Instructor's Resource Binder
Elementary Algebra

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Sample Chapter
Solving Systems of Equations and Inequalities

Note from the Author:

I have always tried to teach algebra in a way that is understandable, interesting, and active. Although you can find activities for teaching algebra to younger students (6^{th} and 7^{th} graders), I have never found a good classroom resource to use with adult students. Even in the resources for younger students, the activities always seem more like busywork, when they could provide an opportunity for students to truly understand difficult concepts or explore the similarities and contrasts that abound in mathematical procedures and ideas.

I have enjoyed taking the time to sit down and write what I hope are interesting and useful resources for teaching algebra to adult students. It has grown from a small idea into a behemoth with a life of its own during the writing process, as the pedagogies described in the Teaching Guides continued to force more activities to be written.

I owe a great big thank you to my faithful mathematics assistant, Megan Arthur, who tirelessly filled in a lot of the necessary (but boring) detailed mathematical work and graphics on these activities all summer – without her time and energy, this work would not be a reality today. Also, I extend a grateful thank you to Maryanne Kirkpatrick, who took on the task of doing every single problem in the Elementary Algebra IRB to minimize the errors in its first printing. Maryanne was my first supervisor when I was a wet-behind-the-ears algebra instructor for LCCC and I'm sure she is amused by this turn of events.

The Instructor's Resource Binder is made up of three main components: Assessments, Teaching Guides, and Activities. Together these three components provide a framework for ensuring a high-quality learning experience in the classroom. *None* of these assessments or activities are meant to be graded. These are tools to help *you* to help students to learn concepts and increase the retention of their mathematical learning.

I hope you, and your students, enjoy using the resources as much as I enjoyed writing them.

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Introduction to the Instructor's Resource Binder

WHY this Resource is Different and Necessary:

To truly understand why these types of activities are so important for mathematical learning to take place, I find it necessary to give a *very brief* summary of two cognitive development theories, Schema Theory and Information Processing Theory.

The tenet of Schema Theory is that students form a schema (plural schemata) which is a "mental representation of what all instances of something have in common[1]." These mental representations serve four functions: they ***categorize*** your experiences, they help you to ***remember*** and ***comprehend*** what you are experiencing, and they are important in developing the ability to ***problem solve***. In fact, successful problem-solvers generally have a large variety of flexible, accurate schemata. It is believed that schemata are formed in the process of abstraction, which can only happen when learners have *multiple* encounters with objects or events in *different* ways. In mathematics, we are fairly good at giving students multiple encounters with events (long homework assignments come to mind), but often this type of repetition does not give the learner encounters in *different* ways. For example, in homework problem sets, we often assign ten of a particular type of problem (thus the same schema is encountered over and over). Mathematical concepts must be encountered in new and interesting ways if the schemata are to be revised and improved.

Information Processing Theory asserts that knowledge is made up of two parts, ***declarative knowledge*** (i.e. a triangle has 3 sides) and ***procedural knowledge*** (i.e. understanding the steps to solve a linear equation). Procedural knowledge is said to be activated when the "if" of an if-then relationship is encountered. Knowledge about linear equations might be stored like this: *If the equation looks linear, follow these steps… to solve it.* Based on this theory, a student who is unable to categorize or recognize a certain problem type would be unable to even take the first step to solve the problem. If you tell them what kind of problem it is (i.e. give them the section number or remind them *when* they did these types of problems), then they are able to do the rest on their own, but without the prompting, they would be stuck. This is especially seen in the classroom on test days. Students who have been able to follow your work in class and did lots of homework from that section in the book get "stuck" staring at a problem, unable to start it. Our students need practice with recognizing and categorizing a wide variety of problem types.

Often, what we see in our mathematics students can be explained by a combination of Schema Theory and Information Processing Theory. For example, a math student, after attending a class, will remember only those aspects of class that are schema-relevant. If they have not begun to form a schema about the material, they are unlikely to remember anything. Leaving class (hopefully) with the ideas that are relevant to their schemata, the student then begins a process called ***gist-extraction***. In this process, the exact wording and details are replaced with the *gist* of what was said and encountered (sometimes accurate, sometimes not). It is the schemata that will be retained by the students and really, we want to be teaching in a way that fine tunes these mathematical schemata created by our students' minds. Often, we see that students develop schemata that have incorrect parts (think of the infamous squaring of a binomial). When this is the case, the only way to alter a schema is to help guide the student through a process called ***restructuring***. This is very difficult to do, but encounters with experiences that are contrary to the existing schema will certainly help.

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[1] Cognitive Development and Learning in Instructional Contexts, Chapter 2, James P. Byrnes.

Introduction to the Instructor's Resource Binder

What's in the Teaching Guide?

- **Pedagogical Resource -** It is often difficult for new instructors in mathematics to figure out *why* students have such difficulty with skills that the instructor finds so easy. The Teaching Guides and Instruction Tips provide a resource for new math instructors, part-time instructors, adjunct instructors, and instructors from disciplines outside mathematics so that they have a "mentor" throughout the book that will help them to provide students with a high-quality learning experience.

- **Coordination with Textbook –** For all instructors, it is helpful to have a quick resource about the new vocabulary for students, types of examples, and topics that are introduced in a particular section. As instructors change from an older edition of a book to a newer one, or change books entirely, it becomes hard to keep track of which book section covers which topics. The Teaching Your Class section of each Teaching Guide provides examples and a framework for how activities can be incorporated into the teaching of the section.

- **Examples –** Examples for the instructor are coordinated with the problem sets in the back of the section so that you can be sure you covered all the variations of problems that you are about to assign for homework. Answers to instructor examples are not provided for simple lecture examples, but notes are made if an example is listed for a specific reason (for example, if the equation has no solution).

- **Guided Learning Activities –** For some topics, it can be quite tedious to go through enough examples (e.g., graphing inequalities on a number line, solving application problems, graphing lines, etc.). For these topics, that are usually time consuming to teach, guided learning activities are provided to help you to get right to the heart of the concepts with your students, without the tedium of drawing out multiple number lines, coordinate grids, etc.

Introduction to the Instructor's Resource Binder

How to use the Assessments:

In higher education today we have to be certain our students are actually learning what we attempt to teach them. It is a time-consuming task to write appropriate assessments for math courses, and the assessments that work in other disciplines are often not applicable to mathematics. Using the materials provided in the IRB, you should be able to perform daily assessments in your classroom. Each chapter has the following assessments:

- **Pretest and Diagnostic Tool –** This tool will help you determine when your students might encounter problems while you are teaching the chapter. The questions on the diagnostic incorporate skills that should have been acquired by students *prior* to the chapter. Use the tally sheet to record the number of incorrect student responses. You can do the assessment quickly and with a minimum of grading by having your students do the assessment at home, and then passing the tally sheet around in class. Students can self-report their incorrect responses with tally marks for the problems they missed. The tally sheet will then provide you with immediate feedback on topics in the chapter that might require a bit more review of previous material.

- **Assessment for Understanding –** Cognitive development theory, specifically information processing theory, tells us that students must learn cognitive monitoring to be successful with attaining learning goals. For our math students, that means that they need to learn to "classify" the type of problem in their mind and be able to make decisions about *which* strategies should be used to solve problems. The mid-chapter and end-of-chapter assessments are provided to help students to grasp the big picture ideas, evaluate the appropriateness of strategies, and help students to learn to categorize the type of problem, so they can then correctly access the type of solving procedure that might be appropriate.

- **Metacognitive Skills Assessment –** Metacognitive skills (another component of cognitive monitoring) refer to the ability to judge how well you have learned something and to effectively direct your own learning and studying. This assessment is a self evaluation tool for students designed to help them better focus their studying, and to improve their ability to evaluate how well they understand their own learning. Students with good metacognitive skills are better able to direct their own learning (a skill that is going to be extremely important in an economy where job responsibilities shift faster than we can measure). In addition to the learning benefits, this assessment also provides a topics list to help students study for the chapter test.

- **Activities –** Although these are not labeled specifically as "assessments," every student activity is really a learning assessment. They are not meant to be graded, and they provide you with immediate feedback about how well each student is learning the new material as you monitor your students' work.

Introduction to the Instructor’s Resource Binder

Why so many Student Activities?

These activities (learning assessments, really) are designed to help students learn in a way that is more interesting and engaging than just watching the instructor do problems on the board. Although activities can be done alone, in pairs, or in groups, they do work best when students have the opportunity to discuss their thoughts and answers with others. If you have a classroom with enough board space, you might consider having students stand up and work in pairs or groups at the boards, bringing in a whole new dimension of kinesthetic learning. The activities are designed to help students:

- Develop schema for *new* mathematical concepts
- Improve on and “exercise” the students’ existing mathematical schemata
- Be attentive to details and not just the gist of the idea
- Create flexible schemata by being able to distinguish between problems that look very similar, but are different
- Restructure faulty schemata by creating confrontation with their existing flawed schema

These activities are meant to *replace* class time that would normally be spent in more passive learning (like taking notes about a lecture). Depending on the depth of coverage and the amount of time that you meet with your students, you will likely *not* be able to use all the activities that are provided. Use the ones you want to use. *You* are the instructor… it is your call.

Student Workbook:

Although instructors will have permission to make classroom copies from the IRB activities and assessments, it might put a strain on your department’s copying budget, and your time, to make so many copies for each class. Consider that at this moment, there are over 400 pages of activities and assessments written for this book. If you copied *half* the activities and assessments for a class of 30 students, then each class would cost your department $240 for copying (assuming 4 cents a page).

For this reason, all of the assessments and activities will be available in a bound student workbook that students can purchase along with their textbook. Although purchasing the workbook does place a small additional cost on the student, consider that a comparable amount of department copying would likely be passed on to the student via an eventual tuition increase. I sincerely believe that if the outcome is a better learning experience for the student, a small additional cost is reasonable.

Group Activities and Key Concepts:

Additional materials from the 3rd edition of the Tussy/Gustafson text have been reformatted in worksheet format for use in the classroom. These materials are included in the IRB as well.

Instructor's Resource Binder: Chapter 4

Table of Contents: *Solving Systems of Equations and Inequalities*

Assessment 4A

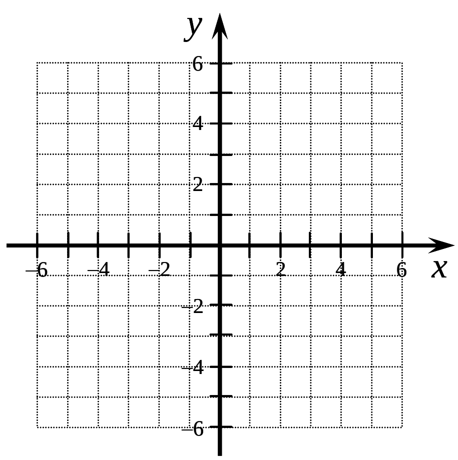
Pretest and Diagnostic Tool: Equations, Inequalities, and Problem Solving

Directions: Complete this assessment without looking back at your notes or your book. **Do not use a calculator on this assessment.**

1. If $x = -2$ and $y = 1$, is the equation $x - 2y = 0$ true or false? _______

2. If $x = 5$, solve the equation $x + 2y = 1$ for y.

3. Graph the equation $y = -3x + 1$.



4. Write the equation $x - 2y = 8$ in slope-intercept form.

5. Solve for x: $3x - 2(x - 4) = 8$.

6. Solve for y: $y - (4y + 10) = -1$.

7. Solve for y: $12\left(\frac{3}{4}y\right) - 4 = y$.

8. If $y = -1$, solve the equation $3x - 2y = 6$ for x.

9. Add the two expressions using the vertical method:

$$\begin{array}{r} 5x - 3y \\ \underline{+\ 2x + 3y} \end{array}$$

10. Add the two expressions using the vertical method:

$$\begin{array}{r} -12x - 4y \\ \underline{+\quad 8x - 4y} \end{array}$$

11. Multiply both sides of the equation $6x + 5y = -1$ by 4.

12. Multiply both sides of the equation $x - 4y = 7$ by -2.

13. Multiply both sides of the equation $\frac{x}{3} - \frac{1}{2} = -2$ by 6.

14. Multiply both sides of the equation $0.05x + 0.06y = 210$ by 100.

15. Find an expression for the interest earned after one year if x dollars earns 4.5% annual interest.

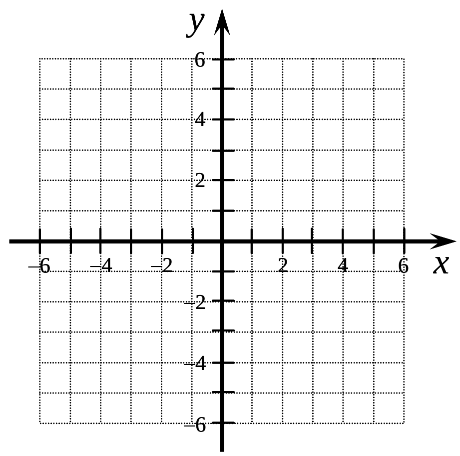
16. Find an expression for the distance traveled by an airplane that travels for 4 hours at a rate of $x+15$ miles per hour.

17. Find an expression for the revenue made by a ticket office if x tickets are sold for $12.00 each and y tickets are sold for $6.00 each.

18. If $x=4$ and $y=-3$, is the inequality $2x+3y<-2$ true or false? _______

19. Is $(2,0)$ a solution of the inequality $y-x\leq -2$? _______

20. Graph the inequality $y<2x-4$.



Assessment 4A: Instructor Tally Sheet
Pretest and Diagnostic Tool: Equations, Inequalities, and Problem Solving

Use this sheet to tally the *incorrect* responses to each question. Each group of questions correlates to specific skills that are needed in this chapter. From the tally sheet, you can see the areas where your students are likely to have more difficulty than others, and thus, where you may need to do additional review before beginning new topics.

Answers:

1. False **2.** $y=-2$

3. See first graph.

4. $y=\frac{1}{2}x-4$ **5.** $x=0$

6. $y=-3$ **7.** $y=\frac{1}{2}$ **8.** $x=\frac{4}{3}$

9. $7x$ **10.** $-4x-8y$

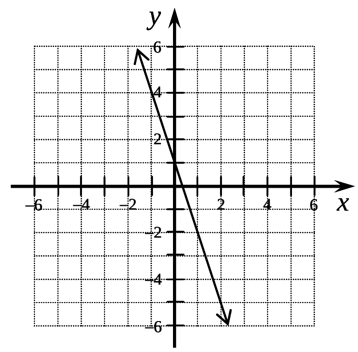
11. $24x+20y=-4$

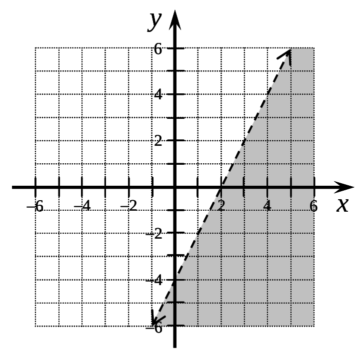
12. $-2x+8y=-14$

13. $2x-3=-12$

14. $5x+6y=21{,}000$ **15.** $0.045x$ **16.** $4(x+15)$ or $4x+60$ **17.** $12x+6y$

18. False **19.** Yes **20.** See second graph.





Skill: *Checking the solution to an equation.*

1.	2.

Skill: *Solving systems of linear equations by graphing.*

3.	4.

Skill: *Solving systems of linear equations using substitution.*

5.	6.	7.	8.

Skill: *Solving systems of linear equations by elimination.*

9.	10.	11.	12.

Skill: *Clearing fractions or decimals from equations.*

13.	14.

Skill: *Problem solving using systems of linear equations.*

15.	16.	17.

Skill: *Solving systems of linear inequalities.*

18.	19.	20.

Teaching Guide

Section 4.1: Solving Systems of Equations by Graphing

■ Preparing for Your Class

Suggested Class Time: 45-60 minutes

Materials Needed: Student activities, guided learning activity, colored pencils, straight-edge, graphing calculator projection technology (if you plan to use Student Activity C)

Vocabulary

- System of equations, solution of the system
- Consistent system, inconsistent system
- Infinitely many solutions
- Independent equations, dependent equations

Instruction Tips

- The edge of a driver's license, student ID, or credit card makes a nice straight-edge that is about the right size for the graphs we draw in math courses (and all the students have one).
- It will be helpful on the board to color-code the two equations in the system with the corresponding lines on the graphs. Make sure that each of your students also has two colored pencils to do this in their notes. This is especially nice when you show an example of a dependent system, as then students can see the two identical equations (overlapping in different colors) much easier than if it is done with only one color.
- In the chapter on linear equations, students learn to examine pairs of equations of lines and categorize them as parallel, perpendicular, or neither. Because of this, students often want to associate these same properties with no solution, one solution, and infinitely many solutions. Of course, a system of two parallel lines *is* associated with no solution. However a system does not have to be made up of perpendicular lines to have one solution. Watch out for this odd student misconception.
- When students solve simple linear equations, the possible solutions options are a) one solution, b) no solution, c) all real numbers are a solution. This might be simplified in their minds as "one, none, all." For this reason, students may think that "infinitely many solutions" is the same as "every point is a solution" unless they are specifically confronted with the difference.
- It is generally easier for students to remember system classifications by memorizing the "exception" and then classifying everything else as "not" that exception. For example, *inconsistent* is used to describe a system with no solution. Every other system is consistent. Likewise, *dependent* is used to describe a system where the equations are really different forms of the same line. Every other set of equations is independent.
- *Equations* are dependent or independent. *Systems* are consistent or inconsistent.

■ Teaching Your Class

Classroom Overhead: *Graphing Grids.* These graphs can be copied to an overhead, projected from a portable video projector, or projected from a computer and ceiling projector to a whiteboard.

Student Activity A: *Targeting solutions.* In this activity, students practice testing ordered pairs to see whether they satisfy the equations in the system that is provided. Each problem has only one solution (one bulls-eye).

Guided Learning Activity: *Graphing to Solve Systems of Equations.* In this guided lecture/discussion, we examine the concept of the *solution* to a system of equations by looking at it algebraically, then graphically. The solution to the first example is (1,4); the instructor should make sure that this gets listed for both equations by guiding the discussion. Problem 3 has no solution and problem 5 has infinitely many solutions.

