36,000 + $200Q + $0.4Q^2)/Q

= $36,000Q^{-1} + $200 + $0.4Q,

it follows that:

MC = AC

$200 + $0.8Q = $36,000Q^{-1} + $200 + $0.4Q

0.4Q = 36,000Q^{-1}

0.4Q^2 = 36,000

Q^2 = 36,000/0.4

Q^2 = 90,000

Q = 300

At Q = 300,

P = $900 - $0.1(300)

= $870

π = -$36,000 + $700(300) - $0.5(300^2)

= $129,000

(Note: This is an average-cost minimum because average cost is rising for Q > 300.)
Given downward sloping demand and marginal revenue curves and a U-shaped, or quadratic, AC function, the profit-maximizing price/output combination will often be at a different price and production level than the average-cost minimizing price-output combination. This stems from the fact that profit is maximized when MR = MC, whereas average cost is minimized when MC = AC. Profits are maximized at the same price/output combination as where average costs are minimized in the unlikely event that MR = MC and MC = AC and, therefore, MR = MC = AC.

It is often true that the profit-maximizing output level differs from the average cost-minimizing activity level. In this instance, expansion beyond Q = 300, the average cost-minimizing activity level, can be justified because the added gain in revenue more than compensates for the added costs. Note that total costs rise by $240,000, from $132,000 to $372,000 as output expands from Q = 300 to Q = 700, as average cost rises from $440 to $531.43. Nevertheless, profits rise by $80,000, from $129,000 to $209,000, because total revenue rises by $320,000, from $261,000 to $581,000. The profit-maximizing activity level can be less than, greater than, or equal to the average-cost minimizing activity level depending on the shape of relevant demand and cost relations.
Chapter 3

Statistical Analysis of Economic Relations

SELF-TEST PROBLEMS & SOLUTIONS

ST3.1  Data Description and Analysis.  Eric Delko, a staff research assistant with Market Research Associates, Ltd., has conducted a survey of households in the Coral Gables part of the Greater Miami area. The focus of Delco’s survey is to gain information on the buying habits of potential customers for a local new car dealership. Among the data collected by Delco is the following information on number of cars per household and household disposable income for a sample of $n = 15$ households:

$\begin{array}{|c|c|}
\hline
\text{Number of Cars} & \text{Income (in$000$)} \\
\hline
1 & 100 \\
3 & 100 \\
0 & 30 \\
2 & 50 \\
0 & 30 \\
2 & 30 \\
2 & 100 \\
0 & 30 \\
2 & 100 \\
2 & 50 \\
3 & 100 \\
2 & 50 \\
1 & 50 \\
1 & 30 \\
2 & 50 \\
\hline
\end{array}$

A. Calculate the mean, median, and mode measures of central tendency for the
number of cars per household and household disposable income. Which measure
does the best job of describing central tendency for each variable?

B. Based on this \( n = 15 \) sample, calculate the range, variance, and standard
deviation for each data series, and the 95 percent confidence interval within
which you would expect to find each variable’s true population mean.

C. Consulting a broader study, Delco found a \$60,000\ mean level of disposable
income per household for a sample of \( n = 196 \) Coral Gables households. Assume
Delco knows that disposable income per household in the Miami area has a
population mean of \$42,500\ and \( \sigma = \$3,000 \). At the 95 percent confidence level,
can you reject the hypothesis that the Coral Gables area has a typical average
income?

ST3.1 SOLUTION

A. The mean, or average number of 1.533 cars per household, and mean household
disposable income of \$60,000\ are calculated as follows:

\[
\text{CARS: } \bar{X} = \frac{1+3+0+2+0+2+2+0+2+2+3+2+1+1+2}{15} = \frac{23}{15} = 1.533 \\
\text{INCOME: } \bar{X} = \frac{100+100+30+50+30+30+100+30+100+50+100+50+50+30+50}{15} = \frac{2000}{15} = 60,000
\]

By inspection of a rank-order from highest to lowest values, the “middle” or median
values are two cars per household and \$50,000\ in disposable income per household. The
mode for the number of cars per household is two cars, owned by seven households.
The distribution of disposable income per household is trimodal with five households
each having income of \$30,000, \$50,000,\ and \$100,000.

In this instance, the median appears to provide the best measure of central
tendency.

B. The range is from zero to three cars per household, and from \$30,000\ to \$100,000\ in
disposable income. For the number of cars per household, the sample variance is 0.98
(cars squared), and the sample standard deviation is 0.9809 cars. For disposable income
per household, the sample variance is 928.42 (dollars squared), and the standard
deviation is \$30.47. These values are calculated as follows:

\[
\text{Cars: } s^2 = [(1-1.533)^2 + (3-1.533)^2 + (0-1.533)^2 + (2-1.533)^2 + (0-1.533)^2 \\
+ (2-1.533)^2 + (2-1.533)^2 + (0-1.533)^2 + (2-1.533)^2 + (0-1.533)^2]
\]

-10-
\[ + (3-1.533)^2 + (2-1.533)^2 + (1-1.533)^2 + (1-1.533)^2 \\
+ (2-1.533)^2] / 14 \\
= 13.733 / 14 \\
= 0.9809 \\
s = \sqrt{s^2} = 0.990 \\
Income: s^2 = [(100-60)^2 + (100-60)^2 + (30-60)^2 + (50-60)^2 + (30-60)^2 + (30-60)^2 + (100-60)^2 + (30-60)^2 + (100-60)^2 + (50-60)^2 + (100-60)^2 + (50-60)^2 + (50-60)^2 + (30-60)^2 + (50-60)^2] / 14 \\
= 13,000 / 14 \\
= 928.42 \\
s = \sqrt{s^2} = $30.470(000) \\

Given the very small sample size involved, the \( t \) test with \( df = n - 1 = 15 - 1 = 14 \) is used to determine the 95 percent confidence intervals within which you would expect to find each variable’s true population mean. The exact confidence intervals are from 0.985 cars to 2.082 cars per household, and from $43,120 to $76,880 in disposable income per household, calculated as follows:

Cars: \( \bar{X} - t(s/\sqrt{n}) = 1.533 - 2.145(0.99/3.873) = 0.985 \) (lower bound) \\
\( \bar{X} + t(s/\sqrt{n}) = 1.533 + 2.145(0.99/3.873) = 2.082 \) (upper bound) \\
Income: \( \bar{X} - t(s/\sqrt{n}) = 60 - 2.145(30.47/3.873) = 43.12 \) (lower bound) \\
\( \bar{X} + t(s/\sqrt{n}) = 60 + 2.145(30.47/3.873) = 76.88 \) (upper bound) \\

Of course, if the rule of thumb of \( t = 2 \) were used rather than the exact critical value of \( t = 2.145 \) (\( df = 14 \)), then a somewhat narrower confidence interval would be calculated.

C. Yes. The \( z \) statistic can be used to test the hypothesis that the mean level of income in Coral Gables is the same as that for the Miami area given this larger sample size because disposable income per household has a known population mean and standard deviation. Given this sample size of \( n = 196 \), the 95 percent confidence interval for the mean level}
of income in Coral Gables is from $58,480 to $61,520 -- both well above the population mean of $42,500:

$$\bar{X} - z(\sigma/\sqrt{n}) = 60,000 - 1.96(3,000/\sqrt{196}) = 59,580 \text{ (lower bound)}$$

$$\bar{X} + z(\sigma/\sqrt{n}) = 60,000 + 1.96(3,000/\sqrt{196}) = 60,420 \text{ (upper bound)}$$

Had the rule of thumb $z = 2$ been used rather than the exact $z = 1.96$, a somewhat wider confidence interval would have been calculated.

The hypothesis to be tested is that the mean income for the Coral Gables area equals that for the overall population, $H_0: \mu = 42,500$, when $\sigma = 3,000$. The test-statistic for this hypothesis is $z = 81.67$, meaning that the null hypothesis can be rejected:

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{60,000 - 42,500}{3,000/\sqrt{196}} = 81.67$$

The probability of finding such a high sample average income when Coral Gables is in fact typical of the overall population average income of $42,500 is less than 5 percent. Coral gables area income appears to be higher than that for the Miami area in general.

**ST3.2 Simple Regression.** The global computer software industry is dominated by Microsoft Corp. and a handful of large competitors from the United States. During the early 2000s, fallout from the government’s antitrust case against Microsoft and changes tied to the Internet caused company and industry analysts to question the profitability and long-run advantages of the industry’s massive long-term investments in research and development (R&D).

The following table shows sales revenue, profit, and R&D data for a $n = 15$ sample of large firms taken from the U.S. computer software industry. Net sales revenue, net income before extraordinary items, and research and development (R&D) expenditures are shown. R&D is the dollar amount of company-sponsored expenditures during the most recent fiscal year, as reported to the Securities and Exchange Commission on Form 10-K. Excluded from such numbers is R&D under contract to others, such as U.S. government agencies. All figures are in $ millions.

<table>
<thead>
<tr>
<th>Company Name</th>
<th>Sales</th>
<th>Net Income</th>
<th>R&amp;D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Microsoft Corp.</td>
<td>22,956.0</td>
<td>9,421.0</td>
<td>3,775.0</td>
</tr>
<tr>
<td>Electronic Arts Inc.</td>
<td>1,420.0</td>
<td>116.8</td>
<td>267.3</td>
</tr>
<tr>
<td>Adobe Systems Inc.</td>
<td>1,266.4</td>
<td>287.8</td>
<td>240.7</td>
</tr>
<tr>
<td>Novell Inc.</td>
<td>1,161.7</td>
<td>49.5</td>
<td>234.6</td>
</tr>
<tr>
<td>Intuit Inc.</td>
<td>1,093.8</td>
<td>305.7</td>
<td>170.4</td>
</tr>
<tr>
<td>Siebel Systems Inc.</td>
<td>790.9</td>
<td>122.1</td>
<td>72.9</td>
</tr>
<tr>
<td>Symantec Corp.</td>
<td>745.7</td>
<td>170.1</td>
<td>112.7</td>
</tr>
<tr>
<td>Networks Associates Inc.</td>
<td>683.7</td>
<td>-159.9</td>
<td>148.2</td>
</tr>
<tr>
<td>Activision Inc.</td>
<td>583.9</td>
<td>-34.1</td>
<td>26.3</td>
</tr>
<tr>
<td>Rational Software Corp.</td>
<td>572.2</td>
<td>85.3</td>
<td>106.4</td>
</tr>
</tbody>
</table>
A simple regression model with sales revenue as the dependent Y variable and R&D expenditures independent X variable shows (t statistics in parentheses):

\[ Sales_i = \$20.065 + \$6.062 \cdot R&D_i \]
\[ R^2 = 99.8\% \]
\[ SEE = 233.75 \]
\[ F = 8460.40 \]
\[ (0.31) \]
\[ (91.98) \]

How would you interpret these findings?

A simple regression model with net income (profits) as the dependent Y variable and R&D expenditures independent X variable shows (t statistics in parentheses):

\[ Profits_i = -$210.31 + $2.538 \cdot R&D_i \]
\[ R^2 = 99.3\% \]
\[ SEE = 201.30 \]
\[ F = 1999.90 \]
\[ (0.75) \]
\[ (7.03) \]

How would you interpret these findings?

Discuss any differences between your answers to parts A and B.

ST3.2 SOLUTION

A.

First of all, the constant in such a regression typically has no meaning. Clearly, the intercept should not be used to suggest the value of sales revenue that might occur for a firm that had zero R&D expenditures. As discussed in the problem, this sample of firms is restricted to large companies with significant R&D spending. The R&D coefficient is statistically significant at the \( \alpha = 0.01 \) level with a calculated t statistic value of 91.98, meaning that it is possible to be more than 99 percent confident that R&D expenditures affect firm sales. The probability of observing such a large t statistic when there is in fact no relation between sales revenue and R&D expenditures is less than 1 percent. The R&D coefficient estimate of $6.062 implies that a $1 rise in R&D expenditures leads to an average $6.062 increase in sales revenue.

The \( R^2 = 99.8\% \) indicates the share of sales variation that can be explained by the variation in R&D expenditures. Note that \( F = 8460.40 > F_{1,13, \alpha = 0.01} = 9.07 \), implying that variation in R&D spending explains a significant share of the total variation in firm sales. This suggests that R&D expenditures are a key determinant of
sales in the computer software industry, as one might expect.

The standard error of the Y estimate or SEE = $233.75 (million) and is the average amount of error encountered in estimating the level of sales for any given level of R&D spending. If the $u_i$ error terms are normally distributed about the regression equation, as would be true when large samples of more than 30 or so observations are analyzed, there is a 95 percent probability that observations of the dependent variable will lie within the range $\hat{Y}_i \pm (1.96 \times \text{SEE})$, or within roughly two standard errors of the estimate. The probability is 99 percent that any given $\hat{Y}_i$ will lie within the range $\hat{Y}_i \pm (2.576 \times \text{SEE})$, or within roughly three standard errors of its predicted value. When very small samples of data are analyzed, as is the case here, “critical” values slightly larger than two or three are multiplied by the SEE to obtain the 95 percent and 99 percent confidence intervals.

Precise critical $t$ values obtained from a $t$ table, such as that found in Appendix B, are $t_{13, 0.05} = 2.160$ (at the 95 percent confidence level) and $t_{13, 0.01} = 3.012$ (at the 99 percent confidence level) for df = 15 - 2 = 13. This means that actual sales revenue $Y_i$ can be expected to fall in the range $\hat{Y}_i \pm (2.160 \times 233.75)$, or $\hat{Y}_i \pm 504.90$, with 95 percent confidence; and within the range $\hat{Y}_i \pm (3.012 \times 233.75)$, or $\hat{Y}_i \pm 704.055$, with 99 percent confidence.

B. As in part A, the constant in such a regression typically has no meaning. Clearly, the intercept should not be used to suggest the level of profits that might occur for a firm that had zero R&D expenditures. Again, the R&D coefficient is statistically significant at the $\alpha = 0.01$ level with a calculated $t$ statistic value of 44.72, meaning that it is possible to be more than 99 percent confident that R&D expenditures affect firm profits. The probability of observing such a large $t$ statistic when there is in fact no relation between profits and R&D expenditures is less than 1 percent. The R&D coefficient estimate of $2.538$ suggests that a $1$ rise in R&D expenditures leads to an average $2.538$ increase in current-year profits.

The $\bar{R}^2 = 99.3$ percent indicates the share of profit variation that can be explained by the variation in R&D expenditures. This suggests that R&D expenditures are a key determinant of profits in the aerospace industry. Again, notice that $F = 1999.90 > F_{1,13, 0.01} = 9.07$, meaning that variation in R&D spending can explain a significant share of profit variation.

The standard error of the Y estimate or SEE = $201.30 (million). This is the average amount of error encountered in estimating the level of profit for any given level of R&D spending. Actual profits $Y_i$ can be expected to fall in the range $\hat{Y}_i \pm (2.160 \times 201.30)$, or $\hat{Y}_i \pm 434.808$, with 95 percent confidence; and within the range $\hat{Y}_i \pm (3.012 \times 201.30)$, or $\hat{Y}_i \pm 606.3156$, with 99 percent confidence.

C. Clearly, a strong link between both sales revenue and profits and R&D expenditures is suggested by a regression analysis of the computer software industry. There appears to be slightly less variation in the sales-R&D relation than in the profits-R&D relation. As
indicated by $R^2$, the linkage between sales and R&D is a bit stronger than the relation between profits and R&D. At least in part, this may stem from the fact that the sample was limited to large R&D intensive firms, whereas no such screen for profitability was included.
Chapter 4

Demand and Supply

SELF-TEST PROBLEMS & SOLUTIONS

ST4.1 Demand and Supply Curves. The following relations describe demand and supply conditions in the lumber/forest products industry

\[ Q_D = 80,000 - 20,000P \quad \text{(Demand)} \]
\[ Q_S = -20,000 + 20,000P \quad \text{(Supply)} \]

where \( Q \) is quantity measured in thousands of board feet (one square foot of lumber, one inch thick) and \( P \) is price in dollars.

A. Set up a spreadsheet to illustrate the effect of price (\( P \)), on the quantity supplied (\( Q_S \)), quantity demanded (\( Q_D \)), and the resulting surplus (+) or shortage (-) as represented by the difference between the quantity supplied and the quantity demanded at various price levels. Calculate the value for each respective variable based on a range for \( P \) from $1.00 to $3.50 in increments of 10¢ (i.e., $1.00, $1.10, $1.20, . . . $3.50).

B. Using price (\( P \)) on the vertical or y-axis and quantity (\( Q \)) on the horizontal or x-axis, plot the demand and supply curves for the lumber/forest products industry over the range of prices indicated previously.

ST4.1 SOLUTION

A. A table or spreadsheet that illustrates the effect of price (\( P \)), on the quantity supplied (\( Q_S \)), quantity demanded (\( Q_D \)), and the resulting surplus (+) or shortage (-) as represented by the difference between the quantity supplied and the quantity demanded at various price levels is as follows:

<table>
<thead>
<tr>
<th>Price</th>
<th>Quantity Demanded</th>
<th>Quantity Supplied</th>
<th>Surplus (+) or Shortage (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.00</td>
<td>60,000</td>
<td>0</td>
<td>-60,000</td>
</tr>
<tr>
<td>1.10</td>
<td>58,000</td>
<td>2,000</td>
<td>-56,000</td>
</tr>
</tbody>
</table>
### Lumber and Forest Industry Supply and Demand Relationships

<table>
<thead>
<tr>
<th>Price</th>
<th>Quantity Demanded</th>
<th>Quantity Supplied</th>
<th>Surplus (+) or Shortage (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.20</td>
<td>56,000</td>
<td>4,000</td>
<td>-52,000</td>
</tr>
<tr>
<td>1.30</td>
<td>54,000</td>
<td>6,000</td>
<td>-48,000</td>
</tr>
<tr>
<td>1.40</td>
<td>52,000</td>
<td>8,000</td>
<td>-44,000</td>
</tr>
<tr>
<td>1.50</td>
<td>50,000</td>
<td>10,000</td>
<td>-40,000</td>
</tr>
<tr>
<td>1.60</td>
<td>48,000</td>
<td>12,000</td>
<td>-36,000</td>
</tr>
<tr>
<td>1.70</td>
<td>46,000</td>
<td>14,000</td>
<td>-32,000</td>
</tr>
<tr>
<td>1.80</td>
<td>44,000</td>
<td>16,000</td>
<td>-28,000</td>
</tr>
<tr>
<td>1.90</td>
<td>42,000</td>
<td>18,000</td>
<td>-24,000</td>
</tr>
<tr>
<td>2.00</td>
<td>40,000</td>
<td>20,000</td>
<td>-20,000</td>
</tr>
<tr>
<td>2.10</td>
<td>38,000</td>
<td>22,000</td>
<td>-16,000</td>
</tr>
<tr>
<td>2.20</td>
<td>36,000</td>
<td>24,000</td>
<td>-12,000</td>
</tr>
<tr>
<td>2.30</td>
<td>34,000</td>
<td>26,000</td>
<td>-8,000</td>
</tr>
<tr>
<td>2.40</td>
<td>32,000</td>
<td>28,000</td>
<td>-4,000</td>
</tr>
<tr>
<td>2.50</td>
<td>30,000</td>
<td>30,000</td>
<td>0</td>
</tr>
<tr>
<td>2.60</td>
<td>28,000</td>
<td>32,000</td>
<td>4,000</td>
</tr>
<tr>
<td>2.70</td>
<td>26,000</td>
<td>34,000</td>
<td>8,000</td>
</tr>
<tr>
<td>2.80</td>
<td>24,000</td>
<td>36,000</td>
<td>12,000</td>
</tr>
<tr>
<td>2.90</td>
<td>22,000</td>
<td>38,000</td>
<td>16,000</td>
</tr>
<tr>
<td>3.00</td>
<td>20,000</td>
<td>40,000</td>
<td>20,000</td>
</tr>
<tr>
<td>3.10</td>
<td>18,000</td>
<td>42,000</td>
<td>24,000</td>
</tr>
<tr>
<td>3.20</td>
<td>16,000</td>
<td>44,000</td>
<td>28,000</td>
</tr>
<tr>
<td>3.30</td>
<td>14,000</td>
<td>46,000</td>
<td>32,000</td>
</tr>
<tr>
<td>3.40</td>
<td>12,000</td>
<td>48,000</td>
<td>36,000</td>
</tr>
<tr>
<td>3.50</td>
<td>10,000</td>
<td>50,000</td>
<td>40,000</td>
</tr>
</tbody>
</table>

B. Using price (P) on the vertical \( Y \) axis and quantity (Q) on the horizontal \( X \) axis, a plot of the demand and supply curves for the lumber/forest products industry is as follows:
ST4.2 **Supply Curve Determination.** Information Technology, Inc., is a supplier of math coprocessors (computer chips) used to speed the processing of data for analysis on personal computers. Based on an analysis of monthly cost and output data, the company has estimated the following relation between the marginal cost of production and monthly output:

\[ MC = 100 + 0.004Q. \]

A. Calculate the marginal cost of production at 2,500, 5,000, and 7,500 units of output.

B. Express output as a function of marginal cost. Calculate the level of output when \( MC = 100, 125, \text{ and } 150. \)

C. Calculate the profit-maximizing level of output if wholesale prices are stable in the industry at \$150 per chip and, therefore, \( P = MR = 150. \)

D. Derive the company’s supply curve for chips assuming \( P = MR. \) Express price as a function of quantity and quantity as a function of price.

