The premiums paid for currency options depend on various factors that must be monitored when anticipating future movements in currency option premiums. Since participants in the currency options market typically take positions based on their expectations of how the premiums will change over time, they can benefit from understanding how options are priced.

**Boundary Conditions**

The first step in pricing currency options is to recognize boundary conditions that force the option premium to be within lower and upper bounds.

**Lower Bounds**

The call option premium ($C$) has a lower bound of at least zero or the spread between the underlying spot exchange rate ($S$) and the exercise price ($X$), whichever is greater, as shown below:

$$C = \text{MAX}(0, S - X)$$

This floor is enforced by arbitrage restrictions. For example, assume that the premium on a British pound call option is $.01, while the spot rate of the pound is $1.62 and the exercise price is $1.60. In this example, the spread ($S - X$) exceeds the call premium, which would allow for arbitrage. One could purchase the call option for $.01 per unit, immediately exercise the option at $1.60 per pound, and then sell the pounds in the spot market for $1.62 per unit. This would generate an immediate profit of $.01 per unit. Arbitrage would continue until the market forces realigned the spread ($S - X$) to be less than or equal to the call premium.

The put option premium ($P$) has a lower bound of zero or the spread between the exercise price ($X$) and the underlying spot exchange rate ($S$), whichever is greater, as shown below:

$$P = \text{MAX}(0, X - S)$$
This floor is also enforced by arbitrage restrictions. For example, assume that the premium on a British pound put option is \(0.02\), while the spot rate of the pound is \$1.60\ and the exercise price is \$1.63\). One could purchase the pound put option for \$0.02\ per unit, purchase pounds in the spot market at \$1.60\, and immediately exercise the option by selling the pounds at \$1.63\ per unit. This would generate an immediate profit of \$0.01\ per unit. Arbitrage would continue until the market forces realigned the spread \((X - S)\) to be less than or equal to the put premium.

**Upper Bounds**

The upper bound for a call option premium is equal to the spot exchange rate \((S)\):

\[
C = S
\]

If the call option premium ever exceeds the spot exchange rate, one could engage in arbitrage by selling call options for a higher price per unit than the cost of purchasing the underlying currency. Even if those call options are exercised, one could provide the currency that was purchased earlier (the call option was covered). The arbitrage profit in this example is the difference between the amount received when selling the premium and the cost of purchasing the currency in the spot market. Arbitrage would occur until the call option’s premium was less than or equal to the spot rate.

The upper bound for a put option is equal to the option’s exercise price \((X)\):

\[
P = X
\]

If the put option premium ever exceeds the exercise price, one could engage in arbitrage by selling put options. Even if the put options are exercised, the proceeds received from selling the put options exceed the price paid (which is the exercise price) at the time of exercise.

Given these boundaries that are enforced by arbitrage, option premiums lie within these boundaries.

**Application of Pricing Models**

Although boundary conditions can be used to determine the possible range for a currency option’s premium, they do not precisely indicate the appropriate premium for the option. However, pricing models have been developed to price currency options. Based on information about an option (such as the exercise price and time to maturity) and about the currency (such as its spot rate, standard deviation, and interest rate), pricing models can derive the premium on a currency option. The currency option pricing model of Biger and Hull\(^1\) is shown below:

\[
C = e^{-rT}S \cdot N(d_1) - e^{-rT}X \cdot N(d_2 - \sigma \sqrt{T})
\]

where

\[ d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - r^* + (\sigma^2/2)\right)T}{\sigma \sqrt{T}} \]

- \(C\) = price of the currency call option
- \(S\) = underlying spot exchange rate
- \(X\) = exercise price
- \(r\) = U.S. riskless rate of interest
- \(r^*\) = foreign riskless rate of interest
- \(\sigma\) = instantaneous standard deviation of the return on a holding of foreign currency
- \(T\) = option's time maturity expressed as a fraction of a year
- \(N(\cdot)\) = standard normal cumulative distribution function

This equation is based on the stock option pricing model (OPM) when allowing for continuous dividends. Since the interest gained on holding a foreign security \((r^*)\) is equivalent to a continuously paid dividend on a stock share, this version of the OPM holds completely. The key transformation in adapting the stock OPM to value currency options is the substitution of exchange rates for stock prices. Thus, the percentage change of exchange rates is assumed to follow a diffusion process with constant mean and variance.

