Inventory represents a significant investment of capital for most companies. Inventory ties up money that could be used more productively elsewhere. Thus, effective inventory management offers the potential for significant cost savings. Furthermore, quality, product engineering, prices, overtime, excess capacity, ability to respond to customers (due-date performance), lead times, and overall profitability are all affected by inventory levels.

There are many ways to manage inventory costs, including the EOQ model, JIT, and the theory of constraints (TOC). All three methods offer ways of reducing inventory costs. The best approach usually depends on the nature of the organization as well as the nature of the inventory itself.

Describing how inventory policy can be used to reduce costs and help organizations strengthen their competitive position is the main purpose of this chapter. First, we review just-in-case inventory management—a traditional inventory model based on anticipated demand. Learning the basics of this model and its underlying conceptual foundation will help us understand where it can still be appropriately applied. Understanding just-in-case inventory management also provides the necessary...
background for grasping the advantages of inventory management methods that are used in the contemporary manufacturing environment. These methods include JIT and the theory of constraints. To fully appreciate the theory of constraints, a brief introduction to constrained optimization (linear programming) is also needed. Although the focus of this chapter is inventory management, the theory of constraints is much more than an inventory management technique, so we also explore what is called constraint accounting.

### Just-in-Case Inventory Management

Inventory management is concerned with managing inventory costs. Three types of inventory costs can be readily identified with inventory: (1) the cost of acquiring inventory (other than the cost of the good itself), (2) the cost of holding inventory, and (3) the cost of not having inventory on hand (stock-out costs) when needed.

If the inventory is acquired from an outside source, then these inventory-acquisition costs are known as ordering costs. Ordering costs are the costs of placing and receiving an order. Examples include the costs of processing an order (clerical costs and documents), insurance for shipment, and unloading costs. If the material or good is produced internally, then the acquisition costs are called setup costs. Setup costs are the costs of preparing equipment and facilities so they can be used to produce a particular product or component. Examples are wages of idled production workers, the cost of idled production facilities (lost income), and the costs of test runs (labour, materials, and overhead). Ordering costs and setup costs are similar in nature—both represent costs that must be incurred to acquire inventory. They differ only in the nature of the prerequisite activity (filling out and placing an order versus configuring equipment and facilities). Thus, in the discussion that follows, any reference to ordering costs can be viewed as a reference to setup costs.

Carrying costs are the costs of holding inventory. Examples include insurance, inventory taxes, obsolescence, the opportunity cost of funds tied up in inventory, handling costs, and storage space.

If demand is not known with certainty, a third category of inventory costs—called stock-out costs—exists. Stock-out costs are the costs of not having a product available when demanded by a customer. Examples are lost sales (both current and future), the costs of expediting (increased transportation charges, overtime, and so on), and the costs of interrupted production.

### Justifying Inventory

Effective inventory management requires that inventory-related costs be minimized. Minimizing carrying costs favours ordering or producing in small lot sizes, whereas minimizing ordering costs favours large, infrequent orders (minimization of setup costs favours long, infrequent production runs). The need to balance these two sets of costs so that the total cost of carrying and ordering can be minimized is one reason organizations choose to carry inventory.

Demand uncertainty is a second major reason for holding inventory. If the demand for materials or products is greater than expected, inventory can serve as a buffer, giving organizations the ability to meet delivery dates (thus keeping customers satisfied). Although balancing conflicting costs and dealing with uncertainty are the two most frequently cited reasons for carrying inventories, other reasons exist.

Inventories of parts and materials are often viewed as necessary because of supply uncertainties. That is, inventory buffers of parts and materials are needed to keep production flowing in case of late deliveries or no deliveries. ( Strikes, bad weather, and bankruptcy are examples of uncertain events that can cause an interruption in supply.) Unreliable production processes may also create a demand for producing extra inventory. For example, a company may decide to produce more units than needed to
meet demand because the production process usually yields a large number of nonconforming units. Similarly, buffers of inventories may be required to continue supplying customers or processes with goods even if a process goes down because of a failed machine. Finally, organizations may acquire larger inventories than normal to take advantage of quantity discounts or to avoid anticipated price increases. Exhibit 18-1 summarizes the reasons typically offered for carrying inventory. It is important to realize that these reasons are given to justify carrying inventories. A host of other reasons can be offered that encourage the carrying of inventories. For example, performance measures such as measures of machine and labour efficiency may promote the buildup of inventories.

**Economic Order Quantity Model**

Of the nine reasons for holding inventory listed in Exhibit 18-1, the first reason is directly concerned with the trade-off between acquisition and carrying costs. Most of the other reasons are concerned directly or indirectly with stock-out costs, with the exception of the last two (which are concerned with managing the cost of the good itself). Initially, we will assume away the stock-out cost problem and focus only on the objective of balancing acquisition costs with carrying costs. To develop an inventory policy that deals with the trade-offs between these two costs, two basic questions must be addressed:

1. How much should be ordered (or produced) to minimize inventory costs?
2. When should the order be placed (or the setup done)?

The first question needs to be addressed before the second can be answered.

**Minimizing Total Ordering and Carrying Costs** Assuming that demand is known, the total ordering (or setup) and carrying cost can be described by the following equation:

\[
TC = O \frac{D}{Q} + \frac{CQ}{2}
\]

\[= \text{Ordering (or setup) cost} + \text{Carrying cost}\]  \hspace{1cm} (18.1)

where

- \(TC\) = The total ordering (or setup) and carrying cost
- \(O\) = Ordering costs, the cost of placing and receiving an order (or the cost of setting up a production run)
- \(Q\) = The number of units ordered each time an order is placed (or the lot size for production)
- \(D\) = The known annual demand (annual sales)
- \(C\) = Carrying costs, the cost of carrying one unit of stock for one year

---

**Traditional Reasons for Carrying Inventory**

1. To balance ordering or setup costs and carrying costs
2. Demand uncertainty
3. Machine failure
4. Defective parts
5. Unavailable parts
6. Late delivery of parts
7. Unreliable production processes
8. To take advantage of discounts
9. To hedge against future price increases
Using Equation 18.1, the cost of inventory can be computed for any organization that carries inventories, although the inventory cost model using setup costs and lot size as inputs pertains only to manufacturers. Ordering or setup cost is the number of orders (or setups), \( \frac{D}{Q} \), multiplied by the ordering or setup cost (\( O \)). Carrying cost for the year is \( \frac{CQ}{2} \), which is simply the average inventory on hand (\( \frac{Q}{2} \)) multiplied by the carrying cost (or setup cost) per unit (\( C \)). Assuming average inventory to be \( \frac{Q}{2} \) is equivalent to assuming that inventory is consumed uniformly.

Equation 18.1 can be used to calculate the total inventory cost for any \( Q \). However, the objective of inventory management is to identify the order quantity (or lot size) that minimizes this total cost (\( TC \)). Thus, the decision variable is the order quantity (or lot size). This quantity that minimizes the total cost is called the economic order quantity (EOQ) and is derived by taking the first derivative of Equation 18.1 with respect to \( Q \) and solving for \( Q \):

\[
Q = \text{EOQ} = \sqrt{\frac{DO}{C}}
\]  

(18.2)

Cornerstone 18-1 illustrates both EOQ and the inventory cost equation.

When to Order or Produce  
Not only must we know how much to order (or produce), but we also must know when to place an order (or to set up for production). Avoiding stock-out costs is a key element in determining when to place an order.

The reorder point is the point in time when a new order should be placed (or setup started). It is a function of the EOQ, the lead time, and the rate at which inventory is depleted. It is expressed in level of inventory; that is, when the inventory of a part reaches a certain level, it triggers the placement of a new order.

Lead time is the time required to receive the economic order quantity once an order is placed or a setup is initiated.

To avoid stock-out costs and to minimize carrying costs, an order should be placed so that it arrives just as the last item in inventory is used. Knowing the rate of usage and lead time allows us to compute the reorder point that accomplishes these objectives:

\[
\text{Reorder point} = \text{Rate of usage} \times \text{Lead time} \quad (18.3)
\]

If the demand for the part or product is not known with certainty, the possibility of stock-out exists. To avoid this problem, organizations often choose to carry safety stock.

Safety stock is extra inventory carried to serve as insurance against fluctuations in demand. Safety stock is computed by multiplying the lead time by the difference between the maximum rate of usage and the average rate of usage. With the presence of safety stock, the reorder point is computed as follows:

\[
\text{Reorder point} = (\text{Average rate of usage} \times \text{Lead time}) + \text{Safety stock} \quad (18.4)
\]

Cornerstone 18-2 illustrates reordering with both certainty and uncertainty.

An Example Involving Setups

The same inventory management concepts apply to settings where inventory is manufactured. To illustrate, consider Expedition Company, a large manufacturer of garden and lawn equipment. One plant in British Columbia produces edgers. The manager of this plant is trying to determine the size of the production runs for the edgers. He is concerned that the current lot size is too large and wants to identify the quantity that should be produced to minimize the sum of the carrying and setup costs. He also wants to avoid stock-outs, since any stock-out would cause problems with the plant’s network of retailers.

\[
1\frac{d(TC)}{dT} = \frac{C}{2} - \frac{DO}{Q^2} = 0; \text{ thus, } Q^2 = 2DO/C \text{ and } Q = \sqrt{2DO/C}.
\]
The HOW and WHY of Calculating the EOQ

Information:
Mantener Corporation does warranty work for a major producer of DVD players. The following values apply for a part used in the repair of the DVD players (the part is purchased from external suppliers):

\[
\begin{align*}
D &= 25,000 \text{ units} \\
Q &= 500 \text{ units} \\
O &= $40 \text{ per order} \\
C &= $2 \text{ per unit}
\end{align*}
\]

Why:
Inventory cost is defined as the sum of ordering cost (or setup cost) plus carrying cost. The quantity ordered (or produced) determines the inventory cost. The quantity that minimizes total inventory cost is called the economic order quantity (EOQ).

Required:
1. For Mantener, calculate the ordering cost, the carrying cost, and the total cost associated with an order size of 500 units.
2. Calculate the EOQ and its associated ordering cost, carrying cost, and total cost. Compare and comment on the EOQ relative to the current order quantity.
3. What if Mantener enters into an exclusive supplier agreement with one supplier who will supply all of the demands with smaller more frequent orders? Under this arrangement, the ordering cost is reduced to $0.40 per order. Calculate the new EOQ and comment on the implications.