**ST4.2 SOLUTION**

A. Marginal production costs at each level of output are:

\[ Q = 2,500: \text{MC} = 100 + 0.004(2,500) = 110 \]

\[ Q = 5,000: \text{MC} = 100 + 0.004(5,000) = 120 \]
Q = 7,500: MC = $100 + $0.004(7,500) = $130

B. When output is expressed as a function of marginal cost:

\[ MC = $100 + 0.004Q \]
\[ 0.004Q = -100 + MC \]
\[ Q = -25,000 + 250MC \]

The level of output at each respective level of marginal cost is:

MC = $100: Q = -25,000 + 250($100) = 0
MC = $125: Q = -25,000 + 250($125) = 6,250
MC = $150: Q = -25,000 + 250($150) = 12,500

C. Note from part B that MC = $150 when Q = 12,500. Therefore, when MR = $150, Q = 12,500 will be the profit-maximizing level of output. More formally:

\[ MR = MC \]
\[ 150 = 100 + 0.004Q \]
\[ 0.004Q = 50 \]
\[ Q = 12,500 \]

D. Because prices are stable in the industry, P = MR, this means that the company will supply chips at the level of output where

\[ MR = MC \]

and, therefore, that

\[ P = 100 + 0.004Q \]

This is the supply curve for math chips, where price is expressed as a function of quantity. When quantity is expressed as a function of price:

\[ P = 100 + 0.004Q \]
\[ 0.004Q = -100 + P \]
Q = -25,000 + 250P
Chapter 5

Demand Analysis and Estimation

SELF-TEST PROBLEMS & SOLUTIONS

ST5.1 Elasticity Estimation. Distinctive Designs, Inc., imports and distributes dress and sports watches. At the end of the company's fiscal year, brand manager J. Peterman has asked you to evaluate sales of the sports watch line using the following data:

<table>
<thead>
<tr>
<th>Month</th>
<th>Number of Sports Watches Sold</th>
<th>Sports Watch Advertising Expenditures</th>
<th>Sports Watch Price, P</th>
<th>Dress Watch Price, P_d</th>
</tr>
</thead>
<tbody>
<tr>
<td>July</td>
<td>4,500</td>
<td>$10,000</td>
<td>26</td>
<td>50</td>
</tr>
<tr>
<td>August</td>
<td>5,500</td>
<td>10,000</td>
<td>24</td>
<td>50</td>
</tr>
<tr>
<td>September</td>
<td>4,500</td>
<td>9,200</td>
<td>24</td>
<td>50</td>
</tr>
<tr>
<td>October</td>
<td>3,500</td>
<td>9,200</td>
<td>24</td>
<td>46</td>
</tr>
<tr>
<td>November</td>
<td>5,000</td>
<td>9,750</td>
<td>25</td>
<td>50</td>
</tr>
<tr>
<td>December</td>
<td>15,000</td>
<td>9,750</td>
<td>20</td>
<td>50</td>
</tr>
<tr>
<td>January</td>
<td>5,000</td>
<td>8,350</td>
<td>25</td>
<td>50</td>
</tr>
<tr>
<td>February</td>
<td>4,000</td>
<td>7,850</td>
<td>25</td>
<td>50</td>
</tr>
<tr>
<td>March</td>
<td>5,500</td>
<td>9,500</td>
<td>25</td>
<td>55</td>
</tr>
<tr>
<td>April</td>
<td>6,000</td>
<td>8,500</td>
<td>24</td>
<td>51</td>
</tr>
<tr>
<td>May</td>
<td>4,000</td>
<td>8,500</td>
<td>26</td>
<td>51</td>
</tr>
<tr>
<td>June</td>
<td>5,000</td>
<td>8,500</td>
<td>26</td>
<td>57</td>
</tr>
</tbody>
</table>

In particular, Peterman has asked you to estimate relevant demand elasticities. Remember that to estimate the required elasticities, you should consider months only when the other important factors considered in the preceding table have not changed. Also note that by restricting your analysis to consecutive months, changes in any additional factors not explicitly included in the analysis are less likely to affect estimated elasticities. Finally, the average arc elasticity of demand for each factor is simply the average of monthly elasticities calculated during the past year.

A. Indicate whether there was or was not a change in each respective independent variable for each month pair during the past year.
### Month-Pair

<table>
<thead>
<tr>
<th>Month-Pair</th>
<th>Sports Watch Advertising Expenditures, A</th>
<th>Sports Watch Price, P</th>
<th>Dress Watch Price, P&lt;sub&gt;D&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>July-August</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>August-September</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>September-October</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>October-November</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>November-December</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>December-January</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>January-February</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>February-March</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>March-April</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>April-May</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>May-June</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**B.** Calculate and interpret the average advertising arc elasticity of demand for sports watches.

**C.** Calculate and interpret the average arc price elasticity of demand for sports watches.

**D.** Calculate and interpret the average arc cross-price elasticity of demand between sports and dress watches.

### ST5.1 SOLUTION

**A.**

<table>
<thead>
<tr>
<th>Month-Pair</th>
<th>Sports Watch Advertising Expenditures, A</th>
<th>Sports Watch Price, P</th>
<th>Dress Watch Price, P&lt;sub&gt;D&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>July-August</td>
<td>No change</td>
<td>Change</td>
<td>No change</td>
</tr>
<tr>
<td>August-September</td>
<td>Change</td>
<td>No change</td>
<td>No change</td>
</tr>
<tr>
<td>September-October</td>
<td>No change</td>
<td>No change</td>
<td>Change</td>
</tr>
<tr>
<td>October-November</td>
<td>Change</td>
<td>Change</td>
<td>Change</td>
</tr>
<tr>
<td>November-December</td>
<td>No change</td>
<td>Change</td>
<td>No change</td>
</tr>
<tr>
<td>December-January</td>
<td>Change</td>
<td>Change</td>
<td>No change</td>
</tr>
<tr>
<td>January-February</td>
<td>Change</td>
<td>No change</td>
<td>No change</td>
</tr>
<tr>
<td>February-March</td>
<td>Change</td>
<td>No change</td>
<td>Change</td>
</tr>
</tbody>
</table>
March-April Change Change Change
April-May No change Change No change
May-June No change No change Change

B. In calculating the arc advertising elasticity of demand, only consider consecutive months when there was a change in advertising but no change in the prices of sports and dress watches:

**August-September**

\[ E_A = \frac{\Delta Q}{\Delta A} \times \frac{A_2 + A_1}{Q_2 + Q_1} \]

\[ = \frac{4,500 - 5,500}{9,200 - 10,000} \times \frac{9,200 + 10,000}{4,500 + 5,500} \]

\[ = 2.4 \]

**January-February**

\[ E_A = \frac{\Delta Q}{\Delta A} \times \frac{A_2 + A_1}{Q_2 + Q_1} \]

\[ = \frac{4,000 - 5,000}{7,850 - 8,350} \times \frac{7,850 + 8,350}{4,000 + 5,000} \]

\[ = 3.6 \]

On average, \( E_A = (2.4 + 3.6)/2 = 3 \) and demand will rise 3%, with a 1% increase in advertising. Thus, demand appears quite sensitive to advertising.

C. In calculating the arc price elasticity of demand, only consider consecutive months when there was a change in the price of sports watches, but no change in advertising nor the price of dress watches:

**July-August**

\[ E_P = \frac{\Delta Q}{\Delta P} \times \frac{P_2 + P_1}{Q_2 + Q_1} \]

\[ = \frac{5,500 - 4,500}{24 - 26} \times \frac{24 + 26}{5,500 + 4,500} \]

\[ = -2.5 \]

**November-December**
\[ E_P = \frac{\Delta Q}{\Delta P} \times \frac{P_2 + P_1}{Q_2 + Q_1} \]
= \frac{15,000 - 5,000}{20 - 25} \times \frac{20 + 25}{15,000 + 5,000}
= -4.5

April-May
\[ E_P = \frac{\Delta Q}{\Delta P} \times \frac{P_2 + P_1}{Q_2 + Q_1} \]
= \frac{4,000 - 6,000}{26 - 24} \times \frac{26 + 24}{4,000 + 6,000}
= -5

On average, \( E_p = \frac{(-2.5) + (-4.5) + (-5)}{3} = -4 \). A 1% increase (decrease) in price will lead to a 4% decrease (increase) in the quantity demanded. The demand for sports watches is, therefore, elastic with respect to price.

D. In calculating the arc cross-price elasticity of demand, we only consider consecutive months when there was a change in the price of dress watches, but no change in advertising nor the price of sports watches:

September-October
\[ E_{PX} = \frac{\Delta Q}{\Delta P_X} \times \frac{P_{X2} + P_{X1}}{Q_2 + Q_1} \]
= \frac{3,500 - 4,500}{46 - 50} \times \frac{46 + 50}{3,500 + 4,500}
= 3

May-June
\[ E_{PX} = \frac{\Delta Q}{\Delta P_X} \times \frac{P_{X2} + P_{X1}}{Q_2 + Q_1} \]
= \frac{5,000 - 4,000}{57 - 51} \times \frac{57 + 51}{5,000 + 4,000}
= 2

On average, \( E_{PX} = \frac{3 + 2}{2} = 2.5 \). Since \( E_{PX} > 0 \), sports and dress watches are
substitutes.

ST5.2 **Cross-Price Elasticity.** Surgical Systems, Inc., makes a proprietary line of disposable surgical stapling instruments. The company grew rapidly during the 1990s as surgical stapling procedures continued to gain wider hospital acceptance as an alternative to manual suturing. However, price competition in the medical supplies industry is growing rapidly in the increasingly price-conscious new millennium. During the past year, Surgical Systems sold 6 million units at a price of $14.50, for total revenues of $87 million. During the current year, Surgical Systems' unit sales have fallen from 6 million units to 3.6 million units following a competitor price cut from $13.95 to $10.85 per unit.

A. Calculate the arc cross price elasticity of demand for Surgical Systems' products.

B. Surgical Systems' director of marketing projects that unit sales will recover from 3.6 million units to 4.8 million units if Surgical Systems reduces its own price from $14.50 to $13.50 per unit. Calculate Surgical Systems' implied arc price elasticity of demand.

C. Assuming the same implied arc price elasticity of demand calculated in part B, determine the further price reduction necessary for Surgical Systems to fully recover lost sales (i.e., regain a volume of 6 million units).

**ST5.2 SOLUTION**

A. 
\[ E_{PX} = \frac{Q_2 - Q_1}{P_2 - P_1} \times \frac{P_2 + P_1}{Q_2 + Q_1} \]
\[ = \frac{3,600,000 - 6,000,000}{10.85 - 13.95} \times \frac{10.85 + 13.95}{3,600,000 + 6,000,000} \]
\[ = 2 \text{ (Substitutes)} \]

B. 
\[ E_p = \frac{Q_2 - Q_1}{P_2 - P_1} \times \frac{P_2 + P_1}{Q_2 + Q_1} \]
\[ = \frac{4,800,000 - 3,600,000}{13.50 - 14.50} \times \frac{13.50 + 14.50}{4,800,000 + 3,600,000} \]
\[ = -4 \text{ (Elastic)} \]
C. 

\[ E_p = \frac{Q_2 - Q_1}{P_2 - P_1} \times \frac{P_2 + P_1}{Q_2 + Q_1} \]

\[-4 = \frac{6,000,000 - 4,800,000}{P_2 - $13.50} \times \frac{P_2 + $13.50}{6,000,000 + 4,800,000} \]

\[-4 = \frac{P_2 + $13.50}{9(P_2 - $13.50)} \]

\[-36P_2 + $486 = P_2 + $13.50 \]

\[37P_2 = $472.50 \]

\[P_2 = $12.77 \]

This implies a further price reduction of 73¢ because:

\[ \Delta P = $12.77 - $13.50 = -$0.73. \]
SELF-TEST PROBLEMS & SOLUTIONS

ST6.1 Gross Domestic Product (GDP) is a measure of overall activity in the economy. It is defined as the value at the final point of sale of all goods and services produced during a given period by both domestic and foreign-owned enterprises. GDP data for the 1950-2004 period shown in Figure 6.3 offer the basis to test the abilities of simple constant change and constant growth models to describe the trend in GDP over time. However, regression results generated over the entire 1950-2004 period cannot be used to forecast GDP over any subpart of that period. To do so would be to overstate the forecast capability of the regression model because, by definition, the regression line minimizes the sum of squared deviations over the estimation period. To test forecast reliability, it is necessary to test the predictive capability of a given regression model over data that was not used to generate that very model. In the absence of GDP data for future periods, say 2005-2010, the reliability of alternative forecast techniques can be illustrated by arbitrarily dividing historical GDP data into two subsamples: a 1950-99 50-year test period, and a 2000-04 5-year forecast period. Regression models estimated over the 1950-99 test period can be used to “forecast” actual GDP over the 2000-04 period. In other words, estimation results over the 1950-99 subperiod provide a forecast model that can be used to evaluate the predictive reliability of the constant growth model over the 2000-04 forecast period.

A. Use the regression model approach to estimate the simple linear relation between the natural logarithm of GDP and time (T) over the 1950-99 subperiod, where

\[ \ln \text{GDP}_t = b_0 + b_1 T_t + u_t \]

and \( \ln \text{GDP}_t \) is the natural logarithm of GDP in year \( t \), and \( T \) is a time trend variable (where \( T_{1950} = 1, T_{1951} = 2, T_{1952} = 3, \ldots, \) and \( T_{1999} = 50 \)); and \( u \) is a residual term. This is called a constant growth model because it is based on the assumption of a constant percentage growth in economic activity per year. How well does the constant growth model fit actual GDP data over this period?

B. Create a spreadsheet that shows constant growth model GDP forecasts over the 2000-04 period alongside actual figures. Then, subtract forecast values from actual figures to obtain annual estimates of forecast error, and squared forecast error, for each year over the 2000-04 period.

Finally, compute the correlation coefficient between actual and forecast
values over the 2000-04 period. Also compute the sample average (or root mean squared) forecast error. Based upon these findings, how well does the constant growth model generated over the 1950-99 period forecast actual GDP data over the 2000-04 period?

ST6.1 SOLUTION

A. The constant growth model estimated using the simple regression model technique illustrates the linear relation between the natural logarithm of GDP and time. A constant growth regression model estimated over the 1950-99 50-year period (t-statistic in parentheses), used to forecast GDP over the 2000-04 5-year period, is:

\[
\ln GDP_t = 5.5026 + 0.0752t, \quad R^2 = 99.2\%
\]

The R\(^2\) = 99.2\% and a highly significant t statistic for the time trend variable indicate that the constant growth model closely describes the change in GDP over the 1950-99 time frame. Nevertheless, even modest changes in the intercept term and slope coefficient over the 2000-04 time frame can lead to large forecast errors.

B. Each constant growth GDP forecast is derived using the constant growth model coefficients estimated in part A, along with values for each respective time trend variable over the 2000-04 period. Remember that T\(_{2000}\) = 51, T\(_{2001}\) = 52, \ldots, and T\(_{2004}\) = 55 and that the constant growth model provides predicted, or forecast, values for ln GDP. To obtain forecast values for GDP, simply take the exponent (antilog) of each predicted ln GDP variable.

The following spreadsheet shows actual and constant growth model GDP forecasts for the 2000-04 forecast period:

<table>
<thead>
<tr>
<th>Year</th>
<th>GDP</th>
<th>ln GDP</th>
<th>Forecast ln GDP</th>
<th>Forecast GDP</th>
<th>Forecast Error (GDP - Forecast)</th>
<th>Squared Forecast Error (GDP - Forecast)(^2)</th>
<th>Time Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>$9,268.4</td>
<td>9.1344</td>
<td>9.3357</td>
<td>$9,441.6</td>
<td>-$173.2</td>
<td>$29,994.1</td>
<td>51</td>
</tr>
<tr>
<td>2001</td>
<td>9,817.0</td>
<td>9.1919</td>
<td>9.4109</td>
<td>10,248.9</td>
<td>-431.9</td>
<td>186,561.8</td>
<td>52</td>
</tr>
<tr>
<td>2002</td>
<td>10,100.8</td>
<td>9.2204</td>
<td>9.4860</td>
<td>11,125.3</td>
<td>-1,024.5</td>
<td>1,049,657.6</td>
<td>53</td>
</tr>
<tr>
<td>2003</td>
<td>10,480.8</td>
<td>9.2573</td>
<td>9.5612</td>
<td>12,076.5</td>
<td>-1,595.7</td>
<td>2,546,191.5</td>
<td>54</td>
</tr>
<tr>
<td>2004</td>
<td>10,987.9</td>
<td>9.3045</td>
<td>9.6364</td>
<td>13,109.2</td>
<td>-2,121.3</td>
<td>4,499,913.7</td>
<td>55</td>
</tr>
<tr>
<td>Average</td>
<td>$10,131.0</td>
<td>9.2217</td>
<td>9.4860</td>
<td>$11,200.3</td>
<td>-$1,069.3</td>
<td>$1,662,463.7</td>
<td></td>
</tr>
</tbody>
</table>

Correlation 99.50%  Mean squared error $\sqrt{1,289.4}$

The correlation coefficient between actual and constant growth model forecast GDP is r\(_{\text{GDP,FGDP}}\) = 99.50%. The sample root mean squared forecast error is $1,298.4 billion (= \sqrt{1,662,463.7}$), or 12.7% of average actual GDP over the 2000-04 period. Thus, despite the fact that the correlation between actual and constant growth forecast
model values is relatively high, forecast error is also very high. Unusually modest economic growth at the start of the new millennium leads to large forecast errors when GDP data from more rapidly growing periods, like the 1950-99 period, are used to forecast economic growth.

ST6.2 Multiple Regression. Branded Products, Inc., based in Oakland, California, is a leading producer and marketer of household laundry detergent and bleach products. About a year ago, Branded Products rolled out its new Super Detergent in 30 regional markets following its success in test markets. This isn't just a “me too” product in a commodity market. Branded Products' detergent contains Branded 2 bleach, a successful laundry product in its own right. At the time of the introduction, management wondered whether the company could successfully crack this market dominated by Procter & Gamble and other big players.