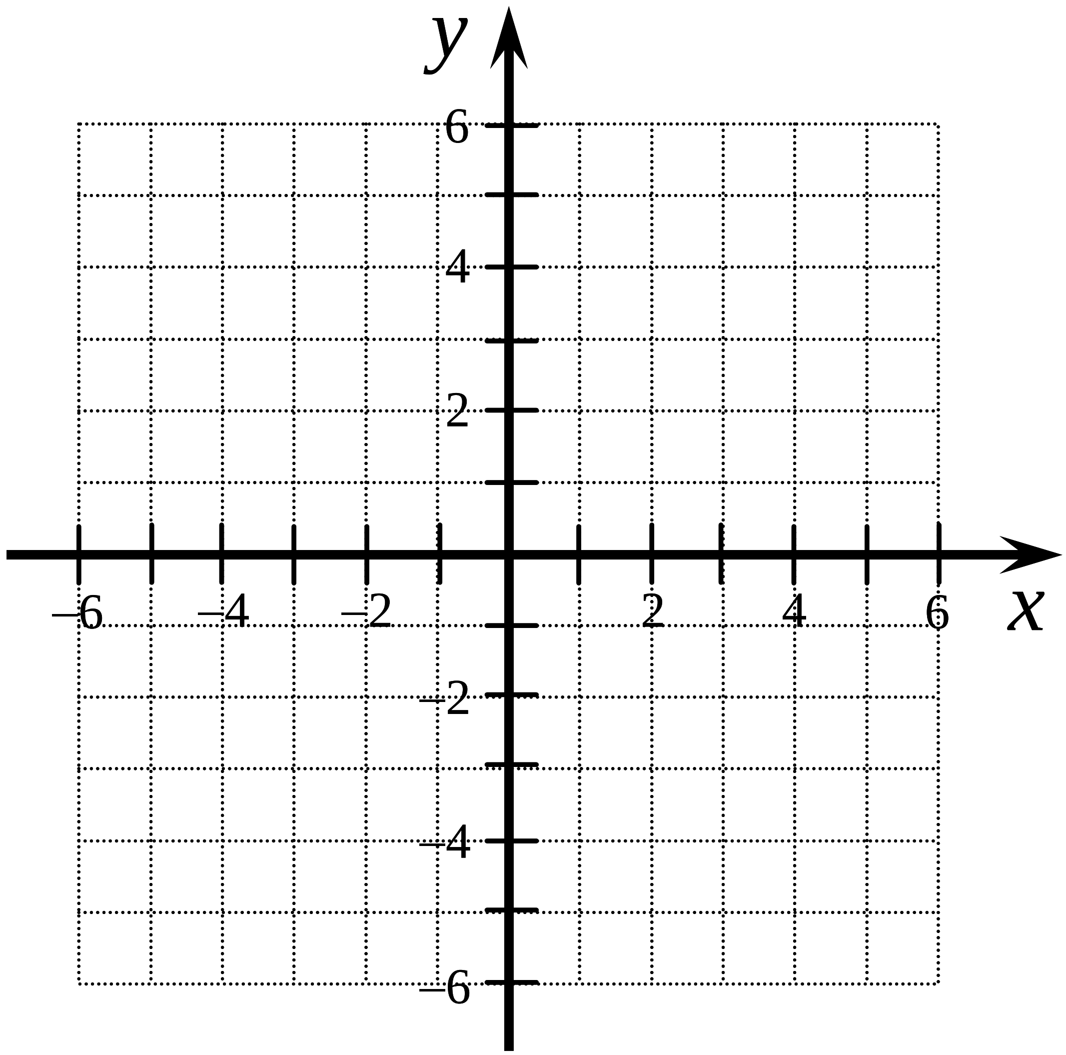
Classifying the solution of a system of equations

- If a system of equations has no solution, it is called an *inconsistent system.* If a system is not inconsistent, then it is a *consistent system* (the system of equations has at least one solution). Go back to the Guided Learning Activity and have the students label each of the six systems as either consistent or inconsistent.
- If the graph of the equations give the same line, they are called *dependent equations.* If a set of equations is is not dependent, then they are called *independent equations* (the graph of the equations were different lines). Go back to the Guided Learning Activity and have the students label each of the six systems as either dependent or independent.
- Ask your class: *How many solutions (intersections) are possible from a system of two linear equations?* and draw a possible graph for each option. Construct a table of possibilities as your discussion proceeds. Although there should only be three columns of possibilities, leave room for doubt (maybe construct a table with four columns) so that students really think about the question. This is a good time to discuss the fact that both perpendicular lines and *other* pairs of lines can have exactly one solution.

How many solutions?	Exactly one solution (the point of intersection)	No solution	Infinitely many solutions (any point *on the line*)
What are the characteristics of the graph of this system?	The lines intersect at one point.	Parallel lines	Identical lines
Consistent or inconsistent?	Consistent	Inconsistent	Consistent
Dependent or independent?	Independent	Independent	Dependent

Student Activity B: *Following the Clues Back to the System of Equations.* This is a challenging activity that asks the students to remember how to find the equation of a line given the graph of the line. After performing this task for two lines, the student will have a system of equations that they can “check” with the intersection point. Often students compartmentalize this equation-finding knowledge and then forget it after a test, so this activity is designed to improve the retention of this concept. You may want to provide a few hints for this activity, depending on how comfortable your students are with finding the equation of a line.

Student Activity C: *Graphing Systems of Equations with a Calculator*. If you allow the use of a graphing calculator in your course, and you want students to get some practice with graphing systems of equations and finding the intersection, here is an activity to help you and your students. You may want to ask students to bring the instruction manual for their calculators to class if they have models you are not familiar with.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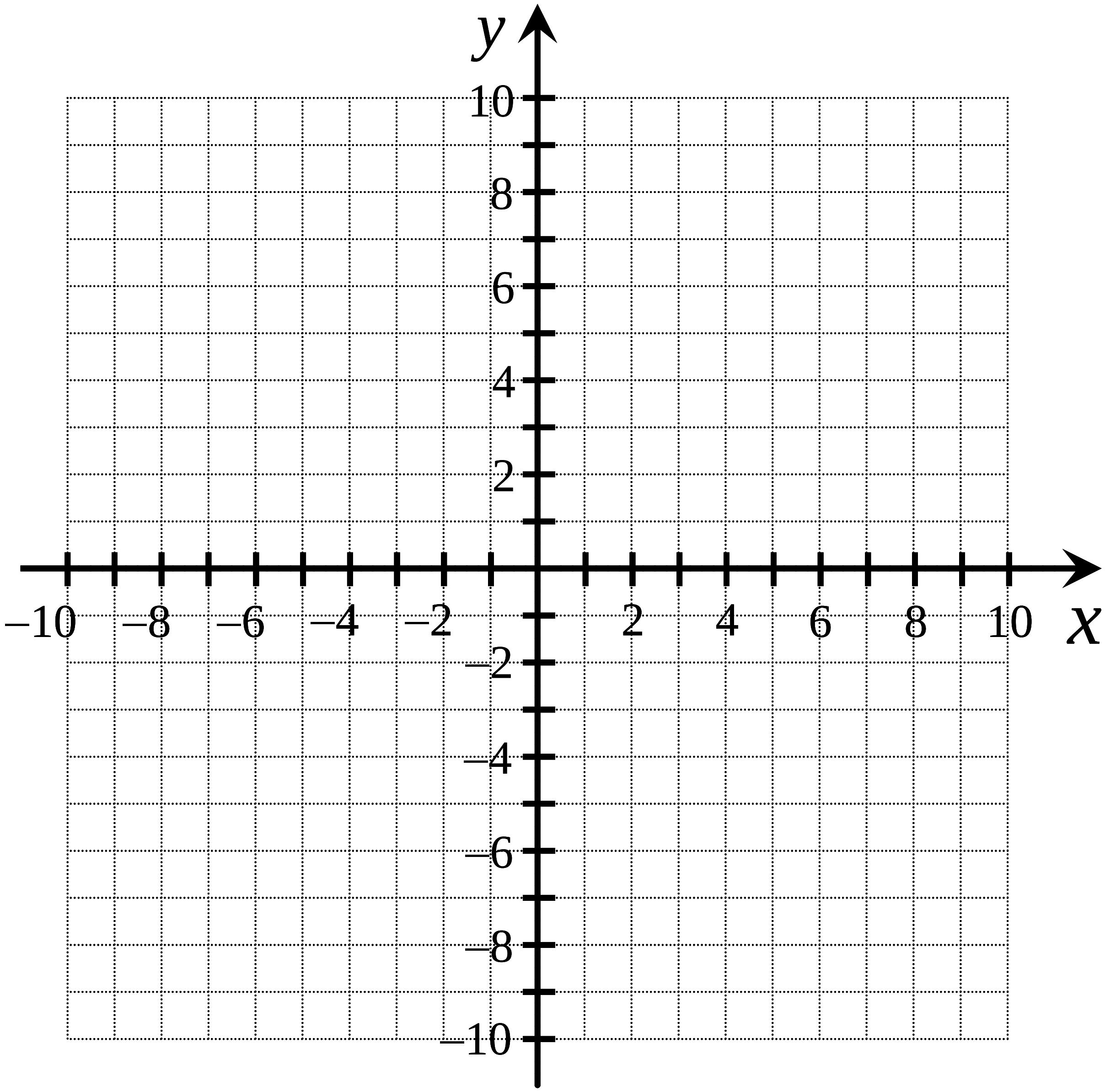
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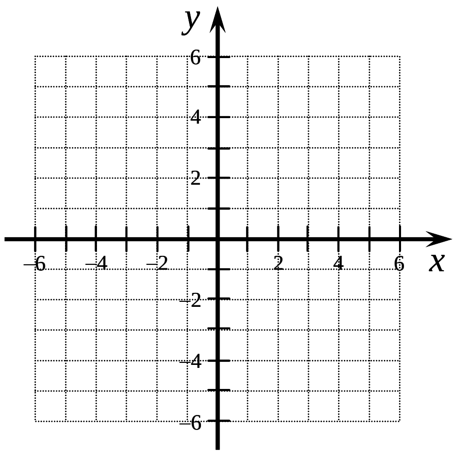
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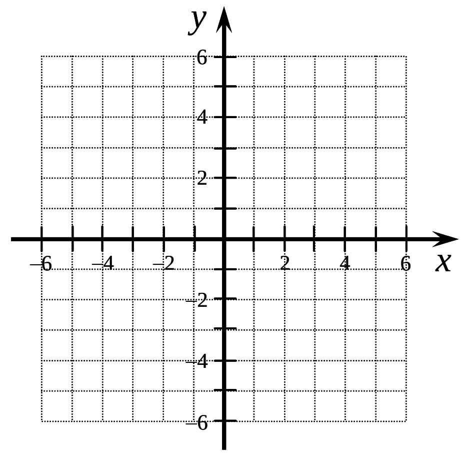
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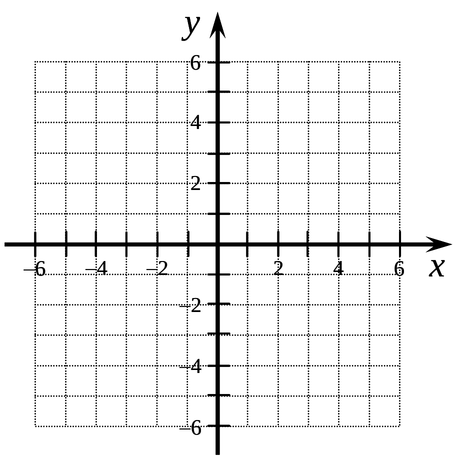
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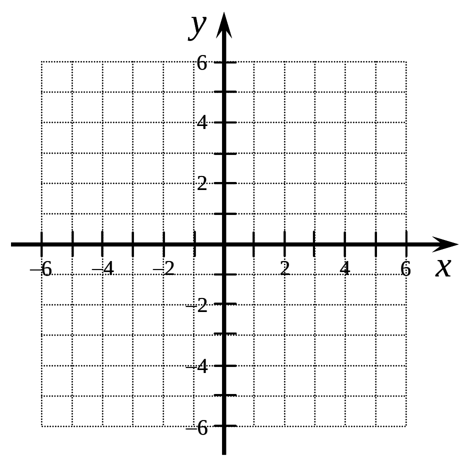
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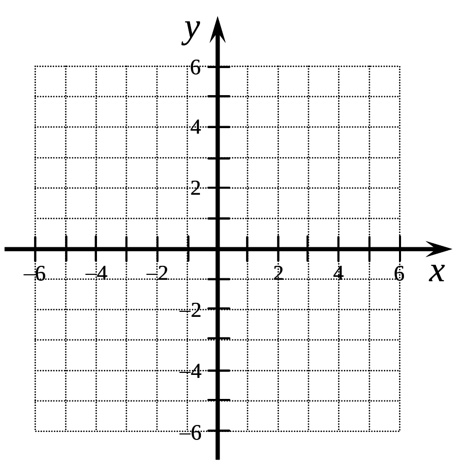
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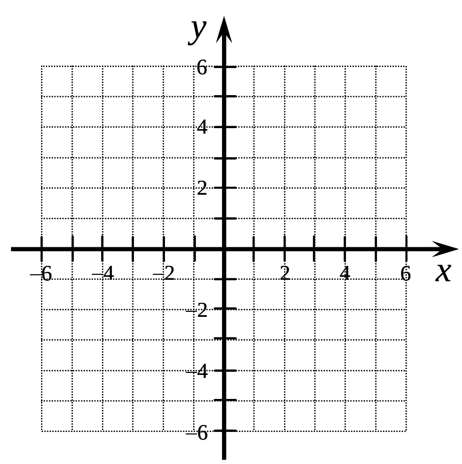
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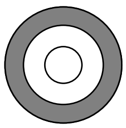
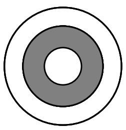
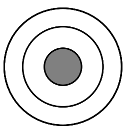
Student Activity A

Section 4.1: Targeting Solutions

Directions: In each problem below, test the ordered pairs that are provided to see if they are possible solutions to either of the equations in the system of equations. After testing, place the ordered pairs into the appropriate region of the “target” for each problem.

 **Outer target region**	The ordered pair IS a solution of the first equation, but not the second equation.
 **Middle target region**	The ordered pair is not a solution of the first equation, but IS a solution of the second equation.
 **Bulls-eye (center target region)**	The ordered pair is a solution of both equations.

1. $\begin{cases} y < 5x + 2 \\ y > -x + 8 \end{cases}$

$(1,6)$

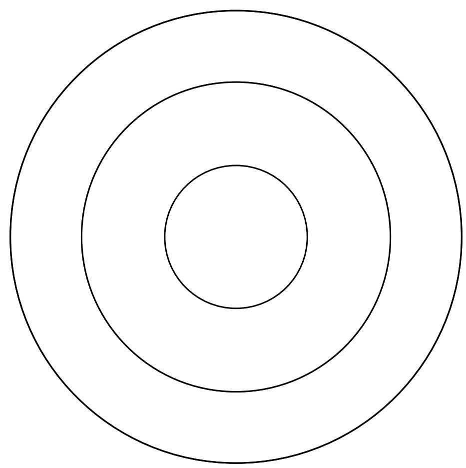
$(4,25)$

$(2,10)$

$(-10,20)$

$(-3,-20)$

$\left(\frac{1}{2},9\right)$



2. $\begin{cases} 4x + 2y = 8 \\ -2x + 4y = 1 \end{cases}$

$(5,-6)$

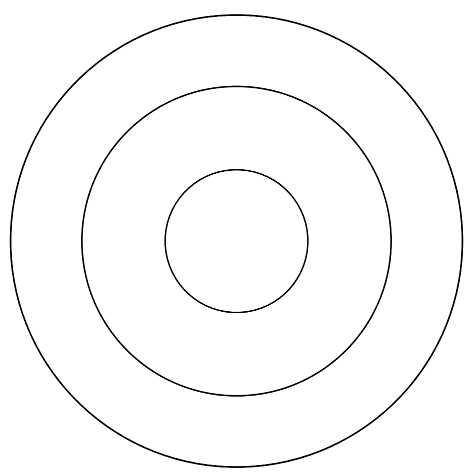
$\left(7,\frac{15}{4}\right)$

$\left(\frac{3}{2},1\right)$

$\left(-4,-\frac{7}{4}\right)$

$(0,4)$

$(-3,10)$



Guided Learning Activity

Section 4.1: Graphing to Solve Systems of Equations

Example 1:

Write some ordered-pair solutions of the equation $x + y = 5$.

(,) (,) (,) (,) (,) (,)

Write some ordered-pair solutions of the equation $x - y = -3$.

(,) (,) (,) (,) (,) (,)

When we write the two equations like this: $\begin{cases} x + y = 5 \\ x - y = -3 \end{cases}$

it is called a *system of equations*. When we *solve* a systems of two equations, we are looking for all of the ordered pairs (x, y) that satisfy *both* equations.

Maybe you found an ordered pair that was a solution of both equations? If so, circle it.

Example 2: What does it mean to be a solution to a system?

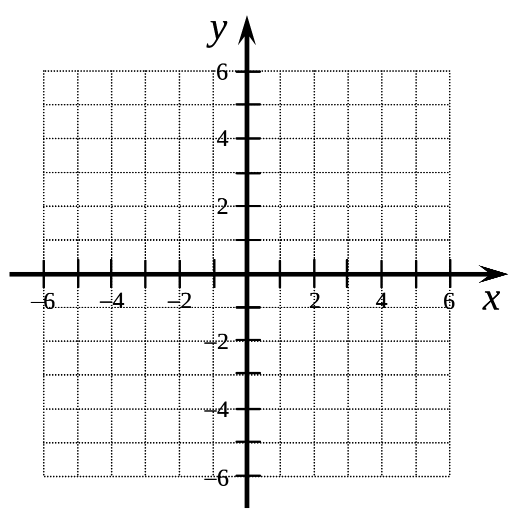
Consider the system of equations given by $\begin{cases} y = 2x + 2 \\ y = x - 1 \end{cases}$ and the ordered pair $(-3, -4)$.

You can see by evaluating each equation for $x = -3$ and $y = -4$ that this ordered pair is a solution of both equations. This means it is a solution of the system of equations.

$y = 2x + 2$	$y = x - 1$
$y = 2(\quad) + 2$	$y = (\quad) - 1$
$-4 \stackrel{?}{=} 2(-3) + 2$	$-4 \stackrel{?}{=} (-3) - 1$
$-4 = -4$	$-4 = -4$

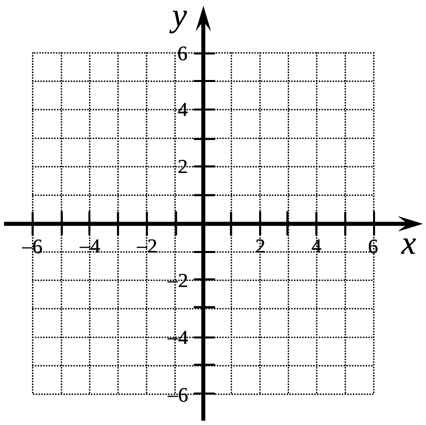
So what does the solution of a system of equations look like? Graph $y = 2x + 2$ and $y = x - 1$ on the graphing grid provided to the right.

What is special about the point $(-3, -4)$ on the graph?

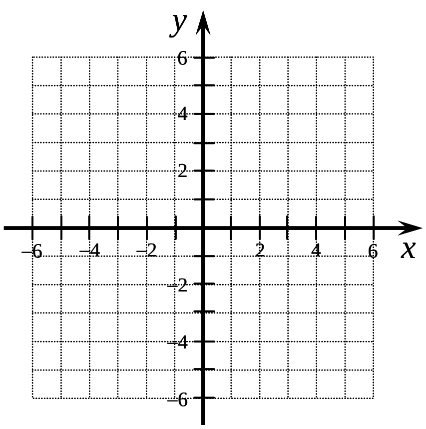


Now solve these systems of equations by graphing.