Solution:
1. Number of orders = \( D/Q = 25,000/500 = 50 \). Ordering cost = \( O \times D/Q = 50 \times $40 = $2,000 \). Carrying cost = \( CQ/2 = ($2 \times 500)/2 = $500 \). 
   \[ TC = $2,000 + $500 = $2,500. \]
2. \[ EOQ = \sqrt{(2 \times 25,000 \times $40)/2} \]
   \[ = \sqrt{1,000,000} \]
   \[ = 1,000 \]
   \[ TC = \text{Ordering cost} + \text{Carrying cost} = ($40 \times 25,000/1,000) \]
   \[ + ([($2 \times 1,000)/2] \]
   \[ = $1,000 + $1,000 = $2,000 \]
   Relative to the current order size, the economic order quantity is larger, with fewer orders placed; however, the total cost is $500 less. Notice that Carrying cost = Ordering cost for EOQ.
3. \[ EOQ = \sqrt{(2 \times 25,000 \times $0.40)/2} \]
   \[ = \sqrt{10,000} \]
   \[ = 100 \]
   \[ TC = \text{Ordering cost} + \text{Carrying cost} = ($0.40 \times 25,000/100) \]
   \[ + ([($2 \times 100)/2] \]
   \[ = $100 + $100 = $200 \]
   Smaller, more frequent orders can dramatically reduce the cost of inventory. This result foreshadows the power of just-in-time purchasing.
To help him in his decision, the controller has supplied the following information:

- Average demand for edgers: 720 per day
- Maximum demand for edgers: 780 per day
- Annual demand for edgers: 180,000
- Unit carrying cost: $4
- Setup cost: $10,000
- Lead time: 22 days

Based on the preceding information, the economic order quantity and the reorder point are computed in Exhibit 18-3. As the computation illustrates, the edgers...
should be produced in batches of 30,000, and a new setup should be started when the supply of edgers drops to 17,160.

**EOQ and Inventory Management**

The traditional approach to managing inventory has been referred to as a *just-in-case system.* In some settings, a just-in-case inventory system is entirely appropriate.

**EOQ and Reorder Point Illustrated**

\[
\text{EOQ} = \sqrt{\frac{2DO}{C}}
\]

\[
= \sqrt{\frac{2 \times 180,000 \times \$10,000}{\$4}}
\]

\[
= \sqrt{900,000,000}
\]

\[
= 30,000 \text{ edgers}
\]

Safety stock:

<table>
<thead>
<tr>
<th>Maximum usage</th>
<th>780</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average usage</td>
<td>(720)</td>
</tr>
<tr>
<td>Difference</td>
<td>60</td>
</tr>
<tr>
<td>Lead time</td>
<td>× 22</td>
</tr>
<tr>
<td>Safety stock</td>
<td>1,320</td>
</tr>
</tbody>
</table>

Reorder point = (Average usage × Lead time) + Safety stock

\[
= (720 \times 22) + 1,320
\]

\[
= 17,160 \text{ edgers}
\]

For example, hospitals need inventories of medicines, drugs, and other critical supplies on hand at all times so that life-threatening situations can be handled. Using an economic order quantity coupled with safety stock would seem eminently sensible in such an environment. Relying on a critical drug to arrive just in time to save a heart attack victim is simply not practical. Furthermore, many smaller retail stores, manufacturers, and services may not have the buying power to command alternative inventory management systems such as just-in-time purchasing.

As the edger example illustrates (Exhibit 18-3), the EOQ model is very useful in identifying the optimal trade-off between inventory carrying costs and setup costs. It also is useful in helping to deal with uncertainty by using safety stock. The historical importance of the EOQ model in many Canadian industries can be better appreciated by understanding the nature of the traditional manufacturing environment. This environment has been characterized by the mass production of a few standardized products that typically have a very high setup cost. The production of the edgers fits this pattern. The high setup cost encouraged a large batch size: 30,000 units. The annual demand of 180,000 units can be satisfied using only six batches. Thus, production runs for these firms tended to be quite long. Furthermore, diversity was viewed as being costly and was avoided. Producing variations of the product can be quite expensive, especially since additional, special features would usually demand even more expensive and frequent setups—the reason for the standardized products.

**JIT Inventory Management**

The manufacturing environment for many of these traditional, large-batch, high-setup-cost firms has changed dramatically in the past two or three decades. For one thing, the competitive markets are no longer defined by national boundaries. Advances in transportation and communication have contributed significantly to the creation of global competition. Advances in technology have contributed to shorter life cycles for products, and product diversity has increased. Foreign firms offering higher-quality, lower-cost products with specialized features have created tremendous pressures for our domestic large-batch, high-setup-cost firms to increase both quality and product diversity while simultaneously reducing total costs. These competitive pressures have led many firms to abandon the EOQ model in favour of a JIT approach. JIT has two strategic objectives: to increase profits and to improve a firm’s competitive position. These two objectives are achieved by controlling costs (enabling better price competition and increased profits), improving delivery performance, and improving quality. JIT offers increased cost efficiency and simultaneously has the flexibility to respond to customer demands for better quality and more variety. Quality, flexibility, and cost efficiency are foundational principles for world-class competition.

**Just-in-time inventory management** represents the continual pursuit of productivity through the elimination of waste. JIT is an essential and integral part of lean manufacturing. Thus, when we speak of JIT practices, they are virtually identical to lean practices. Both have the objective of eliminating waste; lean manufacturing uses JIT concepts in its waste elimination approach. Non-value-added activities are a major source of waste. From Chapter 14, we know that non-value-added activities are either unnecessary or necessary, but inefficient and improvable. Necessary activities are essential to the business and/or are of value to customers. Eliminating non-value-added activities is a major thrust of JIT, but it is also a basic objective of any company following the path of continuous improvement—regardless of whether JIT is being used.

Clearly, JIT is much more than an inventory management system. Inventories, however, are particularly viewed as representing waste. They tie up resources such as cash, space, and labour. They also conceal inefficiencies in production and increase the complexity of a firm’s information system. Thus, even though JIT focuses on more than inventory management, control of inventory is an important ancillary benefit. In this chapter, the inventory dimension of JIT is emphasized. In Chapter 13, other benefits and features of JIT were described. Chapter 14, in particular, focused on non-value-added activity analysis.
A Pull System

JIT is a manufacturing approach that maintains that goods should be pulled through the system by present demand rather than pushed through the system on a fixed schedule based on anticipated demand. Many fast-food restaurants, like Burger King, use a pull system to control their finished goods inventory. When a customer orders a hamburger, it is taken from the rack. When the number of hamburgers gets too low, the cooks make new hamburgers. Customer demand pulls the materials through the system. This same principle is used in manufacturing settings. Each operation produces only what is necessary to satisfy the demand of the succeeding operation. The material or subassembly arrives just in time for production to occur so that demand can be met.

One effect of JIT is to reduce inventories to very low levels. The pursuit of insignificant levels of inventories is vital to the success of JIT. This idea of pursuing insignificant inventories, however, necessarily challenges the traditional reasons for holding inventories (see Exhibit 18-1). These reasons are no longer viewed as valid.

According to the traditional view, inventories solve some underlying problem related to each of the reasons listed in Exhibit 18-1. For example, the problem of resolving the conflict between ordering or setup costs and carrying costs is solved by selecting an inventory level that minimizes the sum of these costs. If demand is greater than expected or if production is reduced by breakdowns and production inefficiencies, then inventories serve as buffers, providing customers with products that otherwise might not have been available. Similarly, inventories can prevent stockouts caused by late delivery of material, defective parts, and failures of machines used to produce subassemblies. Finally, inventories are often the solution to the problem of buying the best materials for the least cost through the use of quantity discounts.

JIT/Lean practices refuse to use inventories as the solution to these problems. In fact, the JIT approach can be seen as substituting information for inventories. Companies must track materials and finished goods more carefully. JIT inventory management offers alternative solutions that do not require high inventories.

The JIT/Lean Approach

JIT takes a radically different approach to minimizing total carrying and setup costs. The traditional approach accepts the existence of setup costs and then finds the order quantity that best balances the two categories of costs. JIT, on the other hand, does not accept setup costs (or ordering costs) as a given; rather, JIT attempts to drive these costs to zero. If setup costs and ordering costs become insignificant, the only remaining cost to minimize is carrying cost, which is accomplished by reducing inventories to very low levels. This approach explains the push for zero inventories in a JIT system.

Long-Term Contracts, Continuous Replenishment, and Electronic Data Interchange

Ordering costs are reduced by developing close relationships with suppliers. Negotiating long-term contracts for the supply of outside materials will obviously reduce the number of orders and the associated ordering costs. Retailers have found a way to reduce ordering costs by adopting an arrangement known as continuous replenishment. Continuous replenishment means a manufacturer assumes the inventory management function for the retailer. The manufacturer tells the retailer when and how much stock to reorder. The retailer reviews the recommendation and approves the order if it makes sense.

Procter & Gamble and Wal-Mart, for example, use this arrangement.3 The arrangement has reduced inventories for Wal-Mart and has also reduced stockout problems. Additionally, Wal-Mart often sells Procter & Gamble’s goods before it has to pay for them. Procter & Gamble, on the other hand, has

become a preferred supplier, has more and better shelf space, and also has less demand uncertainty. The ability to project demand more accurately allows Procter & Gamble to produce and deliver continuously in smaller lots—a goal of JIT manufacturing.

Similar arrangements can be made between manufacturers and suppliers. The process of continuous replenishment is facilitated by electronic data interchange. Electronic data interchange (EDI) allows suppliers access to a buyer’s online database. By knowing the buyer’s production schedule (in the case of a manufacturer), the supplier can deliver the needed parts where they are needed just in time for their use. EDI involves no paper—no purchase orders or invoices. The supplier uses the production schedule, which is in the database, to determine its own production and delivery schedules. When the parts are shipped, an electronic message is sent from the supplier to the buyer that a shipment is en route. When the parts arrive, a bar code is scanned with an electronic wand, and this initiates payment for the goods. Clearly, EDI requires a close working arrangement between the supplier and the buyer—they almost operate as one company rather than two separate companies.

Reducing Setup Times Reducing setup times requires a company to search for new, more efficient ways to accomplish setup. Fortunately, experience has indicated that dramatic reductions in setup times can be achieved. A classic example is that of Harley-Davidson. Upon adopting a JIT system, Harley-Davidson reduced setup time by more than 75 percent on the machines evaluated. In some cases, Harley-Davidson was able to reduce the setup times from hours to minutes. Other companies have experienced similar results.

Due-Date Performance Due-date performance is a measure of a firm’s ability to respond to customer needs. In the past, finished goods inventories have been used to ensure that a firm is able to meet a requested delivery date. JIT solves the problem of due-date performance not by building inventory but by dramatically reducing lead times. Shorter lead times increase a firm’s ability to meet requested delivery dates and to respond quickly to the demands of the market. Thus, the firm’s competitiveness is improved. JIT cuts lead times by reducing setup times, improving quality, and using cellular manufacturing. Moreover, since the JIT approach is a fundamental part of lean manufacturing, JIT practices are essentially lean manufacturing practices. Thus, lean manufacturing is a logical extension and integration of JIT concepts.