The following spreadsheet shows weekly demand data and regression model estimation results for Super Detergent in these 30 regional markets:

<table>
<thead>
<tr>
<th>Regional Market</th>
<th>Demand in Cases, Q</th>
<th>Price per Case, P</th>
<th>Competitor Price, Px</th>
<th>Advertising, Ad</th>
<th>Household Income, I</th>
<th>Estimated Demand, Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,290</td>
<td>$137</td>
<td>$94</td>
<td>$814</td>
<td>$53,123</td>
<td>1,305</td>
</tr>
<tr>
<td>2</td>
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<td>51,749</td>
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<tr>
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<td>149</td>
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<td>4</td>
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<td>117</td>
<td>92</td>
<td>854</td>
<td>43,589</td>
<td>1,326</td>
</tr>
<tr>
<td>5</td>
<td>1,166</td>
<td>135</td>
<td>86</td>
<td>810</td>
<td>42,799</td>
<td>1,185</td>
</tr>
<tr>
<td>6</td>
<td>1,186</td>
<td>143</td>
<td>79</td>
<td>768</td>
<td>55,565</td>
<td>1,208</td>
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<tr>
<td>7</td>
<td>1,293</td>
<td>113</td>
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<td>978</td>
<td>37,959</td>
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<tr>
<td>8</td>
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<td>111</td>
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<tr>
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<td>1,264</td>
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<td>1,024</td>
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<td>846</td>
<td>46,663</td>
<td>1,449</td>
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<tr>
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<td>1,235</td>
<td>140</td>
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<td>768</td>
<td>55,839</td>
<td>1,220</td>
</tr>
<tr>
<td>20</td>
<td>1,367</td>
<td>115</td>
<td>83</td>
<td>856</td>
<td>47,438</td>
<td>1,326</td>
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<tr>
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<tr>
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<tr>
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<td>905</td>
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<td>145</td>
<td>96</td>
<td>996</td>
<td>38,656</td>
<td>1,208</td>
</tr>
<tr>
<td>Regional Market</td>
<td>Demand in Cases, Q</td>
<td>Price per Case, P</td>
<td>Competitor Price, Px</td>
<td>Advertising, Ad</td>
<td>Household Income, I</td>
<td>Estimated Demand, Q</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------------------</td>
<td>------------------</td>
<td>---------------------</td>
<td>----------------</td>
<td>---------------------</td>
<td>---------------------</td>
</tr>
<tr>
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<td>929</td>
<td>46,084</td>
<td>1,291</td>
</tr>
<tr>
<td>27</td>
<td>1,515</td>
<td>116</td>
<td>97</td>
<td>1,000</td>
<td>52,249</td>
<td>1,478</td>
</tr>
<tr>
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<td>148</td>
<td>84</td>
<td>951</td>
<td>50,855</td>
<td>1,226</td>
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<tr>
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<td>1,293</td>
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<td>88</td>
<td>848</td>
<td>54,546</td>
<td>1,314</td>
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<tr>
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<td>87</td>
<td>891</td>
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<td>1,215</td>
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<tr>
<td>Average</td>
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<td>127</td>
<td>87</td>
<td>870</td>
<td>46,788</td>
<td>1,286</td>
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<tr>
<td>Minimum</td>
<td>1,089</td>
<td>103</td>
<td>76</td>
<td>768</td>
<td>37,809</td>
<td>1,024</td>
</tr>
<tr>
<td>Maximum</td>
<td>1,515</td>
<td>149</td>
<td>100</td>
<td>1,000</td>
<td>55,839</td>
<td>1,478</td>
</tr>
</tbody>
</table>

**Regression Statistics**

<table>
<thead>
<tr>
<th></th>
<th>R Square</th>
<th>Standard Error</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>90.4%</td>
<td>34.97</td>
<td>30</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>Coefficients</th>
<th>Standard Error</th>
<th>t Stat</th>
<th>P-value</th>
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<tbody>
<tr>
<td>Intercept</td>
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<tr>
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<td>Competitor Price, Px</td>
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<td>1.006</td>
<td>4.83</td>
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<td>Advertising, Ad</td>
<td>0.328</td>
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</tr>
<tr>
<td>Household Income, I</td>
<td>0.009</td>
<td>0.001</td>
<td>7.99</td>
<td>2.38432E-08</td>
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</tbody>
</table>

**A.** Interpret the coefficient estimate for each respective independent variable.

**B.** Characterize the overall explanatory power of this multiple regression model in light of $R^2$ and the following plot of actual and estimated demand per week.
C. Use the regression model estimation results to forecast weekly demand in five new markets with the following characteristics:

<table>
<thead>
<tr>
<th>Regional Forecast Market</th>
<th>Price per Case, $P$</th>
<th>Competitor Price, $P_x$</th>
<th>Advertising, $Ad$</th>
<th>Household Income, $I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>115</td>
<td>90</td>
<td>790</td>
<td>41,234</td>
</tr>
<tr>
<td>B</td>
<td>122</td>
<td>101</td>
<td>812</td>
<td>39,845</td>
</tr>
<tr>
<td>C</td>
<td>116</td>
<td>87</td>
<td>905</td>
<td>47,543</td>
</tr>
<tr>
<td>D</td>
<td>140</td>
<td>82</td>
<td>778</td>
<td>53,560</td>
</tr>
<tr>
<td>E</td>
<td>133</td>
<td>79</td>
<td>996</td>
<td>39,870</td>
</tr>
<tr>
<td>Average</td>
<td>125</td>
<td>88</td>
<td>856</td>
<td>44,410</td>
</tr>
</tbody>
</table>

ST6.2 SOLUTION

A. Coefficient estimates for the $P$, $P_x$, $Ad$ and $I$ independent X-variables are statistically significant at the 99% confidence level. Price of the product itself ($P$) has the predictably negative influence on the quantity demanded, whereas the effects of competitor price ($P_x$), advertising ($Ad$) and household disposable income ($I$) are positive as expected. The chance of finding such large $t$-statistics is less than 1% if, in fact, there were no relation between each variable and quantity.
B. The $R^2 = 90.4\%$ obtained by the model means that 90.4\% of demand variation is explained by the underlying variation in all four independent variables. This is a relatively high level of explained variation and implies an attractive level of explanatory power. Moreover, as shown in the graph of actual and fitted (estimated) demand, the multiple regression model closely tracks week-by-week changes in demand with no worrisome divergences between actual and estimated demand over time. This means that this regression model can be used to forecast demand in similar markets under similar conditions.

C. Notice that each prospective market displays characteristics similar to those of markets used to estimate the regression model described above. Thus, the regression model estimated previously can be used to forecast demand in each regional market. Forecast results are as follows:

<table>
<thead>
<tr>
<th>Regional Forecast Market</th>
<th>Price per Case, P</th>
<th>Competitor Price, Px</th>
<th>Advertising, Ad</th>
<th>Household Income, I</th>
<th>Forecast Demand, Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>115</td>
<td>90</td>
<td>790</td>
<td>41,234</td>
<td>1,285</td>
</tr>
<tr>
<td>B</td>
<td>122</td>
<td>101</td>
<td>812</td>
<td>39,845</td>
<td>1,298</td>
</tr>
<tr>
<td>C</td>
<td>116</td>
<td>87</td>
<td>905</td>
<td>47,543</td>
<td>1,358</td>
</tr>
<tr>
<td>D</td>
<td>140</td>
<td>82</td>
<td>778</td>
<td>53,560</td>
<td>1,223</td>
</tr>
<tr>
<td>E</td>
<td>133</td>
<td>79</td>
<td>996</td>
<td>39,870</td>
<td>1,196</td>
</tr>
<tr>
<td>Average</td>
<td>125</td>
<td>88</td>
<td>856</td>
<td>44,410</td>
<td>1,272</td>
</tr>
</tbody>
</table>
Chapter 7

Production Analysis and Compensation Policy

SELF-TEST PROBLEMS & SOLUTIONS

ST7.1 Optimal Input Usage. Medical Testing Labs, Inc., provides routine testing services for blood banks in the Los Angeles area. Tests are supervised by skilled technicians using equipment produced by two leading competitors in the medical equipment industry. Records for the current year show an average of 27 tests per hour being performed on the Testlogic-1 and 48 tests per hour on a new machine, the Accutest-3. The Testlogic-1 is leased for $18,000 per month, and the Accutest-3 is leased at $32,000 per month. On average, each machine is operated 25 eight-hour days per month.

A. Describe the logic of the rule used to determine an optimal mix of input usage.

B. Does Medical Testing Lab usage reflect an optimal mix of testing equipment?

C. Describe the logic of the rule used to determine an optimal level of input usage.

D. If tests are conducted at a price of $6 each while labor and all other costs are fixed, should the company lease more machines?

ST7.1 SOLUTION

A. The rule for an optimal combination of Testlogic-1 (T) and Accutest-3 (A) equipment is

\[
\frac{MP_T}{P_T} = \frac{MP_A}{P_A}
\]

This rule means that an identical amount of additional output would be produced with an additional dollar expenditure on each input. Alternatively, an equal marginal cost of output is incurred irrespective of which input is used to expand output. Of course, marginal products and equipment prices must both reflect the same relevant time frame, either hours or months.

B. On a per hour basis, the relevant question is

\[
\frac{27}{\$18,000/(25 \times 8)} \neq \frac{48}{\$32,000/(25 \times 8)}
\]
0.3 $\sqrt{\;}$ 0.3

On a per month basis, the relevant question is

\[
\frac{27 \times (25 \times 8)}{18,000} \quad ? \quad \frac{48 \times (25 \times 8)}{32,000}
\]

0.3 $\sqrt{\;}$ 0.3

In both instances, the last dollar spent on each machine increased output by the same 0.3 units, indicating an *optimal mix* of testing machines.

C. The rule for optimal input employment is

\[
\text{MRP} = MP \times \text{MRQ} = \text{Input Price}
\]

This means that the level of input employment is optimal when the marginal sales revenue derived from added input usage is equal to input price, or the marginal cost of employment.

D. For each machine hour, the relevant question is

**Testlogic-1**

\[
\text{MRP}_T = MP_T \times \text{MRQ} \quad ? \quad P_T
\]

\[
27 \times 6 \quad ? \quad \frac{18,000}{25 \times 8}
\]

\[
\$162 > \$90.
\]

**Accutest-3**

\[
\text{MRP}_A = MP_A \times \text{MRQ} \quad ? \quad P_A
\]

\[
48 \times 6 \quad ? \quad \frac{32,000}{25 \times 8}
\]

\[
\$288 > \$160.
\]
Or, in per month terms:

**Testlogic-1**

\[
\text{MRP}_T = \text{MP}_T \times \text{MR}_Q = P_T
\]

\[
27 \times (25 \times 8) \times $6 = $18,000
\]

\[
$32,400 > $18,000.
\]

**Accutest-3**

\[
\text{MRP}_A = \text{MP}_A \times \text{MR}_Q = P_A
\]

\[
48 \times (25 \times 8) \times $6 = $32,000
\]

\[
$57,600 > $32,000.
\]

In both cases, each machine returns more than its marginal cost (price) of employment, and expansion would be profitable.

**ST7.2 Production Function Estimation**. Washington-Pacific, Inc., manufactures and sells lumber, plywood, veneer, particle board, medium-density fiberboard, and laminated beams. The company has estimated the following multiplicative production function for basic lumber products in the Pacific Northwest market using monthly production data over the past two and one-half years (30 observations):

\[
Q = b_0 L^{b_1} K^{b_2} E^{b_3}
\]

where

- \( Q = \text{output} \)
- \( L = \text{labor input in worker hours} \)
- \( K = \text{capital input in machine hours} \)
- \( E = \text{energy input in BTUs} \)

Each of the parameters of this model was estimated by regression analysis using monthly data over a recent three-year period. Coefficient estimation results were as follows:
\[ \hat{b}_0 = 0.9; \hat{b}_1 = 0.4; \hat{b}_2 = 0.4; \text{ and } \hat{b}_3 = 0.2 \]

The standard error estimates for each coefficient are:

\[ \sigma_{\hat{b}_0} = 0.6; \sigma_{\hat{b}_1} = 0.1; \sigma_{\hat{b}_2} = 0.2; \sigma_{\hat{b}_3} = 0.1 \]

A. Estimate the effect on output of a 1% decline in worker hours (holding \(K\) and \(E\) constant).

B. Estimate the effect on output of a 5% reduction in machine hours availability accompanied by a 5% decline in energy input (holding \(L\) constant).

C. Estimate the returns to scale for this production system.

**ST7.2 SOLUTION**

A. For Cobb-Douglas production functions, calculations of the elasticity of output with respect to individual inputs can be made by simply referring to the exponents of the production relation. Here a 1% decline in \(L\), holding all else equal, will lead to a 0.4% decline in output. Notice that:

\[
\frac{\Delta Q}{Q} = \frac{\Delta Q}{\Delta L} \times \frac{L}{Q} = (b_0 b_1 L^{b_1 - 1} K^{b_2} E^{b_3}) \times L \quad \Rightarrow \quad \frac{\Delta Q}{\Delta L} = b_1 
\]

And because \((\Delta Q/Q)/(\Delta L/L)\) is the percent change in \(Q\) due to a 1% change in \(L\),

\[
\frac{\Delta Q}{Q} = b_1 \times \Delta L/L
\]

\[
= 0.4(-0.01)
\]

\[
= -0.004 \text{ or } -0.4\%
\]
B. From part A it is obvious that:

\[
\frac{\Delta Q}{Q} = b_2(\Delta K/K) + b_3(\Delta E/E) \\
= 0.4(-0.05) + 0.2(-0.05) \\
= -0.03 \text{ or } -3\%
\]

C. In the case of Cobb-Douglas production functions, returns to scale are determined by simply summing exponents because:

\[
Q = b_0L^{b_1}K^{b_2}E^{b_3} \\
hQ = b_0(kL)^{b_1}(kK)^{b_2}(kE)^{b_3} \\
= k^{b_1+b_2+b_3}b_0L^{b_1}K^{b_2}E^{b_3} \\
= k^{b_1+b_2+b_3}Q
\]

Here \(b_1 + b_2 + b_3 = 0.4 + 0.4 + 0.2 = 1\) indicating constant returns to scale. This means that a 1% increase in all inputs will lead to a 1% increase in output, and average costs will remain constant as output increases.
ST8.1 Learning Curves. Modern Merchandise, Inc., makes and markets do-it-yourself hardware, housewares, and industrial products. The company's new Aperture Miniblind is winning customers by virtue of its high quality and quick order turnaround time. The product also benefits because its price point bridges the gap between ready-made vinyl blinds and their high-priced custom counterpart. In addition, the company's expanding product line is sure to benefit from cross-selling across different lines. Given the success of the Aperture Miniblind product, Modern Merchandise plans to open a new production facility near Beaufort, South Carolina. Based on information provided by its chief financial officer, the company estimates fixed costs for this product of $50,000 per year and average variable costs of:

\[ AVC = 0.5 + 0.0025Q, \]

where \( AVC \) is average variable cost (in dollars) and \( Q \) is output.

A. Estimate total cost and average total cost for the projected first-year volume of 20,000 units.

B. An increase in worker productivity because of greater experience or learning during the course of the year resulted in a substantial cost saving for the company. Estimate the effect of learning on average total cost if actual second-year total cost was $848,000 at an actual volume of 20,000 units.

ST8.1 SOLUTION

A. The total variable cost function for the first year is:

\[ TVC = AVC \times Q \]

\[ = (0.5 + 0.0025Q)Q \]

\[ = 0.5Q + 0.0025Q^2 \]

At a volume of 20,000 units, estimated total cost is:
\[ TC = TFC + TVC \]
\[ = \$50,000 + 0.5Q + 0.0025Q^2 \]
\[ = \$50,000 + 0.5(20,000) + 0.0025(20,000^2) \]
\[ = \$1,060,000 \]

Estimated average cost is:

\[ AC = \frac{TC}{Q} \]
\[ = \frac{\$1,060,000}{20,000} \]
\[ = \$53 \text{ per case} \]

**B.** If actual total costs were \$848,000 at a volume of 20,000 units, actual average total costs were:

\[ AC = \frac{TC}{Q} \]
\[ = \frac{\$848,000}{20,000} \]
\[ = \$42.40 \text{ per case} \]

Therefore, greater experience or learning has resulted in an average cost saving of \$10.60 per case since:

\[ \text{Learning effect} = \text{Actual AC} - \text{Estimated AC} \]
\[ = \$42.40 - \$53 \]
\[ = -\$10.60 \text{ per case} \]

Alternatively,

\[ \text{Learning rate} = \left(1 - \frac{AC_2}{AC_1}\right) \times 100 \]
\[ = \left(1 - \frac{\$42.40}{\$53}\right) \times 100 \]
\[ = 20\% \]
Minimum Efficient Scale Estimation. Kanata Corporation is a leading manufacturer of telecommunications equipment based in Ontario, Canada. Its main product is micro-processor controlled telephone switching equipment, called automatic private branch exchanges (PABXs), capable of handling 8 to 3,000 telephone extensions. Severe price cutting throughout the PABX industry continues to put pressure on sales and margins. To better compete against increasingly aggressive rivals, the company is contemplating the construction of a new production facility capable of producing 1.5 million units per year. Kanata's in-house engineering estimate of the total cost function for the new facility is:

\[
TC = 3,000 + 1,000Q + 0.003Q^2,
\]

\[
MC = \frac{\Delta TC}{\Delta Q} = 1,000 + 0.006Q
\]

where \(TC = \text{Total Costs in thousands of dollars, } Q = \text{Output in thousands of units, and } MC = \text{Marginal Costs in thousands of dollars.}\)

A. Estimate minimum efficient scale in this industry.

B. In light of current PABX demand of 30 million units per year, how would you evaluate the future potential for competition in the industry?

ST8.2 SOLUTION

A. Minimum efficient scale is reached when average costs are first minimized. This occurs at the point where \(MC = AC\).

\[
\text{Average Costs } = AC = \frac{TC}{Q} = \frac{(3,000 + 1,000Q + 0.003Q^2)}{Q} = \frac{3,000}{Q} + 1,000 + 0.003Q
\]

Therefore,

\[
MC = AC
\]

\[
1,000 + 0.006Q = \frac{3,000}{Q} + 1,000 + 0.003Q
\]

\[
0.003Q = \frac{3,000}{Q}
\]
\[
\frac{3,000}{Q^2} = 0.003
\]

\[Q^2 = 1,000,000\]

\[Q = 1,000(000)\text{ or } 1\text{ million}\]

(Note: AC is rising for \(Q > 1,000(000)\)).

Alternatively, MES can be calculated using the point cost elasticity formula, since MES is reached when \(\varepsilon_C = 1\).

\[
\varepsilon_C = \frac{\Delta TC}{\Delta Q} \times \frac{Q}{TC}
\]

\[
\frac{($1,000 + $0.006Q)Q}{($3,000 + $1,000Q + $0.003Q^2)} = 1
\]

\[1,000Q + 0.006Q^2 = 3,000 + 1,000Q + 0.003Q^2\]

\[0.003Q^2 = 3,000\]

\[Q^2 = 1,000,000\]

\[Q_{MES} = 1,000(000)\text{ or } 1\text{ million}\]

B. With a minimum efficient scale of 1 million units and total industry sales of 30 million units, up to 30 efficiently sized competitors are possible in Kanata's market.

\[
\text{Potential Number of Efficient Competitors} = \frac{\text{Market Size}}{\text{MES Size}}
\]

\[
= \frac{30,000,000}{1,000,000}
\]

\[= 30\]

Thus, there is the potential for \(N = 30\) efficiently sized competitors and, therefore, vigorous competition in Kanata's industry.
Chapter 9

Linear Programming

**SELF-TEST PROBLEMS & SOLUTIONS**

**ST9.1 Cost Minimization.** Idaho Natural Resources (INR) has two mines with different production capabilities for producing the same type of ore. After mining and crushing, the ore is graded into three classes: high, medium, and low. The company has contracted to provide local smelters with 24 tons of high-grade ore, 16 tons of medium-grade ore, and 48 tons of low-grade ore each week. It costs INR $10,000 per day to operate mine A and $5,000 per day to run mine B. In a day's time, mine A produces 6 tons of high-grade ore, 2 tons of medium-grade ore, and 4 tons of low-grade ore. Mine B produces 2, 2, and 12 tons per day of each grade, respectively. Management's short-run problem is to determine how many days per week to operate each mine under current conditions. In the long run, management wishes to know how sensitive these decisions will be to changing economic conditions.

A report prepared for the company by an independent management consultant addressed the company's short-run operating concerns. The consultant claimed that the operating problem could be solved using linear programming techniques by which the firm would seek to minimize the total cost of meeting contractual requirements. Specifically, the consultant recommended that INR do the following:

Minimize \( \text{Total Cost} = 10,000A + 5,000B \)

subject to

\[
\begin{align*}
6A + 2B & \geq 24 \quad \text{(high-grade ore constraint)} \\
2A + 2B & \geq 16 \quad \text{(medium-grade ore constraint)} \\
4A + 12B & \geq 48 \quad \text{(low-grade ore constraint)}
\end{align*}
\]

\[
\begin{align*}
A & \leq 7 \quad \text{(Mine A operating days in a week constraint)} \\
B & \leq 7 \quad \text{(Mine B operating days in a week constraint)}
\end{align*}
\]

or, in their equality form,

\[
6A + 2B - S_H = 24
\]
\[
\begin{align*}
2A + 2B - S_M &= 16 \\
4A + 12B - S_L &= 48 \\
A + S_A &= 7 \\
B + S_B &= 7
\end{align*}
\]

where

\[A, B, S_{H}, S_{M}, S_{L}, S_A, \text{ and } S_B \geq 0\]

Here, \(A\) and \(B\) represent the days of operation per week for each mine; \(S_{H}, S_{M}, \text{ and } S_{L}\) represent excess production of high-, medium-, and low-grade ore, respectively; and \(S_A\) and \(S_B\) are days per week that each mine is not operated.

A graphic representation of the linear programming problem was also provided. The graph suggests an optimal solution at point \(X\), where constraints 1 and 2 are binding. Thus, \(S_H = S_M = 0\) and

\[
\begin{align*}
6A + 2B - 0 &= 24 \\
\text{minus} \quad 2A + 2B - 0 &= 16 \\
\frac{4A}{4A} &= 8
\end{align*}
\]

\[A = 2 \text{ days per week}\]

Substitute \(A = 2\) into the high-grade ore constraint:

\[
\begin{align*}
6(2) + 2B &= 24 \\
12 + 2B &= 24 \\
2B &= 12 \\
B &= 6 \text{ days per week}
\end{align*}
\]

A minimum total operating cost per week of $50,000 is suggested, because

\[
\begin{align*}
\text{Total Cost} &= 10,000A + 5,000B \\
&= 10,000(2) + 5,000(6) \\
&= 50,000
\end{align*}
\]

The consultant's report did not discuss a variety of important long-run planning issues.
Specifically, INR wishes to know the following, holding all else equal:

A. How much, if any, excess production would result if the consultant's operating recommendation were followed?

B. What would be the cost effect of increasing low-grade ore sales by 50 percent?

C. What is INR's minimum acceptable price per ton if it is to renew a current contract to provide one of its customers with 6 tons of high-grade ore per week?

D. With current output requirements, how much would the cost of operating mine A have to rise before INR would change its operating decision?

E. What increase in the cost of operating mine B would cause INR to change its current operating decision?
Idaho Natural Resources, Ltd.