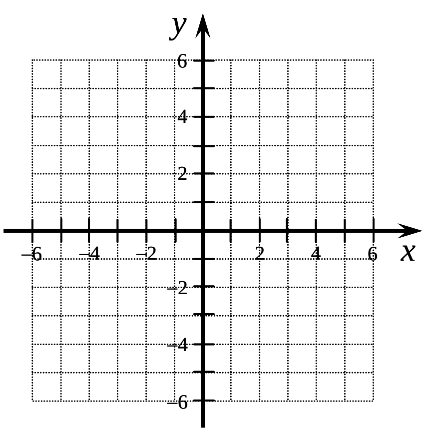
1. $\begin{cases} y = -3x + 6 \\ y = 2x - 4 \end{cases}$



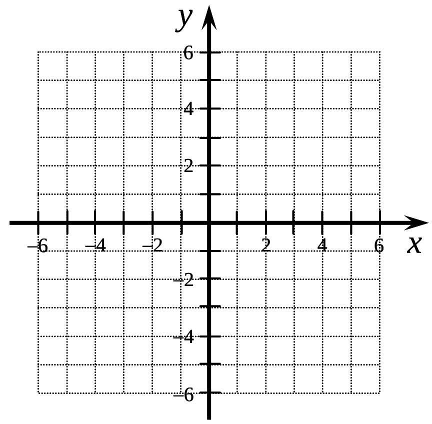
2. $\begin{cases} 5x - y = 6 \\ 3x - 3y = 6 \end{cases}$



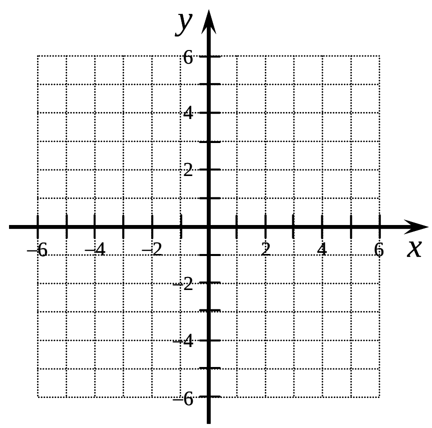
3. $\begin{cases} -4x + 2y = 8 \\ 6x - 3y = 3 \end{cases}$



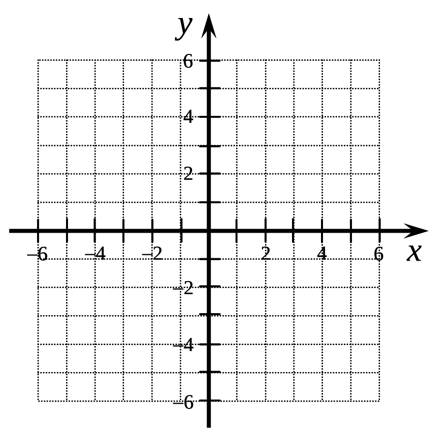
4. $\begin{cases} y = -3x - 2 \\ y = 2x + 3 \end{cases}$



5. $\begin{cases} -2x - 4y = -2 \\ 4x + 8y = 4 \end{cases}$



6. $\begin{cases} y = \frac{3}{4}x - 1 \\ y = -\frac{4}{3}x - 1 \end{cases}$

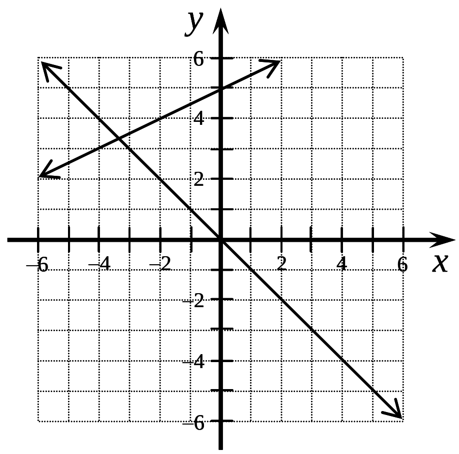


Student Activity B

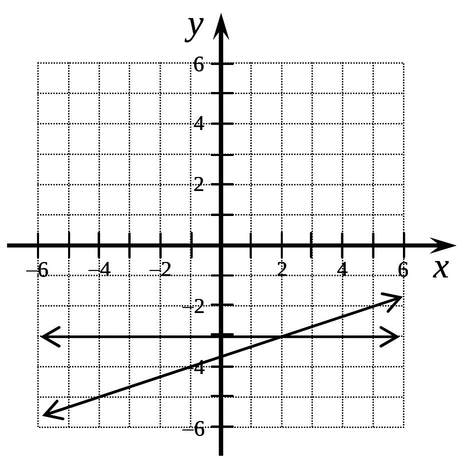
Section 4.1: Following the Clues Back to the System of Equations

Directions: In each "crime-scene" below, you are shown the graph of a system of equations. Use your mathematical powers of reasoning (and detective skills) to determine what the system of equations must have been to result in this graph.

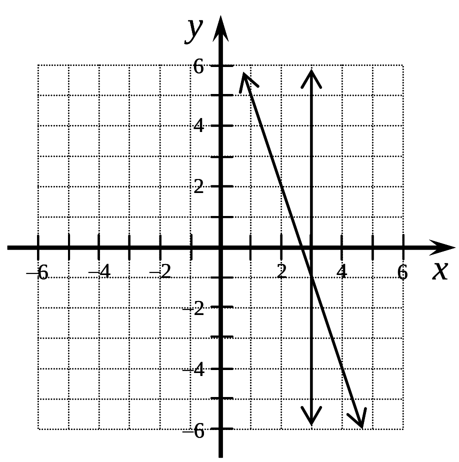
1.



2.



3.

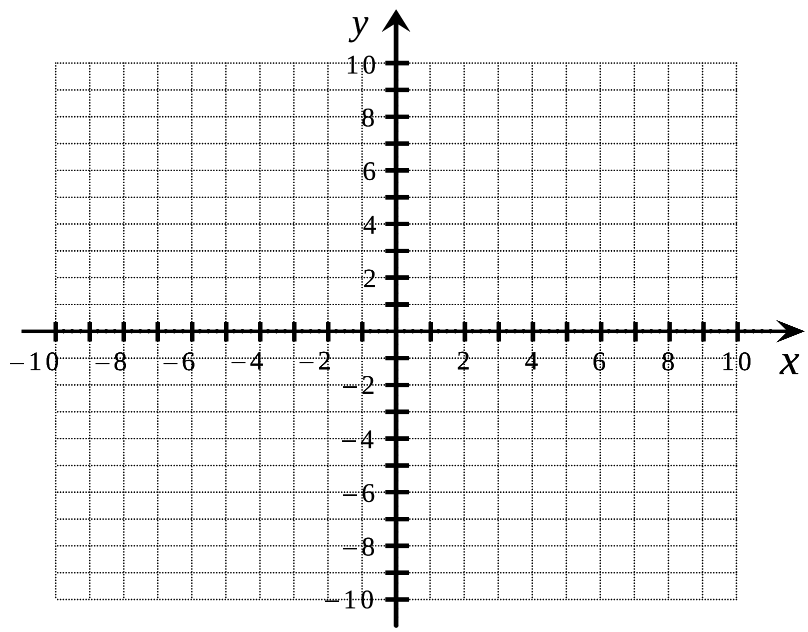


Student Activity C

Section 4.1: Graphing Systems of Equations with a Calculator

You can use your graphing calculator to solve systems of linear equations by graphing. Each model of calculator will use a different set of keystrokes to graph these equations. Before you start to graph any system of equations, it is a good idea to guess what your graph should look like (in case you type something in your calculator incorrectly). The standard zoom on most graphing calculators goes from -10 to 10 on the x-axis and from -10 to 10 on the y-axis.

1. Sketch a graph of the system of equations $\begin{cases} y = 5x - 5 \\ y = -3x + 3 \end{cases}$ on the axes below, and label the solution point.



Note: In Chapter 3 you learned how to graph equations on your calculator (if you have forgotten how, look back at *Section 3.3 Student Activity C: Graphing Linear Equations with a Calculator*).

2. Find the $y =$ screen on your calculator. Place your cursor next to $Y_1 =$ and enter the first equation in your system. Then move your cursor to $Y_2 =$ and enter the second equation. Make sure you are in the standard viewing window and then view the graph. Hopefully this looks like the graph you produced above.

3. In order to find the solution to this system of equations on the calculator, we need to find the point of intersection. **From your graph screen**, find the *Math* or *Calculate* menu. You'll know you have found it when you see a menu that contains the functions *zero (or root)*, *max*, *min*, and *intersect* (it will also have a few others).

Draw the keys or write the steps that were necessary to find this menu.

4. Select *intersect* (or *intersection*) from the menu you just found. Your calculator will give you a series of prompts. At the end of all of the prompts, your calculator should display numerical values labeled $x =$ and $y =$. This should match the solution you found at the beginning of the activity.

Calculator Prompt	**What does it mean? What should you do?**	**Possible Errors**
First Curve?	The calculator is asking you to tell it which curve is the first curve in the system of equations. Place your cursor on one of the lines you graphed using the arrow keys and press *enter*. You will then be asked for the second curve. Move the cursor to the other curve and press *enter*. *Why does it ask this? Well, if you had graphed four lines, it would want to know which lines (curves) were the two that you want to find the intersection of.*	If you give the calculator the **same** curve for both the first and second prompts, you will likely get an error.
Lower Bound?	The calculator is asking you to give it a smaller region to look in to find the intersection point. It is asking you for a value to the left of the point of intersection (lower bound) first. Use your arrow keys to place your cursor to the left of the point of intersection and press *enter* or use the number keypad to enter a value that you know is to the left of the intersection point, and then press *enter*. You will then be asked for the Upper Bound. Repeat the process for a value to the right of the intersection point.	The upper bound must be **above** (to the right of) the lower bound. If you try to give the calculator an upper bound value that is below the lower bound value, you will get an error.
Guess?	It is easier for the calculator to find the exact intersection point if you give it a guess point near the intersection point. Just position your cursor near the intersection point and press enter. *Why does it ask for a guess? If you were graphing more complicated curves (for example polar curves), there might be more than one intersection point in the same lower and upper boundary. In this case, the calculator would want to know which point you wanted to find out exactly.*	You must give the calculator a guess that is **between** the lower and upper bounds. If your guess is outside the region where you told the calculator to look, it will be (understandably) unhappy with you.

Note: If you do not see the intersection values, it could be that you pressed enter one too many times when going through the prompts (inadvertently skipping the solutions). If this happens, repeat the steps a little more slowly, making sure to only press *enter* once for each prompt.

Teaching Guide

Section 4.2: Solving Systems of Equations by Substitution

■ Preparing for Your Class

Suggested Class Time: 45-60 minutes

Materials Needed: Student activities, colored pencils and colored markers

Vocabulary

- The substitution method, the substitution equation

Instruction Tips

- In the English language, we often make "substitutions," only we call them synonyms. If two words or phrases are synonyms, they are equivalent in a sentence, and one can be substituted for another without changing the overall meaning of the sentence. Here are some examples:
  The dress that she wore was really beautiful.
  Substitutes (synonyms) for beautiful: stunning, striking, attractive, gorgeous
  Give me twice that amount of ice cream please.
  Substitutes (synonyms) for twice: two times, double, twofold
  That candy bar cost $1.
  Substitutes for $1: $1 = 100 pennies, $1 = 4 quarters, $1 = 10 dimes, etc.
- It will be helpful on the board to use two different colors for the two equations in the system. When you demonstrate finding the substitution equation, do it in the same color as the starting equation. When you make the substitution, use multi-color to show which part is the **other** equation and which was the substitution. This is demonstrated in the example below with shading/non-shading. Make sure that each of your students also has two colored pencils to do this in their notes.

$$\begin{cases} 2x-3y=9 \\ x+2y=1 \end{cases} \xrightarrow{\text{Solve for } x} \quad \begin{aligned} x+2y&=1 \\ x&=-2y+1 \end{aligned}$$

Make substitution into other equation:

$$2(-2y+1)-3y=9$$
$$-4y+2-3y=9$$
$$-7y+2=9 \quad \text{etc.}$$

- Continually overemphasize that once the students find a substitution equation, it must be used in the **other** equation of the system.
- In the second step of the Substitution Method process, students often get confused if the variable that they solve for results in a value of zero, thinking that this indicates a "no solution" answer. It is possible that this misconception is related to the confusion of the digit 0 and the symbol $\varnothing$ used for no solution in solution set notation. In many applications (i.e. the military, computer passwords, mapping coordinates), a zero with a slash mark is used to keep from confusing the number zero from the capital letter O. Just as likely, students just equate "zero" with "none," and "none" with "no solution." By purposefully choosing examples where one of the solution coordinates is zero, you can quickly see which of your students is falling for this misconception.
- In the third step of the Substitution Method process, you can ask students to find the *easiest* place to make the substitution of the value they have already found (the answer to this question will be the substitution equation). The benefit of having students look for the *easiest* equation, is that this part of the process will not be altered when they go on to solve equations using the elimination method.

■ Teaching Your Class

The Substitution Method

Identify / find a substitution equation. → Substitute into the **other** equation and solve. → Find the missing coordinate of the ordered pair solution. → Check the proposed solution in both of the original equations.

> **Student Activity A:** *Which Egg is Easier to Crack?* In this activity, students gain some expertise on deciding which variable to solve for when they are finding the substitution equation.

Examples (solve by substitution): *The solution is shown to the right of each equation.*

$\begin{cases} 2x-3y=9 \\ 2y=1-x \end{cases}$ $(3,-1)$

$\begin{cases} 4x+y=6 \\ 6y=9x+36 \end{cases}$ $(0,6)$

$\begin{cases} 15x+10y=-20 \\ 3=4x+y \end{cases}$ $(2,-5)$

$\begin{cases} y=2x+6 \\ 2y-4x=12 \end{cases}$ Infinitely many solutions

$\begin{cases} 8x-3y=2 \\ 12x-3=0 \end{cases}$ $\left(\frac{1}{4},0\right)$

$\begin{cases} x+2y=-2 \\ y+\frac{1}{2}x=4 \end{cases}$ No solution

$\begin{cases} 2x+6y=-1 \\ 4x-3y=3 \end{cases}$ $\left(\frac{1}{2},-\frac{1}{3}\right)$

$\begin{cases} \frac{1}{4}x-\frac{1}{2}y=-\frac{1}{2} \\ \frac{1}{2}x=y-1 \end{cases}$ $(-4,-1)$

$\begin{cases} \frac{x}{12}-\frac{y}{4}=\frac{1}{3} \\ \frac{2x}{3}+2y=4 \end{cases}$ $\left(5,\frac{1}{3}\right)$

$\begin{cases} 5(x+y)+2(y+6)=0 \\ 3(x-y)=-5(x+1) \end{cases}$ $(-1,-1)$

- Classify each of the systems above as *consistent* or *inconsistent*, and the equations as *dependent* or *independent.*

> **Student Activity B:** *Clear the Way!* In this activity, students practice clearing the fractions or decimals from an equation. Although decimals are not heavily used in systems of equations until the application problems in Section 4.4, it is good practice for students to work with clearing fractions and decimals at the same time.

> **Student Activity C:** *Double Trouble on Substitution.* In this activity, students learn that there is more than one method to solving a substitution problem and practice with the technique of solving with substitution. There is a self-check built in to the problems, so students should be fairly certain whether their answers are correct or not.

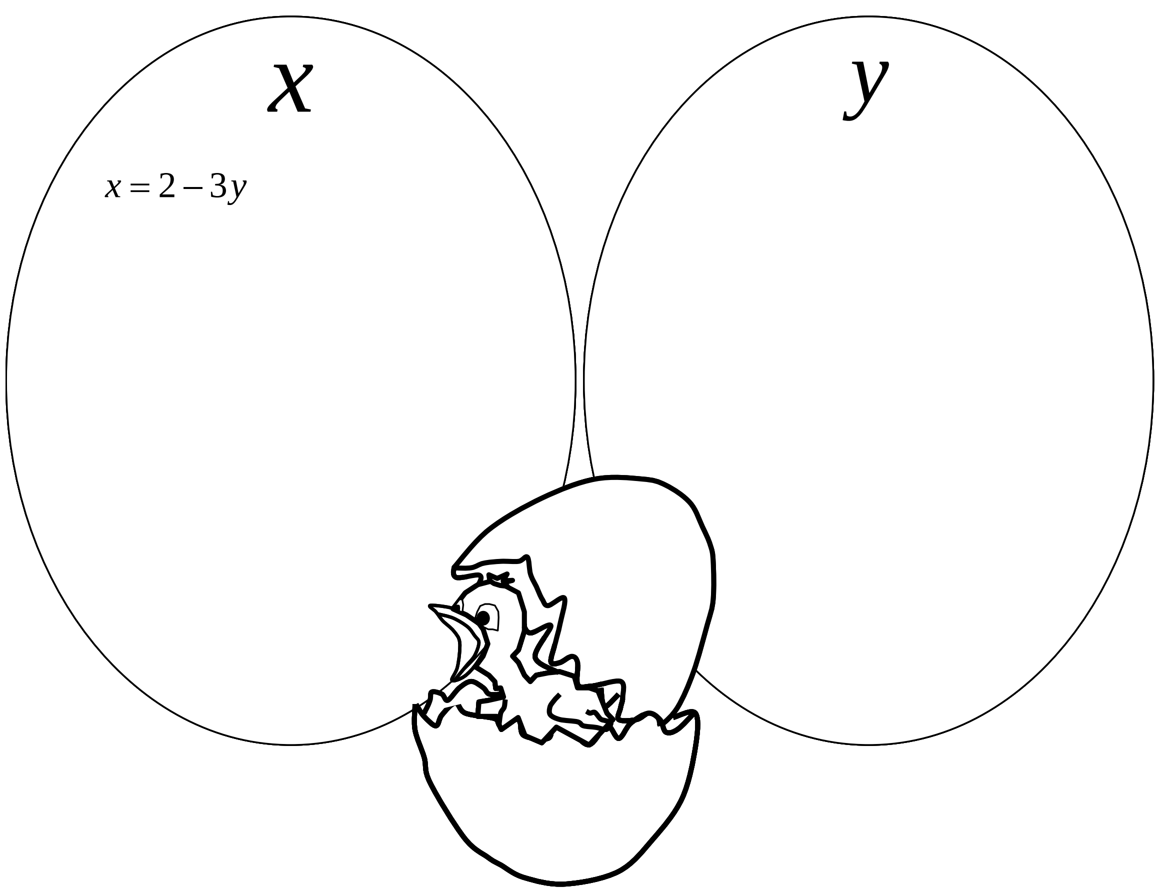
Student Activity A

Section 4.2: Which Egg is Easier to Crack?

When solving systems of equations by substitution, we begin by solving one of the equations for x or y; this is called the *substitution equation*. Sometimes one of the variables is easier to solve for than the other (that is, it will take less steps to solve for one than the other).

Directions: Look at the given equations and find the substitution equation where solving for the variable requires **the least number of steps**. Then place the substitution equation in the egg that corresponds to the variable that was solved for. For example, in the first equation, it would be much easier to solve for x, resulting in the equation $x = 2 - 3y$.

$x + 3y = 2$	$7x + y = 14$	$3y = 2x + 2$	$-2y + x = 18$
$x - 2y = 9$	$15 = -5y + x$	$16x = 16y + 16$	$\frac{2}{3}x + y = \frac{1}{3}$
$25x = 225y + 225$	$12x - y = 0$	$3y = 3x + 6$	$5x + y = 10$
$18x = 9y + 36$	$\frac{1}{10}x + y = 1$	$12x + y = 2$	$4x = 2y + 6$



Student Activity B

Section 4.2: Clear the Way!

Directions: Clear each equation of decimals or fractions and shade in the corresponding square in the grid below. The first one has been done for you. There's a surprise when you're finished!

1. $\frac{1}{10}x - \frac{1}{5}y = 1$

$10\left(\frac{1}{10}x - \frac{1}{5}y\right) = 10(1)$

$\boxed{x - 2y = 10}$

2. $0.1x + 2 = 0.2y$

3. $\frac{1}{100}(3x - 5y) = 3$

4. $0.04(x - 50) = 0.02y$

5. $\frac{1}{10}x + \frac{1}{15}y = \frac{1}{30}$

6. $0.33(x + 1) = 7y$

7. $\frac{1}{8}(4x - 2y) = \frac{1}{16}$

8. $0.001x + 0.0002 = 0.003y$

9. $\frac{2}{3}y + \frac{4}{9} = \frac{1}{18}x$

10. $0.1x + 0.01y = 0.001$

$4x - 2 = 20y$	$4x - 50 = 20y$	$x - 2y = 10$	$3x - 5y = 30$	$x - y = 300$
$4x - 20 = 20y$	$4x - 200 = 2y$	$3x - 5y = 300$	$3x - 5y = 3$	$3x - 500y = 300$
$3x + 2y = 10$	$2x + 3y = 1$	$3x + 2y = 1$	$3x + y = 1$	$x + 2y = 1$
$4x - 2y = 1$	$8x - 2y = 1$	$8x - 4y = 1$	$8x - 4y = 2$	$4x - 2y = 2$
$x + 2 = 2y$	$x + 20 = y$	$x + 20 = 2y$	$x + 20 = 0.2y$	$x - 20 = 2y$
$x + 2 = 3y$	$10x + y = 1$	$33x + 33 = 700y$	$33x + 33 = 7y$	$33x + 1 = 70y$
$10x + 20 = 30y$	$10x + 2 = 30y$	$100x + 10y = 1$	$12y + 8 = x$	$2y + 4 = x$

Student Activity C

Section 4.2: Double Trouble on Substitution

Directions: When you solve a system of equations by substitution, there are *at least* two ways to tackle the problem. For example, in the first system, you could easily solve for y in the first equation, or for x in the second equation to find your substitution equation. **Some choices are easier than others!** For each of the systems below, solve the system of equations two ways as indicated. Your ordered pair solution should be the same for both methods. If they are not, you'll have to go back and look for a mistake. The first one has been started for you.