JIT can also have a dramatic effect on lead time, production schedules, and profits. Dell Computer Corporation has tuned its JIT system so that an order for a customized personal computer that comes in over the Internet at 9 a.m. can be on a delivery truck to the customer by 9 p.m. In addition, Dell’s low-cost production system allows it to underprice its rivals by 10 to 15 percent. This combination has made Dell the envy of the personal computer industry and has enabled the company to grow at five times the industry rate. How does the company’s JIT system deliver lower costs? “While machines from Compaq and IBM can languish on dealer shelves for two months, Dell does not start ordering components and assembling computers until an order is booked. That may sound like no biggie, but the price of PC parts can fall rapidly in just a few months. By ordering right before assembly, Dell figures its parts, on average, are 60 days newer than those in an IBM or Compaq machine sold at the same time. That can translate into a 6% profit advantage in components alone.”


Most companies experience at least a 90 percent reduction in lead times when they implement JIT/lean practices.

**Avoidance of Shutdown and Process Reliability**

Most shutdowns occur for one of three reasons: machine failure, defective material or subassembly, and unavailability of a material or subassembly. Holding inventories is one solution to all three problems.

Those espousing the JIT approach claim that inventories do not solve the problems but cover up or hide them. JIT proponents use the analogy of rocks in a lake. The rocks represent the three problems, and the water represents inventories. If the lake is deep (inventories are high), then the rocks are never exposed, and managers can pretend they do not exist. When inventories are reduced to zero, the rocks are exposed and can no longer be ignored. JIT solves the three problems by emphasizing total preventive maintenance and total quality control in addition to building the right kind of relationship with suppliers.

**Total Preventive Maintenance**  Zero machine failures is the goal of total preventive maintenance. By paying more attention to preventive maintenance, most machine breakdowns can be avoided. This objective is easier to attain in a JIT environment because of the interdisciplinary labour philosophy. It is fairly common for a cell worker to be trained in maintenance of the machines he or she operates. Because of the pull-through nature of JIT, cell workers may have idle manufacturing time. Some of this time, then, can be used productively by having the cell workers involved in preventive maintenance.

**Total Quality Control**  The problem of defective parts is solved by striving for zero defects. Because JIT manufacturing does not rely on inventories to replace defective parts or materials, the emphasis on quality for both internally produced and externally purchased materials increases significantly. The outcome is impressive: the number of rejected parts tends to fall by 75 to 90 percent. Decreasing defective parts also diminishes the justification for inventories based on unreliable processes.

**The Kanban System**  To ensure that parts or materials are available when needed, the Kanban system is employed. This is an information system that controls production through the use of markers or cards. The Kanban system is responsible for ensuring that the necessary products (or parts) are produced (or acquired) in the necessary quantities at the necessary time. It is the heart of the JIT inventory management system.

A Kanban system uses cards or markers, which are plastic, cardboard, or metal plates measuring four inches by eight inches. The Kanban is usually placed in a vinyl sack and attached to the part or a container holding the needed parts.

A basic Kanban system uses three cards: a withdrawal Kanban, a production Kanban, and a vendor Kanban. The first two control the movement of work among the manufacturing processes, while the third controls movement of parts between the processes and outside suppliers. A withdrawal Kanban specifies the quantity that a subsequent process should withdraw from the preceding process. A production Kanban specifies the quantity that the preceding process should produce. A vendor Kanban is used to notify suppliers to deliver more parts; it also specifies when the parts are needed. The three Kanbans are illustrated in Exhibit 18-4.

How Kanban cards are used to control the work flow can be illustrated with a simple example. Assume that two processes are needed to manufacture a product. The first process (CB Assembly) builds and tests printed circuit boards (using a U-shaped manufacturing cell). The second process (Final Assembly) puts eight circuit boards into a subassembly purchased from an outside supplier. The final product is a personal computer.

Exhibit 18-5 provides the plant layout corresponding to the manufacture of the personal computers. Refer to the exhibit as the steps involved in using Kanbans are outlined.

Consider first the movement of work between the two processing areas. Assume that eight circuit boards are placed in a container and that one such container is
located in the CB stores area. Attached to this container is a production Kanban (P-Kanban). A second container with eight circuit boards is located near the Final Assembly line (the withdrawal store) with a withdrawal Kanban (W-Kanban). Now assume that the production schedule calls for the immediate assembly of a computer. The Kanban setups can be described as follows:

The Kanban System

A. Withdrawal Kanban

<table>
<thead>
<tr>
<th>Item No.</th>
<th>15670T07</th>
<th>Preceding Process</th>
<th>Item Name</th>
<th>Circuit Board</th>
<th>CB Assembly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computer Type</td>
<td>TR6547 PC</td>
<td>Subsequent Process</td>
<td>Box Capacity</td>
<td>8</td>
<td>Final Assembly</td>
</tr>
<tr>
<td>Box Type</td>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

B. Production Kanban

<table>
<thead>
<tr>
<th>Item No.</th>
<th>15670T07</th>
<th>Preceding Process</th>
<th>Item Name</th>
<th>Circuit Board</th>
<th>CB Assembly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computer Type</td>
<td>TR6547 PC</td>
<td></td>
<td>Box Capacity</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Box Type</td>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

C. Vendor Kanban

<table>
<thead>
<tr>
<th>Item No.</th>
<th>15670T07</th>
<th>Name of Receiving Company</th>
<th>Item Name</th>
<th>Computer Casting</th>
<th>Electro PC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Box Capacity</td>
<td>8</td>
<td>Receiving Gate</td>
<td>Box Type</td>
<td>A</td>
<td>75</td>
</tr>
<tr>
<td>Time to Deliver</td>
<td>8:30 A.M., 12:30 P.M., 2:30 P.M.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Name of Supplier</td>
<td>Gerry Supply</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. A worker from the Final Assembly line goes to the withdrawal store, removes the eight circuit boards, and places them into production. The worker also removes the withdrawal Kanban and places it on the withdrawal post.
2. The withdrawal Kanban on the post signals that the Final Assembly unit needs an additional eight circuit boards.
3. A worker from Final Assembly (or a material handler called a carrier) removes the withdrawal Kanban from the post and carries it to the CB stores area.
4. At the CB stores area, the carrier removes the production Kanban from the container of eight circuit boards and places it on the production ordering post.
5. The carrier next attaches the withdrawal Kanban to the container of parts and carries the container back to the Final Assembly area. Assembly of the next computer can begin.
6. The production Kanban on the production ordering post signals the workers of CB Assembly to begin producing another lot of circuit boards. The production Kanban is removed and accompanies the units as they are produced.

7. When the lot of eight circuit boards is completed, the units are placed in a container in the CB stores area with the production Kanban attached. The cycle is then repeated.

The use of Kanbans ensures that the subsequent process (Final Assembly) withdraws the circuit boards from the preceding process (CB Assembly) in the necessary quantity at the appropriate time. The Kanban system also controls the preceding process by allowing it to produce only the quantities withdrawn by the subsequent process. In this way, inventories are kept at a minimum, and the components arrive just in time to be used.

Essentially, the same steps are followed for a purchased subassembly. The only difference is the use of a vendor Kanban in place of a production Kanban. A vendor Kanban on a vendor post signals to the supplier that another order is needed. As with the circuit boards, the subassemblies must be delivered just in time for use. A JIT purchasing system requires the supplier to deliver small quantities on a frequent basis. These deliveries could be weekly, daily, or even several times a day. This calls for a close working relationship with suppliers. Long-term contractual agreements tend to ensure supply of materials.

**Discounts and Price Increases**

Traditionally, inventories are carried so that a firm can take advantage of quantity discounts and hedge against future price increases of the items purchased. The objective is to lower the cost of inventory. JIT achieves the same objective without carrying inventories. The JIT solution is to negotiate long-term contracts with a few chosen suppliers located as close to the production facility as possible and to establish more extensive supplier involvement. Suppliers are not selected on the basis of price alone. Performance—the quality of the component and the ability to deliver as needed—and commitment to JIT purchasing are vital considerations. Other benefits of long-term contracts exist. They stipulate prices and acceptable quality levels. Long-term contracts also reduce dramatically the number of orders placed, which helps drive down the ordering cost. Another effect of JIT purchasing is to lower the cost of purchased parts.
JIT’s Limitations

JIT is not simply an approach that can be purchased and plugged in with immediate results. Its implementation should be more of an evolutionary process than a revolutionary process. Patience is needed. JIT is often referred to as a program of simplification—yet this does not imply that it is simple or easy to implement. Time is required, for example, to build sound relationships with suppliers. Insisting on immediate changes in delivery times and quality may not be realistic and may cause difficult confrontations between a company and its suppliers. Partnership, not coercion, should be the basis of supplier relationships. To achieve the benefits that are associated with JIT purchasing, a company may be tempted to redefine unilaterally its supplier relationships. Unilaterally redefining supplier relationships by extracting concessions and dictating terms may create supplier resentment and actually cause suppliers to retaliate. In the long run, suppliers may seek new markets, find ways to charge higher prices (than would exist with a preferred supplier arrangement), or seek regulatory relief. These actions may destroy many of the JIT benefits extracted by the impatient company.

Workers also may be affected by JIT. Studies have shown that sharp reductions in inventory buffers may cause a regimented work flow and high levels of stress among production workers. Some have suggested a deliberate pace of inventory reduction to allow workers to develop a sense of autonomy and to encourage their participation in broader improvement efforts. Forced and dramatic reductions in inventories may indeed reveal problems—but it may cause more problems: lost sales and stressed workers. If the workers perceive JIT as a way of simply squeezing more out of them, then JIT efforts may be doomed. Perhaps a better strategy for JIT implementation is one where inventory reductions follow the process improvements that JIT offers. Implementing JIT is not easy; it requires careful and thorough planning and preparation. Companies should expect some struggle and frustration.

The most glaring deficiency of JIT is the absence of inventory to buffer production interruptions. Current sales are constantly being threatened by an unexpected interruption in production. In fact, if a problem occurs, JIT’s approach consists of trying to find and solve the problem before any further production activity occurs. Retailers who use JIT tactics also face the possibility of shortages. JIT retailers order what they need now—not what they expect to sell—because the idea is to flow goods through the channel as late as possible, hence keeping inventories low and decreasing the need for markdowns. If demand increases well beyond the retailer’s supply of inventory, the retailer may be unable to make order adjustments quickly enough to avoid irked customers and lost sales.

JIT manufacturing systems are vulnerable to unexpected disruptions in supply. A production line can quickly come to a halt if essential parts are unavailable. Toyota, the developer of JIT, found this out the hard way. One Saturday, a fire at Aisin Seiki Company’s plant in Aichi Prefecture stopped the delivery of all brake parts to Toyota. By Tuesday, Toyota had to close down all of its Japanese assembly lines. By the time the supply of brake parts had been restored, Toyota had lost an estimated $15 billion in sales.6

Yet in spite of the downside, many retailers and manufacturers seem to be strongly committed to JIT. Apparently, losing sales on occasion is less costly than carrying high levels of inventory.