Days of Operation of Mine A

- Low-grade ore constraint (3)
- Medium-grade ore constraint (2)
- High-grade ore
- $50,000 Isocost

Days of Operation of Mine B

Maximum days in a week

Flexible Space
ST9.1 \textbf{SOLUTION}

A. If the consultant's operating recommendation of $A = 2$ and $B = 6$ were followed, 32 tons of excess low-grade ore production would result. No excess production of high- or medium-grade ore would occur. This can be shown by solving for $S_H$, $S_M$ and $S_L$ at the recommended activity level. \\

From the constraint equations, we find the following:

\begin{align*}
(1) \quad & 6(2) + 2(6) - S_H = 24, \\
& S_H = 0. \\
(2) \quad & 2(2) + 2(6) - S_M = 16, \\
& S_M = 0. \\
(3) \quad & 4(2) + 12(6) - S_L = 48, \\
& S_L = 32.
\end{align*}

B. There would be a \textit{zero cost impact} of an increase in low-grade ore sales from 48 to 72 tons ($= 1.5 \times 48$). With $A = 2$ and $B = 6$, 80 tons of low-grade ore are produced. A 50\% increase in low-grade ore sales would simply reduce excess production from $S_L = 32$ to $S_L = 8$, because

\begin{align*}
(3') \quad & 4(2) + 12(6) - S_L = 72, \\
& S_L = 8.
\end{align*}

Graphically, the effect of a 50\% increase in low-grade ore sales would be to cause a rightward shift in the low-grade ore constraint to a new constraint line with endpoints $(0B, 3A)$ and $(9B, 0A)$. While such a shift would reduce the feasible space, it would not affect the optimal operating decision of $A = 2$ and $B = 6$ (at Point X).

C. If INR didn't renew a contract to provide one of its current customers with 6 tons of high-grade ore per week, the high-grade ore constraint would fall from 24 to 18 tons per week. The new high-grade ore constraint, reflecting a parallel leftward shift, is written

\begin{align*}
(1') \quad & 6A + 2B - S_H = 18
\end{align*}

and has endpoints $(0B, 3A)$ and $(9B, 0A)$. With such a reduction in required high-grade ore sales, the high-grade ore constraint would no longer be binding and the optimal production point would shift to point W, and $A = 1$ and $B = 7$ (because $S_M = S_B = 0$). At this point, high-grade ore production would equal 20 tons, or 2 tons more than the
new high-grade ore requirement:

\[ 6(1) + 2(7) - S_H = 18, \]

\[ S_H = 2, \]

with operating costs of

\[ \text{Total cost} = 10,000A + 5,000B \]

\[ = 10,000(1) + 5,000(7) \]

\[ = 45,000. \]

Therefore, renewing a contract to provide one of its current customers with 6 tons of high-grade ore per week would result in our earlier operating decision of \( A = 2 \) and \( B = 6 \) and total costs of \( 50,000 \), rather than the \( A = 1 \) and \( B = 7 \) and total costs of \( 45,000 \) that would otherwise be possible. The marginal cost of renewing the 6-ton contract is $5,000, or $833 per ton.

\[
\text{Marginal Cost} = \frac{\text{Change in Operating Costs}}{\text{Number of Tons}}
\]

\[
= \frac{50,000 - 45,000}{6}
\]

\[= 833 \text{ per ton}. \]

D. In general, the isocost relation for this problem is

\[ C_0 = C_A A + C_B B, \]

where \( C_0 \) is any weekly cost level, and \( C_A \) and \( C_B \) are the daily operating costs for mines A and B, respectively. In terms of the graph, A is on the vertical axis and B is on the horizontal axis. From the isocost formula we find the following:

\[ A = C_0/C_A - (C_B/C_A)B, \]

with an intercept of \( C_0/C_A \) and a slope equal to \(-C_B/C_A\). The isocost line will become steeper as \( C_B \) increases relative to \( C_A \). The isocost line will become flatter (slope will approach zero) as \( C_B \) falls relative to \( C_A \).

If \( C_A \) increases to slightly more than $15,000, the optimal feasible point will shift from Point X (6B, 2A) to Point V (7B, 1.67A), because the isocost line slope will then be less than -1/3, the slope of the high-grade ore constraint \( A = 4 - (1/3)B \). Thus, an
increase in CA from $10,000 to at least $15,000, or an increase of at least $5,000, is necessary before the optimal operating decision will change.

E. An increase in CB of at least $5,000 to slightly more than $10,000 will shift the optimal point from Point X to Point Y (2B, 6A), because the isocost line slope will then be steeper than -1, the slope of the medium-grade ore constraint (A = 8 - B).

An increase in CB to slightly more than $30,000 will be necessary before point Z (1.67B, 7A) becomes optimal. With CB ≥ $30,000 and CA = $10,000, the isocost line slope will be steeper than -3, the slope of the low-grade ore constraint, A = 12 - 3B.

As seems reasonable, the greater CA is relative to CB, the more mine A will tend to be employed. The greater CA is relative to CA, the more mine B will tend to be employed.

ST9.2 Profit Maximization. Interstate Bakeries, Inc., is an Atlanta-based manufacturer and distributor of branded bread products. Two leading products, Low Calorie, QA, and High Fiber, QB, bread, are produced using the same baking facility and staff. Low Calorie bread requires 0.3 hours of worker time per case, whereas High Fiber bread requires 0.4 hours of worker time per case. During any given week, a maximum of 15,000 worker hours are available for these two products. To meet grocery retailer demands for a full product line of branded bread products, Interstate must produce a minimum of 25,000 cases of Low Calorie bread and 7,500 cases of High Fiber bread per week. Given the popularity of low-calorie products in general, Interstate must also ensure that weekly production of Low Calorie bread be at least twice that of High Fiber bread.

Low Calorie bread is sold to groceries at a price of $42 per case; the price of High Fiber bread is $40 per case. Despite its lower price, the markup on High Fiber bread substantially exceeds that on Low Calorie bread. Variable costs are $30.50 per case for Low Calorie bread, but only $17 per case for High Fiber bread.

A. Set up the linear programming problem that the firm would use to determine the profit-maximizing output levels for Low Calorie and High Fiber bread. Show both the inequality and equality forms of the constraint conditions.

B. Completely solve the linear programming problem.

C. Interpret the solution values for the linear programming problem.

D. Holding all else equal, how much would variable costs per unit on High Fiber bread have to fall before the production level indicated in part B would change?

ST9.2 SOLUTION

A. First, the profit contribution for Low Calorie bread, QA, and High Fiber bread, QB, must be calculated.
Profit contribution per unit = Price - Variable costs per unit

Thus,

\[ \pi_A = $42 - $30.50 = $11.50 \text{ per case of } Q_A, \]

\[ \pi_B = $40 - $17 = $23 \text{ per case of } Q_B. \]

This problem requires maximization of profits, subject to limitations on the amount of each product produced, the acceptable ratio of production, and available worker hours. The linear programming problem is:

Maximize \[ \pi = 11.50Q_A + 23Q_B \]

Subject to \[
Q_A \geq 25,000 \\
Q_B \geq 7,500 \\
Q_A - 2Q_B \geq 0 \\
0.3Q_A + 0.4Q_B \leq 15,000
\]

In equality form, the constraint conditions are:

(1) \[ Q_A - S_A = 25,000 \] (Low Calorie constraint)

(2) \[ Q_B - S_B = 7,500 \] (High Fiber constraint)

(3) \[ Q_A - 2Q_B - S_R = 0 \] (Acceptable ratio constraint)

(4) \[ 0.3Q_A + 0.4Q_B + S_W = 15,000 \] (Worker hours constraint)

Here, \( Q_A \) and \( Q_B \) are cases of Low Calorie and High Fiber bread, respectively. \( S_A, S_B, S_R, S_W \) are variables representing excess production of Low Calorie and High Fiber bread, respectively. \( S_R \) is the amount by which the production of Low Calorie bread exceeds the minimally acceptable amount, given High Fiber production. \( S_W \) is excess worker capacity.

B. By graphing the constraints and the highest possible isoprofit line, the optimal Point X occurs where \( S_R = S_W = 0 \).
Thus,

\begin{align*}
(1) & \quad Q_A - S_A = 25,000 \\
(2) & \quad Q_B - S_B = 0 \\
(3) & \quad Q_A - 2Q_B = 0 \\
(4) & \quad 0.3Q_A + 0.4Q_B + 0 = 15,000
\end{align*}

From (3), \( Q_A = 2Q_B \). Substituting this value into (4) yields:

\[ 0.3(2Q_B) + 0.4Q_B = 15,000 \]

\[ Q_B = 15,000 \]

From (3),

\[ Q_A - 2(15,000) = 0 \]

\[ Q_A = 30,000 \]

From (1),

\[ 30,000 - S_A = 25,000 \]

\[ S_A = 5,000 \]

From (2),

\[ 15,000 - S_B = 7,500 \]

\[ S_B = 7,500 \]

And the total profit contribution per week is:

\[ \pi = 11.50(30,000) + 23(15,000) \]

\[ = 690,000 \]

C. Solution values can be interpreted as follows:

\[ Q_A = 30,000 \quad \text{Optimal production of Low Calorie bread is 30,000 cases per week.} \]
$Q_B = 15,000$  
Optimal production of High Fiber bread is 15,000 cases per week.

$S_A = 5,000$  
The production of Low Calorie bread exceeds the 25,000 case minimum by 5,000 units.

$S_B = 7,500$  
The production of High Fiber bread exceeds the 7,500 case minimum by 7,500 units.

$S_R = 0$  
The minimally acceptable 2:1 ratio of Low Calorie:High Fiber bread is produced.

$S_W = 0$  
All worker hours are utilized; no excess worker capacity exists.

$\pi = $690,000  
Maximum weekly profit contribution given constraints.

$\bar{\pi} = \bar{\pi}_A Q_A + \bar{\pi}_B Q_B,$

or

$Q_A = (\bar{\pi}/\bar{\pi}_A) - (\bar{\pi}_B/\bar{\pi}_A)Q_B.$

In this specific case, the isoprofit line is:

$Q_A = (\bar{\pi}/$11.50$) - ($23/11.50)Q_B.$

To intersect the feasible space at point Y rather than point X, the slope of this line would have to become slightly less negative than -1.33. To solve for the required level for $\bar{\pi}_B$, note that if:

$$\frac{\bar{\pi}_B}{11.50} < 1.33$$

$\bar{\pi} = \bar{\pi}_A Q_A + \bar{\pi}_B Q_B,$

or

$Q_A = (\bar{\pi}/\bar{\pi}_A) - (\bar{\pi}_B/\bar{\pi}_A)Q_B.$
then

\[ \pi_B < 15.33 \]

Given a price of High Fiber bread of $40 per unit, a profit contribution of $15.33 implies variable costs per unit of $24.67 because:

\[ \pi_B = \text{Price} - \text{Variable costs per unit} \]

\[ = 40 - 24.67 \]

\[ = 15.33 \]

Therefore, to change the optimal production point from point X to point Y, variable costs per unit on High Fiber bread would have to rise by \textit{at least} $7.67 per unit:

\[ \text{Change in variable costs} = \text{New level} - \text{Initial level} \]

\[ = 24.67 - 17 \]

\[ = 7.67 \]
Interstate Bakeries, Inc.

Cases of High Fiber Bread, $Q_B$

Cases of Low Calorie Bread, $Q_A$

High fiber constraint (2)

Acceptable ratio constraint (3)

Low calorie constraint (1)

Worker hours constraint (4)

$Y(7,500, 40,000)$

$X(15,000, 30,000)$

$690,000$ Isoprofit Line

FEASIBLE SPACE
SELF-TEST PROBLEMS & SOLUTIONS

ST10.1 Market Supply. In some markets, cutthroat competition can exist even when the market is dominated by a small handful of competitors. This usually happens when fixed costs are high, products are standardized, price information is readily available, and excess capacity is present. Airline passenger service in large city-pair markets, and electronic components manufacturing are good examples of industries where price competition among the few can be vigorous. Consider three competitors producing a standardized product \((Q)\) with the following marginal cost characteristics:

\[
MC_1 = 5 + 0.0004Q_1 \quad \text{(Firm 1)}
\]

\[
MC_2 = 15 + 0.002Q_2 \quad \text{(Firm 2)}
\]

\[
MC_3 = 1 + 0.0002Q_3 \quad \text{(Firm 3)}
\]

A. Using each firm’s marginal cost curve, calculate the profit-maximizing short-run supply from each firm at the competitive market prices indicated in the following table. For simplicity, assume price is greater than average variable cost in every instance.

**Market Supply is the Sum of Firm Supply Across all Competitors**

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td></td>
<td>(P = MC_1 = 5 + 0.0004Q, ) and (Q_1 = -12,500 + 2,500P)</td>
<td>(P = MC_2 = 15 + 0.002Q_2, ) and (Q_2 = -7,500 + 500P)</td>
<td>(P = MC_3 = 1 + 0.0002Q_3, ) and (Q_3 = -5,000 + 5,000P)</td>
<td>(P = 3.125 + 0.000125P, ) and (Q = Q_1 + Q_2 + Q_3)</td>
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</tbody>
</table>
B. Use these data to plot short-run supply curves for each firm. Also plot the market supply curve.

**ST10.1 SOLUTION**

A. The marginal cost curve constitutes the short-run supply curve for firms in perfectly competitive markets so long as price is greater than average variable cost.

Market Supply is the Sum of Firm Supply Across all Competitors

<table>
<thead>
<tr>
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<tbody>
<tr>
<td></td>
<td>( P = MC_1 = $5 + 0.0004Q_1 ) and ( Q_1 = -12,500 + 2,500P )</td>
<td>( P = MC_2 = $15 + 0.0002Q_2 ) and ( Q_2 = -7,500 + 500P )</td>
<td>( P = MC_3 = $1 + 0.0002Q_3 ) and ( Q_3 = -5,000 + 5,000P )</td>
<td>( P = $3.125 + 0.000125P ) and ( Q = -25,000 + 8,000P ) (( Q_1 = Q_1 + Q_2 + Q_3 ))</td>
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Market Supply is the Sum of Firm Supply Across all Competitors

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</table>

**B.**

**ST10.2 Competitive Market Equilibrium.** Competitive market prices are determined by the interplay of aggregate supply and demand; individual firms have no control over price. Market demand reflects an aggregation of the quantities that customers will buy at each price. Market supply reflects a summation of the quantities that individual firms are willing to supply at different prices. The intersection of industry demand and supply curves determines the equilibrium market price. To illustrate this process, consider the following market demand curve where price is expressed as a function of output:

\[
P = 40 - 0.0001Q_D \quad \text{(Market Demand)}
\]

or equivalently, when output is expressed as a function of price.
\[ Q_D = 400,000 - 10,000P \]

Assume market supply is provided by five competitors producing a standardized product \( Q \). Firm supply schedules are as follows:

\[
\begin{align*}
Q_1 &= 18 + 2P \\
Q_2 &= 12 + 6P \\
Q_3 &= 40 + 12P \\
Q_4 &= 20 + 12P \\
Q_5 &= 10 + 8P
\end{align*}
\]

(Firm 1)
(Firm 2)
(Firm 3)
(Firm 4)
(Firm 5)

A. Calculate optimal supply by each firm at the competitive market prices indicated in the following table. Then, assume there are actually 1,000 firms just like each one illustrated in the table. Use this information to complete the Partial Market Supply and Total Market Supply columns.

<table>
<thead>
<tr>
<th>Price</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>1,000</th>
<th>= Total Market Supply (000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>8</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

B. Sum the individual firm supply curves to derive the market supply curve. Plot the market demand and market supply curve with price as a function of output to illustrate the equilibrium price and level of output. Verify that this is indeed the market equilibrium price-output combination algebraically.

**ST10.2 SOLUTION**
A.

<table>
<thead>
<tr>
<th>Price</th>
<th>1 + 2 + 3 + 4 + 5 = Partial Market Supply × 1,000</th>
<th>= Total Market Supply (000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1</td>
<td>20 18 52 32 18</td>
<td>140</td>
</tr>
<tr>
<td>2</td>
<td>22 24 64 44 26</td>
<td>180</td>
</tr>
<tr>
<td>3</td>
<td>24 30 76 56 34</td>
<td>220</td>
</tr>
<tr>
<td>4</td>
<td>26 36 88 68 42</td>
<td>260</td>
</tr>
<tr>
<td>5</td>
<td>28 42 100 80 50</td>
<td>300</td>
</tr>
<tr>
<td>6</td>
<td>30 48 112 92 58</td>
<td>340</td>
</tr>
<tr>
<td>7</td>
<td>32 54 124 104 66</td>
<td>380</td>
</tr>
<tr>
<td>8</td>
<td>34 60 136 116 74</td>
<td>420</td>
</tr>
</tbody>
</table>

The data in the Table illustrate the process by which an industry supply curve is constructed. First, suppose that each of five firms in an industry is willing to supply varying quantities at different prices. Summing the individual supply quantities of these five firms at each price determines their combined supply schedule, shown in the Partial Market Supply column. For example, at a price of $2, the output supplied by the five firms are 22, 24, 64, 44, and 26 (thousand) units, respectively, resulting in a combined supply of 180(000) units at that price. With a competitive market price of $8, supply quantities would become 34, 60, 136, 116, and 74, for a total supply by the five firms of 420(000) units, and so on.

Now assume that there are 1,000 firms just like each one illustrated in the table. There are actually 5,000 firms in the industry, each with an individual supply schedule identical to one of the five firms illustrated in the table. In that event, the total quantity supplied at each price is 1,000 times that shown under the Partial Market Supply schedule. Because the numbers shown for each firm are in thousands of units, the total market supply column is in thousands of units. Therefore, the number 140,000 at a price of $1 indicates 140 million units, the number 180,000 at a price of $2 indicates 180 million units, and so on.
B. To find the market supply curve, simply sum each individual firm’s supply curve, where quantity is expressed as a function of the market price:

\[
Q_I = Q_1 + Q_2 + Q_3 + Q_4 + Q_5 \\
= 18 + 2P + 12 + 6P + 40 + 12P + 20 + 12P + 10 + 8P \\
= 100 + 40P \text{ (Market Supply)}
\]

Plotting the market demand curve and the market supply curve allows one to determine the equilibrium market price of $6 and the equilibrium market quantity of 340,000,000, or 340 million units.

To find the market equilibrium levels for price and quantity algebraically, simply set the market demand and market supply curves equal to one another so that \( Q_D = Q_S \). To find the market equilibrium price, equate the market demand and market supply curves where quantity is expressed as a function of price:

\[
\text{Demand} = \text{Supply} \\
400,000 - 10,000P = 100,000 + 40,000P \\
50,000P = 300,000
\]
\[ P = \$6 \]

To find the market equilibrium quantity, set equal the market demand and market supply curves where price is expressed as a function of quantity, and \( Q_D = Q_S \):

Demand = Supply

\[
40 - 0.0001Q = -2.5 + 0.000025Q
\]

\[
0.000125Q = 42.5
\]

\[
Q = 340,000(000)
\]

Therefore, the equilibrium price-output combination is a market price of $6 with an equilibrium output of 340,000(000), or 340 million units.
SELF-TEST PROBLEMS & SOLUTIONS

ST11.1 Social Welfare. A number of domestic and foreign manufacturers produce replacement parts and components for personal computer systems. With exacting user specifications, products are standardized and price competition is brutal. To illustrate the net amount of social welfare generated in this hotly competitive market, assume that market supply and demand conditions for replacement tower cases can be described as:

\[ Q_S = -175 + 12.5P \quad \text{(Market Supply)} \]
\[ Q_D = 125 - 2.5P \quad \text{(Market Demand)} \]

where \( Q \) is output in thousands of units and \( P \) is price per unit.

A. Graph and calculate the equilibrium price/output solution.

B. Use this graph to help you algebraically determine the amount of consumer surplus, producer surplus and net social welfare generated in this market.

ST11.1 SOLUTION

A. The market supply curve is given by the equation

\[ Q_S = -175 + 12.5P \]

or, solving for price,

\[ 12.5P = 175 + Q_S \]
\[ P = \$14 + \$0.08Q_S \]

The market demand curve is given by the equation

\[ Q_D = 125 - 2.5P \]

or, solving for price,
Graphically, demand and supply curves appear as follows:

Algebraically, to find the market equilibrium levels for price and quantity, simply set the market supply and market demand curves equal to one another so that $Q_S = Q_D$. To find the market equilibrium price, equate the market demand and market supply curves where quantity is expressed as a function of price:

$$\text{Supply} = \text{Demand}$$

$$-175 + 12.5P = 125 - 2.5P$$

$$15P = 300$$

$$P = \$20$$

To find the market equilibrium quantity, set equal the market supply and market demand curves where price is expressed as a function of quantity, and $Q_S = Q_D$:
Supply = Demand

\[14 + 0.08Q = 50 - 0.4Q\]

\[0.48Q = 36\]

\[Q = 75,000\]

The equilibrium price-output combination is a market price of $20 with an equilibrium output of 75,000 units, as shown in the figure.