	System of Equations	**Solve for a variable in the FIRST equation, then substitute and solve.**	**Solve for a variable in the SECOND equation, then substitute and solve.**
1.	$\begin{cases} 2x+y=5 \\ x-4y=7 \end{cases}$	Solve for y in the 1st equation: $y=-2x+5$ Substitute into 2nd equation and solve: $x-4(-2x+5)=7$ $x+8x-20=7$ $9x-20=7$ $9x=27$ $x=3$ Find the missing coordinate of the ordered pair: $y=-2(3)+5$ $=-6+5=-1$ $(3,-1)$	
2.	$\begin{cases} x+8y=20 \\ 4x-y=14 \end{cases}$		

	System of Equations	Solve for a variable in the FIRST equation, then substitute.	Solve for a variable in the SECOND equation, then substitute.
3.	$\begin{cases} 3x - y = -2 \\ -12x + 4y = 16 \end{cases}$		
4.	$\begin{cases} 3x + 4y = 0 \\ \frac{3}{4}x + \frac{8}{3}y = -\frac{5}{12} \end{cases}$		
5.	$\begin{cases} x - 7y = -1 \\ -\frac{1}{7}x + y = \frac{1}{7} \end{cases}$		

Teaching Guide

Section 4.3: Solving Systems of Equations by Elimination

■ Preparing for Your Class

Suggested Class Time: 45-60 minutes

Materials Needed: Student activities, classroom overhead, colored pencils and colored markers

Vocabulary

- Elimination method, addition method
- Addition property of equality
- Standard form for a linear equation: $Ax + By = C$

Instruction Tips

- It will be helpful on the board to use two different colors for the two equations in the system. When you manipulate one of the equations, write the work in the same color as the equation. When you are working with a combination of the two equations, do not use a colored marker. Make sure that each of your students also has two colored pencils to do this in their notes.

$$\begin{cases} 2x+6y=-1 \\ 4x-3y=3 \end{cases} \xrightarrow[\text{Multiply by 2}]{\text{unchanged}} \begin{cases} 2x+6y=-1 \\ 8x-6y=6 \end{cases}$$

$$2(4x-3y)=2(3)$$
$$8x-6y=6$$

$$10x=5$$
$$x=\tfrac{1}{2}$$

Find y: $4\left(\tfrac{1}{2}\right)-3y=3$

$$2-3y=3$$
$$-3y=1$$
$$y=-\tfrac{1}{3}$$

$$\left(\tfrac{1}{2},-\tfrac{1}{3}\right)$$

- When students have to arrange an equation like $5x-5=0$ in standard form, they often think that $5x-5=0$ *is* in standard form. If students have not paid attention to the *variables* in $Ax+By=C$, then $5x-5=0$ *looks* closer to the correct form than $5x=5$ because it has three terms in it instead of two.
- The most common student mistake is that they do not apply the Multiplication Property of Equality correctly. In other words, they only multiply by a number on *one side* of the equation. Many students try to use some kind of shorthand for multiplication of an equation by a number, like this: $4x-3y=3$ or this $2\cdot(4x-3y=3)$

$$2\cdot 4x-3y=3 \qquad\qquad 8x-6y=6$$
$$8x-6y=6$$

If students instead write the multiplication on *both* sides of the equation, they will be more likely to carry out the multiplication correctly: $4x-3y=3$

$$2(4x-3y)=2(3)$$
$$8x-6y=6$$

- For equations involving both fractions and non-fractions, like $\frac{1}{2}x+3y=\frac{3}{2}$, it is common for students to only multiply the fractions by the desired constant, instead of multiplying all the terms by the constant. In this case, the student is so focused on clearing the *fractions* that they ignore the rest of the non-fraction terms or constants in the problem. Again, reemphasize the Multiplication Property of Equality and also the Distributive Property:

$$\frac{1}{2}x+3y=\frac{3}{2}$$

$$2\left(\frac{1}{2}x+3y\right)=2\left(\frac{3}{2}\right) \quad \text{Addition Property of Equality}$$

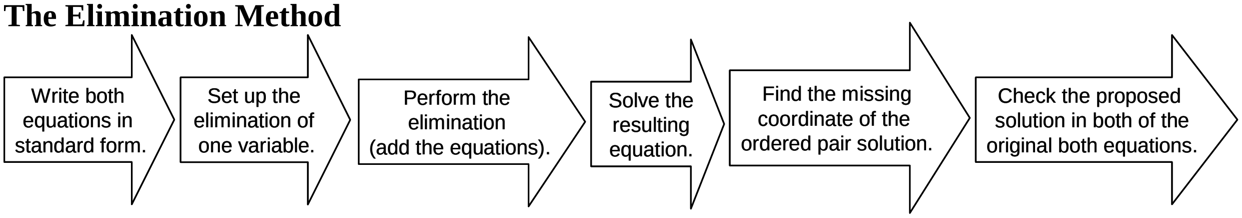
$$2\left(\frac{1}{2}x\right)+2\left(3y\right)=2\left(\frac{3}{2}\right) \quad \text{Distributive Property}$$

$$x+6y=3$$

- Again, students often get confused if the variable that they solve for results in a value of zero, thinking that this indicates a "no solution" answer. See the comment in section 4.2 on this common confusion.
- If your students need to practice the process of clearing fractions again, you might revisit the Classroom Worksheet from Section 2.2 or Student Activity B from Section 4.2.

■ Teaching Your Class

The Elimination Method



> **Student Activity A:** *Forced Elimination.* This activity forces students to practice different elimination strategies than they might have chosen on their own and helps them to gain insight into why certain eliminations are easier than others.

Examples (solve by elimination): *The solution is shown to the right of each equation.*

$\begin{cases} 15x+10y=-20 \\ 3=4x+y \end{cases}$ $(2,-5)$

$\begin{cases} 4x-6y=18 \\ 2x+4y=2 \end{cases}$ $(3,-1)$

$\begin{cases} 5x-10y+10=0 \\ 6y-2x=2 \end{cases}$ $(-4,-1)$

$\begin{cases} 5x+10y=-10 \\ 2y+x=8 \end{cases}$ No solution

$\begin{cases} 3x+4y=15 \\ 5x-3y=25 \end{cases}$ $(5,0)$

$\begin{cases} x+2y=1 \\ 3x+5y=5 \end{cases}$ $(5,-2)$

$\begin{cases} 8y=6x+3 \\ x-\frac{4}{3}y=-\frac{1}{2} \end{cases}$ Infinitely many solutions

$\begin{cases} 2x+6y=-1 \\ 4x-3y=3 \end{cases}$ $\left(\frac{1}{2},-\frac{1}{3}\right)$

$\begin{cases} \frac{1}{4}x-\frac{1}{2}y=-\frac{1}{2} \\ \frac{1}{2}x=y-3 \end{cases}$ No solution

$\begin{cases} \frac{x}{10}-\frac{y}{5}=\frac{1}{4} \\ 6y-6=0 \end{cases}$ $\left(\frac{9}{2},1\right)$

- Classify each of the systems above as *consistent* or *inconsistent*, and the equations as *dependent* or *independent.*

Classroom Overhead: *Solve a System of Two Linear Equations.* This overhead shows all three methods for solving a system of equations side-by-side.

Student Activity B: *Triple the Fun on Systems of Equations.* In this activity, students compare the three methods for solving a system of equations to gain some expertise on deciding which method might be appropriate for a given situation. This also helps students cement the concept that systems of equations can be solved in more than one way, and that regardless of the method, the result should be the same.

- Discuss how you choose a method for solving a system of equations with your class. Although graphing to solve a system of linear equations is not often used, graphing is a well-used method or at least a good starting point for solving harder systems of equations (those involving parabolas, hyperbolas, etc.).

Student Activity C: *Choose Your Tactic.* In this activity, students choose between the methods of substitution or elimination and then describe their first steps.

Student Activity A

Section 4.3: Forced Elimination

Directions: In order to develop some intuition about which variable is easier to eliminate, we will **start** a few problems by trying both of the possible eliminations. Use the elimination method to solve for a variable. **Then circle the elimination that was easier.** The first one has been started for you.

1.

Eliminate *x*	**Eliminate *y***
$\begin{cases} x+3y=7 \\ 2x-3y=-4 \end{cases}$ Multiply the first equation by -2. $\begin{cases} -2x-6y=-14 \\ 2x-3y=-4 \end{cases}$ $-9y=-18$ $y=2$	$\begin{cases} x+3y=7 \\ 2x-3y=-4 \end{cases}$

2.

Eliminate *x*	**Eliminate *y***
$\begin{cases} -10x+y=-5 \\ 10x-2y=0 \end{cases}$	$\begin{cases} -10x+y=-5 \\ 10x-2y=0 \end{cases}$

3.

Eliminate *x*	**Eliminate *y***
$\begin{cases} 5x+7y=12 \\ 10x-3y=7 \end{cases}$	$\begin{cases} 5x+7y=12 \\ 10x-3y=7 \end{cases}$

4.

Eliminate *x*	**Eliminate *y***
$\begin{cases} 3x + y = 2 \\ 4x - 2y = 6 \end{cases}$	$\begin{cases} 3x + y = 2 \\ 4x - 2y = 6 \end{cases}$

Now inspect these systems of equations. For each system, decide which variable is going to easier to eliminate and describe the process you will go through to achieve this elimination. Problem 5 has been done for you.

5. $\begin{cases} 3x + y = 1 \\ 2x + 3y = -11 \end{cases}$ Eliminate *y*.
Multiply the first equation by -3 and then add the equations.

6. $\begin{cases} 5x - y = -5 \\ x + y = 5 \end{cases}$

7. $\begin{cases} x + y = 12 \\ -x + y = 0 \end{cases}$

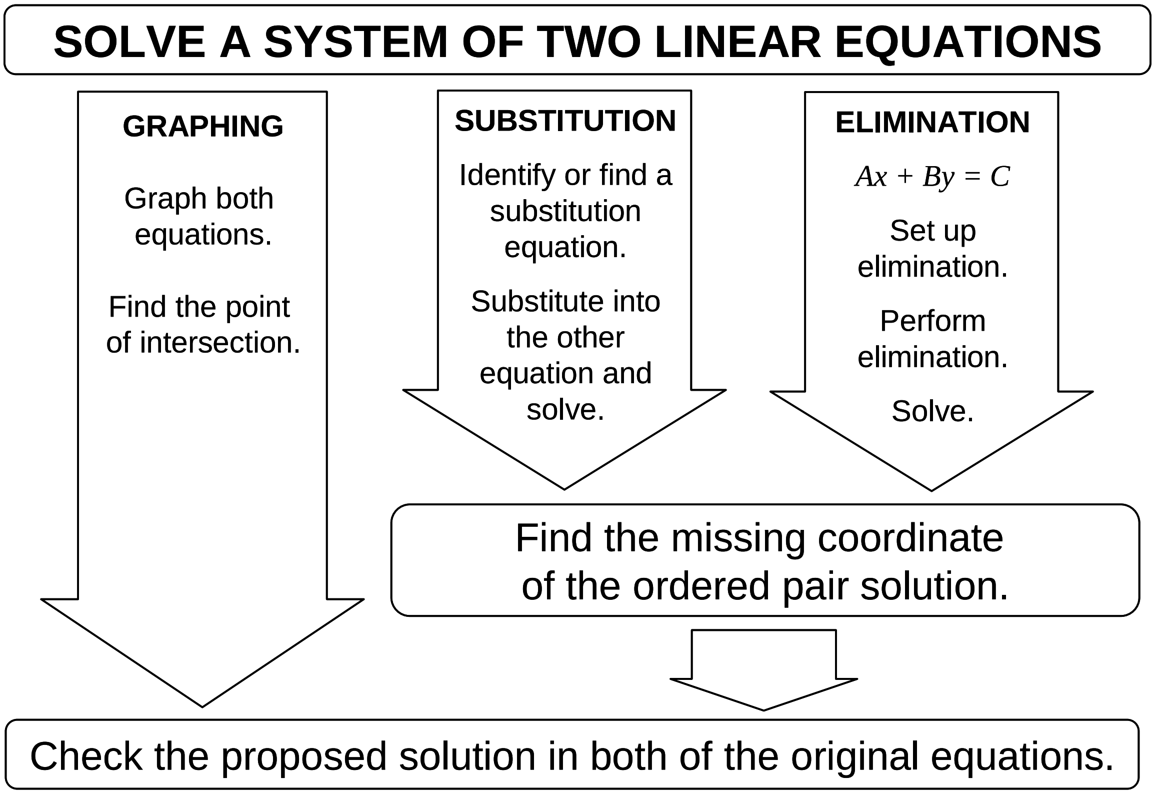
8. $\begin{cases} -4x + 2y = 2 \\ 4x + 6y = 22 \end{cases}$

9. $\begin{cases} 3x + 14y = -7 \\ 2x + 2y = 12 \end{cases}$

10. $\begin{cases} 3x + 8y = 2 \\ -6x - 7y = 5 \end{cases}$

Classroom Overhead
Section 4.3: Solve a System of Two Linear Equations

Here is a comparison of methods for solving a system of two linear equations (assuming the system is consistent and the equations are independent).



Student Activity B

Section 4.3: Triple the Fun on Systems of Equations

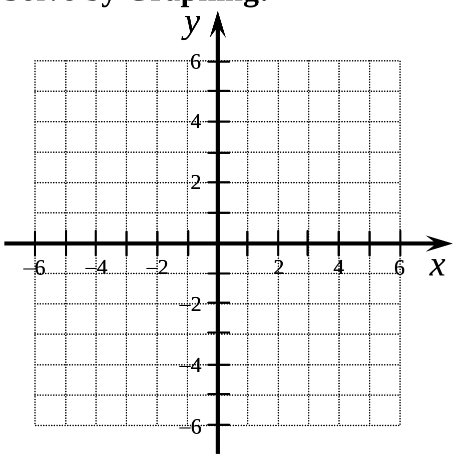
Directions: For each of the problems below, solve the system of equations by graphing, by substitution, and by elimination – your solution should be the same for all three methods methods. If they are not, go back and look for a mistake in your work.

1.

System of Equations:

$$\begin{cases} 10x + 5y = 15 \\ x + 2y = -6 \end{cases}$$

Solve by Graphing:



Solve by Substitution

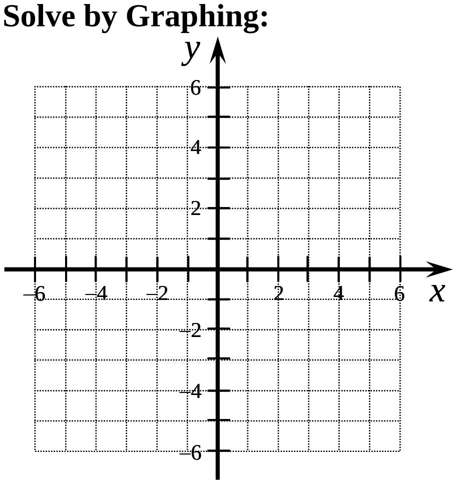
Solve by Elimination

2.

System of Equations:

$$\begin{cases} \frac{1}{4}x - y = -\frac{1}{3} \\ 3x - 12y = -12 \end{cases}$$

Solve by Graphing:



Solve by Substitution

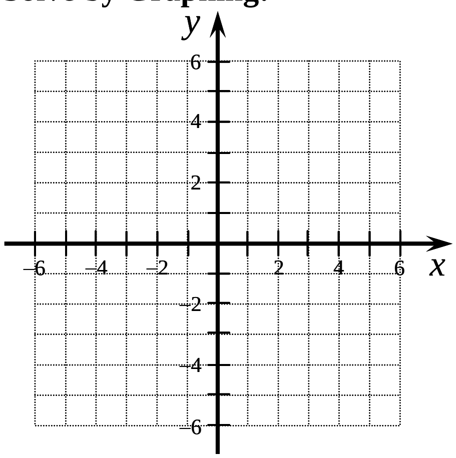
Solve by Elimination

3.

System of Equations:

$$\begin{cases} 3y - 4x = 2 \\ 3x - 4y = -5 \end{cases}$$

Solve by Graphing:



Solve by Substitution

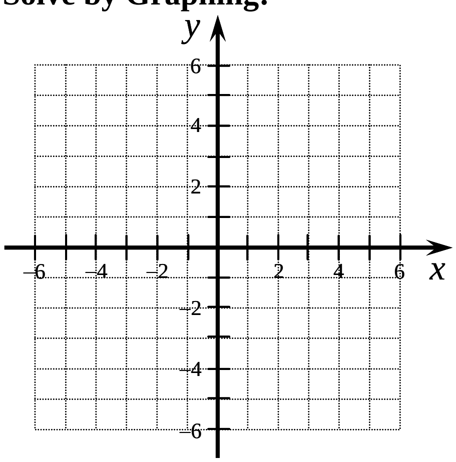
Solve by Elimination

4.

System of Equations:

$$\begin{cases} -4x + y = -5 \\ 12x - 3y = 15 \end{cases}$$

Solve by Graphing:



Solve by Substitution

Solve by Elimination

Student Activity C

Section 4.3: Choose Your Tactic

Directions: For each system of equations below, decide whether it will be easier to solve using substitution or elimination (choose your tactic).

	System of Equations	Tactic (substitution or elimination)	**Tactical Plan** **Substitution** – which variable in which equation will you solve for? **Elimination** – which variable will you eliminate and how?
1.	$\begin{cases} 5x+3y=-9 \\ y=2x+8 \end{cases}$		
2.	$\begin{cases} 2x+y=9 \\ 5x+3y=26 \end{cases}$		
3.	$\begin{cases} y=-x \\ 6x+6y=0 \end{cases}$		
4.	$\begin{cases} 4x+11y=7 \\ 4x+3y=-1 \end{cases}$		
5.	$\begin{cases} 16x-2y=16 \\ 4x=2y-8 \end{cases}$		
6.	$\begin{cases} 0.02x+0.01y=0.1 \\ -2x+3y=-18 \end{cases}$		
7.	$\begin{cases} -\frac{x}{5}-\frac{y}{3}=2 \\ -\frac{3x}{10}+\frac{2y}{10}=\frac{9}{10} \end{cases}$		
8.	$\begin{cases} x=12y-7 \\ 12y-x=12 \end{cases}$		

Assessment 4B

Chapter 4: Mid-chapter Assessment for Understanding

For each of the following, describe the type of problem and the strategies and key steps to remember while doing the problem. You do **not** have to complete the problems.