Even so, we must recognize that a sale lost today is a sale lost forever. Installing a JIT system so that it operates with very little interruption is not a short-run project. Thus, losing sales is a real cost of installing a JIT system. An alternative, and perhaps complementary approach, is the theory of constraints (TOC). In principle, TOC can be used in conjunction with JIT manufacturing. After all, JIT manufacturing environments also have constraints. Furthermore, the TOC approach has the very appealing quality of protecting current sales while also striving to increase future sales by increasing quality, lowering response time, and decreasing operating costs. However,

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6 “Toyota to Rebalance,” International Herald Tribune (February 8, 1997).
before we introduce and discuss the theory of constraints, we need to provide a brief introduction to constrained optimization theory.

**Theory of Constraints—Constrained Optimization**

Manufacturing and service organizations must choose the mix of products that they will produce and sell. Decisions about product mix can have a significant impact on an organization’s profitability. Each mix represents an alternative that carries with it an associated profit level. A manager should choose the alternative that maximizes total profits. The usual approach is to assume that only unit-based variable costs are relevant to the product mix decision. Thus, assuming that non-unit-level costs are the same for different mixes of products, a manager needs to choose the mix alternative that maximizes total contribution margin.

If a firm possesses unlimited resources and the demand for each product being considered is unlimited, then the product mix decision is simple—produce an infinite number of each product. Unfortunately, every firm faces limited resources and limited demand for each product. These limitations are called constraints. **External constraints** are limiting factors imposed on the firm from external sources (such as market demand). **Internal constraints** are limiting factors found within the firm (such as machine or labour time availability). Although resources and demands may be limited, certain mixes may not meet all the demand or use all of the available resources. Constraints whose limited resources are not fully used by a product mix are **loose constraints**. If, on the other hand, a product mix uses all of the limited resources of a constraint, then the constraint is a **binding constraint**. **Constrained optimization** is choosing the optimal mix that maximizes the total contribution margin given the constraints faced by the firm. Some interesting insights are available when at most one internal constraint is allowed.

**One Binding Internal Constraint**

Typically, the constrained optimization problem is modelled by (1) mathematically expressing the objective of maximizing total contribution margin and (2) mathematically expressing both the internal and external constraints. The function to be optimized (maximized in the case of contribution margin) is called the **objective function**. The objective function can be expressed mathematically by multiplying the unit contribution margin by the units to be produced for each product (which is expressed as an unknown variable) and then summing over all products. An internal constraint is expressed as an inequality, where the amount of scarce resource used per unit of product is multiplied by the units to be produced for each product and summed, to obtain the left-hand side of the inequality. The right-hand side of the inequality is simply the total amount of resource available to be used by the products. A similar approach is used for external constraints. **Cornerstone 18-3** illustrates constrained optimization with one internal constraint. The contribution margin per unit of scarce resource is the governing or deciding factor that is used to identify the optimal mix of products.

**Multiple Internal Binding Constraints**

It is possible for an organization to have more than one binding constraint. All organizations face multiple constraints: limitations of materials, limitations of labour inputs, limited machine hours, and so on. The solution of the product mix problem in the presence of multiple internal binding constraints is considerably more complicated and requires the use of a specialized mathematical technique known as **linear programming**.

**Linear Programming** A **linear programming model** expresses a constrained optimization problem as a linear objective function subject to a set of linear
The HOW and WHY of Solving Constrained Optimization Problems with One Internal Constraint

**Information:**
Schaller Company produces two types of machine parts: X and Y, with unit contribution margins of $300 and $600, respectively. Assume initially that Schaller can sell all that is produced of either part. Part X requires one hour of drilling, and Part Y requires three hours of drilling. The firm owns three machines that together provide 120 drilling hours per week.

**Why:**
The objective is to produce the mix of parts that maximizes total contribution margin subject to the constraints faced by the firm. With one internal constraint, the maximum amount of the product with the largest contribution margin per unit of scarce resource should first be produced.

**Required:**
1. Express the objective of maximizing total contribution margin subject to the drilling-hour constraint.
2. Identify the optimal amount that should be produced of each machine part and the total contribution margin associated with this mix.
3. **What if** market conditions are such that Schaller can sell at most 60 units of Part X and 100 units of Part Y? Express the objective function with its associated constraints for this case and identify the optimal mix and its associated total contribution margin.

**Solution:**
1. Objective function: Max $Z = 300X + 600Y$
   subject to: $X + 3Y \leq 120$ (drilling-hour constraint)
2. Contribution margin (CM) per unit of scarce resource for Part X = $300$ ($300$ unit CM/1 drilling hour per unit) and for Part Y = $200$ ($600$ unit CM/3 drilling hours per unit). Thus, 120 units of Part X (120 hours/1 hour per unit) should be produced and sold for a total contribution margin of $36,000$ ($300 \times 120$). None of Part Y should be produced.
3. Max $Z = 300X + 600Y$
   subject to: $X + 3Y \leq 120$ (drilling-hour constraint)
   $X \leq 60$ (demand constraint for Part X)
   $Y \leq 100$ (demand constraint for Part Y)
   The maximum amount of Part X should be produced: 60 units, using 60 drilling hours. This leaves 60 drilling hours so that 20 units of Part Y (60/3) can be produced. Total CM = ($300 \times 60$) + ($600 \times 20$) = $30,000$.

Nonnegativity constraints that simply reflect the reality that negative quantities of a product cannot be produced are usually included in the set of linear constraints. All constraints, taken together, are referred to as the constraint set. A feasible solution is a solution that satisfies the constraints in the linear programming model. The collection of all feasible solutions is called the feasible set of solutions. Linear programming is a method that searches among possible solutions until it finds the optimal solution. The theory of linear programming permits many solutions to be ignored. In fact, all but a finite number of solutions are eliminated by the theory, with the search then limited to the resulting finite set.

When there are only two products, the optimal solution can be identified by graphing. It has been shown in the literature on linear programming that the optimal solution will always be one of the corner points. Thus, once the graph is drawn and
the corner points are identified, finding the solution is simply a matter of computing
the value of each corner point and selecting the one with the greatest value. Four
steps are followed in solving the problem graphically.

1. Graph each constraint.
2. Identify the feasible set of solutions.
3. Identify all corner-point values in the feasible set.
4. Select the corner point that yields the largest value for the objective function.

Graphical solutions are not practical with more than two or three products. Fortunately,
an algorithm called the simplex method can be used to solve larger linear
programming problems. This algorithm has been coded and is available for use on
computers to solve these larger problems.

Since solving a linear programming problem by graphing provides considerable
insight into the way such problems are solved, it is illustrated in Cornerstone 18-4
by expanding the Schaller Company problem.

As Cornerstone 18-4 shows, the linear programming model is an important tool
for making product mix decisions. Requirement 3 of Cornerstone 18-4 illustrates that
when the scarce resource is increased for drilling, a binding constraint, then total
profitability can be increased. The example also produced the per-unit effect ($180
per drilling hour for the Schaller example). This same per-unit information is pro-
duced as a by-product of the simplex method. The simplex method produces what
are called shadow prices. Shadow prices indicate the amount by which contribution
margin will increase for one additional unit of scarce resource. Thus, although the
linear programming model produces an optimal product mix decision, its real mana-
gerial value may be more related to the kinds of inputs that must be generated for
the model to be used and the way these inputs can be managed to create more
favourable outcomes. For example, applying the model forces management to identify
internal and external constraints. Internal constraints relate to how products consume
resources; thus, resource usage relationships must be identified. Once the constrained
relationships are known to management, they can be used by management to identify
means of improving a firm’s performance in a variety of ways, including inventory
management.

Theory of Constraints

The goal of the theory of constraints is to make money now and in the future by
managing constraints. The theory of constraints (TOC) recognizes that the perform-
ance of any organization (system) is limited by its constraints. In operational terms,
every system has at least one constraint that limits its output. The theory of con-
straints develops a specific approach to manage constraints to support the objective of
continuous improvement. TOC, however, focuses on the system-level effects of con-
tinuous improvement. Each company (i.e., system) is compared to a chain. Every
chain has a weakest link that may limit the performance of the chain as a whole. The
weakest link is the system’s constraint and is the key to improving overall organiza-
tional performance. Why? Ignoring the weakest link and improving any other link
costs money and will not improve system performance. On the other hand, by
strengthening the weakest link, system performance can be improved. At some point,
however, strengthening the weakest link shifts the focus to a different link that has
now become the weakest. This next-weakest link is now the key system constraint,
and it must be strengthened so that overall system performance can be improved.
Thus, TOC can be thought of as a systems approach to continuous improvement.

Operational Measures

Given that the goal is to make money, TOC argues that the next crucial step is to
identify operational measures that encourage achievement of the goal. TOC focuses
The HOW and WHY of Solving Linear Programming Problems with Two Variables

Information:
Schaller Company produces two types of machine parts: X and Y, with unit contribution margins of $300 and $600, respectively. See also the following table, which provides the detailed constraint data for this problem.

<table>
<thead>
<tr>
<th>Resource Name</th>
<th>Resource Available</th>
<th>Part X Resource Usage: per Unit</th>
<th>Part Y Resource Usage: per Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grinding</td>
<td>80 grinding hours</td>
<td>One hour</td>
<td>One hour</td>
</tr>
<tr>
<td>Drilling</td>
<td>120 drilling hours</td>
<td>One hour</td>
<td>Three hours</td>
</tr>
<tr>
<td>Polishing</td>
<td>90 labour hours</td>
<td>Two hours</td>
<td>One hour</td>
</tr>
<tr>
<td>Market demand: Part X</td>
<td>60 units</td>
<td>One unit</td>
<td>Zero units</td>
</tr>
<tr>
<td>Market demand: Part Y</td>
<td>100 units</td>
<td>Zero units</td>
<td>One unit</td>
</tr>
</tbody>
</table>

Why:
Two-product linear programming problems can be solved graphically. Constraints are graphed and the feasible corner point that yields the largest value (for maximization problems) is the optimal solution.

Required:
1. Express Schaller Company’s constrained optimization problem as a linear programming model.
2. Using a graphical approach, solve the linear programming model expressed in Requirement 1. Which constraints are binding?
3. What if Schaller Company had five additional drilling hours with all other resources held constant? What is the new optimal mix and associated total contribution margin? What is the incremental benefit per drilling hour caused by the additional five hours, if any?

Solution:
1. Max Z = $300X + $600Y
   subject to
   \[
   \begin{align*}
   Internal\ constraints: & \quad X + Y \leq 80 \text{ (grinding)} \\
   & \quad X + 3Y \leq 120 \text{ (drilling)} \\
   & \quad 2X + Y \leq 90 \text{ (polishing)} \\
   \text{External}\ constraints: & \quad X \leq 60 \\
   & \quad Y \leq 100 \\
   \text{Nonnegativity}\ constraints: & \quad X \geq 0 \\
   & \quad Y \geq 0
   \end{align*}
   \]
2. See Exhibit 18-6, on which the coordinates of A, B, C, and D have been obtained by solving the simultaneous equations of the associated intersecting constraints within the feasible set (Region ABCD, including the frontier).
on three operational measures of systems performance: *throughput*, *inventory*, and *operating expenses*.