B. The value of consumer surplus is equal to the region under the market demand curve that lies above the market equilibrium price of $20. Because the area of a such a triangle is one-half the value of the base times the height, the value of consumer surplus equals:

\[\text{Consumer Surplus} = \frac{1}{2} [75 \times ($50 - $20)]\]

\[= $1,125,000\]

In words, this means that at a unit price of $20, the quantity demanded is 75,000 units, resulting in total revenues of $1,500,000. The fact that consumer surplus equals $1,125,000 means that customers as a group would have been willing to pay an additional $1,125,000 for this level of market output. This is an amount above and beyond the $1,500,000 paid. Customers received a real bargain.

The value of producer surplus is equal to the region above the market supply curve at the market equilibrium price of $20. Because the area of such a triangle is one-half the value of the base times the height, the value of producer surplus equals:

\[\text{Producer Surplus} = \frac{1}{2} [75 \times ($20 - $14)]\]

\[= $225,000\]

At a unit price of $20, producer surplus equals $225,000. Producers as a group received $225,000 more than the absolute minimum required for them to produce the market equilibrium output of 75,000 units. Producers received a real bargain.

In competitive market equilibrium, social welfare is measured by the sum of net benefits derived by consumers and producers. Social welfare is the sum of consumer surplus and producer surplus:

\[\text{Social Welfare} = \text{Consumer Surplus} + \text{Producer Surplus}\]

\[= $1,125 + $225\]

\[= $1,350,000\]
**ST11.2 Price Ceilings.** The local government in a West Coast college town is concerned about a recent explosion in apartment rental rates for students and other low-income renters. To combat the problem, a proposal has been made to institute rent control that would place a $900 per month ceiling on apartment rental rates. Apartment supply and demand conditions in the local market are:

\[
Q_S = -400 + 2P \quad \text{(Market Supply)}
\]

\[
Q_D = 5,600 - 4P \quad \text{(Market Demand)}
\]

where \(Q\) is the number of apartments and \(P\) is monthly rent.

**A.** Graph and calculate the equilibrium price/output solution. How much consumer surplus, producer surplus, and social welfare is produced at this activity level?

**B.** Use the graph to help you algebraically determine the quantity demanded, quantity supplied, and shortage with a $900 per month ceiling on apartment rental rates.

**C.** Use the graph to help you algebraically determine the amount of consumer and producer surplus with rent control.

**D.** Use the graph to help you algebraically determine the change in social welfare and deadweight loss in consumer surplus due to rent control.

**ST11.2 SOLUTION**

**A.** The competitive market supply curve is given by the equation

\[
Q_S = -400 + 2P
\]

or, solving for price,

\[
2P = 400 + Q_S
\]

\[
P = \$200 + \$0.5Q_S
\]

The competitive market demand curve is given by the equation

\[
Q_D = 5,600 - 4P
\]

or, solving for price,

\[
4P = 5,600 - Q_D
\]
\[ P = \$1,400 - 0.25Q_d \]

To find the competitive market equilibrium price, equate the market demand and market supply curves where quantity is expressed as a function of price:

\[
\text{Supply} = \text{Demand} \]
\[-400 + 2P = 5,600 - 4P \]
\[6P = 6,000 \]
\[P = \$1,000 \]

To find the competitive market equilibrium quantity, set equal the market supply and market demand curves where price is expressed as a function of quantity, and \( Q_s = Q_d \):

\[
\text{Supply} = \text{Demand} \]
\[200 + 0.5Q = 1,400 - 0.25Q \]
\[0.75Q = 1,200 \]
\[Q = 1,600 \]

Therefore, the competitive market equilibrium price-output combination is a market price of $1,000 with an equilibrium output of 1,600 units.
The value of consumer surplus is equal to the region under the market demand curve that lies above the market equilibrium price of $1,000. Because the area of a such a triangle is one-half the value of the base times the height, the value of consumer surplus equals:

\[
\text{Consumer Surplus} = \frac{1}{2} [1,600 \times ($1,400 - $1,000)]
\]

\[
= $320,000
\]

In words, this means that at a unit price of $1,000, the quantity demanded is 1,600 units, resulting in total revenues of $1,600,000. The fact that consumer surplus equals $320,000 means that customers as a group would have been willing to pay an additional $320,000 for this level of market output. This is an amount above and beyond the $1,600,000 paid. Customers received a real bargain.

The value of producer surplus is equal to the region above the market supply curve at the market equilibrium price of $1,000. Because the area of a such a triangle is one-half the value of the base times the height, the value of producer surplus equals:

\[
\text{Producer Surplus} = \frac{1}{2} [1,600 \times ($1,000 - $200)]
\]

\[
= $640,000
\]
At a rental price of $1,000 per month, producer surplus equals $640,000. Producers as a group received $640,000 more than the absolute minimum required for them to produce the market equilibrium output of 1,600 units. Producers received a real bargain.

In competitive market equilibrium, social welfare is measured by the sum of net benefits derived by consumers and producers. Social welfare is the sum of consumer surplus and producer surplus:

\[
\text{Social Welfare} = \text{Consumer Surplus} + \text{Producer Surplus}
\]

\[
= \$320,000 + \$640,000
\]

\[
= \$960,000
\]

B. The market demand at the $900 price ceiling is

\[
Q_D = 5,600 - 4(900)
\]

\[
= 2,000 \text{ units}
\]

The market supply at the $900 price ceiling is

\[
Q_S = -400 + 2(900)
\]

\[
= 1,400 \text{ units}
\]

The market shortage created by the $900 price ceiling is

\[
\text{Shortage} = Q_D - Q_S
\]

\[
= 2,000 - 1,400
\]

\[
= 600 \text{ units}
\]

C. Under rent control, the maximum amount of apartment supply that landlords are willing to offer at a rent of $900 per month is 1,400 units. From the market demand curve, it is clear that renters as a group are willing to pay as much as (or have a reservation price of) $1,050 per month to rent 1,400 apartments:

\[
P = \$1,400 - 0.25(1,400)
\]

\[
= \$1,050
\]

Under rent control, the value of consumer surplus has two components. A first component of consumer surplus is equal to the region under the market demand curve
that lies above the price of $1,050 per month. This amount corresponds to uncompensated value obtained by renters willing to pay above the market price all the way up to $1,400 per month. As in the case of an uncontrolled market, the area of such a triangle is one-half the value of the base times the height. A second component of consumer surplus under rent control is the uncompensated value obtained by renters willing to pay as much as $1,050 per month to rent 1,400 apartments, and who are delighted to rent for the controlled price of $900 per month. This amount corresponds to the amount of revenue represented by the rectangle defined by the prices of $1,050 and $900 and the quantity of 1,400 units. Notice that this second component of consumer surplus includes some value privately measured as producer surplus. Under rent control, the total amount of consumer surplus is:

\[
\text{Rent-Controlled Consumer Surplus} = \frac{1}{2} [1,400 \times ($1,400 - $1,050)] \\
+ [1,400 \times ($1,050 - $900)] \\
= $245,000 + $210,000 \\
= $455,000
\]

In this case, consumer surplus rises from $320,000 to $455,000, a gain of $135,000 as a result of rent control.

The value of producer surplus is equal to the region above the market supply curve at the rent-controlled price of $900. Because the area of such a triangle is one-half the value of the base times the height, the value of producer surplus equals:

\[
\text{Producer Surplus} = \frac{1}{2} [1,400 \times ($900 - $200)] \\
= $490,000
\]

At a rent-controlled price of $900 per month, producer surplus falls from $640,000 to $490,000, a loss of $150,000.

D. The change in social welfare caused by rent control is measured by the change in net benefits derived by consumers and producers. The change in social welfare is the change in the sum of consumer surplus and producer surplus:

\[
\text{Social Welfare Change} = \text{Consumer Surplus Change} \\
+ \text{Producer Surplus Change} \\
= $135,000 - $150,000 \\
= -$15,000 \text{ (a loss)}
\]
This $15,000 deadweight loss in social welfare due to rent control has two components. First, there is a deadweight loss of consumer surplus from consumers unable to find a rent-controlled apartment but willing to pay upwards from the prior market equilibrium price of $1,000 per month up to $1,050 per month. This amount is equal to the area shown in the graph as ABD. Because the area of such a triangle is one-half the value of the base times the height, the first component of deadweight loss in consumer surplus equals:

\[
\text{Deadweight Loss in Consumer Surplus} = \frac{1}{2} [(1,600 - 1,400) \times ($1,050 - $1,000)] \\
= $5,000
\]

Second, there is a deadweight loss of producer surplus from landlords forced to rent at the rent-controlled price of $900 per month rather than the market equilibrium price of $1,000 per month. This amount is equal to the area shown in the graph as BCD. Because the area of such a triangle is one-half the value of the base times the height, the second component of deadweight loss in consumer surplus equals:

\[
\text{Deadweight Loss in Producer Surplus} = \frac{1}{2} [(1,600 - 1,400) \times ($1,000 - $900)] \\
= $10,000
\]
Chapter 12

Monopoly and Monopsony

SELF-TEST PROBLEMS & SOLUTIONS

ST12.1 Capture Problem. It remains a widely held belief that regulation is in the public interest and influences firm behavior toward socially desirable ends. However, in the early 1970s, Nobel laureate George Stigler and his colleague Sam Peltzman at the University of Chicago introduced an alternative capture theory of economic regulation. According to Stigler and Peltzman, the machinery and power of the state are a potential resource to every industry. With its power to prohibit or compel, to take or give money, the state can and does selectively help or hurt a vast number of industries. Because of this, regulation may be actively sought by industry. They contended that regulation is typically acquired by industry and is designed and operated primarily for industry's benefit.

Types of state favors commonly sought by regulated industries include direct money subsidies, control over entry by new rivals, control over substitutes and complements, and price fixing. Domestic "air mail" subsidies, Federal Deposit Insurance Corporation (FDIC) regulation that reduces the rate of entry into commercial banking, suppression of margarine sales by butter producers, price fixing in motor carrier (trucking) regulation, and American Medical Association control of medical training and licensing can be interpreted as historical examples of control by regulated industries.

In summarizing their views on regulation, Stigler and Peltzman suggest that regulators should be criticized for pro-industry policies no more than politicians for seeking popular support. Current methods of enacting and carrying out regulations only make the pro-industry stance of regulatory bodies more likely. The only way to get different results from regulation is to change the political process of regulator selection and to provide economic rewards to regulators who serve the public interest effectively.

Capture theory is in stark contrast to more traditional public interest theory, which sees regulation as a government-imposed means of private-market control. Rather than viewing regulation as a "good" to be obtained, controlled, and manipulated, public interest theory views regulation as a method for improving economic performance by limiting the harmful effects of market failure. Public interest theory is silent on the need to provide regulators with economic incentives to improve regulatory performance. Unlike capture theory, a traditional view has been that the public can trust regulators to make a good-faith effort to establish regulatory policy in the public interest.

A. The aim of antitrust and regulatory policy is to protect competition, not to protect competitors. Explain the difference.
B. Starting in the 1970s, growing dissatisfaction with traditional approaches to government regulation led to a global deregulation movement that spurred competition, lowered prices, and resulted in more efficient production. Explain how this experience is consistent with the capture theory of regulation.

C. Discuss how regulatory efficiency could be improved by focusing on output objectives like low prices for cable or telephone services rather than production methods or rates of return.

ST12.1 SOLUTION

A. Entry and exit are common facts of life in competitive markets. Firms that efficiently produce goods and services that consumers crave are able to boost market share and enjoy growing revenues and profits. Firms that fail to measure up in the eyes of consumers will lose market share and suffer declining revenues and profits. The disciplining role of competitive markets can be swift and harsh, even for the largest and most formidable corporations. For example, in August, 2000, Enron Corp. traded in the stock market at an all-time high and was ranked among the 10 most valuable corporations in America. Nevertheless, Enron filed for bankruptcy just 16 months later as evidence emerged of a failed diversification strategy, misguided energy trading, and financial corruption. Similarly, once powerful telecom giant WorldCom quickly stumbled into bankruptcy as evidence came to light concerning the company’s abuse of accounting rules and regulations. In both cases, once powerful corporations were brought to their knees by competitive capital and product markets that simply refused to tolerate inefficiency and corporate malfeasance.

In evaluating the effects of deregulation, and in gauging the competitive implications of market exit by previously regulated firms, it is important to remember that protecting competition is not the same as protecting competitors. Without regulation, it is inevitable that some competitors will fall by the wayside and that concentration will rise in some previously regulated markets. Although such trends must be watched closely for anti-competitive effects, they are characteristics of a vigorously competitive environment. Bankruptcy and exit are the regrettable costs of remedying economic dislocation in competitive markets. Though such costs are regrettable, experience shows that they are much less onerous than the costs of indefinitely maintaining inefficient production methods in a tightly regulated environment.

B. Although it is difficult to pinpoint a single catalyst for the deregulation movement, it is hard to overlook the role played by George Stigler, Sam Peltzman, Alfred E. Kahn, and other economists who documented how government regulation can sometimes harm consumer interests. A study by the Brookings Institution documented important benefits of deregulation in five major industries--natural gas, telecommunications, airlines, trucking, and railroads. It was found that prices fell 4-15% within the first two years after deregulation; within 10 years, prices were 25-50% lower. Deregulation also leads
to service quality improvements. Crucial social goals like airline safety, reliability of
gas service, and reliability of the telecommunications network were maintained or
improved by deregulation. Regulatory reform also tends to confer benefits on most
consumers. Although it is possible to find narrowly defined groups of customers in
special circumstances who paid somewhat higher prices after deregulation, the gains to
the vast majority of consumers far outweighed negative effects on small groups. Finally,
deregulation offers benefits in the sense of permitting greater customer choice.

Although many industries have felt the effects of changing state and local
regulation, changing federal regulation has been most pronounced in the financial,
telemcommunications, and transportation sectors. Since 1975, for example, it has been
illegal for securities dealers to fix commission rates. This broke a 182-year tradition
under which the New York Stock Exchange (NYSE) set minimum rates for each 100-
share ("round lot") purchase. Until 1975, everyone charged the minimum rate approved
by the NYSE. Purchase of 1,000 shares cost a commission of ten times the minimum,
even though the overhead and work involved are roughly the same for small and large
stock transactions. Following deregulation, commission rates tumbled, and, predictably,
some of the least efficient brokerage firms merged or otherwise went out of business.
Today, commission rates have fallen by 90% or more, and the industry is noteworthy
for increasing productivity and innovative new product introductions. It is also worth
mentioning that since brokerage rates were deregulated, the number of sales offices in
the industry, trading volume, employment, and profits have skyrocketed. This has lead
some observers to conclude that deregulation can benefit consumers without causing any
lasting damage to industry. In fact, a leaner, more efficient industry may be one of the
greatest benefits of deregulation.

In Canada, the deregulation movement led to privatization of government-owned
Air Canada. Trucking, historically a regulated industry, also was deregulated.
Specialized telecommunications services industries were deregulated and thrown open
to competition. In other areas where the government considered continued regulation
desirable and necessary, regulatory agencies were pressured to reform and improve the
regulatory decision-making process to reduce inefficiencies, bureaucratic delays, and
administrative red tape.

C. A significant problem with regulation is that regulators seldom have the information or
expertise to specify, for example, the correct level of utility investment, minimum
transportation costs, or the optimum method of pollution control. Because technology
changes rapidly in most regulated industries, only industry personnel working at the
frontier of current technology have such specialized knowledge. One method for
dealing with this technical expertise problem is to have regulators focus on the preferred
outcomes of regulatory processes, rather than on the technical means that industry
adopts to achieve those ends. The FCC’s decision to adopt downward-adjusting price
caps for long-distance telephone service is an example of this developing trend toward
incentive-based regulation. If providers of long-distance telephone service are able to
reduce costs faster than the FCC-mandated decline in prices, they will enjoy an increase
in profitability. By setting price caps that fall over time, the FCC ensures that
consumers share in expected cost savings while companies enjoy a positive incentive to innovate. This approach to regulation focuses on the objectives of regulation while allowing industry to meet those goals in new and unique ways. Tying regulator rewards and regulated industry profits to objective, output-oriented performance criteria has the potential to create a desirable win/win situation for regulators, utilities, and the general public. For example, the public has a real interest in safe, reliable, and low-cost electric power. State and federal regulators who oversee the operations of utilities could develop objective standards for measuring utility safety, reliability, and cost efficiency. Tying firm profit rates to such performance-oriented criteria could stimulate real improvements in utility and regulator performance.

Although some think that there is simply a question of regulation versus deregulation, this is seldom the case. On grounds of economic and political feasibility, it is often most fruitful to consider approaches to improving existing methods of regulation. Competitive forces provide a persistent and socially desirable constraining influence on firm behavior. When vigorous competition is absent, government regulation can be justified through both efficiency and equity criteria. When regulation is warranted, business, government, and the public must work together to ensure that regulatory processes represent the public interest. The unnecessary costs of antiquated regulations dictate that regulatory reform is likely to remain a significant social concern.

ST12.2 Deadweight Loss From Monopoly. The Las Vegas Valley Water District (LVVWD) is a not-for-profit agency that began providing water to the Las Vegas Valley in 1954. The District helped build the city's water delivery system and now provides water to more than one million people in Southern Nevada. District water rates are regulated by law and can cover only the costs of water delivery, maintenance, and facilities. District water rates are based on a four-tier system to encourage conservation. The first tier represents indoor usage for most residential customers. Rate for remaining tiers becomes increasingly higher with the amount of water usage.

To document the deadweight loss from monopoly problem, allow the monthly market supply and demand conditions for water in the Las Vegas Water District to be:

\[ Q_S = 10P \] \hspace{2cm} (Market Supply)

\[ Q_D = 120 - 40P \] \hspace{2cm} (Market Demand)

where \( Q \) is water and \( P \) is the market price of water. Water is sold in units of one thousand gallons, so a $2 price implies a user cost of 0.2 cents per gallon. Water demand and supply relations are expressed in terms of millions of units.

A. Graph and calculate the equilibrium price/output solution. How much consumer surplus, producer surplus, and social welfare is produced at this activity level?

B. Use the graph to help you calculate the quantity demanded and quantity supplied if the market is run by a profit-maximizing monopolist. (Note: If monopoly market
demand is \( P = $3 - $0.025Q \), then the monopolist’s \( MR = $3 - $0.05Q \)

C. Use the graph to help you determine the deadweight loss for consumers and the producer if LVVWD is run as an unregulated profit-maximizing monopoly.

D. Use the graph to help you ascertain the amount of consumer surplus transferred to producers following a change from a competitive market to a monopoly market. How much is the net gain in producer surplus?

**ST12.2 SOLUTION**

A. The market supply curve is given by the equation

\[ Q_S = 10P \]

or, solving for price,

\[ P = 0.1Q_S \]

The market demand curve is given by the equation

\[ Q_D = 120 - 40P \]

or, solving for price,

\[ 40P = 120 - Q_D \]

\[ P = $3 - $0.025Q_D \]

To find the competitive market equilibrium price, equate the market demand and market supply curves where quantity is expressed as a function of price:

\[ \text{Supply} = \text{Demand} \]

\[ 10P = 120 - 40P \]

\[ 50P = 120 \]

\[ P = $2.40 \]

To find the competitive market equilibrium quantity, set equal the market supply and market demand curves where price is expressed as a function of quantity, and \( Q_S = Q_D \):
Supply = Demand

$0.1Q = $3 - $0.025Q

0.125Q = 3

Q = 24 (million) units per month

Therefore, the competitive market equilibrium price-output combination is a market price of $2.40 with an equilibrium output of 24 (million) units.

The value of consumer surplus is equal to the region under the market demand curve that lies above the market equilibrium price of $2.40. Because the area of such a triangle is one-half the value of the base times the height, the value of consumer surplus equals:

Consumer Surplus = ½ [24 ×($3 - $2.40)]

= $7.2 (million) per month

In words, this means that at a unit price of $2.40, the quantity demanded is 24 (million), resulting in total revenues of $57.6 (million). The fact that consumer surplus equals $7.2 (million) means that customers as a group would have been willing to pay an additional $7.2 (million) for this level of market output. This is an amount above and beyond the $57.6 (million) paid. Customers received a real bargain.

The value of producer surplus is equal to the region above the market supply curve at the market equilibrium price of $2.40. Because the area of such a triangle is one-half the value of the base times the height, the value of producer surplus equals:

Producer Surplus = ½ [24 ×($2.40 - $0)]

= $28.8 (million) per month

At a water price of $2.40 per thousand gallons, producer surplus equals $28.8 (million). Producers as a group received $28.8 (million) more than the absolute minimum required for them to produce the market equilibrium output of 24 (million) units of output. Producers received a real bargain.

In competitive market equilibrium, social welfare is measured by the sum of net benefits derived by consumers and producers. Social welfare is the sum of consumer surplus and producer surplus:

Social Welfare = Consumer Surplus + Producer Surplus

= $7.2 (million) + $28.8 (million)
Las Vegas Valley Water District

![Graph showing demand and supply curves, with MC = $0.1Q, Demand P = $3 - $0.025Q, and MR = $3 - $0.05Q.]