	Type of Problem	Strategies and Key Steps
1. Use substitution to solve the system. $\begin{cases} x-3y=-1 \\ 3x-4y=7 \end{cases}$		
2. Is the system below consistent or inconsistent? $\begin{cases} x-4y=6 \\ 2x=8y+10 \end{cases}$		
3. Is $(1,-2)$ a solution to the system? $\begin{cases} 3x+y=1 \\ x-y=-1 \end{cases}$		
4. Use elimination to solve the system. $\begin{cases} x+3y=-2 \\ x+4y=0 \end{cases}$		
5. Solve the system. $\begin{cases} x=y-7 \\ x-2y=14 \end{cases}$		
6. Is the pair of equations dependent or independent? $\begin{cases} x-4y=6 \\ 2x=8y+10 \end{cases}$		
7. Solve the system by graphing. $\begin{cases} y=-2x+4 \\ y=2x \end{cases}$		
8. Solve the system. $\begin{cases} 3x-4y=7 \\ 5x-8y=8 \end{cases}$		

Teaching Guide

Section 4.4: Problem Solving Using Systems of Equations

■ Preparing for Your Class

Suggested Class Time: 60-90 minutes

Materials Needed: Student activities, guided learning activity

Vocabulary

- Complementary, supplementary

Instruction Tips

- Students rarely like doing application problems. Many will simply look at an application problem and give up (often before they even read it). The more practice you can do in class with application problems, letting the *students* guide the problem-solving strategies, the more comfortable your students will feel on their own.
- Often students do not see that there is one distinct advantage associated with application problems: the chance to ask "does this answer make sense?" It is a good habit for students to examine application problems and try to guess what the solutions should look like, before they attempt the problems. This way, they can compare the results they get at the end, and see if these results are realistic.
- Because variables represent the unknown quantities in problems, most of the time your students can determine what the variables should be simply by looking for the question in the problem (which is usually the last sentence). Although this seems obvious to us, students often don't understand this simple truth. It is very difficult to construct a system of equations if the student does not know what the variables should be, and once they practice, it should not be hard for them to correctly declare variables. Student Activity A is designed specifically for students to practice with variable declaration.
- A variety of application problems are included in this section, but you may want to emphasize specific categories of problems to avoid running out of time. Here is a rough classification of the application problems included in this section:
    - Angle problems (supplementary, complementary)
    - Related quantity problems (length-width, salary figures, etc.)
    - Distance-Rate-Time problems (rate of current, rate of wind)
    - Value mixture problems
    - Percent solution problems, percent alloy problems
    - Investment problems
- The majority of this section can be taught with the activities and worksheets.

■ Teaching Your Class

Problem-Solving Strategy

1. Analyze the problem. *What is the given information? What are you trying to solve for? Will a diagram or table help here?*
2. Declare variables. Write equations.
3. Solve the system of equations.
4. State the conclusion. *Don't forget units.*
5. Check the result. *Does it make sense? Is it reasonable?*

Student Activity A: *The Last Sentence.* Students often want to skip the variable declaration step of these application problems, and then they end up with a faulty system of equations. This activity forces students to work with the variable declaration step specifically, using the context of the examples from this section for practice.

Student Activity B: *Working with the Smaller Pieces.* If your students are weak on application problems in general, you may first want to make sure that they understand the concepts that underlie these system-of-equations application problems. In this activity, we go through small problems related to distance-rate-time, investment problems, value mixture problems and solution mixture problems.

Guided Learning Activity: *Using the Problem Solving Strategy.* This worksheet provides three examples, similar to the ones in the book, that the instructor and students can work through together using the problem-solving strategy. Note: Linden dollars (sometimes called "ellz") are the currency of the virtual world Second Life and are pegged to the U.S. Dollar.

Student Activity C: *Setting Up the System.* In this activity, students practice with declaring the variables *and* setting up the system of equations for a variety of application problems. This is typically what students have the most difficulty with in this section. Once the system of equations is written, they will not usually struggle as much with solving it.

Student Activity D: *The Moving Walkway Problem.* Students with little experience of flying planes or driving boats do not necessarily understand the concepts of upstream and downstream or flying with or against the wind. But many students have either seen people-movers in action movies or ridden on a people mover at an amusement park, museum exhibit, zoos, or airport. So here's a new twist on an old problem.

Student Activity A

Section 4.4: The Last Sentence

Directions: What you see below is the last sentence from each of the example problems in this section of the textbook. Just from this last sentence, you can make a fairly good guess about what the two variables should be for the problem. The first one has been done for you.

Example 1: *Find the height of the figure and the height of the base.*

Let f = the height of the figure.
Let b = the height of the base.

Example 2: *Find the measure of each angle.*

Example 3: *Find the length and width of the poster.*

Example 4: *Find the cost of a class picture and the cost of an individual wallet-size picture.*

Example 5: *How much money was invested in each account?*

Example 6: *Find the speed of the boat in still water and the speed of the current.*

Example 7: *How many fluid ounces of each batch should she use?*

Example 8: *How many ounces of each ingredient should be used to create a 20-ounce box of raisin bran cereal that can be sold for $0.15 an ounce?*

Now go back and read all the example problems in this section. Can you add any detail to your variable declarations after reading the complete problems.

Student Activity B

Section 4.4: Working with the Smaller Pieces

Practice with Distance-Rate-Time Problems

Distance, rate and time can be related using the formula: $d = rt$

1. If you travel 8 miles from your house to visit a friend, what is your round-trip distance? ___________

2. Will you run faster when you run *with* the wind or *against* it? __________________

3. How far can you drive in 90 minutes at a rate of 70 miles per hour? _____________

4. If you work 30 miles away from your home, how long will it take you to get there if your average rate is 45 miles per hour? _____________

5. If you drive for 3 hours and travel 210 miles, what was your average rate? ________________

6. If you are in a boat that can travel 4 miles per hour in still water, will you be traveling *faster* or *slower* than 4 miles per hour when traveling upstream? ____________

Practice with Value Mixture and Percent Mixture Problems

7. If a store owner mixes peanuts that cost $3 per pound with cashews that cost $8 per pound, a pound of mixed nuts should cost

a. less than $3 **b.** between $3 and $8 **c.** more than $8

8. Simon has to mix two solutions in his chemistry class. One contains 3% sulfuric acid, and the other contains 10% sulfuric acid. How much sulfuric acid will the resulting mixture contain?

a. less than 3%
b. between 3% and 10%
c. between 10% and 13%
d. more than 13%

9. Lucy has to combine plain water with a solution that contains 15% ammonia. The resulting solution will be

a. less than 15% ammonia **b.** more than 15% ammonia

10. If you wanted to obtain 5 liters of a solution that is 4% nitric acid, which solution could you **not** add?

a. 3 liters of a 3% solution
b. 2 liters of a 5% solution
c. 6 liters of a 2% solution

11. At Meg's coffee shop, frozen coffees cost $3.50 each. If Meg took in a total of $42 on Tuesday, what is the maximum number of frozen coffees she could have sold?________

Practice with Investment Problems

Interest earned, principle invested, interest rate per year, and time invested (years) can be related using the formula: $I = Prt$

12. If you invest $2,500 for one year at an annual rate of 6%, how much interest does the investment earn? _______

13. If you invested a total of $15,000 in two different accounts, and you put $6,000 in the first account, how much did you put in the second account?________

14. If you invest $1,000 in an account that pays 5% annual interest, how much money will you have after one year?_________

15. Martin invests $2,000 in a bank CD. If the interest after one year is $160, what was the annual interest rate? __________

Guided Learning Activity

Section 4.4: Using the Problem Solving Strategy

Problem A: At a scrapbooking party, customers can purchase two special packages, the vacation scrapbook package and the baby album package. Shaude purchases 2 vacation packages and one baby album package for a total of $93.50. Stephan purchases one vacation package and three baby album packages for a total of $138. Find the cost of each of the special packages that were offered at the party.

1. Analyze the problem.

2. Define two variables. Write two equations.

3. Solve the system of equations.

4. State the conclusion.

5. Check the result.

Problem B: Jasper invests a total of 250,000 L$ (Linden dollars). He splits his investment between a 30-day Linden Bank CD paying 7% interest and a 30-day Second Life real estate investment scheme organized by his brother that ends up losing 4% of its value. At the end of 30 days, Jasper makes 2,100 L$. How much did Jasper invest in the CD and how much did he invest in the real estate scheme?

1. Analyze the problem.

2. Define two variables. Write two equations.

3. Solve the system of equations.

4. State the conclusion.

5. Check the result.

Problem C: Technical grade hydrogen peroxide is 35% H_2O_2. Common household hydrogen peroxide is only 3% H_2O_2. Imogen wants to mix 500 mL of a 10% H_2O_2 solution to use in removing a toxic mold in a city building. What amount of the technical grade solution and what amount of the household solution (to the nearest 0.1 mL) should she mix to create the solution she desires?

1. Analyze the problem.

2. Define two variables. Write two equations.

3. Solve the system of equations.

4. State the conclusion.

5. Check the result.

Student Activity C

Section 4.4: Setting Up the System

Directions: Read each problem carefully.
a) **Define the variables** you would use to solve the problem.
b) **Write a system** of two equations that could be used to solve each problem. You do not need to solve the application problems.

1. The elevation in Denver, Colorado, is 13 times greater than that of Waikapu, Hawaii. The sum of their elevations is 5,600 feet. Find the elevation of each city.

2. The owner of a pet food store wants to mix birdseed that costs $1.25 per pound with sunflower seeds that cost $0.75 per pound to make 50 pounds of a mixture that costs $1.00 per pound. How many pounds of each type of seed should he use?

3. An airplane can fly 500 miles in 2 hours when flying with the wind. The same journey takes 4 hours when flying against the wind. Find the speed of the wind and the speed of the plane.

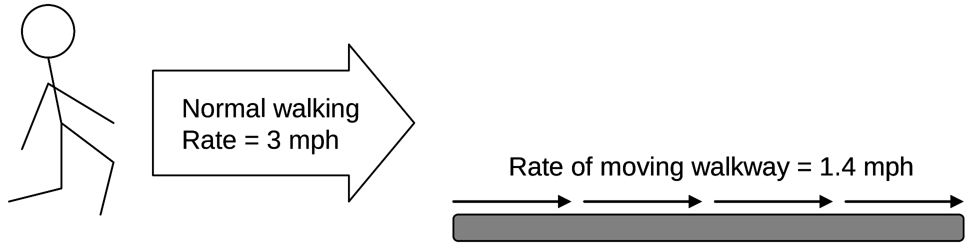
4. Barb invested a total of $25,000 in two accounts. One account has an annual interest rate of 3.5%, while the other has an annual rate of 6%. She earned $1,257 in interest in one year. How much did she invest in each account?

5. Two angles are supplementary. The measure of the first angle is 10 degrees more than three times the second angle. Find the measure of each angle.

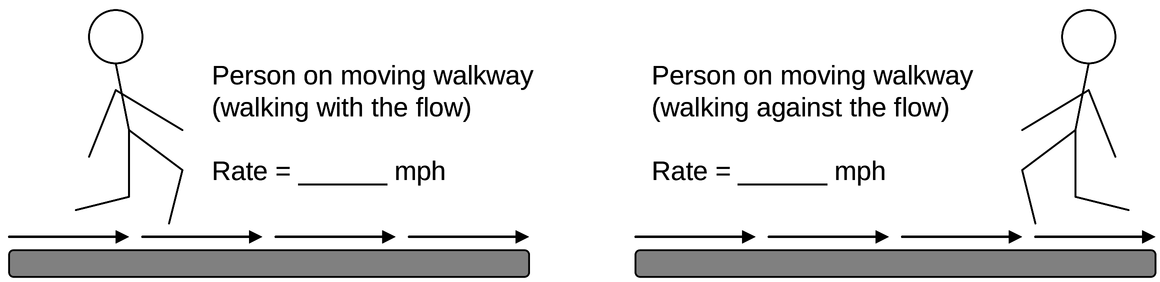
Student Activity D

Section 4.4: The Moving Walkway Problem

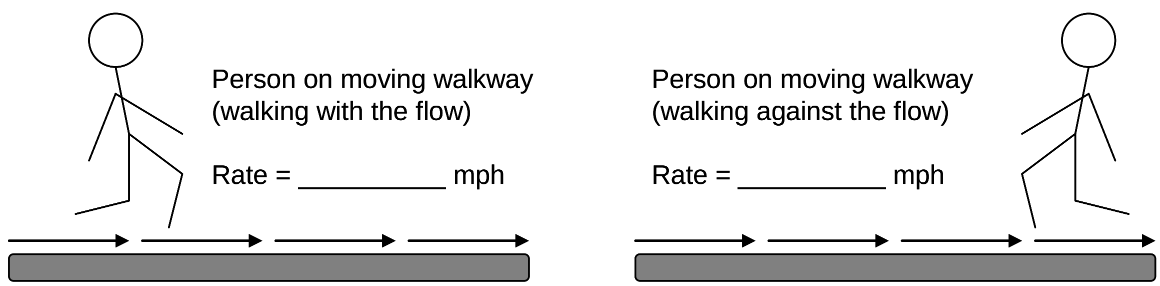
A moving walkway is like an escalator (transporting you from one place to another), only it is a flat belt that you can walk on. Moving walkways can commonly be found now in airports, museums, amusement parks, or zoos. When you walk onto a moving walkway going in your direction and continue to walk, your walking speed is increased by the speed of the walkway.



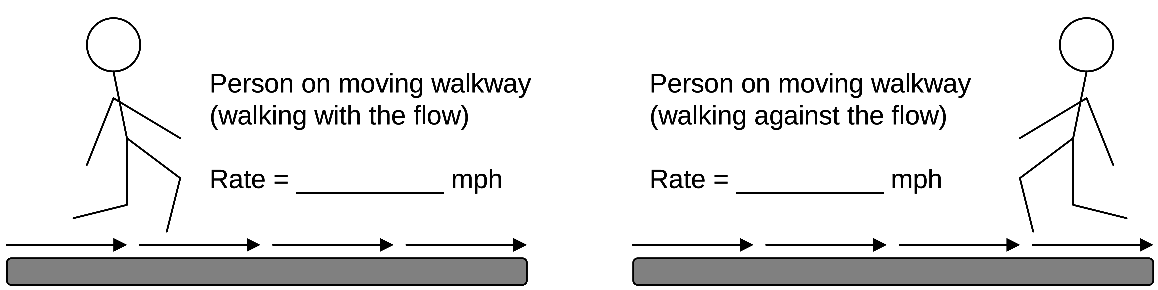
1. Using the information from the diagram above, fill in the missing rates.



2. Often, you do not know one of the rates. In these cases, you have to express the rate as an algebraic expression. Assume the person is walking at a rate of 2.7 mph and the rate of the moving walkway is x mph. Fill in the missing rates.



3. Now the rate of the person is R mph and the rate of the walkway is 2.3 mph. Fill in the missing rates.



4. At the Montparnasse station in Paris, there is a high-speed moving walkway that was designed to help commuters get through the station faster. This moving walkway, which is 0.18 km long, moves at a speed of 9 km/hr (originally the walkway was designed to go 11 km/hr, but there were too many accidents). David walks at his normal pace in the direction of flow on the walkway – it takes 54 seconds to go the full distance of the walkway. How fast was David walking?

5. While on a layover at the Detroit International Airport, a bored math teacher decides to calculate the speed of one of the many moving walkways in the airport. She knows (from years of treadmill walking) that her normal walking speed is 2.5 mph (or 2.2 ft/sec). Walking with the flow of the walkway, it takes 43 seconds to go the distance of the walkway. Walking against the flow of the walkway, it takes 2 minutes and 47 seconds. What is the approximate rate of the moving walkway in feet per second and what is the length of the walkway in feet?

	Rate	Time	Distance
With Flow of Walkway			
Against Flow of Walkway			

6. The Central-Mid-Levels Escalator in Hong Kong is the world's longest covered escalator, stretching 800m (2,625 ft). A trip on the escalator, from top to bottom, takes about 20 minutes when a person is stands still. What is the approximate rate of the escalator in feet per second? How long would the trip take if the person walked downward at a rate of 2 ft/sec?

Teaching Guide

Section 4.5: Solving Systems of Linear Inequalities

■ Preparing for Your Class

Suggested Class Time: 45-60 minutes

Materials Needed: Student activities, straight-edge, guided learning activity, colored markers and colored pencils for your students (SmartBoard or similar technology if available)

Vocabulary

- System of linear inequalities
- Review vocabulary: boundary line, intersection, test point, half-plane

Instruction Tips

- Students generally really enjoy shading in the solution regions using different colors. Make sure to bring in your collection of colored pencils or crayons for this section.
- It's probably been a little while since students have graphed horizontal and vertical lines, you may want to review these concepts.
- Commonly, students make mistakes graphing the boundary lines and then it is extremely difficult for the instructor to decide how to assign points to the shading part of the problem. For this reason, you will probably find it helpful to require that students show the work when they choose a test point and label the boundary lines they graph.
- If you have students that are likely to make mistakes when they rewrite an equation in slope-intercept form, then the intercept method (finding the x-intercept and the y-intercept) can be a quick and relatively safe method for graphing the boundary lines.
- Many of the students will have forgotten the rule: *If you multiply or divide by a negative number, reverse the inequality*. It is best to let them do one of these wrong on their own, then you can "catch" the mistake and correct them in class.
- Students need practice on determining whether an inequality is true or false. You may want to remind them that the *lesser number is on the left* on a number line. Students who like to skip showing any work can be easily fooled by simple inequality statements and will often shade the incorrect side of the boundary line.
- Graphing systems of inequalities on a standard whiteboard with colored markers will inevitably turn out looking like some kind of modern art installation. If you have access to a SmartBoard or similar technology in a classroom, now's the time to schedule your class there and learn how to use the technology. These interactive whiteboards have drawing tools that allow you to create beautiful, colored, semi-transparent shaded regions on the board. The overlap regions then take on the combined colors of the separately-shaded inequality graphs.
- This whole section can be taught with the activities and worksheets.

■ Teaching Your Class

Graphing an Inequality in two variables:

1. Draw the boundary line (remember to draw it either dashed or solid).
2. Using a test point, determine which side of the boundary line to shade.
3. Shade the half-plane that shows the solution of the inequality.

Solving Systems of Linear Inequalities

1. Graph each inequality on the same graphing grid.
2. Use darker shading or another color to highlight the intersection (overlap) of the graphs.
3. Pick a new point from the solution region and verify that it satisfies both inequalities.

Student Activity A: *Tic-tac-toe on Inequalities.* In this quick activity, students practice testing a possible solution point to a system of inequalities. Then students practice testing possible solution points to a graph of a system of inequalities.

Student Activity B: *Testing Test Points.* Sometimes students get so used to just using a single test point that they don't really understand what will happen when they test points in another region of the graph. In this activity, students practice with the specific skill of choosing the correct region to shade by testing a point from every region of the graph.

Guided Learning Activity: *Graphing Systems of Linear Equations.* Using these examples, you can quickly get through graphing the solutions to several systems of linear inequalities with your class. Make sure to clearly denote the test point that you use in each inequality and clearly shade the final region of intersection.

Student Activity C: *Following the Clues Back to the System of Inequalities.* In this challenging activity we'll see if your students are clever enough to remember how to find the equation of a line and to figure out the direction of each inequality. This is a good critical thinking activity, as they know enough mathematics to do these problems, but have not been shown explicitly how to do them. You may want to provide a few hints for this activity, depending on how comfortable your students are with finding the equation of a line.

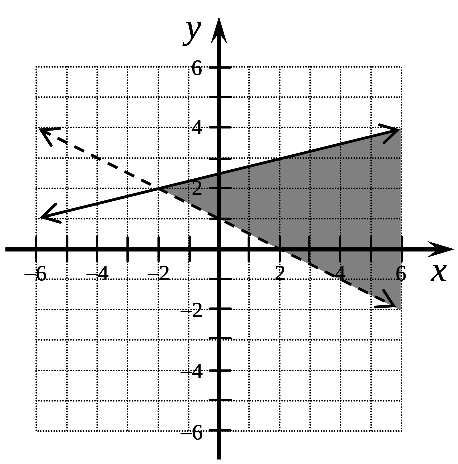
Student Activity A
Section 4.5: Tic-tac-toe on Inequalities

Directions for Tic-tac-toe #1: If the ordered pair in the square **IS** a solution of the system of inequalities shown below, then circle the ordered pair (thus putting an **O** on the square). If it **IS NOT** a solution, then put an **X** over the ordered pair on the square.

$$\begin{cases} y < 4x + 2 \\ 3x - 3y \geq 6 \end{cases}$$

$(-2,-6)$	$(1,1)$	$(4,1)$
$(6,2)$	$(8,6)$	$(4,4)$
$(2,0)$	$(0,2)$	$(0,-2)$

Directions for Tic-tac-toe #2: If the ordered pair in the square **IS** a solution of the graphed system of inequalities, then circle the ordered pair (thus putting an **O** on the square). If it **IS NOT** a solution, then put an **X** over the ordered pair on the square.