Bodurtha and Courtadon\(^2\) have tested the predictive ability of the currency option of the pricing model. They computed pricing errors from the model using 3,326 call options. The model’s average percentage pricing error for call options was \(-6.90\) percent, which is smaller than the corresponding error reported for the dividend-adjusted Black-Scholes stock OPM. Hence, the currency option pricing model has been more accurate than the counterpart stock OPM.

The model developed by Biger and Hull is sometimes referred to as the European model because it does not account for early exercise. European currency options do not allow for early exercise (before the expiration date), while American currency options do allow for early exercise. The extra flexibility of American currency options may justify a higher premium on American currency options than on European currency options with similar characteristics. However, there is not a closed-form model for pricing American currency options. Although various techniques are used to price American currency options, the European model is commonly applied to price American currency options because the European model can be just as accurate.

Bodurtha and Courtadon found that the application of an American currency options pricing model does not improve predictive accuracy. Their average percentage pricing error was \(-7.07\) percent for all sample call options when using the American model.

Given all other parameters, the currency option pricing model can be used to impute the standard deviation \(\sigma\). This implied parameter represents the option’s market assessment of currency volatility over the life of the option.

Pricing Currency Put Options According to Put-Call Parity

Given the premium of a European call option (called \( C \)), the premium for a European put option (called \( P \)) on the same currency and same exercise price (\( X \)) can be derived from put-call parity, as shown below:

\[
P = C + Xe^{-rT} - Se^{-r*T}
\]

where

\[
\begin{align*}
  r & = \text{U.S. riskless rate of interest} \\
  r^* & = \text{foreign riskless rate of interest} \\
  T & = \text{option's time to maturity expressed as a fraction of the year}
\end{align*}
\]

If the actual put option premium is less than what is suggested by the put-call parity equation above, arbitrage can be conducted. Specifically, one could (1) buy the put option, (2) sell the call option, and (3) buy the underlying currency. The purchases are financed with the proceeds from selling the call option and from borrowing at the rate \( r \). Meanwhile, the foreign currency that was purchased can be deposited to earn the foreign rate \( r^* \). Regardless of the scenario for the path of the currency’s exchange rate movement over the life of the option, the arbitrage will result in a profit. First, if the exchange rate is equal to the exercise price such that each option expires worthless, the foreign currency can be converted in the spot market to dollars, and this amount will exceed the amount required to repay the loan. Second, if the foreign currency appreciates and therefore exceeds the exercise price, there will be a loss from the call option being exercised. Although the put option will expire, the foreign currency will be converted in the spot market to dollars, and this amount will exceed the amount required to repay the loan and the amount of the loss on the call option. Third, if the foreign currency depreciates and therefore is below the exercise price, the amount received from exercising the put option plus the amount received from converting the foreign currency to dollars will exceed the amount required to repay the loan. Since the arbitrage generates a profit under any exchange rate scenario, it will force an adjustment in the option premiums so that put-call parity is no longer violated.

If the actual put option premium is more than what is suggested by put-call parity, arbitrage would again be possible. The arbitrage strategy would be the reverse of that used when the actual put option premium was less than what is suggested by put-call parity (as just described). The arbitrage would force an adjustment in option premiums so that put-call parity is no longer violated. The arbitrage that can be applied when there is a violation of put-call parity on American currency options differs slightly from the arbitrage applicable to European currency options. Nevertheless, the concept still holds that the premium of a currency put option can be determined according to the premium of a call option on the same currency and the same exercise price.