**Throughput** is the rate at which an organization generates money through sales. Operationally, throughput is the rate at which contribution dollars come into the organization. Thus, we have the following operational definition:

\[
\text{Throughput} = \frac{\text{Sales revenue} - \text{Unit-level variable expenses}}{\text{Time}}
\]

(18.5)

Typically, the unit-level variable costs acknowledged are materials and power. Direct labour is viewed as a fixed unit-level expense and is not usually included in the definition. With this understanding, throughput corresponds to contribution margin. It is

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**Cornerstone 18-4 (continued)**

<table>
<thead>
<tr>
<th>Corner Point</th>
<th>X-Value</th>
<th>Y-Value</th>
<th>Z = $300X + $600Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>$0</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>40</td>
<td>24,000</td>
</tr>
<tr>
<td>C</td>
<td>30</td>
<td>30</td>
<td>27,000*</td>
</tr>
<tr>
<td>D</td>
<td>45</td>
<td>0</td>
<td>13,500</td>
</tr>
</tbody>
</table>

*Optimal solution.

The binding constraints are drilling and polishing. Drilling: \(30 + 3(30) = 120\); Polishing: \(2(30) + 30 = 90\).

3. The only feasible corner point affected is C. Solving \(X + 3Y = 125\) and \(2X + Y = 90\), simultaneously, we obtain \(X = 29\) and \(Y = 32\), with \(Z = 27,900\).

The incremental benefit is \($900\) (\$27,900 – \$27,000), which is \$180 per drilling hour (total contribution margin increases by \$180 for each additional drilling hour).
also important to note that it is a global measure and not a local measure. Finally, throughput is a rate. It is the contribution earned per unit of time (per day, per month, etc.).

**Inventory** is all the money the organization spends in turning materials into throughput. In operational terms, inventory is money invested in anything that it intends to sell and, thus, expands the traditional definition to include assets such as facilities, equipment (which are eventually sold at the end of their useful lives), fixtures, and computers. In the TOC world, inventory is the money spent on items that do not have to be immediately expensed. Thus, inventory represents the money tied up inside the organization.

**Operating expenses** are defined as all the money the organization spends in turning inventories into throughput and, therefore, represent all other money that an organization spends. This includes direct labour and all operating and maintenance expenses. Thus, throughput is a measure of money coming into an organization, inventory measures the money tied up within the system, and operating expenses represent money leaving the system. Based on these three measures, the objectives of management can be expressed as increasing throughput, minimizing inventory, and decreasing operating expenses.

By increasing these objectives, the following three traditional financial measures of performance will be affected favourably: net income and return on investment will increase and cash flow will improve. Of the three TOC factors, throughput is viewed as being the most important for improving financial performance, followed by inventory, and then by operating expenses. The rationale for this order is straightforward. Operating expenses and inventories can be reduced at most to zero (inventory, though, being the larger amount), while there is virtually no upper limit on throughput. Increasing throughput and decreasing operating expenses have always been emphasized as key elements in improving the three financial measures of performance; the role of minimizing inventory, however, in achieving these improvements has been traditionally regarded as less important than reducing operating expenses.

The theory of constraints, like JIT, assigns inventory management a much more prominent role than does the traditional just-in-case viewpoint. TOC recognizes that lowering inventory decreases carrying costs and, thus, decreases operating expenses and improves net income. TOC, however, argues that lowering inventory helps produce a competitive edge through better products, lower prices, and faster response to customer needs.

**Higher-Quality Products** Better products mean higher quality. It also means that the company is able to improve products and quickly provide these improved products to the market. The relationship between low inventories and quality has been described in the JIT section. Essentially, low inventories allow defects to be detected more quickly and the cause of the problem to be assessed.

Improving products is also a key competitive element. New or improved products need to reach the market quickly—before competitors can provide similar features. This goal is facilitated with low inventories. Low inventories allow new product changes to be introduced more quickly because the company has fewer old products (in stock or in process) that would need to be scrapped or sold before the new product is introduced.

**Lower Prices** High inventories mean more productive capacity is needed, leading to a greater investment in equipment and space. Since lead time and high work-in-process inventories are usually correlated, high inventories may often be the cause of overtime. Overtime, of course, increases operating expenses and lowers profitability. Lower inventories reduce carrying costs, per-unit investment costs, and other operating expenses such as overtime and special shipping charges. By lowering investment and operating costs, the unit margin of each product is increased, providing more flexibility in pricing decisions.

**Improved Delivery Performance** Delivering goods on time and producing goods with shorter lead times than the market dictates are important competitive tools. Delivering goods on time is related to a firm’s ability to forecast the time required to produce and deliver goods. If a firm has higher inventories than its
competitors, then the firm’s production lead time is higher than the industry’s forecast horizon. High inventories may obscure the actual time required to produce and fill an order. Lower inventories allow actual lead times to be more carefully observed, and more accurate delivery dates can be provided. Shortening lead times is also crucial. Shortening lead times is equivalent to lowering work-in-process inventories. A company carrying 10 days of work-in-process inventories has an average production lead time of 10 days. If the company can reduce lead time from 10 to five days, then the company should now be carrying only five days of work-in-process inventories. As lead times are reduced, it is also possible to reduce finished goods inventories. For example, if the lead time for a product is 10 days and the market requires delivery on demand, then the firms must carry, on average, 10 days of finished goods inventory (plus some safety stock to cover demand uncertainty). Suppose that the firm is able to reduce lead time to five days. In this case, finished goods inventory should also be reduced to five days. Thus, the level of inventories signals the organization’s ability to respond. High levels relative to those of competitors translate into a competitive disadvantage. TOC, therefore, emphasizes reduction of inventories by reducing lead times.

Five-Step Method for Improving Performance

The theory of constraints uses five steps to achieve its goal of improving organizational performance:

1. Identify an organization’s constraints.
2. Exploit the binding constraints.
3. Subordinate everything else to the decisions made in step 2.
4. Elevate the organization’s binding constraints.
5. Repeat the process as a new constraint emerges to limit output.

Step 1: Identify an Organization’s Constraints

Step 1 is identical in concept to the process described for linear programming. Internal and external constraints are identified. The optimal product mix is identified as the mix that maximizes throughput subject to all the organization’s constraints. The optimal mix reveals how much of each constrained resource is used and which of the organization’s constraints are binding.

Step 2: Exploit the Binding Constraints

One way to make the best use of any binding constraints is to ensure that the optimal product mix is produced. Making the best use of binding constraints, however, is more extensive than simply ensuring production of the optimal mix. This step is the heart of TOC’s philosophy of short-run constraint management and is directly related to TOC’s goal of reducing inventories and improving performance.

Most organizations have only a few binding resource constraints. The major binding constraint is defined as the drummer. Assume, for example, that there is only one internal binding constraint. By default, this constraint becomes the drummer. The drummer constraint’s production rate sets the production rate for the entire plant. Downstream processes fed by the drummer constraint are naturally forced to follow its rate of production. Scheduling for downstream processes is easy. Once a part is finished at the drummer process, the next process begins its operation. Similarly, each subsequent operation begins when the prior operation is finished. Upstream processes that feed the drummer constraint are scheduled to produce at the same rate as the drummer constraint. Scheduling at the drummer rate prevents the production of excessive upstream work-in-process inventories.

For upstream scheduling, TOC uses two additional features in managing constraints to lower inventory levels and improve organizational performance: buffers and ropes. First, an inventory buffer is established in front of the major binding constraint. The inventory buffer is referred to as the time buffer. A time buffer is the inventory needed to keep the constrained resource busy for a specified time interval. The purpose of a time buffer is to protect the throughput of the organization from any
disruption that can be overcome within the specified time interval. For example, if it takes one day to overcome most interruptions that occur upstream from the drummer constraint, then a two-day buffer should be sufficient to protect throughput from any interruptions. Thus, in scheduling, the operation immediately preceding the drummer constraint should produce the parts needed by the drummer resource two days in advance of their planned usage. Any other preceding operations are scheduled backwards in time to produce so that their parts arrive just in time for subsequent operations.

**Ropes** are actions taken to tie the rate at which material is released into the plant (at the first operation) to the production rate of the constrained resource. The objective of a rope is to ensure that the work-in-process inventory will not exceed the level needed for the time buffer. Thus, the drummer rate is used to limit the rate of material release and effectively controls the rate at which the first operation produces. The rate of the first operation then controls the rates of subsequent operations. The TOC inventory system is often called the **drum-buffer-rope (DBR)** system.

**Step 3: Subordinate Everything Else to the Decisions Made in Step 2** An important TOC principle is that the drummer constraint should set the capacity for the entire plant. All remaining departments should be subordinated to the needs of the drummer constraint. This principle requires many companies to change the way they view things. For example, the use of efficiency measures at the departmental level may no longer be appropriate. Encouraging maximum productive efficiency for the Grinding Department would produce excess work-in-process inventories. The capacity of the Grinding Department is 80 units per week. Assuming the two-day buffer is in place, the Grinding Department would add 20 units per week to the buffer in front of the Drilling Department. Over a period of a year, the potential exists for building very large work-in-process inventories (1,000 units of the two parts would be added to the buffer over a 50-week period).

**Step 4: Elevate the Organization’s Binding Constraints** Once actions have been taken to make the best possible use of the existing constraints, the next step is to embark on a program of continuous improvement by reducing the limitations that the binding constraints place on the organization’s performance. However, if there is more than one binding constraint, which one should be elevated? For example, in the Schaller Company setting, there are two binding constraints: the drilling constraint and the polishing constraint. In this case, the guideline is to increase the resource of the constraint that produces the greatest increase in throughput. Shadow prices can be useful guides for this decision. For the Schaller Company example, the shadow prices for the drilling and polishing resources are $180 and $60, respectively. Thus, Schaller should focus on busting the drilling constraint because it offers the most improvement.

Suppose, for example, that Schaller Company adds a half shift for the Drilling Department, increasing the drilling hours from 120 to 180 per week. Throughput will now be $37,800, an increase of $10,800 ($180 × 60 additional hours). Furthermore, as you can check, the optimal mix is now 18 units of Part X and 54 units of Part Y. Is the half shift worth it? This question is answered by comparing the cost of adding the half shift with the increased throughput. If the cost is labour—say overtime at $50 per hour (for all employees)—then the incremental cost is $3,000, and the decision to add the half shift is a good one.