$(-2,2)$	$(2,0)$	$(2,3)$
$(8,3)$	$(3,4)$	$(0,4)$
$(6,4)$	$(6,-2)$	$(4,0)$

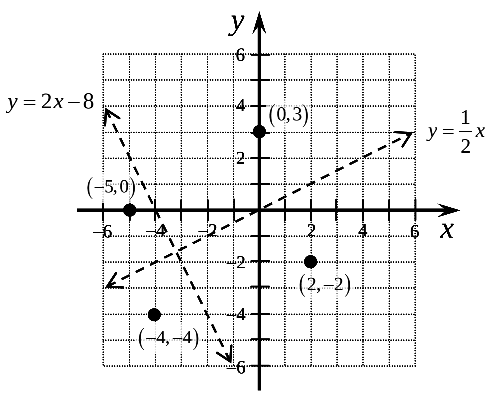
Student Activity B

Section 4.5: Testing Test Points

Directions: In each problem below you are given a system of inequalities, and the graphed inequality boundary lines. In each region separated by the boundary lines, a test point is provided. Test this point in both inequalities to determine if it is true (T) or false (F). Shade the region of the graph where the ordered pair **is** a solution to both inequalities. The first test point has been done for you.

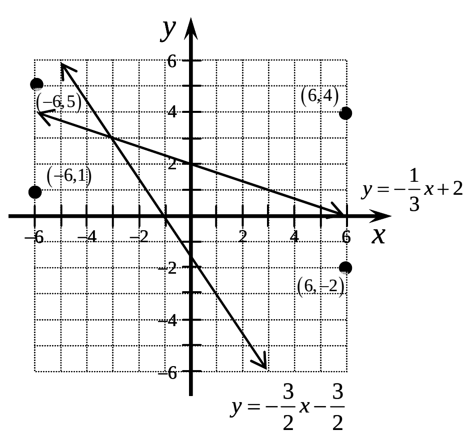
1.

	$y < \frac{1}{2}x$	$y > 2x - 8$
$(0,3)$	F	T
$(2,-2)$		
$(-5,0)$		
$(-4,-4)$		



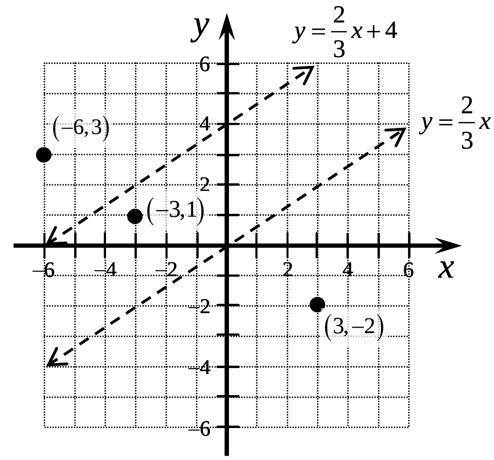
2.

	$y \le -\frac{1}{3}x + 2$	$y \ge -\frac{3}{2}x - \frac{3}{2}$
$(-6,5)$		
$(-6,1)$		
$(6,4)$		
$(6,-2)$		



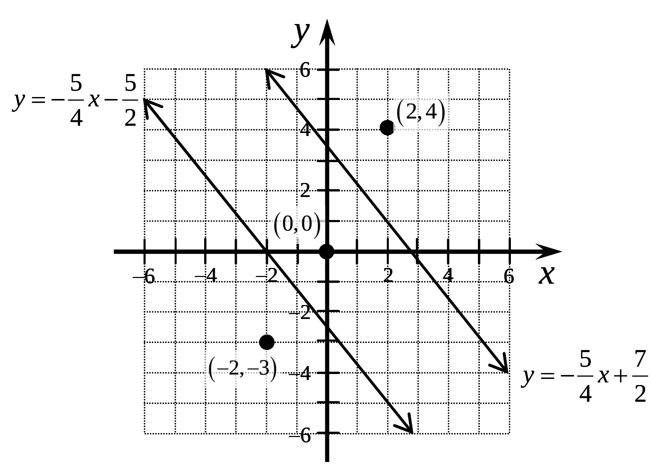
3.

	$y < \frac{2}{3}x + 4$	$y > \frac{2}{3}x$
$(-6,3)$		
$(-3,1)$		
$(3,-2)$		



4.

	$y \ge -\frac{5}{4}x + \frac{7}{2}$	$y \le -\frac{5}{4}x - \frac{5}{2}$
$(-2,-3)$		
$(0,0)$		
$(2,4)$		

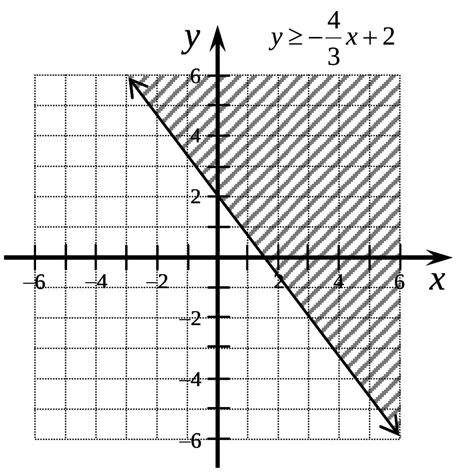


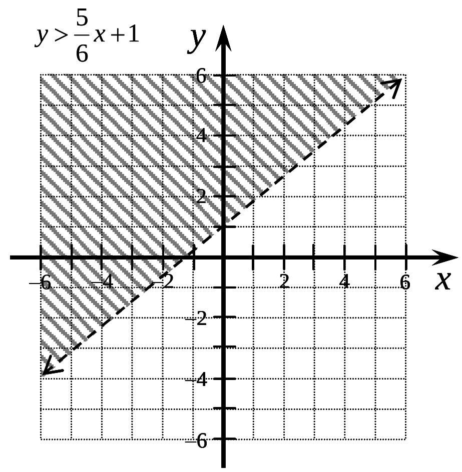
Guided Learning Activity

Section 4.5: Graphing Systems of Linear Inequalities

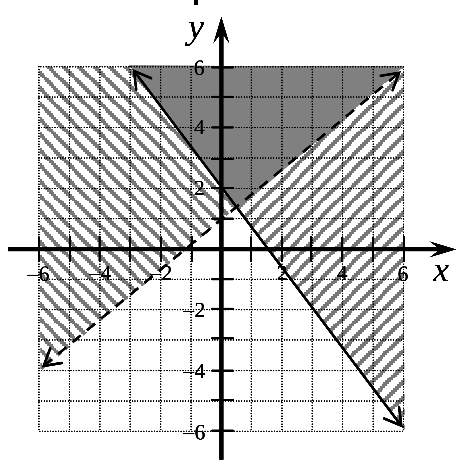
Example: Consider the system of inequalities given by $\begin{cases} y \geq -\dfrac{4}{3}x + 2 \\ y > \dfrac{5}{6}x + 1 \end{cases}$

What does the solution of a system of inequalities look like? When you graph the inequalities on the same set of axes, the solution is the region where the two graphs intersect. Separately, the graphs of our inequalities look like this:

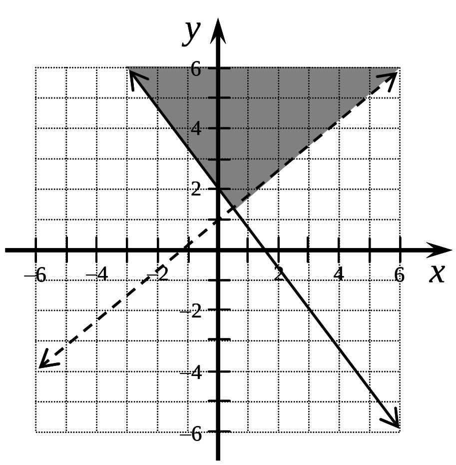




If we overlap these two graphs, we can find the area of intersection (shown here in solid gray).

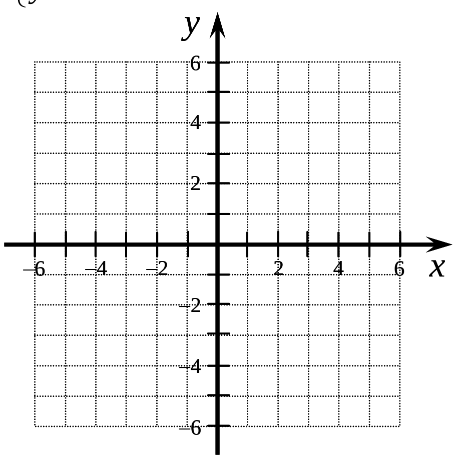


So the solution to the system of inequalities is:

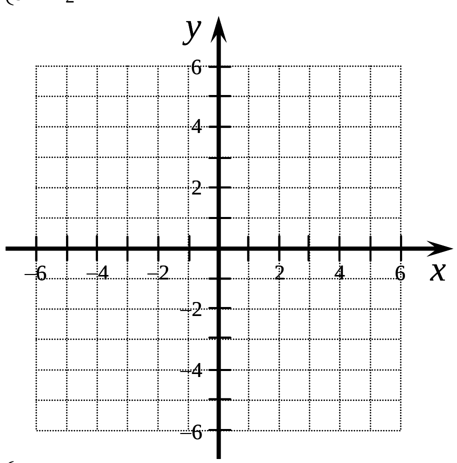


Now we'll solve these systems of inequalities by graphing.

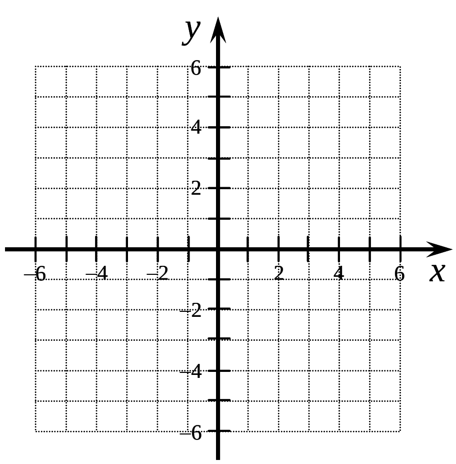
1. $\begin{cases} y < -2x+3 \\ y < 2x-3 \end{cases}$



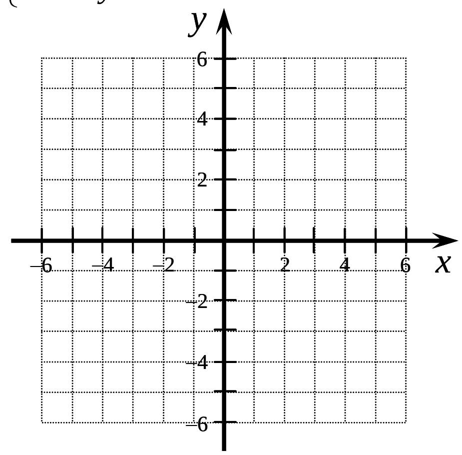
2. $\begin{cases} 2-y \le 0 \\ y-\frac{1}{2}x \le 4 \end{cases}$



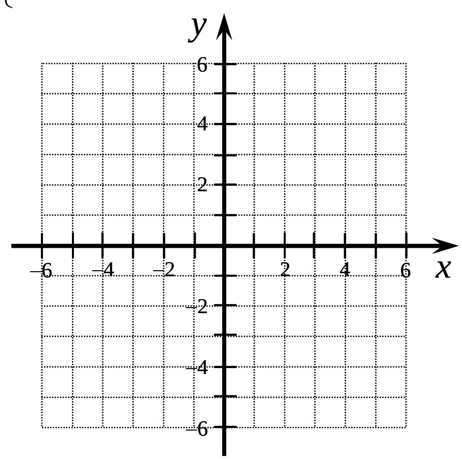
3. $\begin{cases} y > \frac{1}{3}x+1 \\ y < \frac{1}{3}x-3 \end{cases}$



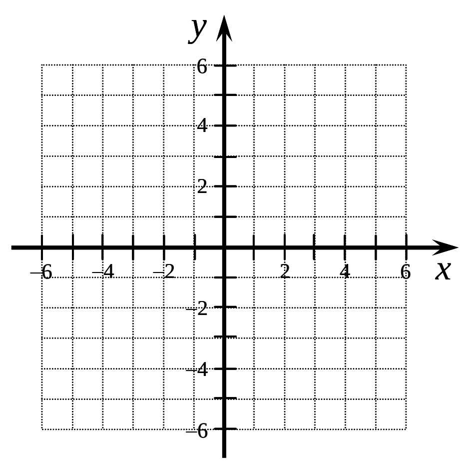
4. $\begin{cases} 3x+y \ge -5 \\ -3x-y \ge -5 \end{cases}$



5. $\begin{cases} y \le 0 \\ x \le 0 \end{cases}$



6. $\begin{cases} y \le -\frac{1}{2}x+2 \\ y > 2x+2 \end{cases}$

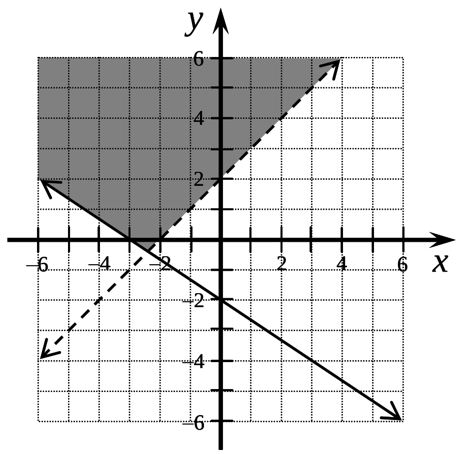


Student Activity C

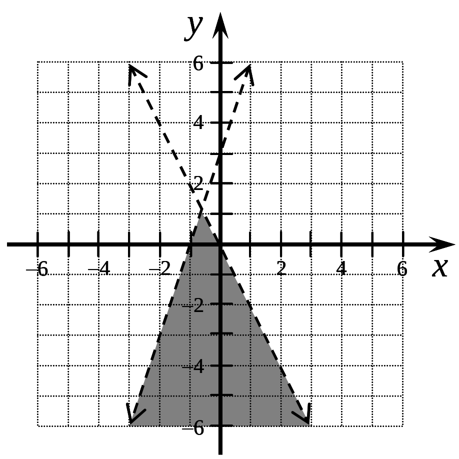
Section 4.5: Following the Clues Back to the System of Inequalities

Directions: In each "crime-scene" below, you are shown the graph of a system of inequalities. Use your mathematical powers of reasoning (and detective skills) to determine what the inequality must have been to result in this graph.

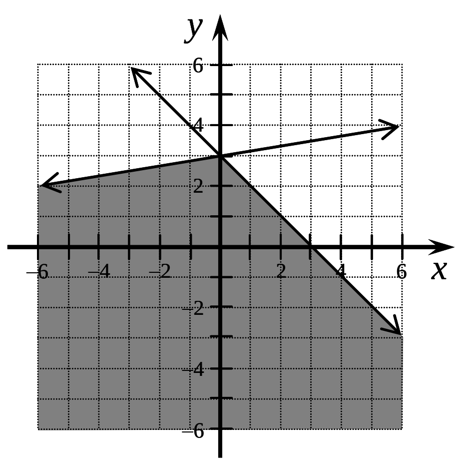
1.



2.



3.



Assessment 4C

Chapter 4: End-of-chapter Assessment for Understanding

For each of the following, describe the type of problem and the strategies and key steps to remember while doing the problem. You do **not** have to complete the problems.

	Type of Problem	Strategies and Key Steps
1. Rafi paddles his canoe upstream 24 miles in 4 hours. If he can make the downstream trip in 3 hours, find the rate that Rafi can paddle and the rate of the current.		
2. Is $(1,-3)$ a solution to the system? $\begin{cases} 3x+y<1 \\ x-y>-1 \end{cases}$		
3. Two angles are supplementary and one angle measures $30°$ more than the other angle. Find the measure of each angle.		
4. Solve the system by graphing. $\begin{cases} y-3x=2 \\ 4x-y=-1 \end{cases}$		
5. Graph the solution to the system. $\begin{cases} x \le 3 \\ y \le \frac{1}{2}x-1 \end{cases}$		
6. Is the pair of equations dependent or independent? $\begin{cases} 3x+y=8 \\ 6x+2y=8 \end{cases}$		
7. Graph the solution to the system. $\begin{cases} 3x+y<4 \\ 2y-x>-2 \end{cases}$		

Assessment 4D

Chapter 4: Metacognitive Skills Assessment

Metacognitive skills refer to the ability to judge how well you have learned something and to effectively direct your own learning and studying. This is a self evaluation tool designed to help you focus your studying and to improve your metacognitive skills with regards to this math class.

Fill the 1st column out before you begin studying.
Fill the 2nd column out after you study and before you take the test.
Go back to this page after your test and circle any of the ratings that you would now change – this identifies the "disconnects" between what you think you know well and what you actually know well.

Use the scale below to assign a number to each topic.
5 *I am confident I can do any problems in this category correctly.*
4 *I am confident I can do most of the problems in this category correctly.*
3 *I understand how to do the problems in this category, but I still make a lot of mistakes.*
2 *I feel unsure about how to do these problems.*
1 *I know I don't understand how to do these problems.*

Topic or Skill	**Before Studying**	**After Studying**
Checking whether an ordered pair is a solution of a system of equations.		
Classifying a system of equations as consistent or inconsistent.		
Classifying a system of equations as dependent or independent.		
Graphing a system of equations to find the solution.		
Understanding what the "solution" to a system of equations is.		
Finding or identifying a substitution equation for a system of equations.		
Solving a system of equations by substitution.		
Clearing the fractions from an equation.		
Identifying a variable to eliminate and performing the steps to set up that elimination in a system of equations.		
Solving a system of equations by elimination.		
Deciding whether substitution or elimination would be an easier method for solving a system of equations.		
Clearing the decimals from an equation.		
Declaring the variables for an application involving a system of equations.		
Writing the system of equations for a problem involving $d = rt$.		
Writing the system of equations for a value-mixture problem or a percent-solution problem.		
Writing the system of equations for an investment problem.		
Solving a system of equations from an application problem.		
Writing the conclusion to an application problem involving a system of equations.		
Checking whether an ordered pair is a solution of a system of inequalities.		
Determining whether the boundary lines for a system of inequalities are dashed or solid.		
Graphing the boundary lines for a system of inequalities.		
Determining the proper region to shade in a system of inequalities.		

Group Activity A
Writing Application Problems

(from Tussy/Gustafson 3rd edition Elementary Algebra)

In Section 4.4, you solved application problems by translating the words of the problem into a system of two equations. In this activity, you will reverse these steps.

Instructions: Form groups of 2 or 3 students. For each type of application, write a problem that could be solved using the given equations. If you need help getting started, refer to the specific problem types in the text. When finished writing the five applications, pick one problem and solve it completely (use another sheet of paper).