B. If the industry is run by a profit-maximizing monopolist, the optimal price-output combination can be determined by setting marginal revenue equal to marginal cost and solving for $Q$:

\[ MR = MC = \text{Market Supply} \]
\[ $3 - $0.05Q = $0.1Q \]
\[ $0.15Q = $3 \]
\[ Q = 20 \text{ (million) units per month} \]

At $Q = 20$,
\[ P = $3 - $0.025Q \]
\[ = $3 - $0.025(20) \]

$= $36 (million) per month
$2.50 per unit

C. Under monopoly, the amount supplied falls to 20 (million) units and the market price jumps to $2.50 per thousand gallons of water. The amount of deadweight loss from monopoly suffered by consumers is given by the triangle bounded by ABD in the figure. Because the area of such a triangle is one-half the value of the base times the height, the value of lost consumer surplus due to monopoly equals:

\[
\text{Consumer Deadweight Loss} = \frac{1}{2} \left[ (24 - 20) \times (\$2.50 - \$2.40) \right]
\]

\[
= \$0.2 \text{ (million) per month}
\]

The amount of deadweight loss from monopoly suffered by producers is given by the triangle bounded by BCD. Because the area of a such a triangle is one-half the value of the base times the height, the value of lost producer surplus equals:

\[
\text{Producer Deadweight Loss} = \frac{1}{2} \left[ (24 - 20) \times (\$2.40 - \$2) \right]
\]

\[
= \$0.8 \text{ (million) per month}
\]

The total amount of deadweight loss from monopoly suffered by consumers and producers is given by the triangle bounded by ACD. The area of a such a triangle is simply the amount of consumer deadweight loss plus producer deadweight loss:

\[
\text{Total Deadweight Loss} = \text{Consumer Loss} + \text{Producer Loss}
\]

\[
= \$0.2 \text{ (million) } + \$0.8 \text{ (million)}
\]

\[
= \$1 \text{ (million) per month}
\]

D. In addition to the deadweight loss from monopoly problem, there is a wealth transfer problem associated with monopoly. The creation of a monopoly results in a significant transfer from consumer surplus to producer surplus. In the figure, this amount is shown as the area in the rectangle bordered by $P_{CM}P_{MAB}$:

\[
\text{Transfer to Producer Surplus} = 20 \times (\$2.50 - \$2.40)
\]

\[
= \$2 \text{ (million) per month}
\]

Therefore, from the viewpoint of the producer, the change to monopoly results in a very favorable net increase in producer surplus:

\[
\text{Net Change in Producer Surplus} = \text{Producer Deadweight Loss} + \text{Transfer}
\]
From the viewpoint of consumers, the problem with monopoly is twofold. Monopoly result in both a significant deadweight loss in consumer surplus ($0.2 million per month), and monopoly causes a significant transfer of consumer surplus to producer surplus ($2 million per month). In this example, the cost of monopoly to consumers is measured by a total loss in consumer surplus of $2.2 million per month. The wealth transfer problem associated with monopoly is seen as an issue of equity or fairness because it involves the distribution of income or wealth in the economy. Although economic profits serve the useful functions of providing incentives and helping allocate resources, it is difficult to justify monopoly profits that result from the raw exercise of market power rather than from exceptional performance.
Chapter 13

Monopolistic Competition and Oligopoly

SELF-TEST PROBLEMS & SOLUTIONS

ST13.1 Price Leadership. Over the last century, The Boeing Co. has grown from building planes in an old, red boathouse to become the largest aerospace company in the world. Boeing’s principal global competitor is Airbus, a French company jointly owned by Eads (80%) and BAE Systems (20%). Airbus was established in 1970 as a European consortium of French, German and later, Spanish and U.K companies. In 2001, thirty years after its creation, Airbus became a single integrated company. Though dominated by Boeing and Airbus, smaller firms have recently entered the commercial aircraft industry. Notable among these is Embraer, a Brazilian aircraft manufacturer. Embraer has become one of the largest aircraft manufacturers in the world by focusing on specific market segments with high growth potential. As a niche manufacturer, Embraer makes aircraft that offer excellent reliability and cost effectiveness.

To illustrate the price leadership concept, assume that total and marginal cost functions for Airbus (A) and Embraer (E) aircraft are as follows:

\[
TC_A = 10,000,000 + 35,000,000Q_A + 250,000Q_A^2 \\
MC_A = 35,000,000 + 500,000Q_A \\
TC_E = 200,000,000 + 20,000,000Q_E + 500,000Q_E^2 \\
MC_E = 20,000,000 + 1,000,000Q_E
\]

Boeing’s total and marginal cost relations are as follows:

\[
TC_B = 4,000,000,000 + 5,000,000Q_B + 62,500Q_B^2 \\
MC_B = \frac{\Delta TC_B}{\Delta Q_B} = 5,000,000 + 125,000Q_B
\]

The industry demand curve for this type of jet aircraft is

\[
Q = 910 - 0.000017P
\]

Assume throughout this problem that the Airbus and Embraer aircraft are perfect substitutes for Boeing’s Model 737-600, and that each total cost function includes a risk-adjusted normal rate of return on investment.
A. Determine the supply curves for Airbus and Embraer aircraft, assuming that the firms operate as price takers.

B. What is the demand curve faced by Boeing?

C. Calculate Boeing’s profit-maximizing price and output levels. (Hint: Boeing’s total and marginal revenue relations are \( TR_B = 50,000,000Q_B - 50,000Q_B^2 \) and \( MR_B = \Delta TR_B/\Delta Q_B = 50,000,000 - 100,000Q_B \).)

D. Calculate profit-maximizing output levels for the Airbus and Embraer aircraft.

E. Is the market for aircraft from these three firms in short-run and in long-run equilibrium?

ST13.1 SOLUTION

A. Because price followers take prices as given, they operate where individual marginal cost equals price. Therefore, the supply curves for Airbus and Embraer aircraft are:

**Airbus**

\[
P_A = MC_A = 35,000,000 + 500,000Q_A
\]

\[
500,000Q_A = -35,000,000 + P_A
\]

\[
Q_A = -70 + 0.000002P_A
\]

**Embraer**

\[
P_E = MC_E = 20,000,000 + 1,000,000Q_E
\]

\[
1,000,000Q_E = -20,000,000 + P_E
\]

\[
Q_E = -20 + 0.000001P_E
\]

B. As the industry price leader, Boeing’s demand equals industry demand minus following firm supply. Remember that \( P = P_B = P_M = P_E \) because Boeing is a price leader for the industry:

\[
Q_B = Q - Q_A - Q_E
\]

\[
= 910 - 0.000017P + 70 - 0.000002P
\]

\[
+ 20 - 0.000001P
\]

\[
= -80 -
\]
To find Boeing’s profit maximizing price and output level, set MR_B = MC_B and solve for Q:

\[ MR_B = MC_B \]
\[ $50,000,000 - $100,000Q_B = $5,000,000 + $125,000Q_B \]
\[ 45,000,000 = 225,000Q_B \]
\[ Q_B = 200 \text{ units} \]
\[ P_B = $50,000,000 - $50,000(200) = $40,000,000 \]

Because Boeing is a price leader for the industry,

\[ P = P_B = P_A = P_E = $40,000,000 \]

Optimal supply for Airbus and Embraer aircraft are:

\[ Q_A = -70 + 0.000002P_A \]
\[ = -70 + 0.000002(40,000,000) \]
\[ = 10 \]

\[ Q_E = -20 + 0.000001P_E \]
\[ = -20 + 0.000001(40,000,000) \]
\[ = 20 \]

Yes. The industry is in short-run equilibrium if the total quantity demanded is equal to total supply. The total industry demand at a price of $40 million is:

\[ Q_D = 910 - 0.000017P \]
\[ = 910 - 0.000017(40,000,000) \]
The total industry supply is:

\[ Q_s = Q_B + Q_A + Q_E \]
\[ = 200 + 10 + 20 \]
\[ = 230 \text{ units} \]

Thus, the industry is in short-run equilibrium. The industry is also in long-run equilibrium provided that each manufacturer is making at least a risk-adjusted normal rate of return on investment. To check profit levels for each manufacturer, note that:

\[ \pi_A = TR_A - TC_A \]
\[ = $40,000,000(10) -$10,000,000 -$35,000,000(10) \]
\[ - $250,000(10^2) \]
\[ = $15,000,000 \]

\[ \pi_E = TR_E - TC_E \]
\[ = $40,000,000(20) -$200,000,000 -$20,000,000(20) \]
\[ - $500,000(20^2) \]
\[ = $0 \]

\[ \pi_B = TR_B - TC_B \]
\[ = $40,000,000(200) -$4,000,000,000 - $5,000,000(200) \]
\[ - $62,500(200^2) \]
\[ = $500,000,000 \]

Boeing and Airbus are both earning economic profits, whereas Embraer, the marginal entrant, is earning just a risk-adjusted normal rate of return. As such, the industry is in long-run equilibrium and there is no incentive to change.

**ST13.2 Monopolistically Competitive Equilibrium.** **Soft Lens, Inc.,** has enjoyed rapid growth in sales and high operating profits on its innovative extended-wear soft contact lenses.
However, the company faces potentially fierce competition from a host of new competitors as some important basic patents expire during the coming year. Unless the company is able to thwart such competition, severe downward pressure on prices and profit margins is anticipated.

A. Use Soft Lens’s current price, output, and total cost data to complete the table:

<table>
<thead>
<tr>
<th>Price ($)</th>
<th>Monthly Output (million)</th>
<th>Total Revenue ($million)</th>
<th>Marginal Revenue ($million)</th>
<th>Total Cost ($million)</th>
<th>Marginal Cost ($million)</th>
<th>Average Cost ($million)</th>
<th>Total Profit ($million)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0</td>
<td>0</td>
<td>$0</td>
<td>19</td>
<td>12</td>
<td>18</td>
<td>42</td>
</tr>
<tr>
<td>19</td>
<td>1</td>
<td>12</td>
<td></td>
<td>18</td>
<td>27</td>
<td>17</td>
<td>42</td>
</tr>
<tr>
<td>18</td>
<td>2</td>
<td>27</td>
<td></td>
<td>16</td>
<td>58</td>
<td>15</td>
<td>75</td>
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<td>3</td>
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<td></td>
<td>14</td>
<td>84</td>
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<td>92</td>
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<tr>
<td>16</td>
<td>4</td>
<td>58</td>
<td></td>
<td>12</td>
<td>96</td>
<td>11</td>
<td>99</td>
</tr>
<tr>
<td>15</td>
<td>5</td>
<td>75</td>
<td></td>
<td>10</td>
<td>105</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Note: Total costs include a risk-adjusted normal rate of return.)

B. If cost conditions remain constant, what is the monopolistically competitive high-price/low-output long-run equilibrium in this industry? What are industry profits?

C. Under these same cost conditions, what is the monopolistically competitive low-price/high-output equilibrium in this industry? What are industry profits?

D. Now assume that Soft Lens is able to enter into restrictive licensing agreements with potential competitors and create an effective cartel in the industry. If demand and cost conditions remain constant, what is the cartel price/output and profit equilibrium?

ST13.2 SOLUTION

A.
<table>
<thead>
<tr>
<th>Price ($</th>
<th>Monthly Output (million)</th>
<th>Total Revenue ($million)</th>
<th>Marginal Revenue ($million)</th>
<th>Total Cost ($million)</th>
<th>Marginal Cost ($million)</th>
<th>Average Cost ($million)</th>
<th>Total Profit ($million)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$20</td>
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<td>$0</td>
<td>---</td>
<td>$0</td>
<td>---</td>
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<td>$0</td>
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<td>19</td>
<td>$19</td>
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<td>$12</td>
<td>$12.00</td>
<td>7</td>
</tr>
<tr>
<td>18</td>
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<td>36</td>
<td>17</td>
<td>27</td>
<td>15</td>
<td>13.50</td>
<td>9</td>
</tr>
<tr>
<td>17</td>
<td>3</td>
<td>51</td>
<td>15</td>
<td>42</td>
<td>15</td>
<td>14.00</td>
<td>9</td>
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<tr>
<td>16</td>
<td>4</td>
<td>64</td>
<td>13</td>
<td>58</td>
<td>16</td>
<td>14.50</td>
<td>6</td>
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<tr>
<td>15</td>
<td>5</td>
<td>75</td>
<td>11</td>
<td>75</td>
<td>17</td>
<td>15.00</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>6</td>
<td>84</td>
<td>9</td>
<td>84</td>
<td>9</td>
<td>14.00</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>7</td>
<td>91</td>
<td>7</td>
<td>92</td>
<td>8</td>
<td>13.14</td>
<td>-1</td>
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<tr>
<td>12</td>
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<td>96</td>
<td>5</td>
<td>96</td>
<td>4</td>
<td>12.00</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>9</td>
<td>99</td>
<td>3</td>
<td>99</td>
<td>3</td>
<td>11.00</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>100</td>
<td>1</td>
<td>105</td>
<td>6</td>
<td>10.50</td>
<td>-5</td>
</tr>
</tbody>
</table>

B. The monopolistically competitive high-price/low-output equilibrium is \( P = AC = $14, Q = 6(000,000) \), and \( \pi = TR - TC = $0 \). Only a risk-adjusted normal rate of return is being earned in the industry, and excess profits equal zero. Because \( \pi = $0 \) and \( MR = MC = $9 \), there is no incentive for either expansion or contraction. Such an equilibrium is typical of monopolistically competitive industries where each individual firm retains some pricing discretion in long-run equilibrium.

C. The monopolistically competitive low-price/high-output equilibrium is \( P = AC = $11, Q = 9(000,000) \), and \( \pi = TR - TC = $0 \). Again, only a risk-adjusted normal rate of return is being earned in the industry, and excess profits equal zero. Because \( \pi = $0 \) and \( MR = MC = $3 \), there is no incentive for either expansion or contraction. This price/output combination is identical to the perfectly competitive equilibrium. (Note that average cost is rising and profits are falling for \( Q > 9 \).)

D. A monopoly price/output and profit equilibrium results if Soft Lens is able to enter into restrictive licensing agreements with potential competitors and create an effective cartel in the industry. If demand and cost conditions remain constant, the cartel price/output and profit equilibrium is at \( P = $17, Q = 3(000,000) \), and \( \pi = $9(000,000) \). There is no incentive for the cartel to expand or contract production at this level of output because \( MR = MC = $15 \).
Chapter 14

Game Theory and Competitive Strategy

SELF-TEST PROBLEMS & SOLUTIONS

ST14.1 Game Theory Strategies. Suppose two local suppliers are seeking to win the right to upgrade the communications capability of the internal “intranets” that link a number of customers with their suppliers. The system quality decision facing each competitor, and potential profit payoffs, are illustrated in the table. The first number listed in each cell is the profit earned by U.S. Equipment Supply; the second number indicates the profit earned by Business Systems, Inc. For example, if both competitors, U.S. Equipment Supply and Business Systems, Inc., pursue a high-quality strategy, U.S. Equipment Supply will earn $25,000 and Business Systems, Inc., will earn $50,000. If U.S. Equipment Supply pursues a high-quality strategy while Business Systems, Inc., offers low-quality goods and services, U.S. Equipment Supply will earn $40,000; Business Systems, Inc., will earn $22,000. If U.S. Equipment Supply offers low-quality goods while Business Systems, Inc., offers high-quality goods, U.S. Equipment Supply will suffer a net loss of $25,000, and Business Systems, Inc., will earn $20,000. Finally, if U.S. Equipment Supply offers low-quality goods while Business Systems, Inc., offers low-quality goods, both U.S. Equipment Supply and Business Systems, Inc., will earn $25,000.

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality Strategy</td>
<td>High Quality</td>
</tr>
<tr>
<td>High Quality</td>
<td>$25,000, $50,000</td>
</tr>
<tr>
<td>Low Quality</td>
<td>-$25,000, $20,000</td>
</tr>
</tbody>
</table>

A. Does U.S. Equipment Supply and/or Business Systems, Inc., have a dominant strategy? If so, what is it?

B. Does U.S. Equipment Supply and/or Business Systems, Inc., have a secure strategy? If so, what is it?

C. What is the Nash equilibrium concept, and why is it useful? What is the Nash equilibrium for this problem?

ST14.1 SOLUTION
A. The dominant strategy for U.S. Equipment Supply is to provide high-quality goods. Irrespective of the quality strategy chosen by Business Systems, Inc., U.S. Equipment Supply can do no better than to choose a high-quality strategy. To see this, note that if Business Systems, Inc., chooses to produce high-quality goods, the best choice for U.S. Equipment Supply is to also provide high-quality goods because the $25,000 profit then earned is better than the $25,000 loss that would be incurred if U.S. Equipment Supply chose a low-quality strategy. If Business Systems, Inc., chose a low-quality strategy, the best choice by U.S. Equipment Supply would again be to produce high-quality goods. U.S. Equipment Supply’s high-quality strategy profit of $40,000 dominates the low-quality payoff for U.S. Equipment Supply of $25,000.

Business Systems, Inc., does not have a dominant strategy. To see this, note that if U.S. Equipment Supply chooses to produce high-quality goods, the best choice for Business Systems, Inc., is to also provide high-quality goods because the $50,000 profit then earned is better than the $22,000 profit if Business Systems, Inc., chose a low-quality strategy. If U.S. Equipment Supply chose a low-quality strategy, the best choice by Business Systems, Inc., would be to produce low-quality goods and earn $25,000 versus $20,000.

B. The secure strategy for U.S. Equipment Supply is to provide high-quality goods. By choosing to provide high-quality goods, U.S. Equipment Supply can be guaranteed a profit payoff of at least $25,000. By pursuing a high-quality strategy, U.S. Equipment Supply can eliminate the chance of losing $25,000, as would happen if U.S. Equipment Supply chose a low-quality strategy while Business Systems, Inc., chose to produce high-quality goods.

The secure strategy for Business Systems, Inc., is to provide low-quality goods. By choosing to provide high-quality goods, Business Systems, Inc., can guarantee a profit payoff of only $20,000. Business Systems, Inc., can be assured of earning at least $22,000 with a low-quality strategy. Thus, the secure strategy for Business Systems, Inc., is to provide low-quality goods.

C. A set of strategies constitute a Nash equilibrium if, given the strategies of other players, no player can improve its payoff through a unilateral change in strategy. The concept of Nash equilibrium is very important because it represents a situation where every player is doing the best possible in light of what other players are doing.

Although useful, the notion of a secure strategy suffers from a serious shortcoming. In the present example, suppose Business Systems, Inc., reasoned as follows: “U.S. Equipment Supply will surely choose its high-quality dominant strategy. Therefore, I should not choose my secure low-quality strategy and earn $22,000. I should instead choose a high-quality strategy and earn $50,000.” A natural way of formalizing the “end result” of such a thought process is captured in the definition of Nash equilibrium.

In the present example, if U.S. Equipment Supply chooses a high-quality strategy, the Nash equilibrium strategy is for Business Systems, Inc., to also choose a high-quality strategy. Similarly, if Business Systems, Inc., chooses a high-quality strategy, the Nash
equilibrium strategy is for U.S. Equipment Supply to also choose a high-quality strategy. Thus, a Nash equilibrium is reached when both firms adopt high-quality strategies.

Although some problems have multiple Nash equilibriums, that is not true in this case. A combination of high-quality strategies for both firms is the only set of strategies where no player can improve its payoff through a unilateral change in strategy.

**ST14.2 Nash Equilibrium.** Assume that IBM and Dell Computer have a large inventory of personal computers that they would like to sell before a new generation of faster, cheaper machines is introduced. Assume that the question facing each competitor is whether or not they should widely advertise a “close out” sale on these discontinued items, or instead let excess inventory work itself off over the next few months. If both aggressively promote their products with a nationwide advertising campaign, each will earn profits of $5 million. If one advertises while the other does not, the firm that advertises will earn $20 million, while the one that does not advertise will earn $2 million. If neither advertises, both will earn $10 million. Assume this is a one-shot game, and both firms seek to maximize profits.

<table>
<thead>
<tr>
<th>IBM</th>
<th>Dell Computer</th>
<th>Advertise</th>
<th>Don’t Advertise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advertise</td>
<td>$5 million, $5 million</td>
<td>$20 million, $2 million</td>
<td></td>
</tr>
<tr>
<td>Don’t advertise</td>
<td>$2 million, $20 million</td>
<td>$10 million, $10 million</td>
<td></td>
</tr>
</tbody>
</table>

**A.** What is the dominant strategy for each firm? Are these also secure strategies?

**B.** What is the Nash equilibrium?

**C.** Would collusion work in this case?

**ST14.2 SOLUTION**

**A.** The dominant strategy for both IBM and Dell is to advertise. Neither could earn higher profits with a “don’t advertise” strategy, irrespective of what the other party chooses to do.

For example, if IBM chooses to advertise, Dell will also choose to advertise and earn $5 million rather than $2 million. If IBM chooses not to advertise, Dell will choose to advertise and earn $20 million rather than $10 million. No matter what IBM decides to do, Dell is better off by advertising. Similarly, if Dell chooses to advertise, IBM will also choose to advertise and earn $5 million rather than $2 million. If Dell chooses not to advertise, IBM will earn $2 million rather than $10 million. If Dell chooses not to advertise, IBM will earn $2 million rather than $10 million. If IBM chooses not to advertise, Dell will earn $2 million rather than $10 million. Therefore, the Nash equilibrium is both firms advertising.
to advertise, IBM will choose to advertise and earn $20 million rather than $10 million. No matter what Dell decides to do, IBM is better off by advertising.