**Step 5: Repeat Process: Does a New Constraint Limit Throughput?** Eventually, the drilling resource constraint will be elevated to a point where the constraint is no longer binding. Suppose, for example, that the company adds a full shift for the drilling operation, increasing the resource availability to 240 hours. The new constraint set is shown in Exhibit 18-7. Notice that the drilling constraint no longer affects the optimal mix decision. The grinding and polishing resource constraints are possible candidates for the new drummer constraint. Once the drummer constraint is
identified, then the TOC process is repeated (step 5). The objective is to continually improve performance by managing constraints. Do not allow inertia to cause a new constraint. Focus now on the next-weakest link.

**Summary of Learning Objectives**

1. **Describe the just-in-case inventory management model.**
   - The just-in-case approach uses inventories to manage the trade-offs between ordering (setup) costs and carrying costs.
   - The traditional approach uses inventories to manage the trade-offs between ordering (setup) costs and carrying costs.
   - Other reasons for inventories:
     - Due-date performance
     - Avoiding shutdowns (protecting throughput)
     - Hedging against future price increases
     - Taking advantage of discounts

2. **Discuss just-in-time (JIT) inventory management.**
   - JIT argues that inventories are costly and are used to cover up fundamental problems that need to be corrected so that the organization can become more competitive.
   - JIT uses long-term contracts, continuous replenishment, and EDI to reduce (eliminate) ordering costs. Engineering efforts are made to reduce setup times drastically.
   - Once ordering costs and setup costs are reduced to minimal levels, then it is possible to reduce carrying costs by reducing inventory levels.
   - JIT carries small buffers in front of each operation and uses a Kanban system to regulate production.

3. **Explain the basic concepts of constrained optimization.**
   - Constrained optimization chooses the optimal mix given the constraints faced by the firm.
Review Problems

I. EOQ
Horizon Inc. uses 15,000 kilograms of plastic each year in its production of plastic cups. The cost of placing an order is $10. The cost of holding one kilogram of plastic for one year is $0.30. Horizon uses an average of 60 kilograms of plastic per day. It takes five days to place and receive an order.

Required:
1. Calculate the EOQ.
2. Calculate the annual ordering and carrying costs for the EOQ.
3. What is the reorder point?

Solution:
1. \[ EOQ = \sqrt{\frac{2DO}{C}} \]
   \[ = \sqrt{\frac{2 \times 15,000 \times $10}{$0.30}} \]
   \[ = \sqrt{1,000,000} \]
   \[ = 1,000 \]
II. JIT, Drum-Buffer-Rope System

Both just-in-case and JIT inventory management systems have drummers—factors that determine the production rate of the plant. For a just-in-case system, the drummer is the excess capacity of the first operation. For JIT, the drummer is market demand.

Required:

1. Explain why the drummer of a just-in-case system is identified as excess demand of the first operation.
2. Explain how market demand drives the JIT production system.
3. Explain how a drummer constraint is used in the TOC approach to inventory management.
4. What are the advantages and disadvantages of the three types of drummers?

Solution:

1. In a traditional inventory system, local efficiency measures encourage the manager of the first operation to keep the department’s workers busy. Thus, materials are released to satisfy this objective. This practice is justified because the inventory may be needed just in case demand is greater than expected, or just in case the first operation has downtime, etc.

2. In a JIT system, when the final operation delivers its goods to a customer, a backward rippling effect triggers the release of materials into the factory. First, the last process removes the buffer inventory from the withdrawal store, and this leads to a P-Kanban being placed on the production post of the preceding operation. This operation then begins production, withdrawing parts it needs from its withdrawal store, leading to a P-Kanban being placed on the production post of its preceding operation. This process repeats itself—all the way back to the first operation.

3. A drummer constraint sets the production rate of the factory to match its own production rate. This is automatically true for succeeding operations. For preceding operations, the rate is controlled by tying the drummer constraint’s rate of production to that of the first operation. A time buffer is also set in front of the drummer constraint to protect throughput in the event of interruptions.

4. The excess capacity drummer typically will build excess inventories. This serves to protect current throughput. However, it ties up a lot of capital and tends to cover up problems such as poor quality, bad delivery performance, and inefficient production. Because it is costly and covers up certain critical productive problems, the just-in-case approach may be a threat to future throughput by damaging a firm’s competitive position. JIT reduces inventories dramatically—using only small buffers in front of each operation as a means to regulate production flow and signal when production should occur. JIT has the significant advantage of uncovering problems and eventually correcting them. However, discovering problems usually means that current throughput will be lost while problems are being corrected. Future throughput tends to be protected because the firm is taking actions to improve its operations. TOC uses time buffers in front of the critical constraints. These buffers are large enough to keep the critical constraints operating while other operations may be down. Once the problem is corrected, the other resource constraints usually have sufficient excess capacity to catch up. Thus, current throughput is protected. Furthermore, future throughput is protected because TOC uses the same approach as JIT—namely, that of uncovering and correcting problems. TOC can be viewed as an improvement on JIT methods—correcting the lost throughput problem while maintaining the other JIT features.

\[
2. \text{Ordering cost} = O \times (D/Q) = (15,000/1,000) \times 10 = 150 \\
\text{Carrying cost} = C \times (Q/2) = (1,000/2) \times 0.30 = 150
\]

\[
3. \text{ROP} = 60 \times 5 = 300 \text{ kilograms (Whenever inventory drops to this level, an order should be placed.)}
\]
Discussion Questions

1. What are ordering costs? What are setup costs? What are carrying costs? Provide examples of each type of cost.
2. Explain why, in the traditional view of inventory, carrying costs increase as ordering costs decrease.
3. Discuss the traditional reasons for carrying inventory.
4. What are stock-out costs?
5. Explain how safety stock is used to deal with demand uncertainty.
6. What is the economic order quantity?
7. What approach does JIT take to minimize total inventory costs?
8. One reason for inventory is to prevent shutdowns. How does the JIT approach to inventory management deal with this potential problem?
9. Explain how the Kanban system helps reduce inventories.
10. Explain how long-term contractual relationships with suppliers can reduce the acquisition cost of materials.
11. What is a constraint? An internal constraint? An external constraint?
12. Explain the procedures for graphically solving a linear programming problem. What solution method is used when the problem includes more than two or three products?
13. Define and discuss the three measures of organizational performance used by the theory of constraints.
14. Explain how lowering inventory produces better products, lower prices, and better responsiveness to customer needs.
15. What are the five steps that TOC uses to improve organizational performance?
Cornerstone Exercises

Cornerstone Exercise 18-1  EOQ
Sterling Corporation produces air conditioning units. The following values apply for a part used in their production (purchased from external suppliers):

\[
\begin{align*}
D & = 6,250 \\
Q & = 250 \\
O & = \$60 \\
C & = \$3
\end{align*}
\]

**Required:**
1. For Sterling, calculate the ordering cost, the carrying cost, and the total cost associated with an order size of 250 units.
2. Calculate the EOQ and its associated ordering cost, carrying cost, and total cost. Compare and comment on the EOQ relative to the current order quantity.
3. **What if** Sterling enters into an exclusive supplier agreement with one supplier who will supply all of the demands with smaller, more frequent orders? Under this arrangement, the ordering cost is reduced to $0.60 per order. Calculate the new EOQ and comment on the implications.

Cornerstone Exercise 18-2  REORDER POINT
Jimbaya Corporation has an EOQ of 2,500 units. The company uses an average of 250 units per day. An order to replenish the part requires a lead time of five days.

**Required:**
1. Calculate the reorder point, using Equation 18.3.
2. Graphically display the reorder point, where the vertical axis is inventory (units) and the horizontal axis is time (days). Show two replenishments, beginning at time zero with the economic order quantity in inventory.
3. **What if** the average usage per day of the part is 250 units but a daily maximum usage of 300 units is possible? What is the reorder point when this demand uncertainty exists?

Cornerstone Exercise 18-3  CONstrained Optimization: One Internal Binding Constraint
Legault Company produces two types of airplane components: Component A and Component B, with unit contribution margins of $200 and $400, respectively. Assume initially that Legault can sell all that is produced of either component. Component A requires two hours of assembly, and B requires five hours of assembly. The firm has 200 assembly hours per week.

**Required:**
1. Express the objective of maximizing total contribution margin subject to the assembly-hour constraint.
2. Identify the optimal amount that should be produced of each airplane component and the total contribution margin associated with this mix.
3. **What if** market conditions are such that Legault can sell at most 50 units of Component A and 40 units of Component B? Express the objective function with its associated constraints for this case and identify the optimal mix and its associated total contribution margin.

Cornerstone Exercise 18-4  CONstrained Optimization: Multiple Internal Constraints
Legault Company produces two types of components for airplanes: A and B, with unit contribution margins of $200 and $400, respectively. The components pass through three sequential processes: cutting, welding, and assembly. Data pertaining to these processes and market demand are given below (weekly data).
Resource & Resource Available & Resource Usage (A) & Resource Usage (B) \\
Cutting & 150 machine hours & Three hours & Five hours \\
Welding & 154 welding hours & Five hours & Three hours \\
Assembly & 200 labour hours & Two hours & Five hours \\
Market demand (A) & 50 & One unit & Zero units \\
Market demand (B) & 40 & Zero units & One unit \\

Required:
1. Express Legault Company’s constrained optimization problem as a linear programming model.
2. Using a graphical approach, solve the linear programming model expressed in Requirement 1. Which constraints are binding?
3. What if Legault Company had five additional machine hours (cutting) with all other resources held constant? What is the new optimal mix and associated total contribution margin? What is the incremental benefit per machine hour caused by the additional five hours, if any?

**Exercises**

**Exercise 18-5 ORDERING AND CARRYING COSTS**
Lamela Inc. uses 160,000 plastic housing units each year in its production of paper shredders. The cost of placing an order is $60. The cost of holding one unit of inventory for one year is $7.50. Currently, Lamela places 40 orders of 4,000 plastic housing units per year.

Required:
1. Compute the annual ordering cost.
2. Compute the annual carrying cost.
3. Compute the cost of Lamela’s current inventory policy. Is this the minimum cost? Why or why not?

**Exercise 18-6 ECONOMIC ORDER QUANTITY**
Refer to the data in Exercise 18-5.

Required:
1. Compute the economic order quantity.
2. Compute the ordering, carrying, and total costs for the EOQ.
3. How much money does using the EOQ policy save the company over the policy of purchasing 4,000 plastic housing units per order?

**Exercise 18-7 ECONOMIC ORDER QUANTITY**
Quezon Company uses 78,125 kilograms of sucrose each year. The cost of placing an order is $18, and the carrying cost for one kilogram of sucrose is $0.45.

Required:
1. Compute the economic order quantity for sucrose.
2. Compute the carrying and ordering costs for the EOQ.