A rectangle problem:

$$\begin{cases} 2l + 2w = 320 \\ l = 40 + w \end{cases}$$

A number-value problem:

$$\begin{cases} 5x + 2y = 23 \\ 3x + 7y = 37 \end{cases}$$

An interest problem:

$$\begin{cases} x + y = 75{,}000 \\ 0.03x + 0.05y = 2{,}750 \end{cases}$$

A with–against the wind problem:

$$\begin{cases} 2(x + y) = 600 \\ 3(x - y) = 600 \end{cases}$$

A liquid mixture problem:

$$\begin{cases} x + y = 36 \\ 0.50x + 0.20y = 0.30(36) \end{cases}$$

Group Activity B

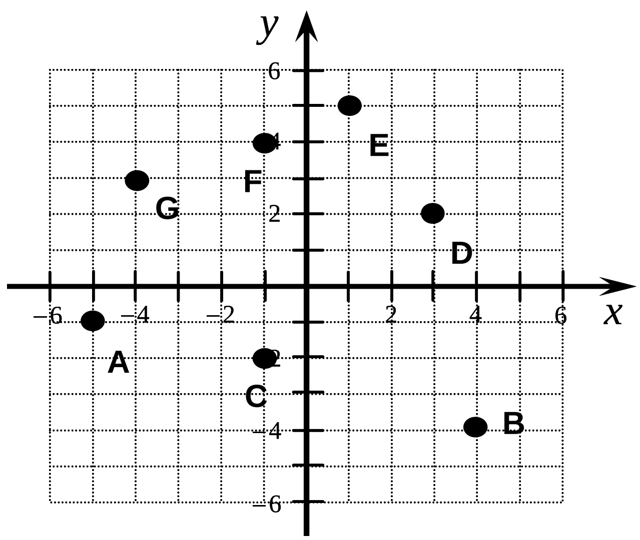
Interpreting Graphs of Linear Inequalities

(from Tussy/Gustafson 3rd edition Elementary Algebra)

In Section 4.5, you learned how to graph systems of linear inequalities. This activity will strengthen your understanding of shading and boundary lines, which are used when graphing the solutions of systems of linear inequalities.

Instructions: Form groups of 2 or 3 students. Find the solutions of the system of linear inequalities by graphing the inequalities on the given coordinate system.

$$\begin{cases} 3x+2y<4 \\ y \le \dfrac{3}{5}x+2 \end{cases}$$

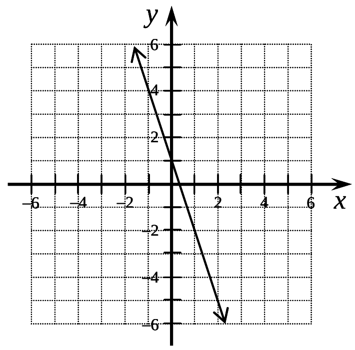


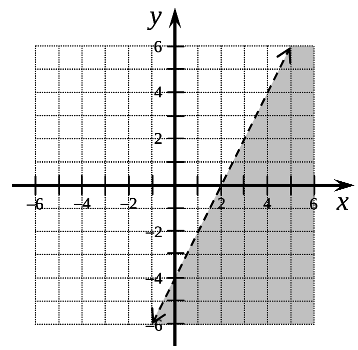
For each point A, B, C, D, E, F, and G on the graph, determine whether its coordinates make the first inequality true or false. Then determine whether each point's coordinates make the second inequality true or false. Write your answers in the table below.

Point	Coordinates	1st inequality	2nd inequality
A			
B			
C			
D			
E			
F			
G			

Solution to Assessment 4A: *Pretest and Diagnostic Tool*

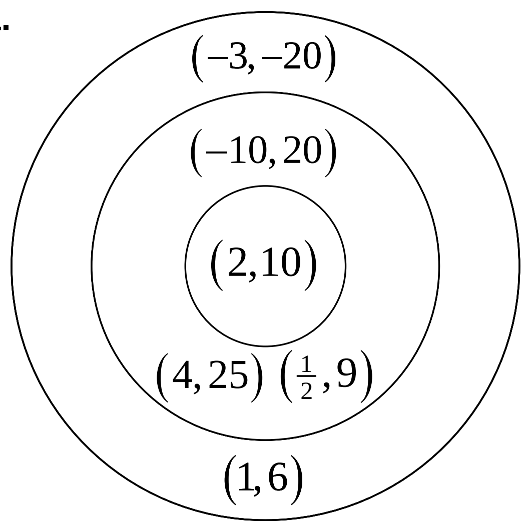
1. False **2.** $y=-2$
3. See first graph.
4. $y=\frac{1}{2}x-4$ **5.** $x=0$
6. $y=-3$ **7.** $y=\frac{1}{2}$ **8.** $x=\frac{4}{3}$
9. $7x$ **10.** $-4x-8y$
11. $24x+20y=-4$
12. $-2x+8y=-14$
13. $2x-3=-12$
14. $5x+6y=21{,}000$ **15.** $0.045x$ **16.** $4(x+15)$ or $4x+60$ **17.** $12x+6y$
18. False **19.** Yes **20.** See second graph.



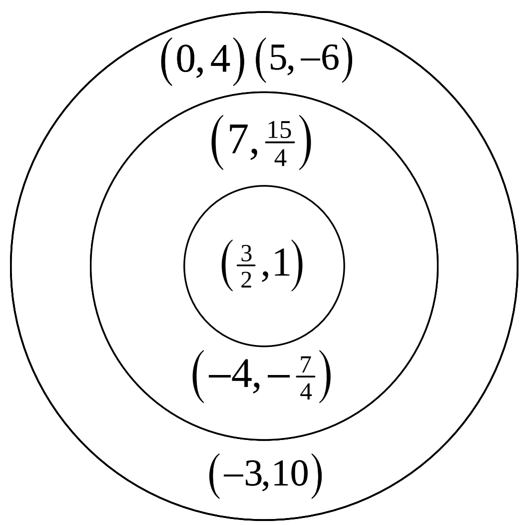


Solution to 4.1 Student Activity A: *Targeting Solutions*

1.



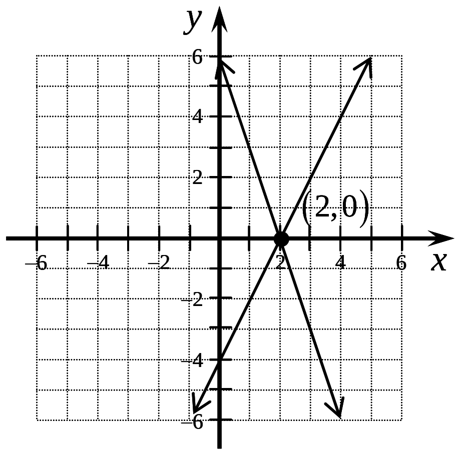
2.



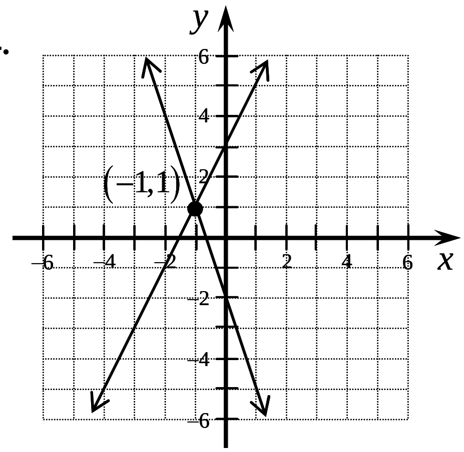
Solution to 4.1 Guided Learning Activity:

Graphing to Solve Systems of Equations

1.



4.



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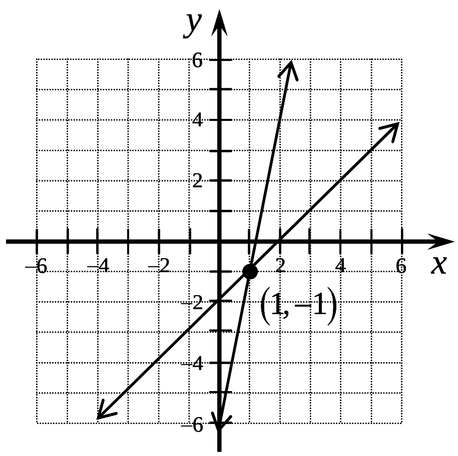
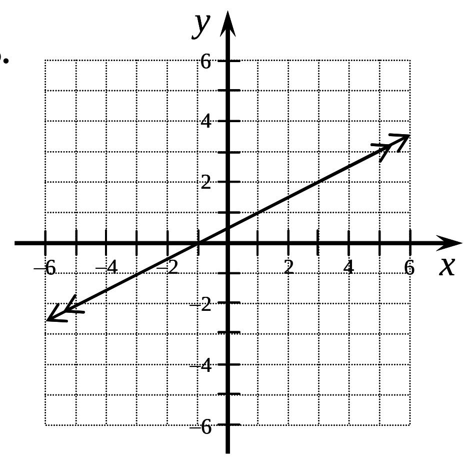
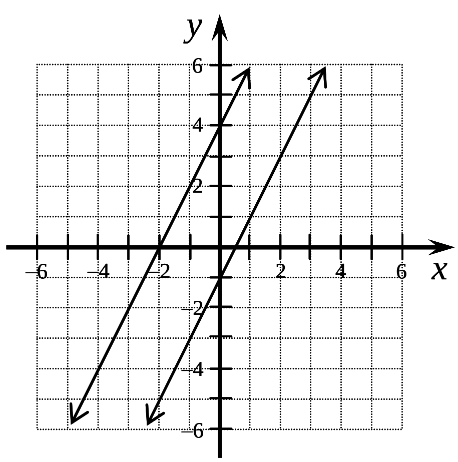


5.



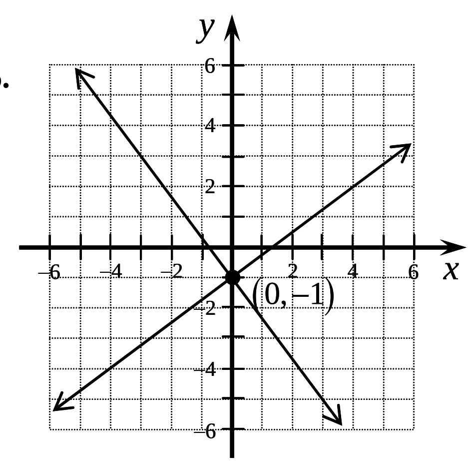
Infinitely many solutions

3.



No Solution

6.

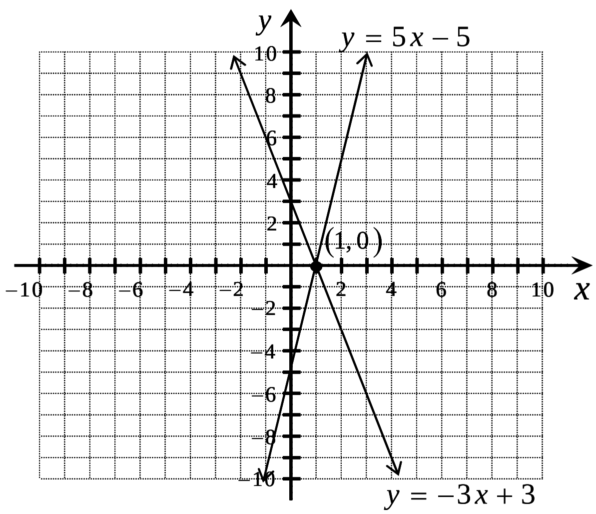


Solution to 4.1 Student Activity B:
Following the Clues Back to the System of Equations

1. $\begin{cases} y = -x \\ y = \frac{1}{2}x + 5 \end{cases}$ **2.** $\begin{cases} y = -3 \\ y = \frac{1}{3}x - \frac{11}{3} \end{cases}$ **3.** $\begin{cases} x = 3 \\ y = -3x + 8 \end{cases}$

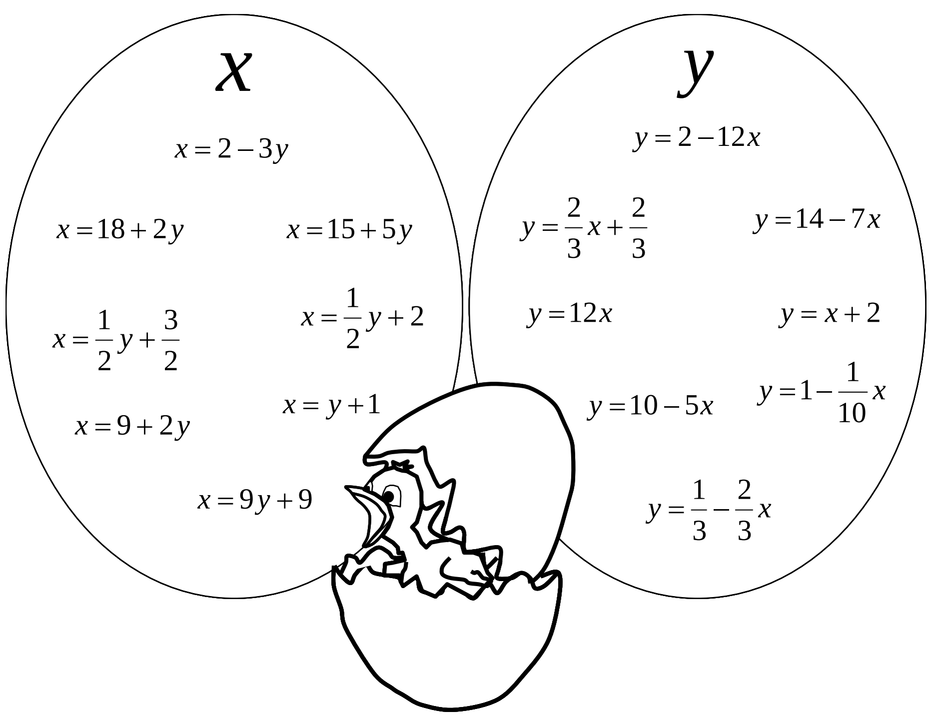
Solution to 4.1 Student Activity C:
Graphing Systems of Equations with a Calculator

1.

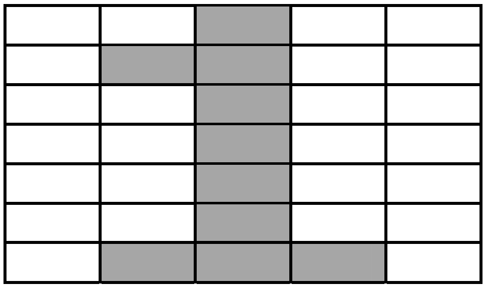


2-4. Answers will vary.

Solution to 4.2 Student Activity A: *Which Egg is Easier to Crack?*



Solution to 4.2 Student Activity B: *Clear the Way!*



Solution to 4.2 Student Activity C: *Double Trouble on Substitution*
Note: Both possibilities for the substitution equation are shown together with the solution for the system of equations.

1.	$\begin{cases} 2x+y=5 \\ x-4y=7 \end{cases}$	Solve for *x*: $x=-\frac{1}{2}y+\frac{5}{2}$. Solve for y: $y=-2x+5$. $(3,-1)$	Solve for x: $x=4y+7$. Solve for y: $y=\frac{1}{4}x-\frac{7}{4}$. $(3,-1)$
2.	$\begin{cases} x+8y=20 \\ 4x-y=14 \end{cases}$	Solve for x: $x=-8y+20$. Solve for y: $y=-\frac{1}{8}x+\frac{5}{2}$. $(4,2)$	Solve for x: $x=\frac{1}{4}y+\frac{7}{2}$. Solve for y: $y=4x-14$. $(4,2)$
3.	$\begin{cases} 3x-y=-2 \\ -12x+4y=16 \end{cases}$	Solve for x: $x=\frac{1}{3}y-\frac{2}{3}$. Solve for y: $y=3x+2$. No Solution	Solve for x: $x=\frac{1}{3}y-\frac{4}{3}$. Solve for y: $y=3x+4$. No Solution
4.	$\begin{cases} 3x+4y=0 \\ \frac{3}{4}x+\frac{8}{3}y=-\frac{5}{12} \end{cases}$	Solve for x: $x=-\frac{4}{3}y$. Solve for y: $y=-\frac{3}{4}x$. $\left(\frac{1}{3},-\frac{1}{4}\right)$	Solve for x: $x=-\frac{32}{9}y-\frac{5}{9}$. Solve for y: $y=-\frac{9}{32}x-\frac{5}{32}$. $\left(\frac{1}{3},-\frac{1}{4}\right)$
5.	$\begin{cases} x-7y=-1 \\ -\frac{1}{7}x+y=\frac{1}{7} \end{cases}$	Solve for x: $x=7y-1$. Solve for y: $y=\frac{1}{7}x+\frac{1}{7}$. Infinitely Many Solutions	Solve for x: $x=7y-1$. Solve for y. $y=\frac{1}{7}x+\frac{1}{7}$. Infinitely Many Solutions

Solution to 4.3 Student Activity A: *Forced Elimination*

1.

Eliminate *x*	**Eliminate *y***
$\begin{cases} x+3y=7 \\ 2x-3y=-4 \end{cases}$ Multiply the first equation by -2. $\begin{cases} -2x-6y=-14 \\ 2x-3y=-4 \end{cases}$ $-9y=-18$ $y=2$	$\begin{cases} x+3y=7 \\ 2x-3y=-4 \end{cases}$ Add the equations. $3x=3$ $x=1$

2.

Eliminate *x*	**Eliminate *y***
$\begin{cases} -10x+y=-5 \\ 10x-2y=0 \end{cases}$ Add the equations. $-y=-5$ $y=5$	$\begin{cases} -10x+y=-5 \\ 10x-2y=0 \end{cases}$ Multiply the first equation by 2. $\begin{cases} -20x+2y=-10 \\ 10x-2y=0 \end{cases}$ $-10x=-10$ $x=1$

3.

Eliminate *x*	**Eliminate *y***
$\begin{cases} 5x+7y=12 \\ 10x-3y=7 \end{cases}$ Multiply the first equation by -2. $\begin{cases} -10x-14y=-24 \\ 10x-3y=7 \end{cases}$ $-17y=-17$ $y=1$	$\begin{cases} 5x+7y=12 \\ 10x-3y=7 \end{cases}$ Multiply the first equation by 3 and the second equation by 7. $\begin{cases} 15x+21y=36 \\ 70x-21y=49 \end{cases}$ $85x=85$ $x=1$

4.

Eliminate *x*	**Eliminate *y***
$\begin{cases} 3x+y=2 \\ 4x-2y=6 \end{cases}$ Multiply the first equation by 4 and the second equation by -3. $\begin{cases} 12x+4y=8 \\ -12x+6y=-18 \end{cases}$ $10y=-10$ $y=-1$	$\begin{cases} 3x+y=2 \\ 4x-2y=6 \end{cases}$ Multiply the first equation by 2. $\begin{cases} 6x+2y=4 \\ 4x-2y=6 \end{cases}$ $10x=10$ $x=1$

5. $\begin{cases} 3x+y=1 \\ 2x+3y=-11 \end{cases}$ Eliminate *y*. Multiply the first equation by -3 and then add the equations.

6. $\begin{cases} 5x-y=-5 \\ x+y=5 \end{cases}$ Eliminate *y*. Add the equations.

7. $\begin{cases} x+y=12 \\ -x+y=0 \end{cases}$ Eliminate *x*. Add the equations.

8. $\begin{cases} -4x+2y=2 \\ 4x+6y=22 \end{cases}$ Eliminate *x*. Add the equations.

9. $\begin{cases} 3x+14y=-7 \\ 2x+2y=12 \end{cases}$ Eliminate *y*. Multiply the second equation by -7 and then add the equations.

10. $\begin{cases} 3x+8y=2 \\ -6x-7y=5 \end{cases}$ Eliminate *y*. Multiply the first equation by 2 and then add the equations.

Solution to 4.3 Student Activity B: *Triple the Fun on Systems of Equations*
Note: All methods should result in the same solution.

1. $(4,-5)$
2. No Solution
3. $(1,2)$
4. Infinitely many solutions

Solution to 4.3 Student Activity C: *Choose Your Tactic*

Note: The solution to each system of equations is provided so that you can assign the completion of the problems for additional practice if your students need it.