These are also secure strategies for each firm because they ensure the elimination of worst outcome payoffs. With an advertising strategy, neither firm is exposed to the possibility of earning only $2 million.

B. A set of strategies constitute a Nash equilibrium if, given the strategies of other players, no player can improve its payoff through a unilateral change in strategy. The concept of Nash equilibrium is very important because it represents a situation where every player is doing the best possible in light of what other players are doing.

In this case, the Nash equilibrium is for each firm to advertise. Although some problems have multiple Nash equilibriums, that is not true in this case. An advertising strategy for both firms is the only set of strategies where no player can improve its payoff through a unilateral change in strategy.

C. Collusion will not work in this case because this is a “one shot” game where moves are taken simultaneously, rather than in sequence. Sequential rounds are necessary with enforcement penalties before successful collusion is possible. If IBM and Dell “agreed” not to advertise in the hope of making $10 million each, both would have an incentive to cheat on the agreement in the hope of making $20 million. Without the possibility for a second round, enforcement is precluded, and collusion isn’t possible.
ST15.1 George Constanza is a project coordinator at Kramer-Seinfeld & Associates, Ltd., a large Brooklyn-based painting contractor. Constanza has asked you to complete an analysis of profit margins earned on a number of recent projects. Unfortunately, your predecessor on this project was abruptly transferred, leaving you with only sketchy information on the firm’s pricing practices.

A. Use the available data to complete the following table:

<table>
<thead>
<tr>
<th>Price</th>
<th>Marginal Cost</th>
<th>Markup on Cost (%)</th>
<th>Markup on Price (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100</td>
<td>$25</td>
<td>300.0</td>
<td>75.0</td>
</tr>
<tr>
<td>240</td>
<td>72</td>
<td></td>
<td></td>
</tr>
<tr>
<td>680</td>
<td>272</td>
<td>150.0</td>
<td>60.0</td>
</tr>
<tr>
<td>750</td>
<td>100.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2,800</td>
<td></td>
<td></td>
<td>40.0</td>
</tr>
<tr>
<td>2,700</td>
<td>33.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3,360</td>
<td>20.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5,800</td>
<td>10.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6,250</td>
<td>5.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10,000</td>
<td>0.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

B. Calculate the missing data for each of the following proposed projects, based on the available estimates of the point price elasticity of demand, optimal markup on cost, and optimal markup on price:

<table>
<thead>
<tr>
<th>Project</th>
<th>Price Elasticity</th>
<th>Optimal Markup on Cost (%)</th>
<th>Optimal Markup on Price (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.5</td>
<td>200.0</td>
<td>66.7</td>
</tr>
<tr>
<td>2</td>
<td>-2.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Project</td>
<td>Price Elasticity</td>
<td>Optimal Markup on Cost (%)</td>
<td>Optimal Markup on Price (%)</td>
</tr>
<tr>
<td>---------</td>
<td>------------------</td>
<td>-----------------------------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>66.7</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td>25.0</td>
</tr>
<tr>
<td>5</td>
<td>-5.0</td>
<td>25.0</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>11.1</td>
<td>10.0</td>
</tr>
<tr>
<td>7</td>
<td>-15.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-20.0</td>
<td></td>
<td>5.0</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td>4.0</td>
</tr>
<tr>
<td>10</td>
<td>-50.0</td>
<td>2.0</td>
<td></td>
</tr>
</tbody>
</table>

**ST15.1 SOLUTION**

A.

<table>
<thead>
<tr>
<th>Price</th>
<th>Marginal Cost</th>
<th>Markup on Cost (%)</th>
<th>Markup on Price (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100</td>
<td>$25</td>
<td>300.0</td>
<td>75.0</td>
</tr>
<tr>
<td>240</td>
<td>72</td>
<td>233.3</td>
<td>70.0</td>
</tr>
<tr>
<td>680</td>
<td>272</td>
<td>150.0</td>
<td>60.0</td>
</tr>
<tr>
<td>750</td>
<td>375</td>
<td>100.0</td>
<td>50.0</td>
</tr>
<tr>
<td>2,800</td>
<td>1,680</td>
<td>66.7</td>
<td>40.0</td>
</tr>
<tr>
<td>3,600</td>
<td>2,700</td>
<td>33.3</td>
<td>25.0</td>
</tr>
<tr>
<td>4,200</td>
<td>3,360</td>
<td>25.0</td>
<td>20.0</td>
</tr>
<tr>
<td>5,800</td>
<td>5,220</td>
<td>11.1</td>
<td>10.0</td>
</tr>
<tr>
<td>6,250</td>
<td>5,938</td>
<td>5.3</td>
<td>5.0</td>
</tr>
<tr>
<td>10,000</td>
<td>10,000</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

B.
<table>
<thead>
<tr>
<th>Project</th>
<th>Price Elasticity</th>
<th>Optimal Markup on Cost (%)</th>
<th>Optimal Markup on Price (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.5</td>
<td>200.0</td>
<td>66.7</td>
</tr>
<tr>
<td>2</td>
<td>-2.0</td>
<td>100.0</td>
<td>50.0</td>
</tr>
<tr>
<td>3</td>
<td>-2.5</td>
<td>66.7</td>
<td>40.0</td>
</tr>
<tr>
<td>4</td>
<td>-4.0</td>
<td>33.3</td>
<td>25.0</td>
</tr>
<tr>
<td>5</td>
<td>-5.0</td>
<td>25.0</td>
<td>20.0</td>
</tr>
<tr>
<td>6</td>
<td>-10.0</td>
<td>11.1</td>
<td>10.0</td>
</tr>
<tr>
<td>7</td>
<td>-15.0</td>
<td>7.1</td>
<td>6.7</td>
</tr>
<tr>
<td>8</td>
<td>-20.0</td>
<td>5.3</td>
<td>5.0</td>
</tr>
<tr>
<td>9</td>
<td>-25.0</td>
<td>4.2</td>
<td>4.0</td>
</tr>
<tr>
<td>10</td>
<td>-50.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>

ST15.2 **Optimal Markup on Price.** TLC Lawncare, Inc., provides fertilizer and weed control lawn services to residential customers. Its seasonal service package, regularly priced at $250, includes several chemical spray treatments. As part of an effort to expand its customer base, TLC offered $50 off its regular price to customers in the Dallas area. Response was enthusiastic, with sales rising to 5,750 units (packages) from the 3,250 units sold in the same period last year.

A. Calculate the arc price elasticity of demand for TLC service.

B. Assume that the arc price elasticity (from Part A) is the best available estimate of the point price elasticity of demand. If marginal cost is $135 per unit for labor and materials, calculate TLC’s optimal markup on price and its optimal price.

ST15.2 **SOLUTION**

A. \[ E_p = \frac{\Delta Q}{\Delta P} \times \frac{P_2 + P_1}{Q_2 + Q_1} \]

\[
= \frac{5,750 - 3,250}{200 - 250} \times \frac{200 + 250}{5,750 + 3,250}
\]

\[= -2.5\]

B. Given \( \epsilon_p = E_p = -2.5 \), the optimal TLC markup on price is:

\[
\text{Optimal Markup on Price} = -\frac{1}{\epsilon_p}
\]
\[
\frac{-1}{-2.5} = 0.4 \text{ or } 40\%
\]

Given \( MC = $135 \), the optimal price is:

\[
\text{Optimal Markup on Price} = \frac{P - MC}{P}
\]

\[
0.4 = \frac{P - $135}{P}
\]

\[
0.4P = P - $135
\]

\[
0.6P = $135
\]

\[
P = $225
\]
ST161 Certainty Equivalent Method. Courteney-Cox, Inc., is a Texas-based manufacturer and distributor of components and replacement parts for the auto, machinery, farm, and construction equipment industries. The company is presently funding a program of capital investment that is necessary to reduce production costs and thereby meet an onslaught of competition from low-cost suppliers located in Mexico and throughout Latin America. Courteney-Cox has a limited amount of capital available and must carefully weigh both the risks and potential rewards associated with alternative investments. In particular, the company seeks to weigh the advantages and disadvantages of a new investment project, project X, in light of two other recently adopted investment projects, project Y and project Z:

<table>
<thead>
<tr>
<th>Year</th>
<th>Project X</th>
<th>Project Y</th>
<th>Project Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>$10,000</td>
<td>$20,000</td>
<td>$0</td>
</tr>
<tr>
<td>2002</td>
<td>10,000</td>
<td>18,000</td>
<td>2,500</td>
</tr>
<tr>
<td>2003</td>
<td>10,000</td>
<td>16,000</td>
<td>5,000</td>
</tr>
<tr>
<td>2004</td>
<td>10,000</td>
<td>14,000</td>
<td>7,500</td>
</tr>
<tr>
<td>2005</td>
<td>10,000</td>
<td>12,000</td>
<td>10,000</td>
</tr>
<tr>
<td>2006</td>
<td>10,000</td>
<td>10,000</td>
<td>12,500</td>
</tr>
<tr>
<td>2007</td>
<td>10,000</td>
<td>8,000</td>
<td>15,000</td>
</tr>
<tr>
<td>2008</td>
<td>10,000</td>
<td>6,000</td>
<td>17,500</td>
</tr>
<tr>
<td>2009</td>
<td>10,000</td>
<td>4,000</td>
<td>20,000</td>
</tr>
<tr>
<td>2010</td>
<td>10,000</td>
<td>2,000</td>
<td>22,500</td>
</tr>
</tbody>
</table>

PV of Cash Flow @ 5%  $91,131  $79,130
Investment Outlay in 2000:  $60,000  $60,000  $50,000

A. Using a 5% risk-free rate, calculate the present value of expected cash flows after tax (CFAT) for the ten-year life of project X.
B. Calculate the minimum certainty equivalent adjustment factor for each project’s CFAT that would justify investment in each project.

C. Assume that the management of Courteney-Cox is risk averse and uses the certainty equivalent method in decision making. Is project X as attractive or more attractive than projects Y and Z?

D. If the company would not have been willing to invest more than $60,000 in project Y nor more than $50,000 in project Z, should project X be undertaken?

ST16.1 SOLUTION

A. Using a 5% risk-free rate, the present value of expected cash flows after tax (CFAT) for the ten-year life of Project X is $77,217, calculated as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Project X</th>
<th>PV of $1 at 5%</th>
<th>PV of CFAT at 5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>$10,000</td>
<td>0.9524</td>
<td>$9,524</td>
</tr>
<tr>
<td>2002</td>
<td>10,000</td>
<td>0.9070</td>
<td>9,070</td>
</tr>
<tr>
<td>2003</td>
<td>10,000</td>
<td>0.8638</td>
<td>8,638</td>
</tr>
<tr>
<td>2004</td>
<td>10,000</td>
<td>0.8227</td>
<td>8,227</td>
</tr>
<tr>
<td>2005</td>
<td>10,000</td>
<td>0.7835</td>
<td>7,835</td>
</tr>
<tr>
<td>2006</td>
<td>10,000</td>
<td>0.7462</td>
<td>7,462</td>
</tr>
<tr>
<td>2007</td>
<td>10,000</td>
<td>0.7107</td>
<td>7,107</td>
</tr>
<tr>
<td>2008</td>
<td>10,000</td>
<td>0.6768</td>
<td>6,768</td>
</tr>
<tr>
<td>2009</td>
<td>10,000</td>
<td>0.6446</td>
<td>6,446</td>
</tr>
<tr>
<td>2010</td>
<td>10,000</td>
<td>0.6139</td>
<td>6,139</td>
</tr>
</tbody>
</table>

PV of Cash Flow @ 5% $77,217

B. To justify each investment alternative, the company must have a certainty equivalent adjustment factor of at least $\alpha_X = 0.777$ for project X, $\alpha_Y = 0.658$ for project Y, and $\alpha_Z = 0.632$ for project Z, because:

$$\alpha = \frac{\text{Certain Sum}}{\text{Expected Risky Sum}} = \frac{\text{Investment Outlay (Opportunity cost)}}{\text{Present Value CFAT}}$$
Project X

\[ \alpha_X = \frac{\$60,000}{\$77,217} = 0.777 \]

Project Y

\[ \alpha_Y = \frac{\$60,000}{\$91,131} = 0.658 \]

Project Z

\[ \alpha_Z = \frac{\$50,000}{\$79,130} = 0.632 \]

In other words, each risky dollar of expected profit contribution from project X must be “worth” at least (valued as highly as) 77.7¢ in certain dollars to justify investment. For project Y, each risky dollar must be worth at least 65.8¢ in certain dollars; each risky dollar must be worth at least 63.2¢ to justify investment in project Z.

C. Given managerial risk aversion, project X is the least attractive investment because it has the highest “price” on each risky dollar of expected CFAT. In adopting projects Y and Z, Courteney-Cox implicitly asserted that it is willing to pay between 63.2¢ (project Z) and 65.8¢ (project Y) per each expected dollar of CFAT.

D. No. If the prices described previously represent the maximum price the company is willing to pay for such risky returns, then project X should not be undertaken.

ST16.2 Project Valuation. The Central Perk Coffee House, Inc., is engaged in an aggressive store refurbishing program and is contemplating expansion of its in-store baking facilities. This investment project is to be evaluated using the certainty equivalent adjustment factor method and the risk-adjusted discount rate method. If the project has a positive value when both methods are employed, the project will be undertaken. The project will not be undertaken if either evaluation method suggests that the investment will fail to increase the value of the firm. Expected cash flow after tax (CFAT) values over the five-year life of the investment project and relevant certainty equivalent adjustment factor information are as follows:
In-store Baking Facilities
Investment Project

<table>
<thead>
<tr>
<th>Time Period (Years)</th>
<th>Alpha</th>
<th>Project E(CFAT)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>($75,000)</td>
</tr>
<tr>
<td>0</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.95</td>
<td>22,500</td>
</tr>
<tr>
<td>2</td>
<td>0.90</td>
<td>25,000</td>
</tr>
<tr>
<td>3</td>
<td>0.85</td>
<td>27,500</td>
</tr>
<tr>
<td>4</td>
<td>0.75</td>
<td>30,000</td>
</tr>
<tr>
<td>5</td>
<td>0.70</td>
<td><strong>32,500</strong></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>$62,500</strong></td>
</tr>
</tbody>
</table>

At the present time, an 8% annual rate of return can be obtained on short-term U.S. government securities; the company uses this rate as an estimate of the risk-free rate of return.

A. Use the 8% risk-free rate to calculate the present value of the investment project.

B. Using this present value as a basis, utilize the certainty equivalent adjustment factor information given previously to determine the risk-adjusted present value of the project.

C. Use an alternative risk-adjusted discount rate method of project valuation on the assumption that a 15% rate of return is appropriate in light of the level of risk undertaken.

D. Compare and contrast your answers to parts B and C. Should the investment be made?

ST16.2 SOLUTION

A. The present value of this investment project can be calculated easily using a hand-held calculator with typical financial function capabilities or by using the tables found in Appendix A. Using the appropriate discount factors corresponding to an 8% risk-free rate, the present value of the investment project is calculated as follows:
### Hot Food Carryout Counter Investment Project

<table>
<thead>
<tr>
<th>Time Period (Years)</th>
<th>Present Value of $1 at 8%</th>
<th>Project E(CFAT)</th>
<th>Present Value of E(CFAT) at 8%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0000</td>
<td>($75,000)</td>
<td>($75,000)</td>
</tr>
<tr>
<td>1</td>
<td>0.9259</td>
<td>22,500</td>
<td>20,833</td>
</tr>
<tr>
<td>2</td>
<td>0.8573</td>
<td>25,000</td>
<td>21,433</td>
</tr>
<tr>
<td>3</td>
<td>0.7938</td>
<td>27,500</td>
<td>21,830</td>
</tr>
<tr>
<td>4</td>
<td>0.7350</td>
<td>30,000</td>
<td>22,050</td>
</tr>
<tr>
<td>5</td>
<td>0.6806</td>
<td>32,500</td>
<td>22,120</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td>$62,500</td>
<td>$33,266</td>
</tr>
</tbody>
</table>

B. Using the present value given in part A as a basis, the certainty equivalent adjustment factor information given previously can be employed to determine the risk-adjusted present value of the project:

### In-store Baking Facilities Investment Project

<table>
<thead>
<tr>
<th>Time Period (Years)</th>
<th>Present Value of $1 at 8%</th>
<th>Project E(CFAT)</th>
<th>Present Value of E(CFAT) at 8%</th>
<th>Alpha</th>
<th>Risk-Adjusted Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0000</td>
<td>($75,000)</td>
<td>($75,000)</td>
<td>1.00</td>
<td>($75,000)</td>
</tr>
<tr>
<td>1</td>
<td>0.9259</td>
<td>22,500</td>
<td>20,833</td>
<td>0.95</td>
<td>19,791</td>
</tr>
<tr>
<td>2</td>
<td>0.8573</td>
<td>25,000</td>
<td>21,433</td>
<td>0.90</td>
<td>19,290</td>
</tr>
<tr>
<td>3</td>
<td>0.7938</td>
<td>27,500</td>
<td>21,830</td>
<td>0.85</td>
<td>18,556</td>
</tr>
<tr>
<td>4</td>
<td>0.7350</td>
<td>30,000</td>
<td>22,050</td>
<td>0.75</td>
<td>16,538</td>
</tr>
<tr>
<td>5</td>
<td>0.6806</td>
<td>32,500</td>
<td>22,120</td>
<td>0.70</td>
<td>15,484</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td>$62,500</td>
<td>$33,266</td>
<td></td>
<td>$14,659</td>
</tr>
</tbody>
</table>

C. An alternative risk-adjusted discount rate method of project valuation based on a 15% rate of return gives the following project valuation:

### In-store Baking Facilities Investment Project

<table>
<thead>
<tr>
<th>Time Period (Years)</th>
<th>Present Value of $1 at 15%</th>
<th>Project E(CFAT)</th>
<th>Present Value of E(CFAT) at 15%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0000</td>
<td>($75,000)</td>
<td>($75,000)</td>
</tr>
<tr>
<td>1</td>
<td>0.8696</td>
<td>22,500</td>
<td>19,566</td>
</tr>
<tr>
<td>2</td>
<td>0.7561</td>
<td>25,000</td>
<td>18,903</td>
</tr>
<tr>
<td>3</td>
<td>0.6575</td>
<td>27,500</td>
<td>18,081</td>
</tr>
<tr>
<td>4</td>
<td>0.5718</td>
<td>30,000</td>
<td>17,154</td>
</tr>
</tbody>
</table>
The answers to parts B and C are fully compatible; both suggest a positive risk-adjusted present value for the project. In part B, the certainty equivalent adjustment factor method reduces the present value of future receipts to account for risk differences. As is typical, the example assumes that money to be received in the more distant future has a greater risk, and hence, a lesser certainty equivalent value. In the risk-adjusted discount rate approach of part C, the discount rate of 15% entails a time-factor adjustment of 8% plus a risk adjustment of 7%. Like the certainty equivalent adjustment factor approach, the risk-adjusted discount rate method gives a risk-adjusted present value for the project. Because the risk-adjusted present value of the project is positive under either approach, the investment should be made.
Chapter 17

Capital Budgeting

SELF-TEST PROBLEMS & SOLUTIONS

ST17.1 NPV and Payback Period Analysis. Suppose that your college roommate has approached you with an opportunity to lend $25,000 to her fledgling home healthcare business. The business, called Home Health Care, Inc., plans to offer home infusion therapy and monitored in-the-home healthcare services to surgery patients in the Birmingham, Alabama, area. Funds would be used to lease a delivery vehicle, purchase supplies, and provide working capital. Terms of the proposal are that you would receive $5,000 at the end of each year in interest with the full $25,000 to be repaid at the end of a ten-year period.

A. Assuming a 10% required rate of return, calculate the present value of cash flows and the net present value of the proposed investment.

B. Based on this same interest rate assumption, calculate the cumulative cash flow of the proposed investment for each period in both nominal and present-value terms.

C. What is the payback period in both nominal and present-value terms?

D. What is the difference between the nominal and present-value payback period? Can the present-value payback period ever be shorter than the nominal payback period?