**Exercise 18-8 REORDER POINT**
Baker Company manufactures backpacks. A heavy-duty strap is one part the company orders from an outside supplier. Information pertaining to the strap is as follows:

<table>
<thead>
<tr>
<th>Economic order quantity</th>
<th>4,200 units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average daily usage</td>
<td>210 units</td>
</tr>
<tr>
<td>Maximum daily usage</td>
<td>250 units</td>
</tr>
<tr>
<td>Lead time</td>
<td>4 days</td>
</tr>
</tbody>
</table>
Required:
1. What is the reorder point assuming no safety stock is carried?
2. What is the reorder point assuming that safety stock is carried?

Exercise 18-9 EOQ WITH SETUP COSTS
Goa Manufacturing produces casings for sewing machines: large and small. To produce the different casings, equipment must be set up. The setup cost per production run is $9,000 for either casing. The cost of carrying small casings in inventory is $3 per casing per year; the cost of large casings is $9 per unit per year. To satisfy demand, the company produces 600,000 small casings and 200,000 large casings.

Required:
1. Compute the number of small casings that should be produced per setup to minimize total setup and carrying costs.
2. Compute the setup, carrying, and total costs associated with the economic order quantity for the small casings.

Exercise 18-10 EOQ WITH SETUP COSTS
Refer to Exercise 18-9.

Required:
1. Compute the number of large casings that should be produced per setup to minimize total setup and carrying costs for this product.
2. Compute the setup, carrying, and total costs associated with the economic order quantity for the large casings.

Exercise 18-11 REORDER POINT
Refer to Exercise 18-9. Assume the economic lot size for small casings is 60,000 and that of the large casings is 20,000. Goa Manufacturing sells an average of 2,400 small casings per workday and an average of 800 large casings per workday. It takes Goa two days to set up the equipment for small or large casings. Once set up, it takes nine workdays to produce a batch of small casings and 10 days for large casings. There are 250 workdays available per year.

Required:
1. What is the reorder point for small casings? Large casings?
2. Using the economic order batch size, is it possible for Goa to produce the amount that can be sold of each casing? Does scheduling have a role here? Explain. Is this a push or pull-through system approach to inventory management? Explain.

Exercise 18-12 SAFETY STOCK
Eureka Manufacturing produces a component used in its production of washing machines. The time to set up and produce a batch of the components is two days. The average daily usage is 800 components, and the maximum daily usage is 875 components.

Required:
Compute the reorder point assuming that safety stock is carried by Eureka Manufacturing. How much safety stock is carried by Eureka?

Exercise 18-13 KANBAN SYSTEM, EDI
Hales Company produces a product that requires two processes. In the first process, a subassembly is produced (subassembly A). In the second process, this subassembly and a subassembly purchased from outside the company (subassembly B) are assembled to
produce the final product. For simplicity, assume that the assembly of one final unit takes the same time as the production of subassembly A. Subassembly A is placed in a container and sent to an area called the subassembly stores (SB stores) area. A production Kanban is attached to this container. A second container, also with one subassembly, is located near the assembly line (called the withdrawal store). This container has attached to it a withdrawal Kanban.

Required:
1. Explain how withdrawal and production Kanban cards are used to control the work flow between the two processes. How does this approach minimize inventories?
2. Explain how vendor Kanban cards can be used to control the flow of the purchased subassembly. What implications does this have for supplier relationships? What role, if any, do continuous replenishment and EDI play in this process?

Exercise 18-14 JIT LIMITATIONS
Many companies have viewed JIT as a panacea—a knight in shining armour that promises rescue from sluggish profits, poor quality, and productive inefficiency. It is often lauded for its beneficial effects on employee morale and self-esteem. Yet JIT may also cause a company to struggle and may produce a good deal of frustration. In some cases, JIT appears to deliver less than its reputation seems to call for.

Required:
Discuss some of the limitations and problems that companies may encounter when implementing a JIT system.

Exercise 18-15 PRODUCT MIX DECISION, SINGLE CONSTRAINT
Behar Company makes three types of stainless steel frying pans. Each of the three types of pans requires the use of a special machine that has total operating capacity of 182,000 hours per year. Information on each of the three products is as follows:

<table>
<thead>
<tr>
<th></th>
<th>Basic</th>
<th>Standard</th>
<th>Deluxe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selling price</td>
<td>$12.00</td>
<td>$17.00</td>
<td>$32.00</td>
</tr>
<tr>
<td>Unit variable cost</td>
<td>$7.00</td>
<td>$11.00</td>
<td>$12.00</td>
</tr>
<tr>
<td>Machine hours required</td>
<td>0.10</td>
<td>0.20</td>
<td>0.50</td>
</tr>
</tbody>
</table>

The marketing manager has determined that the company can sell all that it can produce of each of the three products.

Required:
1. How many of each product should be sold to maximize total contribution margin? What is the total contribution margin for this product mix?
2. Suppose that Behar can sell no more than 300,000 units of each type at the prices indicated. What product mix would you recommend, and what would be the total contribution margin?

Exercise 18-16 DRUM-BUFFER-ROPE SYSTEM
Duckstein Inc. manufactures two types of aspirin: plain and buffered. It sells all it produces. Recently, Duckstein implemented a TOC approach for its Fort George plant. One binding constraint was identified, and the optimal product mix was determined. The diagram on the following page reflects the TOC outcome:
Materials for 2,000 bottles per day
Plain aspirin: 1,500; Buffered aspirin: 500

Mixing Process

A

B
750 units
Plain aspirin
250 units
Buffered aspirin

Tableting Process

C

Finished Goods
1,500 bottles of plain aspirin per day
500 bottles of buffered aspirin per day

Bottling Process

Required:
1. What is the daily production rate? Which process sets this rate?
2. How many days of buffer inventory is Duckstein carrying? How is this time buffer determined?
3. Explain what the letters A, B, and C in the exhibit represent. Discuss each of their roles in the TOC system.

Problems

Problem 18-17 EOQ, SAFETY STOCK, LEAD TIME, BATCH SIZE, AND JIT

Quan Company produces helmets for drivers of motorcycles. Helmets are produced in batches according to model and size. Although the setup and production time vary for each model, the smallest lead time is six days. The most popular model, Model HA2, takes two days for setup, and the production rate is 750 units per day. The expected annual demand for the model is 36,000 units. Demand for the model, however, can reach 45,000 units. The cost of carrying one HA2 helmet is $3 per unit. The setup cost is $6,000. Quan chooses its batch size based on the economic order quantity criterion. Expected annual demand is used to compute the EOQ.

Recently, Quan has encountered some stiff competition—especially from foreign sources. Some of the foreign competitors have been able to produce and deliver the helmets to retailers in half the time it takes Quan to produce. For example, a large retailer recently requested a delivery of 12,000 Model HA2 helmets with the stipulation that the helmets be delivered within seven working days. Quan had 3,000 units of HA2 in stock.
Quan informed the potential customer that it could deliver 3,000 units immediately and the other 9,000 units in about 14 working days—with the possibility of interim partial orders being delivered. The customer declined the offer indicating that the total order had to be delivered within seven working days so that its stores could take advantage of some special local conditions. The customer expressed regret and indicated that it would accept the order from another competitor who could satisfy the time requirements.

**Required:**

1. Calculate the optimal batch size for Model HA2 using the EOQ model. Was Quan’s response to the customer right? Would it take the time indicated to produce the number of units wanted by the customer? Explain with supporting computations.

2. Upon learning of the lost order, the marketing manager grumbled about Quan’s inventory policy. “We lost the order because we didn’t have sufficient inventory. We need to carry more units in inventory to deal with unexpected orders like these.” Do you agree or disagree? How much additional inventory would have been needed to meet customer requirements? In the future, should Quan carry more inventory? Can you think of other solutions?

3. Fenton Gray, the head of industrial engineering, reacted differently to the lost order. “Our problem is more complex than insufficient inventory. I know that our foreign competitors carry much less inventory than we do. What we need to do is decrease the lead time. I have been studying this problem, and my staff have found a way to reduce setup time for Model HA2 from two days to 1.5 hours. Using this new procedure, setup cost can be reduced to about $94. Also, by rearranging the plant layout for this product—creating what are called manufacturing cells—we can increase the production rate from 750 units per day to about 2,000 units per day. This is done simply by eliminating a lot of move time and waiting time—both non-value-added activities.” Assume that the engineer’s estimates are on target. Compute the new optimal batch size (using the EOQ formula). What is the new lead time? Given this new information, would Quan have been able to meet the customer’s time requirements? Assume that there are eight hours available in each workday.

4. Suppose that the setup time and cost are reduced to 0.5 hour and $10, respectively. What is the batch size now? As setup time approaches zero and the setup cost becomes negligible, what does this imply? Assume, for example, that it takes five minutes to set up, and costs are $0.864 per setup.

---

**Problem 18-18 PRODUCT MIX DECISIONS, MULTIPLE CONSTRAINTS**

Cardin Company produces two types of gears: Model 12 and Model 15. Market conditions limit the number of each gear that can be sold. For Model 12, no more than 15,000 units can be sold, and for Model 15, no more than 40,000 units. Each gear must be notched by a special machine. Cardin owns eight machines that together provide 40,000 hours of machine time per year. Each unit of Model 12 requires two hours of machine time, and each unit of Model 15 requires one half-hour of machine time. The unit contribution for Model 12 is $30 and for Model 15 is $15. Cardin wants to identify the product mix that will maximize total contribution margin.

**Required:**

1. Formulate Cardin’s problem as a linear programming model.
2. Solve the linear programming model in Requirement 1.
3. Identify which constraints are binding and which are loose. Also, identify the constraints as internal or external.

---

**Problem 18-19 PRODUCT MIX DECISION, SINGLE AND MULTIPLE CONSTRAINTS**

Taylor Company produces two industrial cleansers that use the same liquid chemical input: Plasma and Maza. Plasma uses two litres of the chemical for every unit produced, and Maza uses five litres. Currently, Taylor has 6,000 litres of the material in inventory. All of the material is imported. For the coming year, Taylor plans to import 6,000 litres...
to produce 1,000 units of Plasma and 2,000 units of Maza. The detail of each product’s unit contribution margin is as follows:

<table>
<thead>
<tr>
<th></th>
<th>Plasma</th>
<th>Maza</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selling price</td>
<td>$81</td>
<td>$139</td>
</tr>
<tr>
<td>Less variable expenses:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct materials</td>
<td>(20)</td>
<td>(50)</td>
</tr>
<tr>
<td>Direct labour</td>
<td>(21)</td>
<td>(14)</td>
</tr>
<tr>
<td>Variable overhead</td>
<td>(10)</td>
<td>(15)</td>
</tr>
<tr>
<td>Contribution margin</td>
<td>$30</td>
<td>$60</td>
</tr>
</tbody>
</table>

Taylor Company has received word that the source of the material has been shut down by embargo. Consequently, the company will not be able to import the 6,000 litres it planned to use in the coming year’s production. There is no other source of the material.

**Required:**

1. Compute the total contribution margin that the company would earn if it could import the 6,000 litres of the material.

2. Determine the optimal usage of the company’s inventory of 6,000 litres of the material. Compute the total contribution margin for the product mix that you recommend.