1.	$\begin{cases} 5x+3y=-9 \\ y=2x+8 \end{cases}$	Substitution $(-3,2)$	In the second equation, y is already isolated.
2.	$\begin{cases} 2x+y=9 \\ 5x+3y=26 \end{cases}$	Elimination $(1,7)$	Eliminate y by multiplying the first equation by -3.
3.	$\begin{cases} y=-x \\ 6x+6y=0 \end{cases}$	Substitution Infinitely many solutions.	In the first equation, y is already isolated.
4.	$\begin{cases} 4x+11y=7 \\ 4x+3y=-1 \end{cases}$	Elimination $(-1,1)$	Eliminate x by multiplying either equation by -1.
5.	$\begin{cases} 16x-2y=16 \\ 4x=2y-8 \end{cases}$	Substitution $(2,8)$	Solve for x in the second equation by dividing both sides by 4. Elimination of *y*'s is also an arguably easy tactic here.
6.	$\begin{cases} 0.02x+0.01y=0.1 \\ -2x+3y=-18 \end{cases}$	Elimination $(6,-2)$	Eliminate x by multiplying the first equation by 100.
7.	$\begin{cases} -\frac{x}{5}-\frac{y}{3}=2 \\ -\frac{3x}{10}+\frac{2y}{10}=\frac{9}{10} \end{cases}$	Elimination $(-5,-3)$	Eliminate x by multiplying the first equation by -15 and the second equation by 10.
8.	$\begin{cases} x=12y-7 \\ 12y-x=12 \end{cases}$	Substitution No Solution	In the first equation, x is already isolated.

Solution to Assessment 4A: *Mid-chapter Assessment for Understanding*

Section numbers are included below as a reference for the instructor. The assessment is designed to foster points of discussion in class; answers will vary.

1. Use substitution to solve the system. $\begin{cases} x-3y=-1 \\ 3x-4y=7 \end{cases}$	Solve a system using substitution. (4.2)	It would be easiest to solve for x in the first equation. Then substitute into the second equation.
2. Is the system consistent or inconsistent? $\begin{cases} x-4y=6 \\ 2x=8y+10 \end{cases}$	Categorize a system of equations. (4.1, 4.2, 4.3)	A system is inconsistent if there is no solution. Determine whether the system has a solution by solving the system.
3. Is $(1,-2)$ a solution to the system? $\begin{cases} 3x+y=1 \\ x-y=-1 \end{cases}$	Check the solution to a system of equations. (4.1, 4.2 or 4.3)	Evaluate both equations for $x=1$ and $y=-2$. To be a solution to the system, both equations must be true.

4. Use elimination to solve the system. $\begin{cases} x+3y=-2 \\ x+4y=0 \end{cases}$	Solve a system using elimination. (4.3)	Multiply either the first or second equation by -1, then add the equations to eliminate x.
5. Solve the system. $\begin{cases} x=y-7 \\ x-2y=14 \end{cases}$	Solve a system using substitution. (4.2)	This equation is easier to solve with substitution because the first equation is already a substitution equation.
6. Is the pair of equations dependent or independent? $\begin{cases} x-4y=6 \\ 2x=8y+10 \end{cases}$	Categorize a system of equations. (4.1, 4.2, 4.3)	A pair of equations is dependent if the equations describe the same line. Determine whether the equations are the same by solving the system.
7. Solve the system by graphing. $\begin{cases} y=-2x+4 \\ y=2x \end{cases}$	Solve a system of equations by graphing. (4.1)	Draw a set of coordinate axes. Graph the two lines and look for a point of intersection.
8. Solve the system. $\begin{cases} 3x-4y=7 \\ 5x-8y=8 \end{cases}$	Solve the system using elimination. (4.3)	This equation is easier to solve with elimination because both equations are written in standard form. Begin by multiplying the first equation by -2.

Solution to 4.4 Student Activity A: *The Last Sentence*
Note: Information in parentheses is from going back to read the complete problem.

Example 2: *Find the measure of each angle.*
Let x = degree measure of the first angle
Let y = degree measure of the second angle

Example 3: *Find the length and width of the poster.*
Let L = the length of the poster (in inches)
Let W = the width of the poster (in inches)

Example 4: *Find the cost of a class picture and the cost of an individual wallet-size picture.*
Let c = cost of a class picture
Let w = cost of a wallet-size picture

Example 5: *How much money was invested in each account?*
Let x = money invested in first account (Swiss bank account paying 8%)
Let y = money invested in second account (Cayman Islands account paying 7%)

Example 6: *Find the speed of the boat in still water and the speed of the current.*
Let b = speed of the boat in still water (in miles per hour)
Let c = speed of the current (in miles per hour)

Example 7: *How many fluid ounces of each batch should she use?*

Let x = fluid ounces of first batch (weak batch, 40% alcohol)

Let y = fluid ounces of second batch (strong batch, 60% alcohol)

Example 8: *How many ounces of each ingredient should be used to create a 20-ounce box of raisin bran cereal that can be sold for $0.15 an ounce?*

Let $x =$ ounces of first ingredient (raisins)

Let $y =$ ounces of second ingredient (bran flakes)

Solution to 4.4 Student Activity B: *Working with the Smaller Pieces*

1. 16 miles
2. with the wind
3. 105 miles
4. 40 minutes
5. 70 miles per hour
6. slower
7. b
8. b
9. a
10. c
11. 12 frozen coffees
12. $150
13. $9,000
14. $1,050
15. 8%

Solution to 4.4 Guided Learning Activity: *Using the Problem Solving Strategy*

Note: For each problem, the solution process is just roughly outlined below.

Problem A: Let x = Price of vacation package

y = Price of baby album package

$$\begin{cases} 2x + y = \$93.50 \\ x + 3y = \$138 \end{cases}$$

$x = \$28.50$

$y = \$36.50$

Problem B: Let x = Amount invested in 30-day CD

y = Amount invested in real estate scheme

$$\begin{cases} x + y = 250{,}000 \text{ L\$} \\ 0.07x + (-0.04)y = 2{,}100 \text{ L\$} \end{cases}$$

$x = 110{,}000$ L$

$y = 140{,}000$ L$

Problem C: Let x = Amount of 35% H_2O_2 solution (in mL)

y = Amount of 3% H_2O_2 solution (in mL)

$$\begin{cases} x + y = 500 \text{ mL} \\ 0.35x + 0.03y = 0.10(500 \text{ mL}) \end{cases}$$

$x \approx 109.4$ mL

$y \approx 390.6$ mL

Solution to 4.4 Student Activity C: *Setting Up the System*

1. $d =$ elevation of Denver, Colorado (in feet)
$w =$ elevation of Waikapu, Hawaii (in feet)
$d = 13w$
$d + w = 5600$

2. $b =$ pounds of birdseed
$s =$ pounds of sunflower seeds
$\$1.25b + \$0.75s = \$1.00(50)$
$b + s = 50$

3. $p =$ speed of plane in miles per hour
$w =$ speed of wind in miles per hour
$500 = 2p + 2w$
$500 = 4p - 4w$

4. $x =$ amount invested at 3.5%
$y =$ amount invested at 6%
$0.035x + 0.06y = \$1,275$
$x + y = \$25,000$

5. $x =$ degree measure of first angle
$y =$ degree measure of second angle
$x + y = 180°$
$x = 3y + 10$

Solution to 4.4 Student Activity D: *The Moving Walkway Problem*

1. With: 4.4 mph; Against: 1.6 mph
2. With: $2.7 + x$; Against: $2.7 - x$
3. With: $R + 2.3$; Against: $R - 2.3$
4. 3 km/hr
5. Walkway speed $\approx$ 1.3 ft/sec
Walkway length $\approx$ 150.5 ft
6. Escalator speed $\approx$ 2.2 ft/sec
Time to bottom when walking
$\approx$ 625 sec or 10.4 min

Solutions to 4.5 Student Activity A: *Tic-tac-toe on Inequalities*

1.

X	X	O
O	O	X
O	X	O

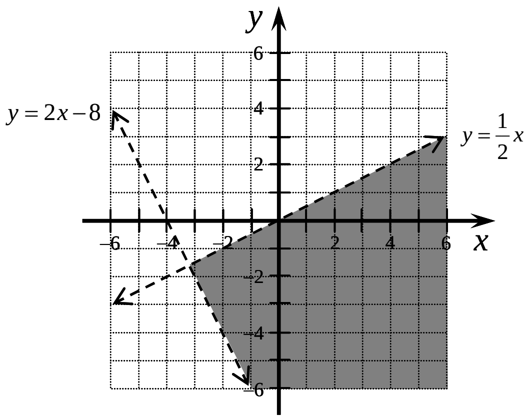
2.

X	X	O
O	X	X
O	X	O

Solutions to 4.5 Student Activity B: *Testing Test Points*

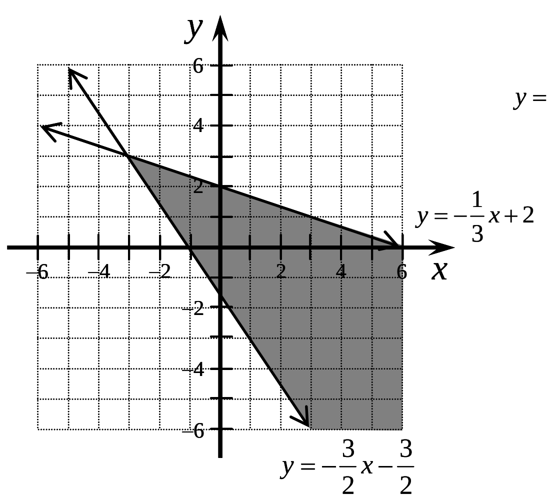
1.

	$y<\frac{1}{2}x$	$y>2x-8$
$(0,3)$	F	T
$(2,-2)$	T	T
$(-5,0)$	F	T
$(-4,-4)$	T	F



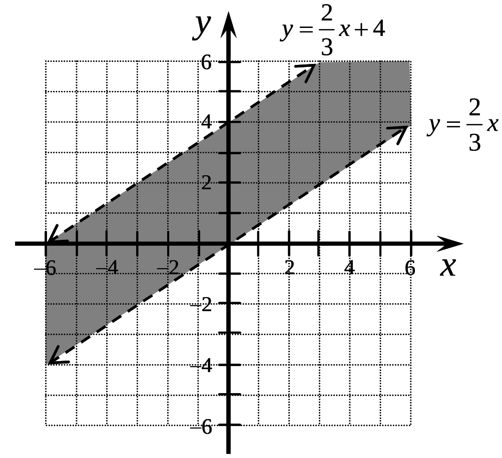
2.

	$y\le-\frac{1}{3}x+2$	$y\ge-\frac{3}{2}x-\frac{3}{2}$
$(-6,5)$	F	F
$(-6,1)$	T	F
$(6,4)$	F	T
$(6,-2)$	T	T



3.

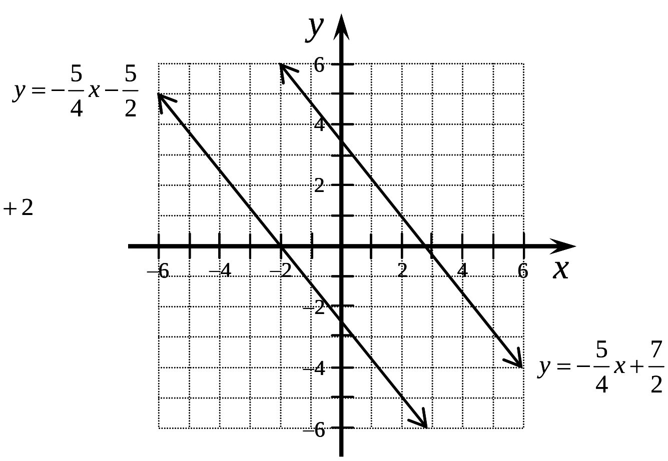
	$y<\frac{2}{3}x+4$	$y>\frac{2}{3}x$
$(-6,3)$	F	T
$(-3,1)$	T	T
$(3,-2)$	T	F



4.

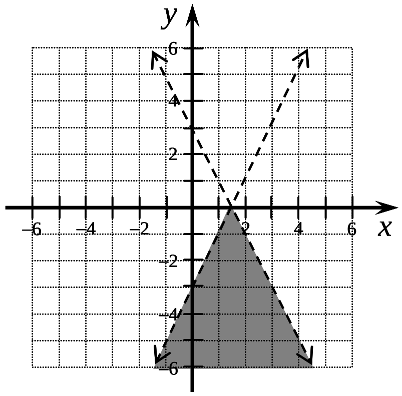
	$y\ge-\frac{5}{4}x+\frac{7}{2}$	$y\le-\frac{5}{4}x-\frac{5}{2}$
$(-2,-3)$	F	T
$(0,0)$	F	F
$(2,4)$	T	F

No Solution



Solution to 4.5 Guided Learning Activity: *Graphing Systems of Linear Inequalities*

1.



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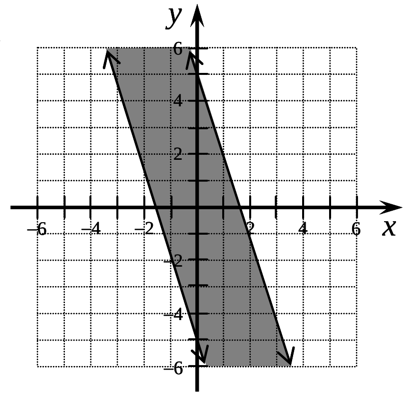

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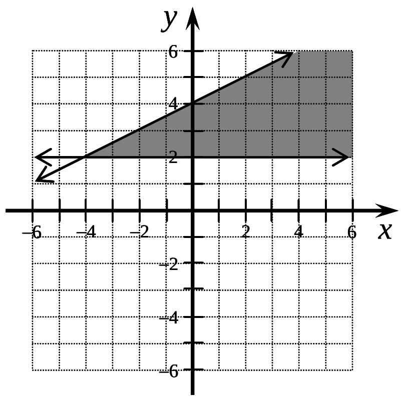

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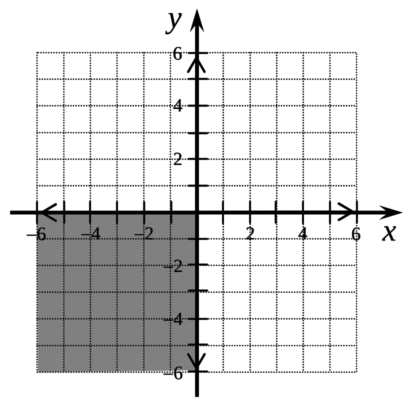
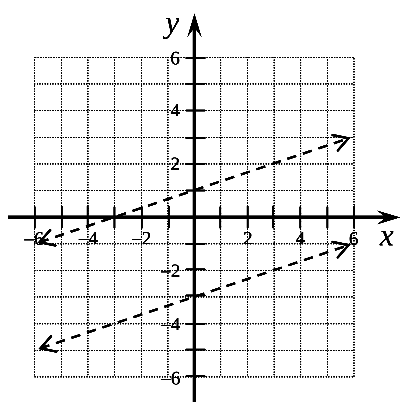
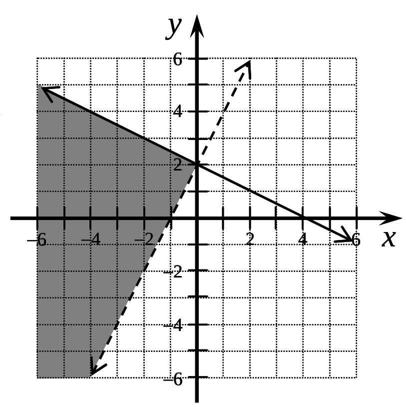


3.



No Solution

6.



Solution to 4.5 Student Activity C:

Following the Clues Back to the System of Inequalities

1. $\begin{cases} y \geq -\frac{2}{3}x - 2 \\ y > x + 2 \end{cases}$ **2.** $\begin{cases} y < -2x \\ y < 3x + 3 \end{cases}$ **3.** $\begin{cases} y \leq -x + 3 \\ y \leq \frac{1}{6}x + 3 \end{cases}$

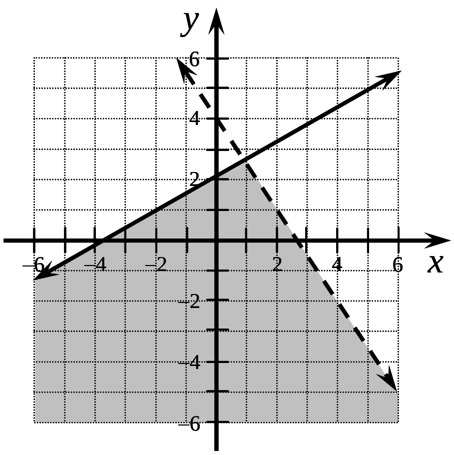
Solution to Assessment 4C: *End-of-chapter Assessment for Understanding*
Section numbers are included below as a reference for the instructor. The assessment is designed to foster points of discussion in class; answers will vary.

1. Rafi paddles his canoe upstream 24 miles in 4 hours. If he can make the downstream trip in 3 hours, find the rate that Rafi can paddle and the rate of the current.	Problem solving using systems of equations. (4.4)	This problem could be set up with a table and the formula $d = rt$. Note that the distance traveled is the same in both directions. Let x be the rate that Rafi can paddle and y be the rate of the current.
2. Is $(1,-3)$ a solution to the system below? $\begin{cases} 3x + y < 1 \\ x - y > -1 \end{cases}$	Checking the solution to a system of inequalities. (4.5)	Evaluate both inequalities at $x = 1$ and $y = -3$. For this to be a solution, it must satisfy both inequalities.
3. Two angles are supplementary and one angle measures $30°$ more than the other angle. Find the measure of each angle.	Problem solving using systems of equations. (4.4)	Supplementary angles sum to 180°. Let x be the first angle and y be the second angle. The equations would be $x + y = 180$ and $x + 30 = y$.
4. Solve the system below by graphing. $\begin{cases} y - 3x = 2 \\ 4x - y = -1 \end{cases}$	Solve a system of equations by graphing. (4.1)	Draw a set of coordinate axes. Graph the two lines and look for a point of intersection.
5. Graph the solution to the system below. $\begin{cases} x \le 3 \\ y \le \frac{1}{2}x - 1 \end{cases}$	Graph a system of inequalities. (4.5)	Both boundary lines are solid. The first boundary line is a vertical line. Use a test point to determine the shaded region for the solution.
6. Is the pair of equations dependent or independent? $\begin{cases} 3x + y = 8 \\ 6x + 2y = 8 \end{cases}$	Categorize a system of equations. (4.1, 4.2, 4.3)	A pair of equations is dependent if the equations describe the *same* line. Determine whether the equations are the same by solving the system.
7. Graph the solution to the system below. $\begin{cases} 3x + y < 4 \\ 2y - x > -2 \end{cases}$	Graph a system of inequalities. (4.5)	Both boundary lines are dashed. Write each boundary line equation in slope-intercept form to make them easier to graph. Use a test point to determine the shaded region for the solution.

Solution to Chapter 4 Group Activity A: *Writing Application Problems*
Note: The problems that students write will vary.

A rectangle problem: $l = 100,\ w = 60$
A number-value problem: $x = 3,\ y = 4$
An interest problem: $x = 50{,}000,\ y = 25{,}000$
A with–against the wind problem: $x = 250,\ y = 50$
A liquid mixture problem: $x = 12,\ y = 24$

Solution to Chapter 4 Group Activity B:
Interpreting Graphs of Linear Inequalities



Point	Coordinates	1st inequality	2nd inequality
A	(-5, -1)	True	True
B	(4, -4)	False	True
C	(-1, -2)	True	True
D	(3, 2)	False	True
E	(1, 5)	False	False
F	(-1, 4)	False	False
G	(-4, 3)	True	False