ST17.1 SOLUTION

A. The present value of cash flows and the net present value of the proposed investment can be calculated as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash Flow</th>
<th>Interest Factor</th>
<th>Present Value Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>($25,000)</td>
<td>1.0000</td>
<td>($25,000)</td>
</tr>
<tr>
<td>1</td>
<td>5,000</td>
<td>0.9091</td>
<td>4,545</td>
</tr>
<tr>
<td>2</td>
<td>5,000</td>
<td>0.8264</td>
<td>4,132</td>
</tr>
<tr>
<td>3</td>
<td>5,000</td>
<td>0.7513</td>
<td>3,757</td>
</tr>
</tbody>
</table>
B. The cumulative cash flow of the proposed investment for each period in both nominal and present-value terms is:

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash Flow</th>
<th>Present Value Interest Factor</th>
<th>Present Value Cash Flow</th>
<th>Cumulative Cash Flow</th>
<th>Cumulative PV Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>($25,000)</td>
<td>1.0000</td>
<td>($25,000)</td>
<td>($25,000)</td>
<td>($25,000)</td>
</tr>
<tr>
<td>1</td>
<td>5,000</td>
<td>0.9091</td>
<td>4,545</td>
<td>(20,000)</td>
<td>(20,455)</td>
</tr>
<tr>
<td>2</td>
<td>5,000</td>
<td>0.8264</td>
<td>4,132</td>
<td>(15,000)</td>
<td>(16,322)</td>
</tr>
<tr>
<td>3</td>
<td>5,000</td>
<td>0.7513</td>
<td>3,757</td>
<td>(10,000)</td>
<td>(12,566)</td>
</tr>
<tr>
<td>4</td>
<td>5,000</td>
<td>0.6830</td>
<td>3,415</td>
<td>(5,000)</td>
<td>(9,151)</td>
</tr>
<tr>
<td>5</td>
<td>5,000</td>
<td>0.6209</td>
<td>3,105</td>
<td>0</td>
<td>(6,046)</td>
</tr>
<tr>
<td>6</td>
<td>5,000</td>
<td>0.5645</td>
<td>2,822</td>
<td>5,000</td>
<td>(3,224)</td>
</tr>
<tr>
<td>7</td>
<td>5,000</td>
<td>0.5132</td>
<td>2,566</td>
<td>10,000</td>
<td>(658)</td>
</tr>
<tr>
<td>8</td>
<td>5,000</td>
<td>0.4665</td>
<td>2,333</td>
<td>15,000</td>
<td>1,675</td>
</tr>
<tr>
<td>9</td>
<td>5,000</td>
<td>0.4241</td>
<td>2,120</td>
<td>20,000</td>
<td>3,795</td>
</tr>
<tr>
<td>10</td>
<td>5,000</td>
<td>0.3855</td>
<td>1,928</td>
<td>25,000</td>
<td>5,723</td>
</tr>
</tbody>
</table>

Payback Period 5 years
Present Value Payback Period 8.28 years (= 8 + $658/$2,333).

C. Based on the information provided in part B, it is clear that the cumulative cash flow in nominal dollars reached $0 at the end of Year 5. This means that the nominal payback period is 5 years. The cumulative cash flow in present-value dollars exceeds $0 when the Year 8 interest payment is received. This means that the present-value payback
period is roughly 8 years. If cash flows were received on a continuous basis, the present-value payback period would be 8.28 years (= $658/$2,333).

D. Assuming a positive rate of interest, the present-value payback period is always longer than the nominal payback period. This stems from the fact that present-value dollars are always less than nominal dollars, and it therefore takes longer to receive a fixed dollar amount back in terms of present-value dollars rather than in nominal terms.

ST17.2 Decision Rule Conflict. Bob Sponge has been retained as a management consultant by Square Pants, Inc., a local specialty retailer, to analyze two proposed capital investment projects, projects X and Y. Project X is a sophisticated working capital and inventory control system based upon a powerful personal computer, called a system server, and PC software specifically designed for inventory processing and control in the retailing business. Project Y is a similarly sophisticated working capital and inventory control system based upon a powerful personal computer and general-purpose PC software. Each project has a cost of $10,000, and the cost of capital for both projects is 12%. The projects’ expected net cash flows are as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Project X</th>
<th>Project Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>($10,000)</td>
<td>($10,000)</td>
</tr>
<tr>
<td>1</td>
<td>6,500</td>
<td>3,500</td>
</tr>
<tr>
<td>2</td>
<td>3,000</td>
<td>3,500</td>
</tr>
<tr>
<td>3</td>
<td>3,000</td>
<td>3,500</td>
</tr>
<tr>
<td>4</td>
<td>1,000</td>
<td>3,500</td>
</tr>
</tbody>
</table>

A. Calculate each project’s nominal payback period, net present value (NPV), internal rate of return (IRR), and profitability index (PI).

B. Should both projects be accepted if they are interdependent?

C. Which project should be accepted if they are mutually exclusive?

D. How might a change in the cost of capital produce a conflict between the NPV and IRR rankings of these two projects? At what values of k would this conflict exist? (Hint: Plot the NPV profiles for each project to find the crossover discount rate k.)

E. Why does a conflict exist between NPV and IRR rankings?

ST17.2 SOLUTION
A. **Payback:**

To determine the nominal payback period, construct the cumulative cash flows for each project:

<table>
<thead>
<tr>
<th>Year</th>
<th>Project X</th>
<th>Project Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>($10,000)</td>
<td>($10,000)</td>
</tr>
<tr>
<td>1</td>
<td>(3,500)</td>
<td>(6,500)</td>
</tr>
<tr>
<td>2</td>
<td>(500)</td>
<td>(3,000)</td>
</tr>
<tr>
<td>3</td>
<td>2,500</td>
<td>500</td>
</tr>
<tr>
<td>4</td>
<td>3,500</td>
<td>4,000</td>
</tr>
</tbody>
</table>

\[
\text{Payback}_X = 2 + \frac{500}{3,000} = 2.17 \text{ years.}
\]

\[
\text{Payback}_Y = 2 + \frac{3,000}{3,500} = 2.86 \text{ years.}
\]

**Net Present Value (NPV):**

\[
\text{NPV}_X = -$10,000 + \frac{6,500}{(1.12)^1} + \frac{3,000}{(1.12)^2} + \frac{3,000}{(1.12)^3} + \frac{1,000}{(1.12)^4}
\]

\[
= $966.01.
\]

\[
\text{NPV}_Y = -$10,000 + \frac{3,500}{(1.12)^1} + \frac{3,500}{(1.12)^2} + \frac{3,500}{(1.12)^3} + \frac{3,500}{(1.12)^4}
\]

\[
= $630.72.
\]

**Internal Rate of Return (IRR):**

To solve for each project’s IRR, find the discount rates that set NPV to zero:

\[
\text{IRR}_X = 18.0\%.
\]

\[
\text{IRR}_Y = 15.0\%.
\]

**Profitability Index (PI):**
\[
\text{PI}_X = \frac{\text{PV Benefits}}{\text{PV Costs}} = \frac{\$10,966.01}{\$10,000} = 1.10.
\]
\[
\text{PI}_Y = \frac{\$10,630.72}{\$10,000} = 1.06.
\]

B. Using all methods, project X is preferred over project Y. Because both projects are acceptable under the NPV, IRR, and PI criteria, both projects should be accepted if they are interdependent.

C. Choose the project with the higher NPV at \( k = 12\% \), or project X.

D. To determine the effects of changing the cost of capital, plot the NPV profiles of each project. The crossover rate occurs at about 6% to 7%. To find this rate exactly, create a project \( \Delta \), which is the difference in cash flows between projects X and Y:

<table>
<thead>
<tr>
<th>Year</th>
<th>Project X - Project Y = Project ( \Delta ) Net Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$0</td>
</tr>
<tr>
<td>1</td>
<td>3,000</td>
</tr>
<tr>
<td>2</td>
<td>(500)</td>
</tr>
<tr>
<td>3</td>
<td>(500)</td>
</tr>
<tr>
<td>4</td>
<td>(2,500)</td>
</tr>
</tbody>
</table>

Then find the IRR of Project \( \Delta \):

\[
\text{IRR}_\Delta = \text{Crossover Rate} = 6.2\%.
\]

Thus, if the firm’s cost of capital is less than 6.2%, a conflict exists, because \( NPV_Y > NPV_X \) but \( IRR_X > IRR_Y \).

Graphically, the crossover discount rate is illustrated as follows:
E. The basic cause of conflict is the differing reinvestment rate assumptions between NPV and IRR. The conflict occurs in this situation because the projects differ in their cash flow timing.
Chapter 18

Government in the Market Economy

SELF-TEST PROBLEMS & SOLUTIONS

ST18.1 Pollution Control Costs. Anthony Soprano is head of Satriale Pork Producers, Inc., a family-run pork producer with a hog-processing facility in Musconetcong, New Jersey. Each hog processed yields both pork and a render by-product in a fixed 1:1 ratio. Although the by-product is unfit for human consumption, some can be sold to a local pet food company for further processing. Relevant annual demand and cost relations are as follows:

\[ P_p = 110 - 0.00005Q_p, \]

(Demand for pork)

\[ MRP = \Delta TR_p/\Delta Q_p = 110 - 0.0001Q_p, \]

(Marginal revenue from pork)

\[ P_b = 10 - 0.0001Q_b, \]

(Demand for render by-product)

\[ MR_b = \Delta TR_b/\Delta Q_b = 10 - 0.0002Q_b, \]

(Marginal revenue from render by-product)

\[ TC = 10,000,000 + 60Q, \]

(Total cost)

\[ MC = \Delta TC/\Delta Q = 60. \]

(Marginal cost)

Here, \( P \) is price in dollars, \( Q \) is the number of hogs processed (with an average weight of 100 pounds), and \( Q_p \) and \( Q_b \) are pork and render by-product per hog, respectively; both total and marginal costs are in dollars. Total costs include a risk-adjusted normal return of 15% on a $50 million investment in plant and equipment.
Currently, the city allows the company to dump excess by-product into its sewage treatment facility at no charge, viewing the service as an attractive means of keeping a valued employer in the area. However, the sewage treatment facility is quickly approaching peak capacity and must be expanded at an expected operating cost of $3 million per year. This is an impossible burden on an already strained city budget.

A. Calculate the profit-maximizing price/output combination and optimal total profit level for Satriale.

B. How much by-product will the company dump into the Musconetcong sewage treatment facility at the profit-maximizing activity level?

C. Calculate output and total profits if the city imposes a $35 per unit charge on the amount of by-product Satriale dumps.

D. Calculate output and total profits if the city imposes a fixed $3-million-per-year tax on Satriale to pay for the sewage treatment facility expansion.

E. Will either tax alternative permit Satriale to survive in the long run? In your opinion, what should the city of Musconetcong do about its sewage treatment problem?

**ST18.1 SOLUTION**

A. Solution to this problem requires that one look at several production and sales options available to the firm. One option is to produce and sell equal quantities of pork (P) and by-product (B). In this case, the firm sets relevant MC = MR.

\[
MC = MR_p + MR_B = MR
\]

\[
\begin{align*}
$60 &= $110 - 0.0001Q + $10 - 0.0002Q \\
0.0003Q &= 60 \\
Q &= 200,000 \text{ hogs}
\end{align*}
\]

Thus, the profit-maximizing output level for production and sale of equal quantities of P and B would be 200,000 hogs. However, the marginal revenues of both products must be positive at this sales level for this to be an optimal activity level.

Evaluated at 200,000 hogs:

\[
MR_p = $110 - 0.0001(200,000)
\]
Because the marginal revenue for B is negative, and Satriale can costlessly dump excess production, the sale of 200,000 units of B is suboptimal. This invalidates the entire solution developed above because output of P is being held down by the negative marginal revenue associated with B. The problem must be set up to recognize that Satriale will stop selling B at the point where its marginal revenue becomes zero because, given production for P, the marginal cost of B is zero.

Set:

\[ \text{MR}_B = \text{MC}_B \]

\[ 10 - 0.0002Q_B = 0 \]

\[ 0.0002Q_B = 10 \]

\[ Q_B = 50,000 \text{ units} \]

Thus, 50,000 units of B are the maximum that would be sold. Any excess units will be dumped into the city’s sewage treatment facility. The price for B at 50,000 units is:

\[ P_B = 10 - 0.0001Q_B \]

\[ = 10 - 0.0001(50,000) \]

\[ = 5 \]

To determine the optimal production of P (pork), set the marginal revenue of P equal to the marginal cost of hog processing because pork production is the only motive for processing more than 50,000 units:

\[ \text{MR}_p = \text{MC}_p = \text{MC}_Q \]

\[ 110 - 0.0001Q_p = 60 \]

\[ 0.0001Q_p = 50 \]

\[ Q_p = 500,000 \text{ units} \]
(Remember \( Q_p = Q \))

and

\[
P_p = \$110 - \$0.00005Q_p
\]

\[
= 110 - 0.00005(500,000)
\]

\[
= \$85
\]

Excess profits at the optimal activity level for Satriale are:

\[
\text{Excess profits} = \pi = \text{TR}_p + \text{TR}_B - TC
\]

\[
= P_p \times Q_p + P_B \times Q_B - TC_Q
\]

\[
= \$85(500,000) + \$5(50,000) - \$10,000,000 - \$60(500,000)
\]

\[
= \$2,750,000
\]

Because total costs include a normal return of 15% on $50 million in investment,

\[
\text{Total profits} = \text{Required return} + \text{Excess profits}
\]

\[
= 0.15(\$50,000,000) + \$2,750,000
\]

\[
= \$10,250,000
\]

B. With 500,000 hogs being processed, but only 50,000 units of B sold, dumping of B is:

\[
\text{Units B dumped} = \text{Units produced} - \text{Units sold}
\]

\[
= 500,000 - 50,000
\]

\[
= 450,000 \text{ units}
\]

C. In part A, it is shown that if all P and B produced is sold, an activity level of \( Q = 200,000 \) results in \( \text{MR}_B = -\$30 \). A dumping charge of \$35 per unit of B will cause Satriale to prefer to sell the last unit of B produced (and lose \$30) rather than pay a \$35 fine. Therefore, this fine, as does any fine greater than \$30, will eliminate dumping and cause Satriale to reduce processing to 200,000 hogs per year. This fine structure would undoubtedly reduce or eliminate the need for a new sewage treatment facility.

Although eliminating dumping is obviously attractive in the sense of reducing sewage treatment costs, the \$35 fine has the unfortunate consequence of cutting output
substantially. Pork prices rise to \( P_p = \$110 - \$0.00005(200,000) = \$100 \), and by product prices fall to \( P_B = \$10 - \$0.0001(200,000) = \$-10 \). This means Satriale will pay the pet food company \$10 per unit to accept all of its by-product sludge. Employment will undoubtedly fall as well. In addition to these obvious short-run effects, long-run implications may be especially serious. At \( Q = 200,000 \), Satriale’s excess profits are:

\[
\text{Excess profits} = TR_p + TR_B - TC \\
= \$110Q - \$0.00005Q^2 + \$10Q - \$0.0001Q^2 - \$10,000,000 - \$60Q \\
= \$110(200,000) - \$0.00005(200,000^2) + \$10(200,000) \\
- \$0.0001(200,000^2) - \$10,000,000 - \$60(200,000) \\
= \$4,000,000 (a loss)
\]

This means that total profits are:

\[
\text{Total profits} = \text{Required return} + \text{Excess profits} \\
= 0.15(\$50,000,000) + (-\$4,000,000) \\
= \$3,500,000
\]

This level of profit is insufficient to maintain investment. Although a \$35 dumping charge will eliminate dumping, it is likely to cause the firm to close down or move to some other location. The effect on employment in Musconetcong could be disastrous.

D. In the short run, a \$3 million tax on Satriale has no effect on dumping, output or employment. At the \( Q = 500,000 \) activity level, a \$3 million tax would reduce Satriale’s total profits to \$7,250,000, or \$250,000 below the required return on investment. However, following imposition of a \$3 million tax, the firm’s survival and total employment would be imperiled in the long run.

E. No. Satriale is not able to bear the burden of either tax alternative. Obviously, there is no single best alternative here. The highest fixed tax the company can bear in the long run is \$2.75 million, the full amount of excess profits. If the city places an extremely high priority on maintaining employment, perhaps a \$2.75 million tax on Satriale plus \$250,000 in general city tax revenues could be used to pay for the new sewage system treatment facility.

ST20.2 Benefit-cost Analysis Methodology. The benefit-cost approach is not new. The concept first surfaced in France in 1844. In this century, benefit-cost analysis has been widely used in the evaluation of river and harbor projects since as early as 1902. In the United
States, the 1936 Flood Control Act authorized federal assistance in developing flood-control programs “if the benefits to whomsoever they may accrue are in excess of the estimated costs.” By 1950, federal agency practice required the consideration of both direct and indirect benefits and costs and that unmeasured intangible influences be listed. Despite this long history of widespread use, it has only been since 1970 that public-sector managers have sought to broadly apply the principles of benefit-cost analysis to the evaluation of agricultural programs, rapid transit projects, highway construction, urban renewal projects, recreation facility construction, job training programs, health-care reform, education, research and development projects, and defense policies.

A. Briefly describe major similarities and differences between public-sector benefit-cost analysis and the private-sector capital budgeting process.

B. What major questions must be answered before meaningful benefit-cost analysis is possible?

C. Although the maximization of society’s wealth is the primary objective of benefit-cost analysis, it is important to recognize that constraints often limit government’s ability to achieve certain objectives. Enumerate some of the common economic, political, and social constraints faced in public-sector benefit-cost analysis.

D. In light of these constraints, discuss some of the pluses and minuses associated with the use of benefit-cost analysis as the foundation for a general approach to the allocation of government-entrusted resources.

ST20.2 SOLUTION

A. Benefit-cost analysis is a method for assessing the desirability of social programs and public-sector investment projects when it is necessary to take a long view of the public and private repercussions of such expenditures. As in the case of private-sector capital budgeting, benefit-cost analysis is frequently used in cases where the economic consequences of a program or project are likely to extend beyond 1 year in time. Unlike capital budgeting, however, benefit-cost analysis seeks to measure both direct private effects and indirect social implications of public-sector investment decisions and policy changes.

B. Before meaningful benefit-cost analysis is possible, a number of important policy questions must be answered. Among these policy questions are

- What is the social objective function that is to be maximized?
- What constraints are placed on the decision-making process?
What marginal social benefits and marginal social costs are to be included, and how are they to be measured?

What social investment criterion should be used?

What is the appropriate social rate of discount?

C. A number of constraints impinge upon society’s ability to maximize the social benefits derived from public expenditures. Among these constraints are

- **Physical constraints.** Program alternatives are limited by the available state of technology and by current production possibilities. For example, it is not yet possible to cure AIDS. Therefore, major emphasis for public policy in this area must be directed toward prevention, early detection and treatment, and research.

- **Legal constraints.** Domestic laws and international agreements place limits on property rights, the right of eminent domain, due process, constitutional limits on a particular agency’s activities, and so on. These legal constraints often play an important role in shaping the realm of public policy.

- **Administrative constraints.** Effective programs require competent management and execution. Qualified individuals must be available to carry out social objectives. Even the best-conceived program is doomed to failure unless managers and workers with the proper mix of technical and administrative skill are available.

- **Distributional constraints.** Social programs and public-sector investment projects affect different groups in different ways. The “gainers” are seldom the same as “losers.” When distributional impacts of public policy are of paramount concern, the objective of benefit-cost analysis might maximize subject to the constraint that equity considerations be met.

- **Political constraints.** That which is optimal may not be feasible because of slowness and inefficiency in the political process. Often what is best is tempered by what is possible, given the existence of strong competing special-interest groups.

- **Budget constraints.** Public agencies often work within the bounds of a predetermined budget. As a result, virtually all social programs and public-sector investment projects have some absolute financial ceiling above which the program cannot be expanded, irrespective of social benefits.

- **Social or religious constraints.** Social or religious constraints may limit the
range of feasible program alternatives. It is futile to attempt to combat teen pregnancy with public support for family planning if religious constraints prohibit the use of modern birth control methods.

D. An important potential use of benefit-cost analysis is as the structure for a general philosophy of government resource allocation. As such, the results of benefit-cost studies have the potential to serve as a guide for resource-allocation decisions within and among government programs and investment projects in agriculture, defense, education, healthcare, welfare, and other areas. The objective of such a comprehensive benefit-cost approach to government would be to maximize the net present-value of the difference between the marginal social benefits and the marginal social costs derived from all social programs and public-sector investment projects.

Although a benefit-cost approach to evaluating all levels and forms of government is conceptually appealing on an efficiency basis, it suffers from a number of serious practical limitations. Perhaps most importantly, the measurement of marginal social benefits and marginal social costs for goods and services that are not or cannot be provided by the private sector is often primitive, at best. Measurement systems have not been sufficiently refined or standardized to permit meaningful comparisons among the social net present-value of “Star Wars” defense systems, the guaranteed student loan program for college students, funding for AIDS research, Medicare, and Medicaid. A further problem arises because benefit-cost analysis is largely restricted to a consideration of the efficiency objective; equity-related considerations are seldom accorded full treatment in benefit-cost analysis. In addition, as discussed previously, a number of important economic, political, and social constraints limit the effectiveness of benefit-cost analysis. As a result, significant problems arise when a given social program or public-sector investment project is designed to meet efficiency and equity-related objectives.

For these reasons, benefit-cost analysis is traditionally viewed within the narrow context of a decision technique that is helpful in focusing interest on the economic consequences of proposed social programs and public-sector investment projects. Its greatest use is in comparing programs and projects that are designed to achieve the same or similar objectives and as a tool for focusing resources on the best use of resources intended to meet a given social objective.