3. Assume that Plasma uses three direct labour hours for every unit produced and that Maza uses two hours. A total of 6,000 direct labour hours is available for the coming year.
   a. Formulate the linear programming problem faced by Taylor Company. To do so, you must derive mathematical expressions for the objective function and for the materials and labour constraints.
   b. Solve the linear programming problem using the graphical approach.
   c. Compute the total contribution margin produced by the optimal mix.

**Problem 18-20 PRODUCT MIX DECISION, SINGLE AND MULTIPLE CONSTRAINTS, BASICS OF LINEAR PROGRAMMING**

Dynamic Products Inc. produces cornflakes and branflakes. The manufacturing process is highly mechanized; both products are produced by the same machinery by using different settings. For the coming period, 200,000 machine hours are available. Management is trying to decide on the quantities of each product to produce. The following data are available:

<table>
<thead>
<tr>
<th></th>
<th>Cornflakes</th>
<th>Branflakes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Machine hours per unit</td>
<td>1.00</td>
<td>0.50</td>
</tr>
<tr>
<td>Unit selling price</td>
<td>$2.50</td>
<td>$3.00</td>
</tr>
<tr>
<td>Unit variable cost</td>
<td>$1.50</td>
<td>$2.25</td>
</tr>
</tbody>
</table>

**Required:**

1. Determine the units of each product that should be produced in order to maximize profits.

2. Because of market conditions, the company can sell no more than 150,000 boxes of cornflakes and 300,000 boxes of branflakes. Do the following:
   a. Formulate the problem as a linear programming problem.
   b. Determine the optimal mix using a graph.
   c. Compute the maximum contribution margin given the optimal mix.

**Problem 18-21 IDENTIFYING AND EXPLOITING CONSTRAINTS, CONSTRAINT ELEVATION**

Berry Company produces two different metal components used in medical equipment (Component X and Component Y). The company has three processes: moulding, grinding, and finishing. In moulding, moulds are created, and molten metal is poured into the shell. Grinding removes the gates that allowed the molten metal to flow into the mould’s cavities. In finishing, rough edges caused by the grinders are removed by small, handheld
pneumatic tools. In moulding, the setup time is one hour. The other two processes have no setup time required. The demand for Component X is 300 units per day, and the demand for Component Y is 500 units per day. The minutes required per unit for each product are as follows:

<table>
<thead>
<tr>
<th>Product</th>
<th>Moulding</th>
<th>Grinding</th>
<th>Finishing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Component X</td>
<td>5</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>Component Y</td>
<td>10</td>
<td>15</td>
<td>20</td>
</tr>
</tbody>
</table>

The company operates one eight-hour shift. The moulding process employs 12 workers (who each work eight hours). Two hours of their time, however, are used for setups (assuming both products are produced). The grinding process has sufficient equipment and workers to provide 200 grinding hours per shift.

The Finishing Department is labour intensive and employs 35 workers, who each work eight hours per day. The only significant unit-level variable costs are materials and power. For Component X, the variable cost per unit is $40, and for Component Y, it is $50. Selling prices for X and Y are $90 and $110, respectively. Berry’s policy is to use two setups per day: an initial setup to produce all that is scheduled for Component X and a second setup (changeover) to produce all that is scheduled for Component Y. The amount scheduled does not necessarily correspond to each product’s daily demand.

Required:
1. Calculate the time (in minutes) needed each day to meet the daily market demand for Component X and Component Y. What is the major internal constraint facing Berry Company?
2. Describe how Berry should exploit its major binding constraint. Specifically, identify the product mix that will maximize daily throughput.
3. Assume that manufacturing engineering has found a way to reduce the moulding setup time from one hour to 10 minutes. Explain how this affects the product mix and daily throughput.

OBJECTIVE

Problem 18-22 THEORY OF CONSTRAINTS, INTERNAL CONSTRAINTS

Prague Company produces two replacement parts for a popular line of Blu-ray disc players: Part A and Part B. Part A is made up of two components, one manufactured internally and one purchased from external suppliers. Part B is made up of three components, one manufactured internally and two purchased from suppliers. The company has two processes: fabrication and assembly. In fabrication, the internally produced components are made. Each component takes 20 minutes to produce. In assembly, it takes 30 minutes to assemble the components for Part A and 40 minutes to assemble the components for Part B. Prague Company operates one shift per day. Each process employs 100 workers who each work eight hours per day.

Part A earns a unit contribution margin of $20, and Part B earns a unit contribution margin of $24 (calculated as the difference between revenue and the cost of materials and energy). Prague can sell all that it produces of either part. There are no other constraints. Prague can add a second shift of either process. Although a second shift would work eight hours, there is no mandate that it employ the same number of workers. The labour cost per hour for fabrication is $15, and the labour cost per hour for assembly is $12.

Required:
1. Identify the constraints facing Prague, and graph them. How many binding constraints are possible? What is Prague’s optimal product mix? What daily contribution margin is produced by this mix?
2. What is the drummer constraint? How much excess capacity does the other constraint have? Assume that a 1.5-day buffer inventory is needed to deal with any
production interruptions. Describe the drum-buffer-rope concept using the Prague data to illustrate the process.

3. Explain why the use of local labour efficiency measures will not work in Prague’s TOC environment.

4. Suppose Prague decides to elevate the binding constraint by adding a second shift of 50 workers (labour rates are the same as those of the first shift). Would elevation of Prague’s binding constraint improve its system performance? Explain with supporting computations.

**Problem 18-23 TOC, INTERNAL AND EXTERNAL CONSTRAINTS**

Bountiful Manufacturing produces two types of bike frames (Frame X and Frame Y). Frame X passes through four processes: cutting, welding, polishing, and painting. Frame Y uses three of the same processes: cutting, welding, and painting. Each of the four processes employs 10 workers who work eight hours each day. Frame X sells for $40 per unit, and Frame Y sells for $55 per unit. Materials is the only unit-level variable expense. The materials cost for Frame X is $20 per unit, and the materials cost for Frame Y is $25 per unit. Bountiful’s accounting system has provided the following additional information about its operations and products:

<table>
<thead>
<tr>
<th>Resource Name</th>
<th>Resource Available</th>
<th>Frame X Resource Usage per Unit</th>
<th>Frame Y Resource Usage per Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutting labour</td>
<td>4,800 minutes</td>
<td>15 minutes</td>
<td>10 minutes</td>
</tr>
<tr>
<td>Welding labour</td>
<td>4,800 minutes</td>
<td>15 minutes</td>
<td>30 minutes</td>
</tr>
<tr>
<td>Polishing labour</td>
<td>4,800 minutes</td>
<td>15 minutes</td>
<td>—</td>
</tr>
<tr>
<td>Painting labour</td>
<td>4,800 minutes</td>
<td>10 minutes</td>
<td>15 minutes</td>
</tr>
<tr>
<td>Market demand:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frame X</td>
<td>200 per day</td>
<td>One unit</td>
<td>—</td>
</tr>
<tr>
<td>Frame Y</td>
<td>100 per day</td>
<td>—</td>
<td>One unit</td>
</tr>
</tbody>
</table>

Bountiful’s management has determined that any production interruptions can be corrected within two days.

**Required:**

1. Assuming that Bountiful can meet daily market demand, compute the potential daily profit. Now, compute the minutes needed for each process to meet the daily market demand. Can Bountiful meet daily market demand? If not, where is the bottleneck? Can you derive an optimal mix without using a graphical solution? If so, explain how.

2. Identify the objective function and the constraints. Then, graph the constraints facing Bountiful. Determine the optimal mix and the maximum daily contribution margin (throughput).

3. Explain how a drum-buffer-rope system would work for Bountiful.

4. Suppose that the Engineering Department has proposed a process design change that will increase the polishing time for Frame X from 15 to 23 minutes per unit and decrease the welding time from 15 minutes to 10 minutes per unit (for Frame X). The cost of process redesign would be $10,000. Evaluate this proposed change. What step in the TOC process does this proposal represent?

**CMA Problem**

**CMA Problem 18-1 PRODUCT MIX DECISIONS**

Calen Company manufactures and sells three products in a factory of three departments. Both labour and machine time are applied to the products as they pass through each department. The nature of the machine processing and of the labour skills required in each department is such that neither machines nor labour can be switched from one department to another.
Calen’s management is attempting to plan its production schedule for the next several months. The planning is complicated by the fact that labour shortages exist in the community and some machines will be down several months for repairs.

Following is information regarding available machine and labour time by department and the machine hours and direct labour hours required per unit of product. These data should be valid for at least the next six months.

<table>
<thead>
<tr>
<th>Department</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labour hours available</td>
<td>3,700</td>
<td>4,500</td>
<td>2,750</td>
</tr>
<tr>
<td>Machine hours available</td>
<td>3,000</td>
<td>3,100</td>
<td>2,700</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Product</th>
<th>Input per Unit Produced</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Labour hours</td>
<td>Machine hours</td>
</tr>
<tr>
<td>401</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>402</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>403</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Calen believes that the monthly demand for the next six months will be as follows:

<table>
<thead>
<tr>
<th>Product</th>
<th>Units Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>401</td>
<td>500</td>
</tr>
<tr>
<td>402</td>
<td>400</td>
</tr>
<tr>
<td>403</td>
<td>1,000</td>
</tr>
</tbody>
</table>

Inventory levels will not be increased or decreased during the next six months. The unit cost and price data for each product are as follows:

<table>
<thead>
<tr>
<th>Product</th>
<th>Unit costs:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Direct material</td>
<td>Direct labour</td>
</tr>
<tr>
<td>401</td>
<td>$7</td>
<td>$66</td>
</tr>
<tr>
<td>402</td>
<td>$13</td>
<td>$38</td>
</tr>
<tr>
<td>403</td>
<td>$17</td>
<td>$51</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Product</th>
<th>Total unit cost</th>
<th>Unit selling price</th>
</tr>
</thead>
<tbody>
<tr>
<td>401</td>
<td>$118</td>
<td>$196</td>
</tr>
<tr>
<td>402</td>
<td>$83</td>
<td>$123</td>
</tr>
<tr>
<td>403</td>
<td>$129</td>
<td>$167</td>
</tr>
</tbody>
</table>

**Required:**

1. Calculate the monthly requirement for machine hours and direct labour hours for producing Products 401, 402, and 403 to determine whether or not the factory can meet the monthly sales demand.
2. Determine the quantities of 401, 402, and 403 that should be produced monthly to maximize profits. Prepare a schedule that shows the contribution to profits of your product mix.
3. Assume that the machine hours available in Department 3 are 1,500 instead of 2,700. Calculate the optimal monthly product mix using the graphing approach to linear programming. Prepare a schedule that shows the contribution to profits from this optimal mix. *(CMA adapted)*

*The Collaborative Learning Exercises can be found on the product support site at www.hansen1ce.nelson.com.*